Money in monetary policy design: Monetary cross-checking in the New-Keynesian Model†

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1 Introduction

The notion that inflation and deflation are monetary phenomena ultimately caused by monetary policy is a central tenet of monetary economics. Closely related is the so-called quantity theory. This theory, which is well-grounded in empirical observation, states that sustained increases or decreases in the overall price level eventually coincide with faster or slower growth rates of monetary aggregates adjusted for long-run output and velocity trends. This notion has at times played a key role in monetary policy design, for example when FOMC Chairman Paul Volcker set out to overcome the great inflation in the United States in 1979, or earlier on when the Bundesbank successfully fought back the inflationary impetus of oil price shocks in Germany. However, the attempt to implement monetary targets on a short-run basis ran into difficulties in light of the fluctuations arising from variations in money demand and financial innovations as perhaps best illustrated by the Bundesbank's experience in the 1990s.

While the majority of central banks still refer to monetary information as an important ingredient in the conduct of policy — most prominently the ECB which stands with a separate, monetary pillar in its strategy[1] — it is by no means clear how to incorporate monetary information in policy strategies based on today's standard model of monetary policy analysis. For example, Woodford (2006) answers the question ”How important is money in the conduct of monetary policy?”[2] as follows:

> ”*I believe that a serious examination of the reasons given thus far for assigning a prominent role to monetary aggregates in (policy) deliberations provides little support for a continued emphasis on those aggregates.*”

As to continued efforts to better understand monetary dynamics Woodford concludes:

> ”.. *There is at present little reason ... to devote much attention to questions such as the construction of improved measures of the money supply or improved econometric models of money demand. For there is little intelligible connection between those questions and the kinds of uncertainty about the effects of monetary policy that are the actual obstacles to the development of more effective, more reliable and more transparent ways of conducting policy.*

[1]The ECB's former chief economist, Otmar Issing, wrote on the monetary pillar: “*In line with the argument of a closer relationship between money and inflation at lower frequencies, the function ascribed to the monetary pillar is to reveal medium-term risks to price stability* ..” but “... *there is no mechanical monetary policy reaction to deviations of M3 growth from the reference value”* and “... *cross-checking the information from the economic analysis with the information from the monetary pillar is ... a crucial element underpinning the robustness and medium-term policy orientation.*”

[2]This is the title of the paper prepared for the Fourth ECB Central Banking Conference, The Role of Money: Money and Monetary Policy in the Twenty-First Century, November 9-10, 2006.

These conclusions are based on the New-Keynesian model of monetary policy. This model as laid out by Rotemberg and Woodford (1997) and Goodfriend and King (1997) and developed in detail in Woodford (2003) has quickly become the principal workhorse model in monetary economics. Requiring only a small number of equations and variables, the model has proved very useful in deriving several important principles for the conduct of monetary policy (see for example Clarida et al. (1999) and King (2000)).

But not all macroeconomists share these conclusions. For example, Lucas (2007), writes

> *"Events since 1999 have not tested the importance of the (ECB's) second, monetary pillar, ... I am concerned that this encouraging but brief period of success will foster the opinion, already widely held, that the monetary pillar is superfluous, and lead monetary policy analysis back to the kind of muddled eclecticism that brought us the 1970s inflation.*"

Lucas identifies New-Keynesian-style research as one of the possible culprits stating

> *"One source of this concern is the increasing reliance of central bank research on New-Keynesian modeling. New-Keynesian models define monetary policy in terms of a choice of money market rate and so make direct contact with central banking practice. Money supply measures play no role in the estimation, testing or policy simulation of these models. A role for money in the long run is sometimes verbally acknowledged, but the models themselves are formulated in terms of deviations from trends that are themselves determined somewhere off stage.*"

He then proceeds to suggest strategies for research and policy, respectively:

> "*It seems likely that these models could be reformulated to give a unified account of trends, including trends in monetary aggregates, and deviations about trend but so far they have not been. This remains an unresolved issue on the frontier of macroeconomic theory. Until it is resolved, monetary information should continue be used as a kind of add-on or cross-check, just as it is in the ECB policy formulation today.*

In this paper we extend the New-Keynesian model to allow for persistent central bank misperceptions regarding unobservables such as (flexible-price) equilibrium output or equilibrium interest rates. Taking into account such misperceptions we are able to give a unified account of trends in inflation and monetary aggregates as requested by Lucas (2007). Furthermore, we find that adding a cross-check regarding long-run money growth to the optimal interest-rate policy under perfect information provides superior inflation control in the presence of persistent central bank misperceptions. Specifically, we utilize the formulation of monetary cross-checking suggested by Beck and Wieland (2007). In this earlier paper, we have defined a simple formula for including cross-checking in a formal, statistical sense in a systematic monetary policy rule. We illustrated the potential benefits of such cross-checking in a traditional Keynesian-style model for the case of a strict-inflation targeting central bank.

The present paper contains three substantial innovations relative to Beck and Wieland (2007). As noted above we show that central bank misperceptions can be a source of monetary and inflationary trends in the New-Keynesian model. Furthermore, we find that monetary cross-checking provides better inflation performance from the perspective of a strict inflation targeting central bank. And finally, we extend the analysis to the case of flexible inflation targeting, i.e. when the central bank also cares about output stabilization. In this case, the central bank accepts a certain degree of endogenous inflation persistence that is absent from the analysis under strict inflation targeting. Even so, we confirm that monetary cross-checking still improves inflation control. Finally, we note that our definition and analysis of monetary cross-checking is different from but complementary to another very interesting approach that identifies beneficial effects from monetary targeting in conjunction with Taylor-style interest rate rules (cf. Christiano and Rostagno (2001), Christiano et al. (2006)).

