

Financial Intermediation and Delegated Security Design

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Abstract

This paper explains the existence of a financial intermediary based on its advantage over firms in designing *low market risk* debt securities and *high market risk* equity securities when markets are incomplete. The firm's objective in the model is to choose a financial structure in order to maximize the value of its securities. Investors are either risk-neutral or risk-averse and require a premium for market risk. When markets are complete, the value of a firm's securities is independent of how they are structured or sold. When markets are incomplete, firms can increase the total value of their securities by choosing the optimal mix of debt and equity, as shown by Allen and Gale (1988). In this paper it is shown that firms can further increase the value of their securities by delegating security design to a financial intermediary.

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1 Introduction

Individual investors are limited in their ability to optimize their portfolios by buying and selling securities among themselves. One important restriction is that they usually cannot issue or short sell securities because this is not allowed or because this would be extremely costly. As a consequence, individual investors optimize their portfolios by investing in appropriate securities issued by firms and/or financial intermediaries. The question arises if financial intermediaries are needed for an optimal allocation of risks and returns.

This paper explains the existence of a financial intermediary based on its advantages over firms in designing *low market risk* debt securities and *high market risk* equity securities when markets are incomplete. The firm's objective in the model is to choose a financial structure in order to maximize the value of its securities. When markets are complete, the value of a firm's securities is independent of how they are structured or sold. When markets are incomplete and investors have heterogeneous risk preferences, as assumed in this model, it is possible to increase the total value of securities by choosing the optimal mix of debt and equity, as shown by Allen and Gale (1988). In this paper the option for firms to set up a financial intermediary is introduced. It is then shown that it is possible to further increase the value of securities by delegating security design to a financial intermediary. This means that firms sell their securities to a financial intermediary who then builds a pool of securities and chooses the optimal structure of debt and equity for refinancing. It is assumed that investors are either risk-averse or risk neutral; firms' returns are imperfectly correlated; investors can fully diversify at no costs; firms and investors have the same information and expectations about the future; markets are incomplete in two regards: 1) state-contingent securities that purely reflect market risk or diversifiable risk do not exist, 2) investors cannot issue securities.

Research Question. Do financial intermediaries have advantages over firms in designing securities and risk-sharing under incomplete markets? The underlying puzzle is how splitting cash flows (security design) and diversification (pooling) play together and how this effects risk characteristics of securities issued by firms and financial intermediaries.

It is assumed that firms and investors have the same information and expectations about the future because it is the focus of this paper to analyze financial intermediation under incomplete markets. As a consequence, techniques to overcome asymmetric information such as monitoring or signalling are not needed. Of course,

asymmetric information is an important explanation for the existence of security design and financial intermediation. As shown in the literature review, it has been dealt with extensively.

Security Design. Security design means splitting the cash flow of an asset across separate securities with different risk characteristics. The goal is to maximize the value of securities. Separate securities are ranked in seniority, such that holders of a senior security have a priority claim on the cash flow versus holders of a junior security. The role of security design in my model is to split a cash flow into a *low market risk* (low beta) debt security that reflects mostly diversifiable risk and a *high market risk* (high beta) equity security that reflects mostly market risk. Risk averse investors value most a portfolio with low market risk. Risk neutral investors are indifferent about the portion of market risk in their portfolio.

The rationale for security design is based on the assumption that a complete set of state-contingent securities does not exist, and in particular that the diversifiable risk portion and the market risk portion of a cash flow are not separately contractible. Contracts such as debt contracts and equity contracts must be written on the composite cash flow of a firm.

Alternative Financial Structures. The first structure I consider is *direct contracting by firms*. This is a structure without a financial intermediary. Security design is done by firms who decide on the optimal mix of debt and equity and who issue securities directly to investors. Diversification (pooling) is done by investors who invest in a high number of securities. This structure implies first tranching and then pooling. The second structure I consider is *financial intermediation*. This structure represents a typical service of a financial intermediary: a bank or fund manager invests in many firms, pools the claims on its balance sheet, and issues senior securities (deposits and other senior debt) and junior securities (equity) for refinancing. This structure implies first pooling and then tranching.

Results. The main result of this paper is that a financial intermediary may arise endogenously as constituent of the optimal financial structure a firm can choose when markets are incomplete. A financial intermediary can create a large pool of assets and, based on the law of large numbers, design equity securities that reflect purely market risk and design debt securities that reflect purely diversifiable risk. In other words, a financial intermediary can make market risk and diversifiable risk separately contractible. A single firm can *not* do this and will therefore delegate

security design to a financial intermediary if this implies an increase in value of its securities that is above its delegation costs. The optimal financial structure in this model depends on risk-characteristics of firms, risk preferences of investors and firms' costs for delegating security design. This paper adds to theories on financial intermediation that are based on transaction costs and asymmetric information.

