How Does Skill-Biased Technological Change Affect Human Capital Accumulation?

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Abstract

The paper studies the effects of skill-biased technological change (SBTC) on human capital accumulation (HCA). Two particular issues are addressed: (i) the effects on the accumulation path in the short- and long-run of a given economy; (ii) the extent of the effects in high- and low-fertility countries. To study these questions, I employ an OLG-model where the capital market is imperfect. The analysis shows that, in both the short- and long-run, SBTC affects HCA negatively when the economy is in low development stages and positively in high development stages. As for countries with different characteristics, high-fertility countries are more likely to be negatively affected in the long-run while low-fertility countries converge to steady-state with higher human capital accumulation after the occurrence of SBTC.

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1 Introduction

Human capital is one major input factor in the production sector of modern economies. Therefore, the output and income level in a society is influenced, beside other factors, by human capital accumulation (HCA). Shocks may affect the process of HCA; and they may influence the evolution of output and income int the society via HCA. The question arises: does a society have to fear such a shock or, to put it differently, is human capital growth and consequently output and income growth higher or lower after the shock occurred? I am concentrating on skill-biased technological change (SBTC) as one example of shocks which may affect HCA. The link between SBTC and HCA arises from the effect of the technology change on wage levels and wage inequality. I later show that any change in wage levels and wage inequality will also affect education decisions of individuals in the present and future periods.

Taking a more detailed look on the link between SBTC and HCA, we firstly find that SBTC changes the marginal product of workers in current and future periods where the new technology is in use. In particular, SBTC increases the marginal product of higher skilled workers more than that of less skilled workers. Therefore, wage levels will change and firms give higher remuneration to high-skilled workers relative to low-skilled workers. Secondly, these changes in current wage levels and future wage inequality play an important role in determining education decision of a household. On the one hand, household's expenses on education investment depend on current wage levels because borrowing is constrained under the assumption of imperfect capital market. On the other hand, these expenses depend on return of education investment given by the future wage inequality between high- and low-skilled workers. This implies that the education decision itself is therefore affected by SBTC via changes in wages. Furthermore, not only the current education decisions are influenced by SBTC but also all future education decisions. As a result, the sequence of human capital accumulation will change after introduction of a new skill-biased technology.

To study the SBTC-HCA relationship, two questions are of particular interest: (i) For an economy with given characteristics, what are effects of SBTC on HCA in the short- and long-run? (ii) Does SBTC affect equally or not countries with different characteristics? I distinguish the cases of countries with high and low fertility rates.

Concerning dynamic effects of SBTC, short-run analysis studies the evolution of human capital growth rate and identifies whether human capital growth accelerates or slows down. Long-run analysis focuses on how SBTC affects the steady-state of human capital accumulation where human capital is measured in terms of average human capital of an economy.

To study these questions, I employ an overlapping-generations model where parents invest in education of their children. Capital market is assumed to be imperfect where the assumption of imperfection is taken to the extreme of no borrowing. To simplify the analysis, I consider only two types of adults, the skilled ones who were educated in previous period, and the unskilled ones who were not.

Results of the analysis are the following. With regard to SBTC effects in an economy with given characteristics, I find that, in both short- and long-run, SBTC affects the HCA negatively when the economy is in low development stages and positively in high development stages. As for countries with different characteristics, high-fertility countries are more likely to be negatively affected in the long-run while low-fertility countries converge to steady-state with higher human capital after the occurrence of SBTC.

The analysis of SBTC effects on HCA is recent in the literature though there are many contributions that have been studying some other factors affecting human capital accumulation. Moaz and Moav (1999), for example, study the effects of changes in wages on the human capital accumulation process for a given level of technology. They find that wage levels and future wage inequality are important determinants for education decisions and human capital accumulation. As an extension of their work, I take into account the possibility of technology change, in particular SBTC, as one of the possible shocks affecting wages, which in turn affect HCA.

Maoz and Moav (1999) is based on the model introduced by Galor and Zeira (1993) who analyse the link between income distribution and education investments. They take additionally into account that the accumulation of human capital via education decisions has an effect on future income levels and distributions since the supply of skilled/unskilled labour affects marginal products and remunerations for input factors.