The remainder of this paper is structured as follows. In section 2 we review the optimal policy under discretion in the New-Keynesian model in order to present the case for monetary policy design without money. Section 3 reviews the long-run link between money and inflation and the resulting definition of monetary cross-checking as characterized by Beck and Wieland (2007). In section 4 we introduce persistent central bank misperceptions and noise in the New-Keynesian model. Such misperceptions are shown to generate long-term, and parallel movements in inflation and money growth trends. Monetary cross-checking is shown to introduce a self-correcting mechanism that tends to offset after some time persistent policy mistakes. Section 5 extends the analysis to the more realistic case of a central bank that aims to stabilize output gaps and inflation. Section 6 concludes.

2 Optimal monetary policy in the New-Keynesian model

The monetary transmission mechanism in the New-Keynesian model works as follows: The nominal interest rate affects the real interest rate due to price rigidity, the real rate influences the output gap via aggregate demand and the output gap impacts on inflation via a forward-looking Phillips curve. Thus, monetary aggregates play no direct role in the transmission of policy from nominal interest rates to inflation. Of course, money supply is influenced by open-market operations intended to keep the nominal interest rate at the target level chosen by the central bank. Money growth is then determined recursively from a money demand equation depending on the intended level of the interest rate, the current price level and current income.

We illustrate this well-established finding using the New-Keynesian model as presented in Clarida et al. (1999) (henceforth denoted by CGG). In its simplest, log-linear form the model consists of a forward-looking Phillips curve derived from the firms' pricing problem under monopolistic competition and Calvo-style price rigidity and an aggregate demand relation, the so-called forward-looking IS curve, that is derived from the households' intertemporal Euler equation:

$$\pi_t = \lambda x_t + \beta \pi^e_{t+1} + u_t, \tag{1}$$

$$x_t = -\varphi \left(i_t - \pi^e_{t+1} \right) + x^e_{t+1} + g_t. \tag{2}$$

$\pi_t = \Delta \ln P_t$ denotes inflation, and $x_t = y_t - z_t$ denotes the output gap, i.e. the percentage deviation of (log) actual output (y_t) from (log) potential or natural output (z_t). The superscript ^{e} refers to market expectations, which we assume to be rational. i_t denotes the short-term nominal interest rate that is under full control of the central bank. The parameter λ is a decreasing function of the discount factor β and the share of firms that do not adjust their prices in any given period.[3] (u_t, g_t) stand for zero-mean cost-push and demand shocks with variances (σ^2_u, σ^2_g). Following CGG, we assume that these shocks are observable[4] and follow AR(1) processes, i.e.

$$g_t = \mu g_{t-1} + \eta^g_t, \tag{4}$$

$$u_t = \nu u_{t-1} + \eta^u_t \tag{5}$$

with $0 \leq \mu, \nu < 1$ and (η^g_t, η^u_t) representing mean-zero, i.i.d. disturbances. Also following CGG we assume that money demand[5] is given by

$$m_t - p_t = \gamma_y y_t - \gamma_i i_t + e_t. \tag{6}$$

[3]More precisely, the parameter λ is given by the following expression:

$$\lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta}, \tag{3}$$

where θ denotes the proportion of firms which is not allowed to adjust their prices in a given period.

[4]This unrealistic assumption will be relaxed later on once we introduce output gap misperceptions and cross-checking into the model.

[5]As noted by CGG it is possible to motivate this specification of the money demand function from first principles assuming that utility is separable in consumption and real money balances and that consumption is the only type of good (see also Woodford (2003)). Theoretical foundations for direct effects of money on aggregate demand and inflation can be obtained from micro-founded models that allow for non-separability of money and consumption in household utility. Empirical implementations, however, have failed to detect strong direct effects (cf. Ireland (2004) and Andres et al. (2006)). Thus, we decided against making use of such effects in order to motivate a policy strategy that gives a special role to monetary aggregates.

Here, γ_y denotes the income elasticity and γ_i the semi-interest rate elasticity of money demand. Money demand shocks are assumed to be normally distributed with mean zero and variance σ^2_{md}.

A strict inflation-targeting central bank[6] would set the nominal interest rate i_t in order to minimize expected discounted inflation deviations from target

$$\min_{i_t, i_{t+1}, \ldots} E_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} (\pi_s - \pi^\star)^2 \right\}, \tag{7}$$

where π^* denotes the central bank's inflation target and δ is its discount factor. For simplicity we set $\pi^* = 0$.

Under discretion, the central bank optimizes every period taking the public's expectations, for example regarding next period's inflation rate, π^e_{t+1}, as given. In doing so it chooses the inflation rate, the output gap, and the nominal interest rate in order to minimize equation (7) subject to the Phillips curve (1) and the aggregate demand curve (2).

Under the optimum the central bank will set the nominal interest rate such that current inflation and the current output gap take the following values:

$$\pi_t^{opt} = 0 \tag{8}$$

$$x_t^{opt} = -\frac{1}{\lambda} u_t \tag{9}$$

where the superscript 'opt' refers to 'optimal'.
Next period's expected values of the two variables are given by:

$$\pi_{t+1}^{e,opt} = 0 \tag{10}$$

$$x_{t+1}^{e,opt} = -\frac{1}{\lambda} \rho u_t \tag{11}$$

The interest rate that achieves these values is obtained by plugging the optimal values for inflation and the output gap into the aggregate demand curve and solving for i_t. This yields:

$$i_t^{opt} = \frac{1-\rho}{\lambda\varphi} u_t + \frac{1}{\varphi} g_t. \tag{12}$$

Thus, the central bank acts to fully offset the effects of both demand and cost-push shocks on inflation under a strict-inflation targeting regime.

To be clear, the central bank achieves the desired interest rate setting by conducting open-market operations that influence the money supply. Thus, the money supply is

[6] Following Svensson (1997) we use the expression 'strict inflation targeting' to characterize a monetary policy where the central bank only cares for inflation stabilization and puts no weight on output stabilization. The case of flexible-inflation targeting where the central bank also puts some weight on output stabilization will be analyzed in Section 5.

determined according to the money demand equation (6) consistently with the desired policy rate, current output and the price level. However, money does not appear as a variable in the central bank's optimal interest rate rule and the remainder of the economy is automatically insulated from money demand shocks. Thus, the case for monetary policy design without money directly follows from the New-Keynesian model.