Organization of the Paper. The paper proceeds as follows. Section 2 contains a literature review. Section 3 contains the assumption of the model (3.1), the rationale for financial intermediation (3.2), considerations for an equilibrium (3.3) and a characterization of the first best (3.4). Section 4 analyzes the optimal financial structure in a setting when firms make direct contracts with investors (4.1), and in a setting with financial intermediation (4.2). The Section concludes with an example (4.3). Finally, Section 5 contains concluding remarks.

2 Related Literature

Security design and financial intermediation are not needed in perfect markets. The following literature gives a rationale for the existence of security design and financial intermediation. The literature review starts with the paper by Allen and Gale (1988) as the model in this paper builds on their results.

Imperfect markets. Allen and Gale (1988) analyze optimal security design for firms who seek financing. Key assumptions of their model are differences in endowments and investors' risk preferences, constraint of short-selling securities and the existence of transaction costs. This implies the absence of full risk-sharing possibilities. Allen and Gale (1988) show that a firm's cash flow should be split so that all cash flows are allocated to the security held by investors who value it most. Their example with risk-averse and risk-neutral investors leads to the issuance of debt and equity by firms. The risk-free portion of the cash flow is allocated to debt, and the risky portion of the cash flow is allocated to equity. Allen and Gale do not consider a financial intermediary in their paper. In a related paper, Allen and Gale (1990) discuss incentives to set up an options exchange which issues derivative securities under incomplete markets.

Transaction costs. The earliest explanation for the existence of financial intermediation is based on transaction costs. Gurley and Shaw (1960) are among the

first who stressed that financial intermediaries have an advantage over individuals in reducing transaction costs.

Asymmetric information. The main field of the literature that explains security design and financial intermediation is based on asymmetric information. Two basic ideas developed in this literature to overcome asymmetric information are *signalling* and *monitoring*.

Leland and Pyle (1977) develop a model where the entrepreneur has private information about a project and can signal a high quality of this project by keeping a large investment in the project. The entrepreneur is risk-averse and a large investment is costly because it prevents better diversification. DeMarzo (2005) examines the impact of pooling and tranching on signalling costs. The model assumes that signalling costs come from an above market discount rate of the issuer. Results are that pooling may increase or decrease signalling costs of the entrepreneur depending on the specifics of the information asymmetries, which DeMarzo interprets as *information destruction* effect and *risk diversification* effect. DeMarzo shows that tranching reduces signalling costs as it creates a low-risk and highly liquid security.

Diamond (1984) focusses on monitoring to overcome asymmetric information. He explains the existence of financial intermediaries based on their advantages over individuals in issuing unmonitored debt. Gorton and Pennacchi (1990) are the first to suggest splitting cash flows to address asymmetric information. They develop a model where banks issue bank debt (deposits) that is risk-free and liquid and therefore protects uninformed investors. Their model focusses on a financial intermediary, but they state explicitly that firms could also split risky cash flows as an alternative to bank intermediation. Boot and Thakor (1993) focus on incentives of investors to perform monitoring. They show that a firm or a financial intermediary can create an *informationally sensitive* security by splitting cash flows and issuing debt (informationally insensitive) and equity (informationally sensitive). Security design makes informed trade more profitable for equity holders, and thereby also benefits the issuer. Under the assumptions that investors receive a noisy signal about the quality of the assets and that the noise in the signal is unsystematic, Boot and Thakor (1993) also show why firms pool individual assets into a portfolio and *then* split the portfolio cash flows: diversification creates potential *information diversification* gains. Plantin (2004) combines the ideas of Gorton and Pennacchi (1990) and Boot and Thakor (1993) in a model and shows that splitting cash flows into several securities is a value-maximizing arrangement for the issuer.

For a survey on financial intermediation, see Allen and Santomero (1998), Allen

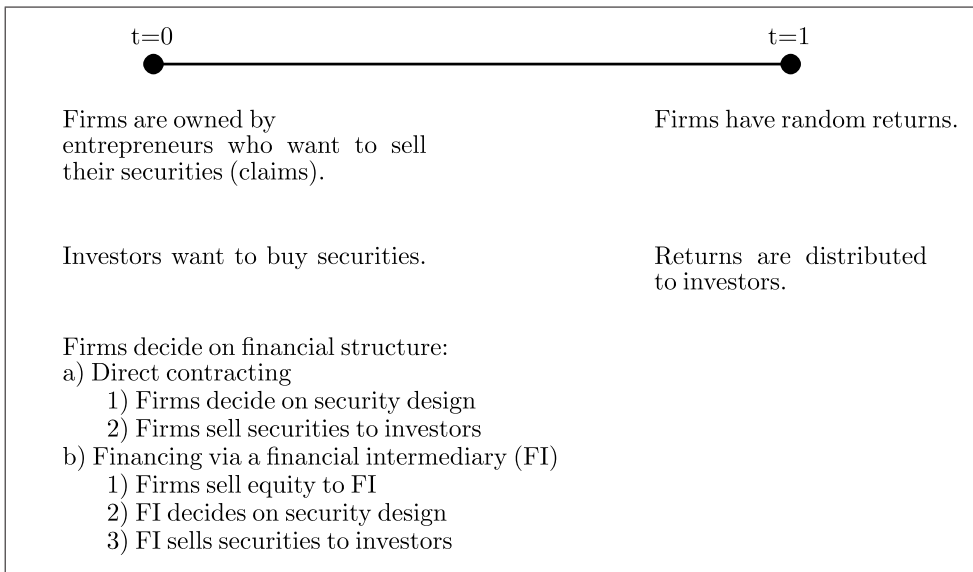
(2001) or Gorton and Winton (2003). Hellwig (1998) discussed the role of banks and markets in the allocation of risks in an economy. Krahen and Wilde (2008) build a Monte Carlo Simulation and analyze the allocation of credit risk to different tranches of a CDO transaction when the underlying asset returns are driven by a common macro factor and an idiosyncratic component.