However, there are mainly two explanations in the literature regarding changes in wages: (i) skill-biased technological change and (ii) increasing international trade or globalisation effects. Both arguments are raised to explain the increase in wage inequality in the USA during the 1980s (see for example empirical studies of Murphy and Welch, 1989; Levy and Murnane, 1992). The SBTC explanation argues that SBTC increases the demand for skilled labour relative to unskilled labour and as a consequence, raises wage inequality between skilled and unskilled workers.[1] Empirical evidence founded by Mincer (1991) and Autor, Katz and Krueger (1998) shows that introduction of computers in workplaces, which is seen as SBTC, can explain most of the observed changes in wage inequality. Nevertheless, Card and DiNardo (2002) argue that not all skill-biased technologies would have effects on wages but only a certain type. International trade is another explanation of changes in domestic wage levels (see Wood, 1995; Burtless, 1995; and Wood, 1998). This explanation is based on the Stolper-Samuelson theorem which predicts changes in domestic input factor rewards when final good prices on the world market change.[2] To identify which one of these two arguments has stronger explanatory power, Acemoglu (2002) studies empirically the case of the USA and Winchester and Greenaway (2007) have done the case of the UK. Both studies find that SBTC is the major source for the observed increase in wage inequality in these countries.

Therefore, I concentrate in this paper on SBTC effects on wages which in turn affect human capital accumulation. The remaining part of the paper is organised as follows. Section 2 describes the model specification. Section 3 derives the human capital accumulation path. Section 4 discusses the effect of skill-biased technological change on human capital accumulation. Section 5 concludes the paper.

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[1] See Acemoglu (1998), Bound and Johnson (1992) for the USA; Machin and van Reenen (1998) for a comparison of the USA and other OECD countries.

[2] See Stolper and Samuelson (1941).

2 The Model Specification

To analyse effects of SBTC on HCA, I employ an overlapping-generations model over an infinite time horizon. The setup is similar to Maoz and Moav (1999) and Galor and Zeira (1993). In every period, the economy produces a single homogeneous good using all types of human capital as input factors. The supply of human capital is determined by individuals' decision on the level of education investment in each child in the preceding period. The capital market is assumed to be imperfect where this assumption is driven to the extreme of no borrowing possibilities whereas the labour market is characterised by perfect competition.

2.1 Individuals

Individuals live for two periods labelled child- and adulthood respectively. In childhood, individuals are passive, i.e., do not consume, and may receive education. Later in life, adult individuals supply labour, consume, and invest in their children's education.

For simplification reason, I assume further that parents' fertility decision is exogenous.[3] A version of the model with both endogenous education and endogenous fertility decisions would lead to the same qualitative results comparing to the here presented version with endogenous education and exogenous fertility decisions.[4] Once parental education decision has been made, it leads to two different skill levels of grown-up children in the next period, i.e., an adult can be either skilled or unskilled depending on his parent's education decision in the past.

Each parent shares the same utility function which is defined by his own consumption and quality of his children. The latter is represented by expected average

[3]However, it is largely argued in the literature that there is a link between parents' fertility decision and their decision on children's education plan. This interdependency was pointed out first by Becker (1960) and Becker and Lewis (1973); it is also taken into account in Galor and Zeira (1993).

[4]When fertility choice is endogenous as well, I take into account that both decisions are normally not made at the same time. Parents decide first on the number of children and some years later on their education. Due to uncertainty, for instance in the actual evolution of personal income, both decisions cannot be perfectly adjusted to each other. This explains why inner solutions for the education decision can occur in this setup while many contributions in the literature arrive at corner solutions [as for instance Galor and Zeira (1993)].

income of children. Formally, this utility is given by

$$U_t^i = \ln[c_t^i] + \beta \ln \left[w_{t+1}^u + \lambda_t^i \Big(w_{t+1}^s - w_{t+1}^u \Big) \right] \tag{1}$$

where i denotes parent's skill level, with $i = u$ in the case of unskilled parent and $i = s$ in the case of skilled parent. Variable c_t^i denotes parent's consumption, β represents degree of altruism, and w_{t+1}^s (resp. w_{t+1}^u) denotes wage income of his skilled (resp. unskilled) grown-up children in the next period. The proportion of children from parent i who receive education is denoted by λ_t^i, where $\lambda_t^i \in [0, 1]$. This proportion is endogenously determined by the parental education decision.