3 Monetary cross-checking and the long-run link between money and inflation

Researchers and policymakers that are in favor of assigning a special role to money in monetary policy design typically refer to the well-established long-run link between money and inflation, or in other words the quantity theory. This long-run relation is also present in the New-Keynesian model summarized in the preceding section. Taking first differences and re-arranging the money demand equation (6) we first obtain a short-run link between money and inflation:

$$\Delta p_t = \Delta m_t - \gamma_y \Delta y_t + \gamma_i \Delta i_t - \Delta e_t. \tag{13}$$

Δ is the first-difference operator. Long-run equilibrium values (superscript *) can then be determined as follows. In the long-run, money demand shocks would average to zero, and the nominal interest rate would settle down to its steady state level. Thus, the change in the interest rate would similarly converge to zero. The long-run link between inflation, money growth and output growth then corresponds to:

$$\Delta p_t^* = \Delta m_t^* - \gamma_y \Delta y_t^*. \tag{14}$$

Note, that this relationship also incorporates the long-run trend in velocity.[7] Thus, long-run inflation is proportional to long-run money growth adjusted for output and velocity trends:[8]

Recent studies obtained empirical support for this long-run relationship using various filters or frequency-specific estimation. Even more interestingly, they have found money growth to lead inflation at this frequency. To give an example,[9] Gerlach (2004) uses the following filter

$$\mu_t^f = \mu_{t-1}^f + \omega\left(\mu_t - \mu_{t-1}^f\right) \tag{15}$$

[7]Specifically, with velocity defined as $v_t \equiv -m_t + p_t + y_t$ and money demand determined by equation (6) the long-run trend in velocity corresponds to $\Delta v_t^* = (1-\gamma_y)\Delta y_t^*$.

[8]Changes in the trend in velocity may arise from changes in potential output growth Δy_t^*, changes in the income elasticity γ_y and possibly from other sources such as financial innovations (see Orphanides and Porter (2001) and Masuch et al. (2001)).

[9]Other examples include Benati (2005) and Pill and Rautananen (2006).

to approximate long-run values of inflation and money growth. In his work, μ_t may alternatively stand for money growth, Δm_t, or money growth adjusted for output growth. In our paper we will follow equation (14) and adjust money growth using the estimate of the income-elasticity of money demand, i.e.

$$\mu_t^f = \Delta m_t^f - \gamma_y \Delta y_t^f. \tag{16}$$

Given a long-run relationship between money and inflation that is consistent with the New-Keynesian model we can now proceed to formulate a strategy for setting interest rates that involves monetary cross-checking as proposed by Beck and Wieland (2007) for the New-Keynesian model. This strategy consistes of two components:

$$i_t^{CC} = i_t^{NK} + i_t^M \tag{17}$$

Here the superscript *CC* refers to cross-checking, *NK* to the optimal interest rate in the New-Keynesian model and *M* to an additive adjustment in interest rate setting due to monetary information.

The optimal interest rate decision derived in the preceding section takes into account expected future inflation as well as current and expected future output gaps. In solving the optimal these expected conditions turn out to be functions of the observed cost-push and aggregate demand shocks.

$$i_t^{NK} = i_t^{opt} \qquad \text{as defined in equation (12).} \tag{18}$$

This interest rate setting should ensure that inflationary risks based on a forward-looking Phillips curve and excess aggregate demand are perfectly controlled.[10] However, this component relies on knowledge of unobservables, namely natural rates of output and real interest rates that may be subject to large and persistent misperceptions on behalf of the central bank decision makers.

Thus, the second, novel component, of the interest-rate rule proposed in Beck and Wieland (2007), i_t^M, aims to capture the idea of cross-checking using the long-run relationship between money and inflation. This component is additive and persistent. We assume that the central bank regularly tests whether filtered and adjusted money growth, μ^f, still averages around the inflation target. Thus, the central bank computes

[10]These conditions and determinants of inflation may well be covered by the ECB's "economic analysis" describe as follows on the ECB's website: "*Economic analysis assesses the short to medium-term determinants of price developments. The focus is on real activity and financial conditions in the economy. The economic analysis takes account of the fact that price developments over those horizons are influenced largely by the interplay of supply and demand in the goods, services and factor markets.*"

the normally-distributed test statistic,

$$\kappa = \frac{\mu_t^f - \pi^*}{\sigma_{\mu^f}}, \tag{19}$$

and checks whether κ deviates from a critical value κ^{crit}. σ_{μ^f} denotes the standard deviation when $i_t^{EA} = i^{opt}$ is implemented under full information and the mean of μ^f corresponds to π^*. If the central bank obtains successive signals of a sustained deviation from target, i.e. ($\kappa > \kappa^{crit}$ for N periods) or ($\kappa < -\kappa^{crit}$ for N periods), it responds by adjusting interest rates accordingly.[11]

$$i_t^M = \begin{cases} i_{t-1}^M + (\frac{1}{\varphi\lambda})(\mu_{t-1}^f - \pi^*) & \text{if } \kappa > \kappa^{crit} \text{ or } \kappa < -\kappa^{crit} \text{ for } N \text{ periods} \\ \\ i_{t-1}^M + 0 & \text{else} \end{cases} \tag{20}$$

As long as $i_t^{NK} = i_t^{opt}$ is implemented under full information as in section 2 cross-checking with regard to i_t^M will almost never lead to an adjustment in interest rates. Under imperfect knowledge, however, cross-checking may once in a while have a very important effect on interest rate policy that is investigated in the next section.[12]

In our earlier paper, we noted the similarity between monetary cross-checking as defined above and the ECB's claim that it bases its interest-rate decisions not exclusively on short- to medium run determinants of inflation, but also includes a separate, long-run oriented monetary analysis. The ECB website, for example, states:

> "*Monetary analysis focuses on a longer-term horizon than the economic analysis. It exploits the long-run link between money and prices. The monetary analysis mainly serves as a means of cross-checking, from a medium to long-term perspective, the short to medium-term indications for monetary policy coming from the economic analysis.*"

However, the formal rule presented above is solely due to the authors as the ECB has refrained from providing a formal, mathematical exposition of its strategy. Our aim in this paper, is to show the usefulness of monetary cross-checking as defined by equation (20) in its own right in today's most standard model of monetary policy analysis.