3 The Model

3.1 Assumptions and basic considerations

Figure 1 shows an overview of the model. The main idea of the model is that entrepreneurs want to maximize the value of their securities (claims) and therefore choose the optimal financial structure. When markets are complete, the value of their securities is independent of how they are sold. When markets are incomplete, it may be possible to increase the value by choosing the optimal financial structure.

Figure 1: Overview of the model



(A.1) Basic setup. One period model. Symmetric information. No transaction costs.

(A.2) Firms. The total number of firms is n . Firms yield random returns C_i in $t = 1$, where $i = 1..n$. All returns are nonnegative. Returns have no risk-free fraction. All firms have returns with identical probability density functions. Each firm is initially owned by an entrepreneur who only wants to consume in $t = 0$.

The effect of this assumption is to ensure that the entrepreneur wants to maximize the firm's value at $t = 0$. In order to introduce correlation between returns in a simple way, it is supposed that $C_i = C_i^m + C_i^d$, where C_i^m represents the return that depends on *market risk* and C_i^d represents the return that depends on *diversifiable risk*. Market risk means that the sensitivity of C_i^m to the return of the overall market is $\beta_i^m = 1$. Diversifiable risk means that the sensitivity of C_i^d to the return of the overall market is $\beta_i^d = 0$. It follows that the sensitivity of C_i to the return of the overall market is given by the portion of a firm's expected cash flow that reflects market risk, i.e. $\beta_i = \frac{E(C_i^m)}{E(C_i^m) + E(C_i^d)}$.

(A.3) Investor types. Two types of investors are considered: *risk-neutral* and *risk-averse*. Risk-averse investors are mean-variance-optimizing. They are fully diversified which implies that they only care about market risk, not about diversifiable risk. The number of risk-averse investors is m . Risk-neutral investors don't care about market risk or diversifiable risk. The number of risk-neutral investors is arbitrary.

(A.4) Budget of investors. The budget of risk-neutral investors, denoted as B^N , is not sufficient to buy all securities. Otherwise the allocation problem would be trivial. The fraction of the budget of risk-neutral investors and the expected value of all firms is given by $\alpha = \frac{B^N}{E[\sum_{i=1}^n C_i]}$ with $\alpha \in (0, 1)$. The budget of risk-averse investors is denoted as B^A . The total budget of risk-neutral and risk-averse investors is sufficient to buy all securities, i.e. $B^N + B^A \geq E[\sum_{i=1}^n C_i]$,¹ which is necessary for market clearance.

(A.5) Government security. Besides investing (directly or indirectly) in a firm, investors can also invest in a government security that yields a risk-free return r_f . For simplicity it is assumed that $r_f = 0$.

(A.6) Security design. Security design means that the firm or a financial intermediary may issue more than one security. A financial structure with one security represents equity. The variety of securities in this model is restricted to debt and equity. The defining characteristic of debt is that the promised payment that is given by the face value of debt is the same in all states. In states where the return of the firm is below the face value of debt, debt investors receive the entire return. The defining characteristic of equity is that equity investors get the return above the face value of debt. A face value of zero or a face value equal to or above the maximum return of the firm implies that only equity is issued. A face value between zero and the maximum return of the firm implies a financial structure with debt and equity. All securities are divisible.

¹It is reasonable to assume that the price of a security is not above its expected value.

(A.7) Market incompleteness. The diversifiable risk fraction C_i^d and the market risk fraction C_i^m are not separately contractible. Contract such as debt or equity securities must be written on the composite cash flow of a firm C_i .

(A.8) Issuance of securities. Securities may be issued by firms or a financial intermediary. Issuance of securities by individual investors is not allowed.

(A.9) Direct contracting by firms. Under a financial structure with direct contracting by firms, securities are issued by firms directly to investors. A debt security of a firm is given by $X_i^D = \min(F, C_i)$, where F is the face value of debt. The risk premium of debt (in %) is denoted as r_F^D . An equity security of a firm is given by $X_i^E = C_i - X_i^D$. The risk premium of equity (in %) is denoted as r_F^E . The market value of a firm is given by $MV_F = \frac{E(X^D)}{1+r_F^D} + \frac{E(X^E)}{1+r_F^E}$, where $E(X^D)$ represents the expected value of the returns of the debt security, and $E(X^E)$ represents the expected value of the returns of the equity security.