Note that no expectation operator is contained in equation (1) although future or expected wages of children are determinants of parent's utility. Here, individuals have perfect foresight over future wages because I suppose that there is no uncertainty in the model and that individuals form rational expectations over future variables.

Note also that the number of children does not enter equation (1) though parents may also value the number of children. This is due to the assumption that the number of children is constant and exogenously given. In fact, including n would have only a multiplier effect on children's average wage income and would not change the results qualitatively.

To form his education decision, parent i will maximise his utility subjected to his budget constraint which is given by

$$w_t^i = c_t^i + \lambda_t^i n e_t \tag{2}$$

where w_t^i denotes parent's i wage income and e_t denotes education cost per child.

Education cost per child is assumed to follow the equation

$$e_t = \eta w_t^s.$$

It is the proportion η of the wage of a skilled adult. This is because only skilled adults can work as "teachers" and one teacher can transfer skill and knowledge to more than one child at the same time.[5] For plausible education cost per child, I suppose $\eta \in (0, 1)$.

Now that we have defined education cost per child, it is important to know whether this cost is affordable for parents or not. To give an idea of how much this cost represents in parent's income, I introduce relative education cost for skilled and unskilled parents, $\frac{e_t}{w_t^i}$. For skilled parents, relative education cost is determined by parameter η and is independent of the wage ratio between skilled and unskilled workers. For unskilled parents, relative education cost is given by $\eta \frac{w_t^s}{w_t^u}$ and thus, depends on the wage ratio.

2.2 Firms

Firms produce a single consumption good employing both skilled and unskilled labour in production. Output Y_t follows the constant returns to scale production function

$$Y_t = \left[b(W_t^s)^\gamma + (W_t^u)^\gamma \right]^{\frac{1}{\gamma}} \tag{3}$$

where W_t^s (resp. W_t^u) denotes the number of skilled (resp. unskilled) workers employed in the production process. Input factors are weighted by b, which is positive and exogenously given. Exponent $\gamma \in (0, 1)$ determines the elasticity of substitution between both labour inputs.[6] This particular formulation of the production function in equation (3) allows SBTC occurrence which can be simulated by an exogenous rise in b.[7]

Wages are determined on the labour market where supply of and demand for labour meet. Current supply of labour is given by education decisions in the last period. Current demand for labour depends on the production function which describes the technology currently used by firms. In a perfectly competitive labour market, wages are determined by marginal product of skilled and unskilled work-

[5]Maoz and Moav (1999), de la Croix and Doepke (2003, 2004) instead assume that education cost depends on average human capital in a society since this is the average human capital level of teachers. In their models with skilled and unskilled labour being perfect substitutes to each other, this assumption means that education cost depends on average wage. For simplification reason, I assume here dependency of the education cost on the skilled wage rate only. The results of this paper do not change when education cost per child would depend on average wage.

[6]The assumption on γ ensures that both input factors are substitutes in this production function.

[7]This production function is similar to the one in Acemoglu (2002).

ers. The level of marginal product for each labour type in turn is, in the case of CRS production functions, uniquely determined by the input factor ratio $\frac{W_t^s}{W_t^u}$. To solve for skilled and unskilled wages for given production function and labour supply, we therefore have to identify the input factor ratio.

Since all adults may not work according to their skill level, i.e., not every skilled one works as skilled worker, the input factor ratio is not necessarily equal to the ratio between skilled and unskilled adults in the population. The reason is that a skilled adult has high knowledge allowing him to apply for jobs requiring either high or low skills. Thus, he will choose the type of work which offers him the higher remuneration. Unlike a skilled adult, an unskilled adult can only apply for jobs which requires less skill.

However, it follows that adults will only work according to their skill level as long as the following wage condition is fulfilled: $w_t^s \geq w_t^u$. Otherwise, when $w_t^u > w_t^s$, a skilled adult would have an incentive to apply for an unskilled job. This decision would lead to the readjustment process of the wage condition, i.e., skilled wage would increase and unskilled wage would decrease. At the end, skilled adults will cease to choose to work as unskilled workers when skilled and unskilled wages are equalised.