[11] The response coefficient on inflation deviations from target will be explained in the next section.

[12] The two parameters of i_t^M, κ^{crit} and N play different roles. κ^{crit} reflects the probability that an observed deviation of μ^f from $\pi^\star$ is purely accidental (for example a 5% or 1% significance level). N defines the number of successive deviations in excess of this critical value. Thus, the greater N the longer the central bank waits to accumulate evidence of a sustained policy bias. For example, if κ^{crit} is set to the 1% critical value for the normal distribution (2.575) and the critical number of periods of sustained deviations N is set to 4, the probability of such an event in the absence of policy misperceptions would be less than 10^{-8}.

4 The consequences of persistent policy misperceptions and monetary cross-checking in the New-Keynesian model

We are now ready to relax the assumption of full information (on behalf of the central bank and market participants) in the New-Keynesian model. In doing so we will treat the central bank and market participants symmetrically. In other words, we will assume that they share the same beliefs regarding macroeconomic aggregates. We introduce two sources of imperfect knowledge. First, we do away with the unrealistic assumption that economic shocks are perfectly observed in real time. Instead, we assume that demand, cost-push and money demand shocks (g_t, u_t, e_t) are observed only with noise. That is, we have

$$u_t = u_t^e + \varepsilon_t^u \tag{21}$$

$$g_t = g_t^e + \varepsilon_t^g \tag{22}$$

$$e_t = e_t^e + \varepsilon_t^m \tag{23}$$

where the superscript e above a variable denotes the value of the respective shock perceived by the policy maker (and the public). The innovation processes ε_t^u, ε_t^g, and ε_t^m are Gaussian white noise processes.

Secondly, we allow for misperceptions regarding unobservable equilibrium variables such as potential output. The presence of noise renders the estimation of, in principle, unobservable variables such as potential output nontrivial. In the absence of noise, one set of observations would be sufficient to deduce the level of potential output. In the presence of noise, however, the central bank will remain forever uncertain about such equilibrium variables.

Recent research exploiting data on historical revisions to real-time estimates of the output gap has identified very persistent policy misperceptions.[13] The persistence of measurement errors arises primarily from biased estimates of unobservable potential output, since revisions to actual output decline more rapidly than those to the output gap. Thus, if a central bank relies on potential output measures in policy design, its policy stance may be biased for a sustained period of time.

To account for this possibility we now allow the central bank and the public to make mistakes in estimating the natural output level. Denoting the available estimate by z_t^e we thus assume that

$$z_t^e = z_t + \text{bias}_t \tag{24}$$

where bias_t denotes the error that the central bank and public make when estimating z_t. Given this estimate of z_t the central bank's perception of the output gap, denoted by x_t^e,

[13] See Orphanides (2003) and Orphanides et al. (2000) who estimate a worst-case process of misperceptions with a near unit root (0.96) and standard deviation of 3.77% using quarterly revisions from 1966 to 1994. For German data on output gap revisions see Gerberding et al. (2006).

is

$$x_t^e = y_t - z_t^e = y_t - z_t - \text{bias}_t = x_t - \text{bias}_t. \tag{25}$$

Optimal policy under discretion in the New-Keynesian model then implies that the central bank sets the nominal interest rate in each period such that the perceived level of the output gap will be equal to the perceived optimal value derived in the preceding section. In other words, since the optimization problem is linear-quadratic certainty-equivalence applies. Thus, the central bank sets the nominal interest rate such that:

$$x_t^e = -\frac{1}{\lambda} u_t^e. \tag{26}$$

The interest rate that achieves this goal can be derived from the forward-looking IS curve with x_t replaced by x_t^e. Next period's expected output gap, however, takes the some value as in the case with full information, because the central bank is not aware that the output gap estimate is incorrect.

The optimal interest rate that is set by the central bank conditional on the perceived output gap and shocks is thus given by:

$$i_t^{opt} = \frac{1}{\varphi} \left[\frac{1-\rho}{\lambda} u_t^e - \text{bias}_t + g_t^e \right]. \tag{27}$$

This equation shows that the chosen interest rate will be too low (too high) if the policy maker overestimates (underestimates) the level of natural output, i.e. if bias_t is positive (negative), and therefore underestimates (overestimates) the output gap. The resulting inflation rate will be above (below) the target of zero inflation and is given by:

$$\pi_t = \lambda \text{bias}_t + \lambda \varepsilon_t^g + \varepsilon_t^u. \tag{28}$$

In order to show that introducing persistent central bank misperceptions into the New-Keynesian model is sufficient to explain similar trend movements in money and inflation — the challenge posed by Lucas – we calibrate and simulate the model under imperfect knowledge. The calibrated parameter values are summarized in Table 1. Regarding the perceived output gap we choose to study a specific series of misperceptions:

$$\begin{array}{ll} \text{for } t = (1,10) & bias(t) = 0 \\ \text{for } t = (11,12,13,14) & bias(t) = (1,2,3,4) \\ \text{for } t = (15,100) & bias(t) = 4 \\ \text{for } t = (101,102,103) & bias(t) = (3,2,1) \\ \text{for } t = (104,200) & bias(t) = 1 \end{array} \tag{29}$$

The central bank's initial estimate of potential output is assumed to coincide with the true value. In periods 11 to 14 the central bank overestimates potential output growth by