(A.10) Financial intermediation. Under a financial structure with financial intermediation, the financial intermediary buys all securities issued by firms and then issues debt and equity for refinancing. Financing via a financial intermediary, or, in other words *delegating security design*, costs D per firm. A debt security of a financial intermediary is given by $Y^D = \min(G, \sum_{i=1}^n C_i)$, where G is the face value of debt. The risk premium of debt (in %) is denoted as r_I^D . An equity security of a financial intermediary is given by $Y^E = \sum_{i=1}^n C_i - Y^D$. The risk premium of equity (in %) is denoted as r_I^E . The market value of a firm (before delegation costs) is given by $MV_I = \frac{E(C_i)}{1+r_I}$, where $E(C_i)$ represents the expected value of the firm's returns, and r_I is the risk premium required by the financial intermediary, who in turn pays risk premia to its debt and equity investors. The risk premium r_I therefore depends on r_I^D and r_I^E .

3.2 Rationale for financial intermediation

Delegated security design pays, if

$$MV_I - D \geq MV_F, \quad (1)$$

where MV_I is the market value of a firm with delegation, D is the costs per firm for setting up a financial intermediary, and MV_F is the market value of a firm with direct contracting.² Under the assumptions of this model, the market value of

²The rationale for delegated security design in this model is obviously similar to the rationale of *delegated monitoring* (Diamond 1984) that pays, if $K + D \leq \min[E_{\bar{y}}, (m \cdot K)]$, where K = monitoring costs per monitor, D = delegation costs per firm, $E_{\bar{y}}$ = expected bankruptcy penalty

a firm completely depends on the risk premium required by risk-averse investors, which in turn depends on the portion of market risk that a security adds to the portfolio of risk-averse investors. The rationale for financial intermediation can therefore be transformed as follows: *Delegated security design* pays, if

$$\Delta\beta \leq \beta_F - \beta_I, \tag{2}$$

where β_F is the market sensitivity of the securities sold to risk-averse investors under direct contracting by firms, β_I is the market sensitivity of the securities sold to risk-averse investors under financial intermediation, and $\Delta\beta$ is a threshold level that depends on the specific risk-characteristics of firms, risk preferences of investors and delegation costs.

3.3 Equilibrium conditions and first best

Three conditions define an equilibrium:

- (E.1) Entrepreneurs choose a financial structure that maximizes the net value of their securities (objective function).
- (E.2) Investors maximize their utility.
- (E.3) Markets clear.

The first best solution maximizes the value of securities through optimal risk-sharing under complete markets. It serves as a benchmark for the analysis in the following sections. A sufficient condition for the first best is either a complete set of Arrow securities (which is prohibited via A.7, the *market incompleteness* assumption) or issuance of securities by individual investors (which is prohibited via A.8, the *issuance of securities* assumption). Suppose that a firm can offer investors a security that reflects purely market risk and a security that reflects purely diversifiable risk.

In order to maximize the value of the firm (equilibrium condition E.1), it is straight forward that each security should be allocated to investors who value it most (discount it least). It follows that the security representing market risk should be allocated to risk-neutral investors, and the security representing diversifiable risk should be allocated to risk-averse investors.

In order to satisfy that investors maximize their utilities (equilibrium condition E.2) and to separate between the demand from risk-neutral and risk-averse investors such

per firm, and $m =$ number of investors per firm.

that risk-neutral investors maximize utility by investing in equity and risk-averse investors maximize utility by investing in debt, it must hold that $r^E = r^D + \epsilon$, where ϵ is positive and sufficiently small.

Because markets must clear (equilibrium condition E.3), it is not possible to allocate all market risk to risk-neutral investors and to allocate all diversifiable risk to risk-averse investors. The first best is characterized as follows:

Lemma 1. *Risk-neutral investors hold as much market risk as possible given their limited budget. Risk-averse investors hold as little market risk as possible.*

Correspondingly, if the budget of risk-neutral investors is equal to or greater than the expected value of market risk of all firms, $B^N \geq E(\sum_{i=1}^n C_i^m)$, risk-neutral investors hold *all* market risk. Risk-averse investors hold purely diversifiable risk. The sensitivity of the portfolio of risk-averse investors to the return of the overall market, denoted as β_P , is zero, and consequently, no risk premium for debt or equity is required by investors. If the budget of risk-neutral investors is smaller than the present value of market risk of all firms, $B^N < E(\sum_{i=1}^n C_i^m)$, risk-neutral investors hold no diversifiable risk and *a fraction* of all market risk. Risk-averse investors hold both diversifiable risk and market risk. The sensitivity of the portfolio of risk-averse investors to the return of the overall market is positive ($\beta_P > 0$), and consequently a risk premium for debt $r^D > 0$ and for equity $r^E = r^D + \epsilon > 0$ is required by investors.