In the case of $w_t^s \geq w_t^u$, where adults work according to their skill level[8], wages are formally given by

$$w_t^s = b\left[b + \Big(\frac{1-\alpha_t}{\alpha_t}\Big)^{\gamma}\right]^{\frac{1-\gamma}{\gamma}} \equiv w^s(\alpha_t, b), \tag{4}$$

$$w_t^u = \left[b\Big(\frac{\alpha_t}{1-\alpha_t}\Big)^{\gamma} + 1\right]^{\frac{1-\gamma}{\gamma}} \equiv w^u(\alpha_t, b), \tag{5}$$

which in turn implies

$$\alpha_t \leq \frac{1}{1+b^{\frac{1}{\gamma-1}}} \equiv \widehat{\alpha}. \tag{6}$$

Variable α_t denotes the proportion of skilled adults in the adult population and is

[8]Formally, according to their skill level means $W_t^s = L_t^s$ and $W_t^u = L_t^u$.

derived as

$$\alpha_t = \frac{L_t^s}{L_t^s + L_t^u}.$$

Considering this definition of α_t, condition (6) implies that the current proportion of skilled adults must be low enough to end up in the case where all adults work according to their skill level and $w_t^s \geq w_t^u$.

Note that type-specific wages in period t can be expressed as functions of α_t and b because this formulation of wages will be useful in the later analysis. The wage ratio in this case is computed as

$$\frac{w_t^s}{w_t^u} = b\left(\frac{1-\alpha_t}{\alpha_t}\right)^{1-\gamma}$$

and depends also on the current skill formation α_t and technology b.

In the case where a large number of skilled adults in the population exist, i.e., $\alpha_t > \widehat{\alpha}$, w_t^u would be larger than w_t^s if all adults work according to their skill level. The readjustment process would take place such that in the end the input factor ratio $\frac{W_t^s}{W_t^u}$ equals to $\widehat{\alpha}$ and wages are equalised at the level

$$w_t^s = w_t^u = \left[b^{\frac{1-\gamma}{\gamma}} + 1\right]^{\frac{1-\gamma}{\gamma}}$$

for all values of α_t. Note that after the readjustment process the wage ratio in this case is always equal to one.

For a graphical illustration of the wage determination for given skill formations in the economy described by α_t, see figure 1 where wages are displayed as functions $w^i(\alpha_t, b)$ with $i = s, u$. Depending on the current proportion of skilled adults, dotted lines in figure 1 display marginal products and solid lines wages. Note that to the left of the threshold $\widehat{\alpha}$, adults work according to their skill level. To the right of $\widehat{\alpha}$ in contrast, the readjustment process takes place which in the end leads to equalised wages.

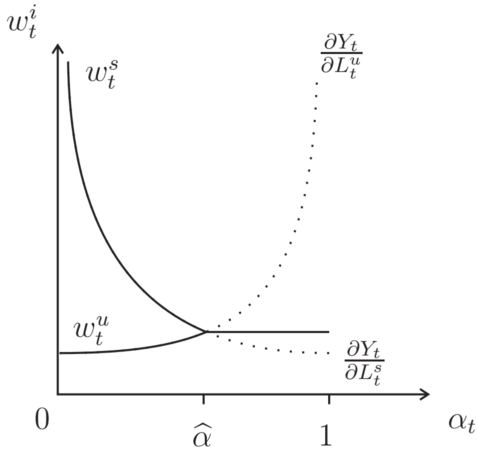

Figure 1: Skilled and unskilled wages for given proportion of skilled adults α_t.

3 The Dynamics of Human Capital Accumulation

This section derives the human capital accumulation path for the modelled economy. A proper measure for human capital is the level of average human capital in each period since the aggregate stock of human capital depends also on the population size. Average human capital in turn is uniquely determined by the proportion of skilled adults, denoted by α_t, assuming that skill levels are constant. Therefore, I will concentrate on the dynamics of α_t in the remaining part of the paper. This section shows the derivation of the dynamics of α_t. A detailed description of the accumulation path can be found in appendix A.1.

Economy's dynamic evolution is formally given by the dynamic equation for the proportion of skilled adults:

$$\alpha_{t+1} = \alpha_t \lambda_t^{s*} + (1 - \alpha_t)\lambda_t^{u*}. \tag{7}$$

This equation states that the next period's proportion of skilled adults α_{t+1} is determined by the current optimum education decision of skilled adults λ_t^{s*} and that of unskilled adults λ_t^{u*} weighted by their proportion in the actual adult population.