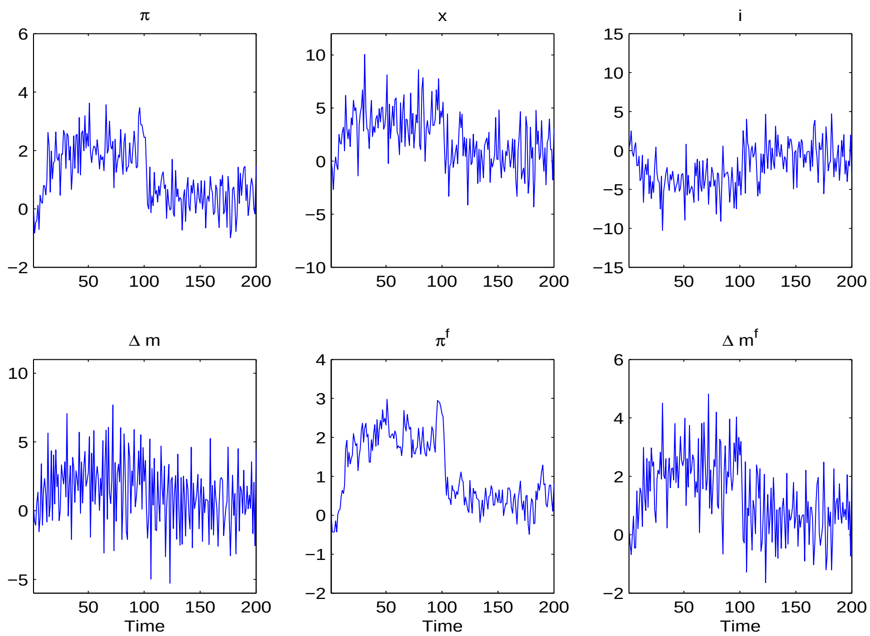
Table 1: Calibration

Parameter	Value	Economic interpretation
β	0.99	Discount factor of the policy maker.
$-\varphi$	-1	Real interest rate elasticity of aggregate demand (in line with Andres et al. (2006) and Ireland (2004)).
θ	0.5	Proportion of firms that adjust its prices in a given period (based on Bils and Klenow (2004)).
γ_y	0.1	Income elasticity of money demand (in line with Andres et al. (2006) and Ireland (2004)).
$-\gamma_i$	-0.4	Interest rate elasticity of money demand (in line with Andres et al. (2006) and Ireland (2004)).
ω	0.2	Weighting parameter of filter (broadly in line in Gerlach (2004))
$\Delta y_t^*, \pi^\star$	2	Equilibrium real interest rate, potential output growth and inflation target
$\sigma_g, \sigma_u, \sigma_e$	0.8	Standard deviation of cost-push, demand and money demand shocks
μ, ν	0	Persistence of cost-push and aggregate demand shocks
σ_{η^u}	0.6	Standard deviation of noise of cost-push shocks
$\sigma_{\eta^g}, \sigma_{\eta^e}$	0.4	Standard deviation of noise of demand shocks
σ_{η^e}	0.1	Standard deviation of noise of money demand shocks
σ_{μ^f}	0.54	Standard deviation of μ^f
κ^{crit}	5%	Critical value for the cross-checking rule.
N	4	Number of periods required for a sustained deviation in the cross-checking rule.

1 percentage point per period. As a consequence, the central bank's (and the public's) estimate of the output gap is 4 percentage points lower than the true output gap from period 15 onwards. Given an interest rate-elasticity of aggregate demand equal to unity as in the sources noted in Table 1, the mistake (bias) on the output gap of 4 percentage points translates into an interest-rate setting under the optimal policy under discretion, (27), that is four percentage points lower than needed to keep output at potential and inflation on target. Ultimately, this policy bias (caused by central bank misperceptions) will induce an increase in average inflation of about 2 ($= \lambda \text{bias}_t$) percentage points. Accordingly, money growth and the long-run level of nominal interest rates will also rise by 2 percentage points. From period 100 onwards the central bank's overestimate of potential output declines to 1 percentage point and the resulting deviation in average inflation to 0.5 percentage points.

A simulation of the consequences of output gap misperceptions under the optimal policy in the New-Keynesian model is shown in Figure 1 for a single draw of normally-distributed cost-push, demand and money demand shocks and noise terms. Due to an interest rate policy that is more accommodative than the central bank believes to be the case, money growth and inflation rise. This increase is persistent due to persistent central bank misperceptions regarding potential output. Over time, also the filtered measures of inflation, π^f, and money growth, Δm^f, increase, thus depicting a trend change in nominal

Figure 1: Output gap Misperceptions and the Money-Inflation Link



variables. This trend change is mirrored by a sustained increase in nominal interest rates - a consequence of the Fisher effect.

The simulation reported in Figure 1 suggests that the introduction of imperfect knowledge and persistent central bank misperceptions is sufficient to provide a unified account of monetary and inflation trends – the unresolved issue on the frontier of macroeconomic theory noted by Robert E. Lucas in the quote cited in the introduction. Of course, one may interject that the series of misperceptions simulated above is chosen in an ad hoc manner. However, empirical observations of central bank misperceptions are available for the United States and Germany (see Orphanides (2003) and Gerberding et al. (2006). In future research we plan to investigate this question further on an empirical basis.[14]