4 Analysis

First, the optimal security design with direct contracting is analyzed. Second, the optimal security design with financial intermediation is analyzed. From the comparison of the corresponding risk prima of securities it follows under which financial structure firms can maximize the value of their securities. Finally, results from the analysis are illustrated with an example.

4.1 Contracting by firms

Under this setting firms issue securities directly to investors. First, the optimal contracting by firms is considered. The optimization problem is the following:

$$\max_F MV_F = \frac{E(X^D)}{1 + r_F^D} + \frac{E(X^E)}{1 + r_F^E},$$

where F is the face value of debt F that determines X^D and X^E , and subject to investors maximizing their utility (E.2) and markets clearing (E.3).

Lemma 2. *It is optimal for firms to offer risk premia for debt and equity that separate between demand of risk-neutral investors and demand of risk-averse investors. The risk premium for equity r_F^E satisfies $r_F^E = r_F^D + \epsilon$, where ϵ is sufficiently small and positive.*

Proof: Lemma 2 is proofed by contradiction. By definition equity is more risky than debt. If risk-neutral and risk-averse investors buy both equity and debt, the required risk-premium r_F^E increases because risk-averse investors need to be compensated for the higher risk of equity. The risk-premium r_F^D does not decrease because risk-averse investors also buy debt and need to be compensated for any risk incorporated in debt relative to a risk-free government security. An equilibrium where risk-neutral investors only buy debt and risk-averse investors only buy equity is not feasible. \square

As long as ϵ is sufficiently small, risk averse investors maximize utility by investing in debt (ϵ must not compensate risk-averse investors for the higher risk of equity vs. debt). As long as ϵ is positive, risk-neutral investors maximize utility by investing in equity.

Next, it is shown that the value of equity securities should match B^N , the budget of risk-neutral investors. Security design is done according to this criterion.

Lemma 3. *The optimal security design which minimizes risk premia is given by equity and debt with face value F^* , where F^* is the solution to*

$$\frac{E[\sum_{i=1}^n X_i^E]}{1 + r_F^E} = \frac{E[\sum_{i=1}^n (C_i - \min(F^*, C_i))]}{1 + r_F^E} \stackrel{!}{=} B^N$$

Proof: If F above F^* , debt becomes larger and more risky. The risk premia r_o^D and $r_F^E = 0$ rise. Consequently, the market value MV_F falls. Risk-neutral investors invest their extra budget in debt or a government security. If F below F^* , debt becomes smaller and less risky. The risk premium r_o^D decreases. However, to achieve market clearance, risk-averse investors need to buy part of the equity. The risk premium r_F^E then reflects the required risk premium of risk-averse investors and rises as equity is more risky than debt. The market value MV_F falls. Risk-neutral investors enjoy a rent, which corresponds to losses of the firm relative to F^* . A solution exists because $\frac{E[\sum_{i=1}^n X_i^E]}{1+r_F^E}$ is strictly decreasing in F . \square

Second, properties of debt securities issued by firms are considered. The sensitivity

of a debt security's return to the return of the overall market, denoted as β_F , is measured as the percent change in the return of the debt security, given by $\frac{X_i^D}{E(X_i^D)}$, for a 1% change in the return of the market, given by $\frac{C^m}{E(C^m)}$:

$$\beta_F = \frac{COV(\frac{X_i^D}{E(X_i^D)}, \frac{C^m}{E(C^m)})}{VAR(\frac{C^m}{E(C^m)})}$$

By assumption, market risk and diversifiable risk are not separately contractible. It follows that securities issued by firms always contain market risk. A firm can *not* implement a financial structure that makes market risk and diversifiable risk separately contractible.

Lemma 4. *The sensitivity of a debt security's return to the return of the market, β_F , is always positive.*

Proof: As defined earlier, the debt security is given by $X_i^D = \min(F, C_i)$. For $F > 0$, we know that the probability that C_i^d is below F is positive, $P(C_i^d < F) > 0$, and the probability that C_i^m is above zero is positive, $P(C_i^m > 0) > 0$. Given that C_i^d and C_i^m are independent, it follows that $P(C_i^d < F \cap C_i^m > 0) > 0$, which means that the return from X_i^D also depends on C_i^m . \square

Third, properties of portfolios of investors are considered.

Proposition 1. *Under a financial structure with direct contracting by firms, risk-averse investors always hold a portfolio with market risk, i.e. $\beta_P^{RA} > 0$, and require a risk premium. This solution is weakly dominated by the first best.*

Proof: Under the assumptions of the model investors can not issue securities. This implies that the final portfolio of investors consists purely of securities issued by firms. It is known from lemma 4 that securities issued by firms contain market risk, and it follows directly that portfolios of investors also contain market risk. \square

4.2 Financial intermediation

Under this setting, the financial intermediary buys all securities issued by firms and then issues debt and equity for refinancing. *Delegating security design* costs D per firm. Again, it is optimal to offer risk premia for debt and equity that separate between demand of risk-neutral investors and demand of risk-averse investors. The risk premium for equity therefore satisfies $r_I^E = r_I^D + \epsilon$, where ϵ is sufficiently small

and positive. r_I^D compensates risk-averse investors for any potential risk included in debt relative to a risk-free government security.