Optimum education decisions are the result of the following maximisation problems:

$$\max_{\lambda_t^i} U_t^i \quad \text{s.t.} \quad w_t^i = c_t^i + \lambda_t^i n e_t \tag{8}$$

with $i = s, u$. An adult with skill level i maximises utility subject to his budget constraint over the fraction of children who shall receive education.

For a moment, we take expected next period's wages w_{t+1}^i as given and derive the main determinants of optimal education decisions. These decisions can be derived as

$$\lambda_t^{i*} = \begin{cases} 0 & \text{if } \frac{1}{1+\beta}\,[.] < 0, \\ 1 & \text{if } \frac{1}{1+\beta}\,[.] > 1, \\ \frac{1}{1+\beta}\left[\beta\frac{w_t^i}{ne_t} - \frac{w_{t+1}^u}{w_{t+1}^s - w_{t+1}^u}\right] & \text{else.} \end{cases} \tag{9}$$

That is, adults with skill level i may not invest in education of any child, may invest in education of all children, or may invest in education of some of their children.

From equation (9), we can infer the dependency of optimal education decisions on fertility rate, current wages, and next period's wages. Higher fertility rate n implies that, ceteris paribus and comparing to situations with lower fertility rate, investing in education of the same proportion of children is more costly for the parent, simply due to the larger number of children per parent and hence the larger number of children who shall receive education. Note that the optimal education decision depends positively on parental wage income w_t^i, which in turn implies that skilled parents never invest less in education of their children comparing to unskilled ones since $w_t^s \geq w_t^u$. Note also that the term $\frac{w_t^i}{e_t}$ in the first summand of the inner solution reflects the inverse relative education cost for parents with skill level i. Furthermore, the term $\frac{w_{t+1}^u}{w_{t+1}^s - w_{t+1}^u}$ reflects the inverse of next period's wage inequality between skilled and unskilled workers relative to the unskilled wage (relative wage inequality in the following). These findings have two implications. On the one hand, higher relative education cost $\frac{e_t}{w_t^i}$ would lower the chosen proportion of educated children since it is more costly for parents to invest in children's education. On the other hand, higher relative wage inequality in the next period $\frac{w_{t+1}^s - w_{t+1}^u}{w_{t+1}^u}$ would affect positively λ_t^{i*} because this means higher incentive to invest in education in order to become skillful in the next period and receive higher wage. These findings shown that education decisions depend on current and next period's wages and wage inequality.

So far, we took next period's wages w_{t+1}^i as given. We will now take into account that next period's wages are endogenously determined in the model and will analyse how optimum education decisions change. The only change will be that optimum education decisions cannot be explicitly derived as in the derivation with given next period's wages presented above.

We know from equations (4) and (5) that wages depend on the skill formation in the respective period, that is $w_t^i = w(\alpha_t, b)$. Therefore, next period's wages depend on the skill formation α_{t+1} and current wages depend on α_t. Accounting for these facts, optimum education decisions in equation (9) can be rewritten as

$$\lambda_t^{i*}(\alpha_t, \alpha_{t+1}, b) = \begin{cases} 0 & \text{if } \frac{1}{1+\beta}\,[.] < 0, \\ 1 & \text{if } \frac{1}{1+\beta}\,[.] > 1, \\ \frac{1}{1+\beta}\left[\beta \frac{w^i(\alpha_t,b)}{n\eta w^s(\alpha_t,b)} - \frac{w^u(\alpha_{t+1},b)}{w^s(\alpha_{t+1},b) - w^u(\alpha_{t+1},b)}\right] & \text{else} \end{cases} \tag{10}$$

where

$$\alpha_{t+1} = \alpha_t \lambda_t^{s*}(\alpha_t, \alpha_{t+1}, b) + (1-\alpha_t)\lambda_t^{u*}(\alpha_t, \alpha_{t+1}, b). \tag{11}$$

Here, optimum education decisions are no longer explicitly given by equation (9) because optimal education decisions in t depend on future wages and hence on future skill formation α_{t+1} where the latter is determined by the optimal education decisions in t themselves. But optimal education decisions are implicitly defined by equation (10) in conjunction with equation (11). An explicit solution cannot be derived though the model structure is relatively simple. However, optimum education decisions λ_t^{i*} and consequently next period's skill formation α_{t+1} are uniquely determined by the current skill formation α_t for given parameter values.