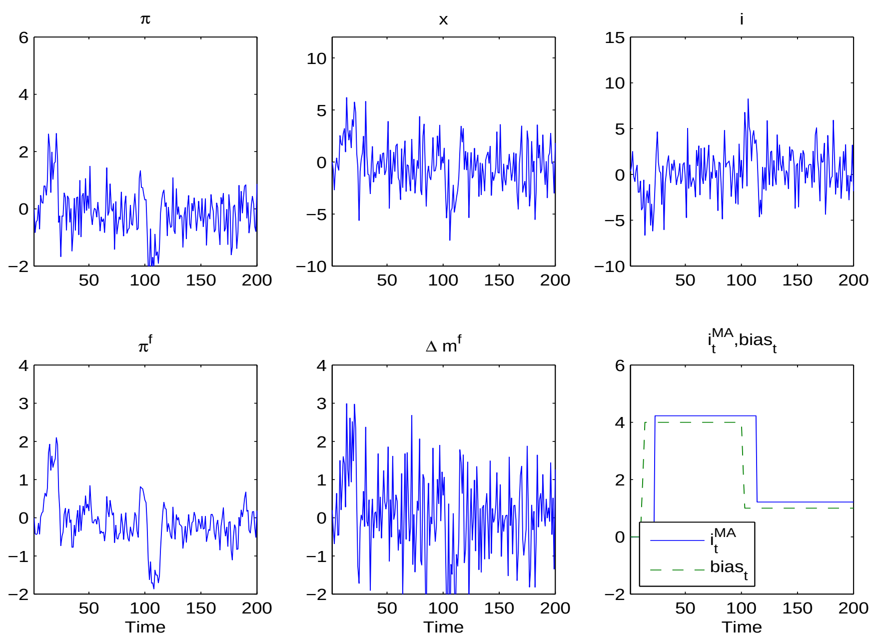
Returning to the preceding simulation one may ask why the central bank does not realize that its perceptions are biased, raises interest rates to an even higher level and thereby ensures that inflation returns to target relatively quickly. The reason is simple. Optimal monetary policy takes account of the best available forecast for inflation. This forecast, which is based on an incorrect output gap, states that inflation will return to target due to the belief that aggregate demand is consistent with price stability. If the central bank were to raise interest rates further its own forecast would indicate a recession and undershooting of its inflation target. The persistent bias in the forecast implies that the

[14] A short-cut that has been used in estimating New-Keynesian models over episodes with trending inflation is to assume that the central bank's inflation target follows a random walk. Our explanation with a constant inflation target but persistent policy mistakes due to persistence misperceptions regarding unobservables offers an alternative that can be grounded in empirical observation and does not require unobserved random changes in the central bank's price objective.

central bank is attributing successive periods of high inflation to a sequence of unfavorable shocks rather than a mistaken output gap estimate. This example is not without parallel in reality considering accounts of the 1970s inflation in the United States (see Orphanides (2003)).

In the next step of the analysis we show that augmenting the optimal interest rate policy from the New-Keynesian model,$i^{NK} = i_t^{opt}$ with monetary cross-checking as defined by equation (20) provides a convenient and effective avenue for correcting the central bank's policy bias that lead to the sustained increase in filtered money growth and inflation in the preceding simulation. To this end, we repeat the simulation using the policy with cross-checking, i_t^{CC}. This policy incorporates an additive and persistent adjustment in the event of sustained deviations of filtered (adjusted) money growth from target. The outcome is reported in Figure 2. We have dropped the panel with actual money growth, Δm, and have instead included a panel reporting the bias in the central bank's estimate of potential output, $bias_t$, and the adjustment in interest rates due to monetary cross-checking. This adjustment corresponds to i_t^M as defined in equations (20) and (19). The

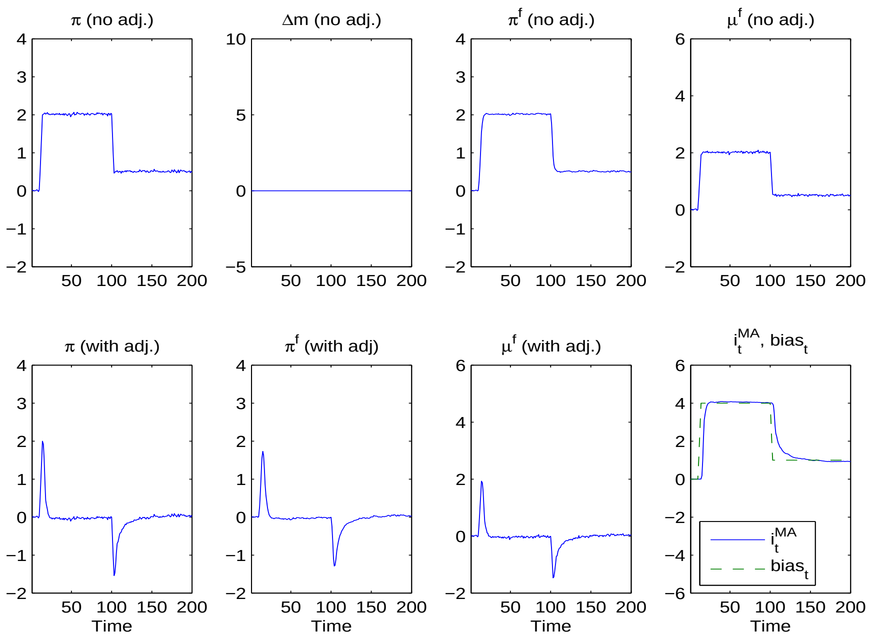
Figure 2: Output Gap Misperceptions and Monetary Cross-Checking



policy with cross-checking responds to the increase in filtered money growth, μ_t^f, fairly quickly after the policy bias has arisen. The interest rate adjustment of $(\frac{1}{\lambda\varphi})(\mu_t^f - \pi^*)$ almost perfectly offsets the policy bias arising from potential output, (bias_t). Once the misperception of potential output declines after period 100 cross-checking soon leads to another adjustment of interest rates.

To assess the sensitivity of our findings we draw 1000 series of shocks of 200 periods length from a normal distribution and use them to conduct a set of alternative simulations. Some of the findings are reported in Figure 3. The bottom right panel of Figure 3

Figure 3: Sensitivity Analysis: Averages over 1000 Simulations



reports the average path of the interest rate adjustment due to monetary cross-checking, i.e. i_t^M, over 1000 simulations under the same parameter settings as in the single simulation displayed in Figure 2. This panel confirms that, on average, cross-checking leads to the appropriate interest rate adjustments offsetting the policy bias due to output gap misperceptions.