First, optimal contracting by a financial intermediary is considered.

Lemma 5. *The optimal security design which minimizes risk premia is given by debt with face value G^* , where G^* is the solution to*

$$\frac{E[Y^E]}{1 + r_I^E} = \frac{E[\sum_{i=1}^n C_i - \min(G^*, \sum_{i=1}^n C_i)]}{1 + r_I^E} \stackrel{!}{=} B^N$$

Proof: Equivalent to proof of lemma 3.

Second, properties of debt securities issued by a financial intermediary are considered. Each risk-averse investor invests in a portion $\frac{1}{m}$ of the debt security Y^D . The sensitivity of a debt security's return *per investor* to the return of the market is measured as the percent change in the return of the debt security per investor, given by $\frac{\frac{Y^D}{m}}{E(\frac{Y^D}{m})}$, for a 1% change in the return of the market, given by $\frac{C^m}{E(C^m)}$:

$$\beta_I = \frac{COV(\frac{\frac{Y^D}{m}}{E(\frac{Y^D}{m})}, \frac{C^m}{E(C^m)})}{VAR(\frac{C^m}{E(C^m)})}$$

Lemma 6. *For a pool with a large number of firms and a large number of risk-averse investors, $n, m \rightarrow \infty$, the portion of market risk per investor included in a debt security issued by a financial intermediary equals that of the first best allocation. The sensitivity of a debt security's return per investor to the return of the overall market, β_I , is virtually zero, if the budget of risk-neutral investors is equal to or greater than the expected value of market risk of all firms, i.e. $B^N \geq E(\sum_{i=1}^n C_i^m)$.*

Proof: The law of large numbers (Jacob Bernoulli) says that the sample average $\frac{\sum_{i=1}^n C_i^d}{n}$ converges to the expected value $E(C_i^d)$, where C_1^d, C_2^d, \dots is an infinite sequence of i.i.d. random variables with finite expected values. The portion of diversifiable risk *per debt investor*, $\frac{\sum_{i=1}^n C_i^d}{n} (\frac{n}{m}) = \frac{\sum_{i=1}^n C_i^d}{m}$, becomes virtually risk-free.³ This allows the financial intermediary to write a debt contract that separates between diversifiable risk and market risk equal to the first best risk allocation. \square

The variance of the cash flows of a single firm implies an *allocation inefficiency*. This allocation inefficiency becomes negligible when security design is done for a large pool and a large number of investors. Note that the so-called *fallacy of large numbers* (Samuelson (1963)) is not relevant here, as it is the variance of diversifiable

³Note that $\frac{n}{m}$ is the proportion of firms to risk-averse investors.

risk per risk-averse investor (that decreases in n for a constant proportion of $\frac{n}{m}$) and not the variance of the debt security (that increases in n) that is of interest.

Proposition 2. *When markets are incomplete, it may be possible for firms to increase the total net value of their securities by delegating security design to a financial intermediary, depending on the risk-characteristics of firms, risk preferences of investors, and firms' costs for delegating security design.*

Proof: It follows directly from lemma 4 and lemma 6 that beta of debt under financial intermediation may be lower than beta of debt under direct contracting. It follows that depending on the threshold value $\Delta\beta$, *delegated security design* might pay. \square

Proposition 2 reflects the rationale for financial intermediation and implies that a financial intermediary may arise endogenously as constituent of the optimal financial structure a firm can choose.

5 Example

5.1 Two firms and two investors

The example considers two firms and two investors. Each firm has returns $C_i = C_i^d + C_i^m$, where the diversifiable risk portion C_i^d and the market risk portion C_i^m are defined as

$$C_i^d = \begin{cases} 0 & : \text{ with } f(0) = \frac{1}{2} \\ 1 & : \text{ with } f(1) = \frac{1}{2} \end{cases} \quad \text{and} \quad C_i^m = \begin{cases} 0 & : \text{ with } f(0) = \frac{1}{2} \\ 1 & : \text{ with } f(1) = \frac{1}{2} \end{cases}$$

The function $f(\cdot)$ represents a probability density function. The expected value of each firm is 1. The *unlevered* beta of each firm is $\beta_i = \frac{1}{2}$, which follows from the earlier definition that C^d has a beta of zero and C^m has a beta of one.

Furthermore, it is assumed that one investor is risk-averse and has a budget of 1. The second investor is risk-neutral and also has a budget of 1. Besides investing in securities issued by firms or a financial intermediary, investors can also invest in a risk-free government security.