Proposition 1. *For every combination of α_t, b, β, γ, η and n, there exist unique solutions for the maximisation problem of skilled and unskilled parents, i.e., for λ_t^{s*} and λ_t^{u*}. Consequently, the skill formation in the next period α_{t+1} is uniquely determined.*

Proof. See appendix A.2. □

Given the dynamic equation for α_t and having solved for optimum education decisions, the evolution of economy's average human capital over time can be described now. Due to the lack of explicit solutions for the optimum education decisions and for the dynamics of α_t, human capital accumulation over time is derived by employing a numerical example.[9] Figure 2 displays the accumulation

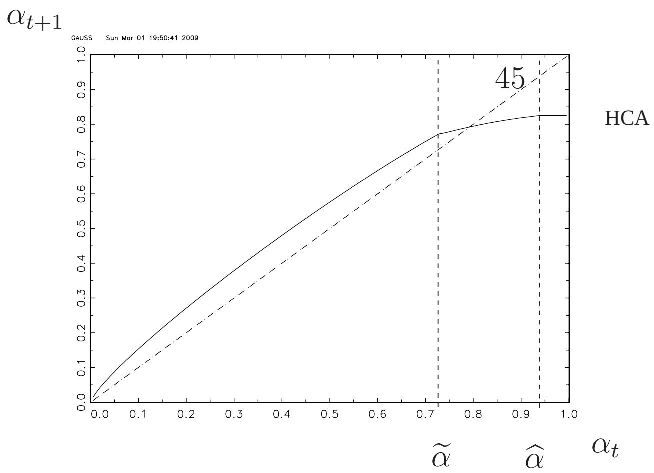

Figure 2: The Human capital accumulation path.

path for an example economy. The accumulation path is concave which implies low overall investments in children's education in development stages with currently low average human capital and higher investments in stages with higher average human capital. We can also deduct from the figure that the economy converges in the long-run to a unique steady-state which is given by the intersection of the HCA-curve with the $45°$-line.[10] Note also that the accumulation path can be divided into three segments (divided by $\widetilde{\alpha}$ and $\widehat{\alpha}$) which are described in detail in appendix A.1.

Comparing HCA in economies with different characteristics, figure 3 depicts accumulation paths for economies which differ with respect to their fertility rate. Fertility is higher in the economy described by the lower accumulation path, i.e., $n_{high-fertility} > n_{low-fertility}$.[11] The reason why economies with higher fertility face lower HCA is based on the implication of higher fertility on the education decisions. Higher fertility rate induces parents to invest in education of a lower

[9]To simulate the model, I use the following parameter values: $\beta = 1$, $\gamma = 0.5$, $\eta = 0.22$, $b = 4$ and $n = 1.6$. In the choice of γ, I follow Acemoglu (2002) who proposes a level of $\gamma = 0.5$ which fits empirical data on the elasticity of substitution in the production function (See Acemoglu, 2002, page 20).

[10]Beside the positive steady-state mentioned here, there exists a second but trivial steady-state at $\alpha_t = 0$ which is not covered in the following analysis.

[11]In the simulation of the depicted HCA processes, I choose the following fertility rates: $n_{high-fertility} = 2.5$ and $n_{low-fertility} = 1.6$.

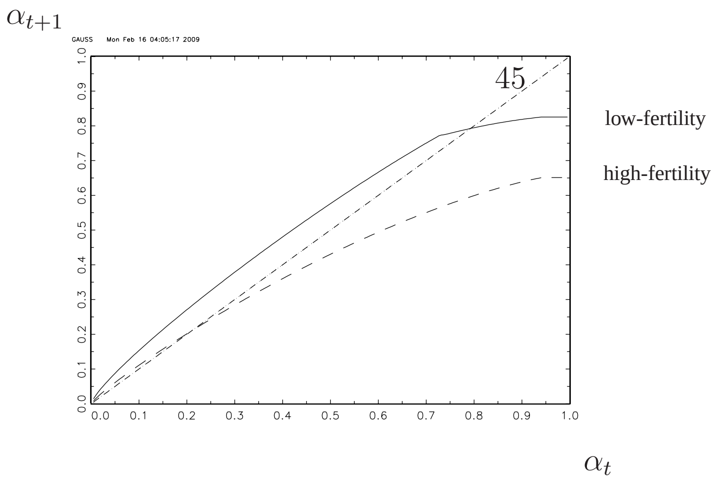

Figure 3: Human capital accumulation path in the case of a high- and a low-fertility country.

proportion of children. Therefore, given that two countries differ in n only, higher fertility negatively affects the accumulation path.