5 Flexible-inflation targeting

So far, we have restricted our study to the analysis of a strict inflation-targeting central bank. In this section, we extend the analysis to allow for flexible inflation targeting, or in other words, for a positive weight α on output gap stabilization in the central bank's objective function:

$$\min \frac{1}{2} E_t \left\{ \sum_{i=0}^{\infty} \beta^i \left[\alpha x_{t+i}^2 + \pi_{t+i}^2 \right] \right\} \tag{30}$$

The central bank's optimization problem consists of choosing current and future interest rates such that the loss function is minimized given the forward-looking Phillips curve and IS curve presented previously (equations (1) and (2)). If the central bank acts under perfect knowledge about the shocks and the level of potential output, then the nominal interest rate consistent with optimal policy under discretion is chosen such that period t

inflation and output gap are given by:[15]

$$\pi_t^{opt} = \frac{\alpha}{\lambda^2 + \alpha(1-\beta\rho)} u_t = \alpha q u_t. \tag{31}$$

$$x_t^{opt} = -\frac{\lambda}{\lambda^2 + \alpha(1-\beta\rho)} u_t = -\lambda q u_t. \tag{32}$$

The interest rate that achieves this goal corresponds to:

$$i_t^{opt} = \gamma_\pi \pi_{t+1}^e + \frac{1}{\varphi} g_t = \left(1 + \frac{\lambda(1-\rho)}{\alpha\varphi\rho}\right) \alpha q \rho u_t + \frac{1}{\varphi} g_t. \tag{33}$$

The relationship between the optimal interest rate setting and the central bank's inflation forecast is directly apparent from the equation. As CGG point out the central bank's optimal policy is characterized by gradualism in the following sense. While policy moves to fully offset the effect of an aggregate demand shock on inflation, the consequences of cost-push shocks are at first only partially offset in order to cushion the negative consequences for output stabilization. As a result, the inflation rate can deviate for some time from the zero inflation target. These deviations can be quite persistent depending on the degree of serial correlation in the cost-push shocks. The long-run or unconditional mean of the inflation rate still remains equal to the (zero) long-run inflation target, however. As shown previously under strict-inflation targeting, the money supply continues to be determined according to the money demand equation (equation (6)) and plays no role in the conduct of monetary policy when the level of potential output is known.

Again, we extend the analysis to consider imperfect knowledge regarding economic shocks and unobservables such as potential output. Specifically, the shocks (g_t, u_t, and e_t) are only observed with noise[16] and the central bank and public's perceived estimate of the output gap may deviate from its true value for a sustained period of time. The central bank's optimal choice of the perceived output gap is then given by:

$$x_t^{opt,e} = -\frac{\lambda}{\lambda^2 + \alpha(1-\beta\rho)} u_t^e = -\lambda q u_t^e. \tag{34}$$

In the presence of output gap misperceptions the actual output gap will deviate from the desired one in the following way:

$$x_t = x_t^e + \text{bias}_t = -\lambda q u_t^e + \text{bias}_t \tag{35}$$

where bias_t again denotes the mistake that the central bank makes when estimating the level of potential output. The interest rate that the central bank chooses will accordingly deviate from its optimal level where the size of the deviation will depend on the extent

[15] See Clarida et al. (1999) for the derivation of these results.

[16] See equations (21) to (23) for the specification of the shocks.

of the output gap misperception.

The nominal interest rate that the flexible inflation-targeting cental bank will set under imperfect knowledge is then given by:

$$i_t^{NK} = i_t^{opt} = \frac{1}{\varphi}\left[\left(1+\frac{\lambda(1-\rho)}{\alpha\varphi\rho}\right)\alpha q\rho u_t^e - \text{bias}_t + g_t^e\right]. \tag{36}$$

Again, the central bank will choose an interest rate that is too low (high) to achieve the optimal level of the output gap and the inflation rate when it overestimates (underestimates) the level of potential output. The resulting inflation rate is given by:

$$\pi_t = \alpha q u_t^e + \lambda \text{bias}_t + \lambda \varepsilon_t^g + \varepsilon_t^u = \pi_t^{opt} + \lambda \text{bias}_t + \lambda \varepsilon_t^g + \varepsilon_t^u. \tag{37}$$

Finally, we also consider augmenting the central bank's interest policy with monetary cross-checking based on the strategy with two components discussed previously. It is useful to consider for a moment if it is necessary to modify the original definition of this strategy for the case of flexible inflation targeting. Of course, the first component, that is i_t^{NK} now corresponds to the policy defined by equation (36), but what about the second, long-run monetary component? Interestingly, it need not necessarily be revised. The reason is that the long-run mean of inflation (if the target is achieved) remains unchanged from the strict-inflation targeting case. Flexible inflation-targeting simply implies that inflation returns to target more slowly after disturbances (and in the absence of sustained misperceptions).

Thus, the central bank continues to compute every period the normally-distributed test statistic,