First best. Under the first best, market risk and diversifiable risk can be allocated directly to risk-neutral and risk-averse investors. Table 1 shows the corresponding

values. All diversifiable risk ($X_1^a + X_2^a$) with an expected value of 1 is allocated to risk-averse investors. All market risk ($X_1^b + X_2^b$) with an expected value of 1 is allocated to risk-neutral investors. The sensitivity of the portfolio of risk-averse investors to the return of the overall market is zero ($\beta_P^{RA} = 0$), and consequently, no risk premium for debt or equity (besides ϵ) is required by investors.

Table 1: first best

state s	probability(s)	$C_1^d + C_1^m$	X_1^a	X_1^b	$C_2^d + C_2^m$	X_2^a	X_2^b
1	$P = \frac{1}{8}$	1+1	1	1	1+1	1	1
2	$P = \frac{1}{8}$	1+1	1	1	0+1	0	1
3	$P = \frac{1}{8}$	0+1	0	1	1+1	1	1
4	$P = \frac{1}{8}$	0+1	0	1	0+1	0	1
5	$P = \frac{1}{8}$	1+0	1	0	1+0	1	0
6	$P = \frac{1}{8}$	1+0	1	0	0+0	0	0
7	$P = \frac{1}{8}$	0+0	0	0	1+0	1	0
8	$P = \frac{1}{8}$	0+0	0	0	0+0	0	0
EV		1	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{1}{2}$
beta		$\frac{1}{2}$	0	1	$\frac{1}{2}$	0	1

Direct contracting. Suppose direct contracting between firms and investors takes place. Both firms issue debt with a face value of $F = \frac{2}{3}$ and equity. Table 2 shows the corresponding values. The beta of each debt security (X_1^D and X_2^D) is $\frac{1}{3}$. A risk-averse investor who invests in debt issued by both firms holds a portfolio with an expected value of 1 and a beta of $\beta_P^{RA} = \frac{1}{3}$. The risk-averse investor thus requires a risk-premium in order to be compensated for the additional risk vs. a risk-free government security. Risk-averse investors have to spend $\frac{1}{1+r_F^D} < 1$ for $X_1^D + X_2^D$ and could invest their remaining budget in government securities. Risk-neutral investors have to spend $\frac{1}{1+r_F^E} < 1$ for $X_1^E + X_2^E$ and could invest their remaining budget in debt securities.

Financial intermediation. Suppose both firms sell their claims to a financial intermediary who then issues debt and equity. As shown by Table 3, the debt security (Y^D) has a beta of $\frac{1}{5}$ and the equity security (Y^E) has a beta of $\frac{4}{5}$. The risk-averse investor who buys the debt security will require a risk-premium, that is, however, *lower than the risk premium the risk-averse investor requires under direct contracting*. Risk-averse investors pay $\frac{1}{1+r_F^D} < 1$ for Y^D and could invest their remaining budget in government securities. Risk-neutral investors have to spend $\frac{1}{1+r_F^E} < 1$ for Y^E and could invest their remaining budget in debt securities.

Table 2: direct contracting with debt ($F = \frac{2}{3}$) and equity

state s	probability(s)	$C_1^d + C_1^m$	X_1^D	X_1^E	$C_2^d + C_2^m$	X_2^D	X_2^E
1	$P = \frac{1}{8}$	1+1	$\frac{2}{3}$	$\frac{4}{3}$	1+1	$\frac{2}{3}$	$\frac{4}{3}$
2	$P = \frac{1}{8}$	1+1	$\frac{2}{3}$	$\frac{4}{3}$	0+1	$\frac{2}{3}$	$\frac{1}{3}$
3	$P = \frac{1}{8}$	0+1	$\frac{2}{3}$	$\frac{1}{3}$	1+1	$\frac{2}{3}$	$\frac{4}{3}$
4	$P = \frac{1}{8}$	0+1	$\frac{2}{3}$	$\frac{1}{3}$	0+1	$\frac{2}{3}$	$\frac{1}{3}$
5	$P = \frac{1}{8}$	1+0	$\frac{2}{3}$	$\frac{1}{3}$	1+0	$\frac{2}{3}$	$\frac{1}{3}$
6	$P = \frac{1}{8}$	1+0	$\frac{2}{3}$	$\frac{1}{3}$	0+0	0	0
7	$P = \frac{1}{8}$	0+0	0	0	1+0	$\frac{2}{3}$	$\frac{1}{3}$
8	$P = \frac{1}{8}$	0+0	0	0	0+0	0	0
EV		1	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{1}{2}$
beta		$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$

 Table 3: financial intermediation with debt ($F = \frac{5}{6}$) and equity

state s	probability(s)	$C_1^d + C_1^m + C_2^d + C_2^m$	Y^D	Y^E
1	$P = \frac{1}{8}$	1+1+1+1	$\frac{6}{5}$	$\frac{14}{5}$
2	$P = \frac{1}{8}$	1+1+0+1	$\frac{6}{5}$	$\frac{9}{5}$
3	$P = \frac{1}{8}$	0+1+1+1	$\frac{6}{5}$	$\frac{9}{5}$
4	$P = \frac{1}{8}$	0+1+0+1	$\frac{6}{5}$	$\frac{4}{5}$
5	$P = \frac{1}{8}$	1+0+1+0	$\frac{6}{5}$	$\frac{4}{5}$
6	$P = \frac{1}{8}$	1+0+0+0	1	0
7	$P = \frac{1}{8}$	0+0+1+0	1	0
8	$P = \frac{1}{8}$	0+0+0+0	0	0
EV		2	1	1
beta		$\frac{1}{2}$	$\frac{1}{5}$	$\frac{4}{5}$

5.2 Many firms and many investors

In this section it is assumed that the number of firms and investors is $n, m \rightarrow \infty$. Furthermore, for illustration purposes, it is assumed that the required threshold level that reflects delegation costs and preferences of investors is $\Delta\beta = 0.1$.

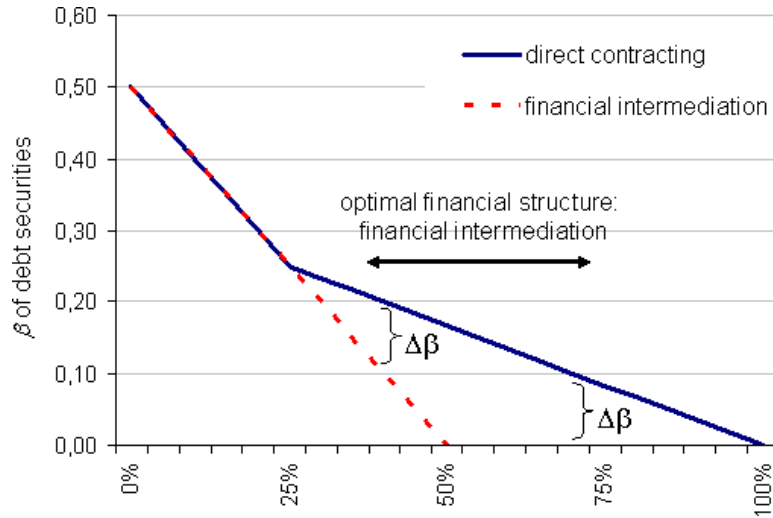
As discussed in section 3.2, *delegated security design* pays, if the difference between beta of debt under direct contracting, denoted as β_F , and beta of debt under financial intermediation, denoted as β_I , is above some threshold level, denoted as $\Delta\beta$, which means $\Delta\beta \leq \beta_F - \beta_I$. Remember that the lower β_F or β_I , the lower the risk-premium requested by risk-averse investors. β_F depends on risk-characteristics of firms and α , the fraction of funds of risk-neutral investors. β_I depends on risk-characteristics of firms, on α and also on the number of firms available for pooling.

The required threshold level $\Delta\beta$ depends on delegation costs and risk-preferences of investors.

Figure 2 shows the result of the analysis. The x-axis shows α , the fraction of funds from risk-neutral investors. The y-axis shows the values of beta of debt under a *direct contracting* financial structure (solid line) and under a *financial intermediation* financial structure (dashed line). Beta of debt is decreasing in α under both financial structures, as the higher the fraction of funds of risk-neutral investors, the lower the investment of risk-averse investors and the value of debt. If $\alpha = 0$, all securities are allocated to risk-averse investors such that $\beta_F = \beta_I = \beta_i = \frac{1}{2}$. If $\alpha = 1$, all securities are allocated to risk-neutral investors such that $\beta_F = \beta_I = 0$. In this example *delegated security design* pays, if the difference between β_F and β_I is above a threshold value $\Delta\beta = 0.1$, which is the case for the region between $\alpha = 40\%$ and $\alpha = 70\%$. For $\alpha < 40\%$ and $\alpha > 70\%$, the difference between β_F and β_I is not sufficient to make *delegated security design* a value maximizing financial structure.

Figure 2: beta of debt

The x-axis shows α , the fraction of funds from risk-neutral investors. The solid line shows the values of beta of debt under *direct contracting*, denoted as β_F . The dotted line shows the values of beta of debt under *financial intermediation*, denoted as β_I .



6 Conclusion

This paper shows that *delegated security design* matters. Security design by firms does not achieve separation of diversifiable risk and market risk. Only financial intermediaries can design risk-free securities based on a large portfolio and a large number of investors.

The main result is that a financial intermediary may arise endogenously as constituent of the optimal financial structure a firm can choose when markets are incomplete. A financial intermediary can create a large pool of assets and, based on the law of large numbers, design equity securities that reflect as much market risk as possible (in the limit purely market risk) and debt securities that reflect as much diversifiable risk as possible (in the limit purely diversifiable risk). In other words, a financial intermediary can make market risk and diversifiable risk separately contractible. A single firm can *not* do this and will therefore delegate security design to a financial intermediary if this implies an increase in value of its securities that is above its delegation costs. The optimal financial structure in this model depends on risk-characteristics of firms, risk preferences of investors and firms' costs for delegating security design. This paper adds to theories on financial intermediation that are based on transaction costs and asymmetric information.

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