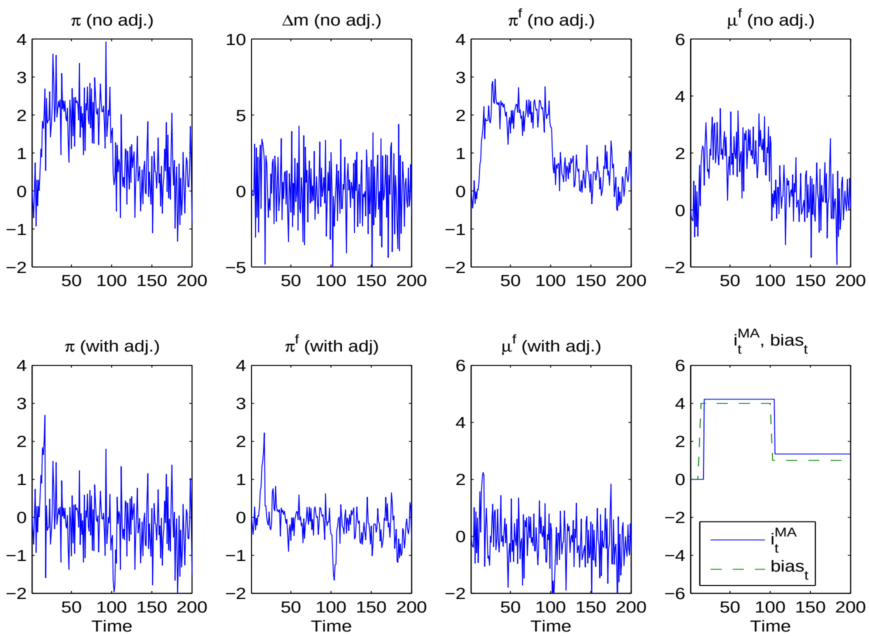
$$\kappa = \frac{\mu_{t-1}^f - \pi^*}{\sigma_{\mu^f}}, \tag{38}$$

and checks whether κ deviates from a critical value κ^{crit}. Again, if the central bank obtains successive signals of a sustained deviation from target, i.e. $(\kappa > \kappa^{crit}$ for N periods$)$ or $(\kappa < -\kappa^{crit}$ for N periods$)$, it responds by adjusting interest rates accordingly.

$$i_t^M = \begin{cases} i_{t-1}^M + (\frac{1}{\varphi\lambda})(\mu_{t-1}^f - \pi^*) & \text{if } \kappa > \kappa^{crit} \text{ or } \kappa < -\kappa^{crit} \text{ for } N \text{ periods} \\ \\ i_{t-1}^M + 0 & \text{else} \end{cases} \tag{39}$$

The consequences of persistent policy misperceptions concerning the output gap under interest rate policy with and without monetary cross-checking are simulated in Figure 4. To conduct such simulations we had to calibrate the New-Keynesian model for the case of flexible targeting. We chose a value of the relative output weight α of 0.5 and set the persistence of cost-push shocks to 0.5. Otherwise, we use the same specification as in the analysis of strict inflation targeting. Furthermore, to keep the simulation comparable to previous ones we use the same values for the process of misperceptions as in the preceding section. The results depicted in the upper four panels of Figure 4 again confirm that persistent central bank misperceptions cause similar trend movements in money growth and inflation.

Figure 4: Flexible-Inflation Targeting: Output Gap Misperceptions and Monetary Cross-Checking



The lower four panels of Figure 4 illustrate the behavior of inflation as well as filtered measures of adjusted money-growth (μ^f) inflation (π^f), when the central bank's interest rate policy incorporates monetary cross-checking, i_t^{CC}, as defined in equation (39). The lower right panel of Figure 4 reports the bias in the central bank's estimate of potential output and the adjustment in the nominal interest rate due to cross-checking. As can be seen in Figure 4 the cross-checking rule also work very well for the flexible-inflation targeting case. The interest rate responds relatively fast to the increase in long-run adjusted money growth, and largely offsets the bias arising from the misperception of the output gap. As a consequence the inflation rate returns to its target after the adjustment and fluctuates around it in the subsequent periods.

6 Conclusions and Outlook

In conclusion we summarize the contributions of this paper to the current debate on the role of money in monetary policy design as follows:

We have endeavored to follow the research strategy suggested by Lucas namely to extend the New-Keynesian model in order to give a unified account of trends, including trends in monetary aggregates. We found that allowing for imperfect knowledge and persistent central bank misperceptions regarding key unobservables such as potential output (or correctly stated flexible-price equilibrium output) is sufficient to account for persistent and similar movements of trend inflation and trend money growth. So far, the movements of trend money growth and inflation in our model occur more or less contemporaneously. However, in empirical work (cf. Gerlach (2003) and others) trend money growth is reported to lead trend inflation. Thus, in future work we plan to investigate potential modifications of the base-line New-Keynesian model that would make it possible to replicate this empirical fact.

Building on Beck and Wieland (2007) we show that a strategy which uses monetary information as a cross-check improves inflation control in the presence of persistent central bank misperceptions. Essentially, our novel characterization of a policy rule with cross-checking acts to ensure that observed, past deviations of money growth and inflation from target are eventually corrected. This correction occurs even though the available inflation forecasts at the time suggest that demand is sufficiently weak to return inflation on track by itself.

Our definition of a rule with cross-checking would also allow incorporating cross-checks regarding other information than monetary aggregates. However, given the well-established quantity relation between inflation and money growth adjusted for output and velocity trends, money growth is an obvious candidate. Using past inflation itself may be an odd candidate for cross-checking, precisely because in this case inflation forecasters using all available information and modeling theory would predict an under- or (over-)shooting of inflation given the output gap estimate. For this reason, one can also speculate whether such an insistence on reacting to past outcomes is perhaps better institutionalized separately from those in charge of inflation forecasting in a central bank, even though our proposed rule combines both elements mathematically.

Furthermore, we have shown that the above findings not only apply to the case of a central bank that cares exclusively about inflation control but also to the case of central bank that implements flexible inflation targeting. In the latter case, the central bank's concern for output stabilization leads to more persistent inflation dynamics in the event of cost-push shocks. As a result, cross-checking simply against the long-run link between money and inflation becomes more difficult. Nevertheless, we find that the improved inflation control under monetary cross-checking remains apparent in our simulations of the baseline New-Keynesian model under imperfect knowledge.

In future work we plan to explore in detail to what extent available historical series of output gap revisions from the U.S.A and Germany would fill our simple examples with empirical content. In particular, we plan to investigate to what extent output gap revisions in the U.S.A. and Germany may account for inflation surges in the 1970s and whether attention to money growth would have helped to improve inflation control. Furthermore, we plan to investigate to what extent the beneficial effects of monetary cross-checking for inflation control may be the possibility of longer-run shifts in velocity.

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