Note that in figure 3 the long-run equilibrium is again given by the intersection between the accumulation path and the 45-line for each displayed economy. We can therefore deduct that in the long-run high-fertility economies will converge to a steady-state with lower proportion of skilled adults comparing to low-fertility economies which converge to a steady-state with higher proportion of skilled adults. Considering that lower proportion of skilled adults implies lower average income level, we can think of the high-fertility economy as a poor country. Since the low-fertility economy consists of relatively many skilled adults in the long-run equilibrium, we can think of this economy as a rich country. This negative relationship between fertility and average income level is also empirically observable.[12]

[12]Birdsall (1988) presents more details on population growth and the negative relationship between income level and fertility rate in recent decades. The negative relationship is explained in the literature by quality-quantity trade-off models which were introduced by Becker (1960) and Becker and Lewis (1973) as mentioned above. Their results are empirically proven, for instance, by Hanushek (1992).

4 The Effects of Skill-Biased Technological Change on Human Capital Accumulation

In the previous section, we analysed human capital accumulation over time for an economy with constant technology. This section analyses firstly the effects of an unexpected skill-biased technological change on the human capital accumulation path and compares secondly these effects between two economies which are different in their fertility rate.

4.1 General Effects of SBTC on HCA

Regarding SBTC effects on HCA in general, I start with analysing the effects of SBTC on wages and wage ratios, followed by the analysis of how changes in wages affect education decisions, and finally, derive the net effect of wage changes on the HCA.

SBTC is simulated by raising parameter b which weights skilled and unskilled labour input in the production process. Due to this change in technology, marginal products and consequently wages are affected. As shown formally in equations (12) to (14), marginal product and wage for each skill level in period t rise after the introduction of a new skill-biased technology for a given skill formation α_t, where $t \in [k, \infty]$ and k denotes the period of SBTC occurrence.

$$\frac{\partial w_t^s}{\partial b} = \left[b + \left(\frac{1-\alpha_t}{\alpha_t}\right)^{\gamma}\right]^{\frac{1-\gamma}{\gamma}} + b\frac{1-\gamma}{\gamma}\left[b + \left(\frac{1-\alpha_t}{\alpha_t}\right)^{\gamma}\right]^{\frac{1-2\gamma}{\gamma}} > 0 \quad (12)$$

$$\frac{\partial w_t^u}{\partial b} = \frac{1-\gamma}{\gamma}\left[b\left(\frac{\alpha_t}{1-\alpha_t}\right)^{\gamma} + 1\right]^{\frac{1-2\gamma}{\gamma}}\left(\frac{\alpha_t}{1-\alpha_t}\right)^{\gamma} > 0 \quad (13)$$

$$\frac{\partial \frac{w_t^s}{w_t^u}}{\partial b} = \left(\frac{1-\alpha_t}{\alpha_t}\right)^{1-\gamma} > 0 \quad (14)$$

From equation (14) we can see that the skilled wage rises faster than the unskilled wage due to the skill-biased character of the technology change. This implies, in current and subsequent periods after SBTC occurrence, a larger ratio between skilled and unskilled wage.

Two effects on current and future education decisions are caused by the increased wage ratios. It is helpful for identifying these effects to rearrange the optimal education decision equation for each parental skill level. Concentrating on inner solutions, skilled and unskilled parents' education decision are derived as

$$\lambda_t^{u*} = \frac{1}{1+\beta}\left[\frac{\beta}{n}\frac{w_t^u}{\eta w_t^s} - \frac{1}{\frac{w_{t+1}^s}{w_{t+1}^u}-1}\right] \text{ and } \tag{15}$$

$$\lambda_t^{s*} = \frac{1}{1+\beta}\left[\frac{\beta}{n}\frac{1}{\eta} - \frac{1}{\frac{w_{t+1}^s}{w_{t+1}^u}-1}\right] \tag{16}$$

respectively where $t \in [k, \infty]$. These two equations allow us to see how education decisions are affected by changes in current and next period's wage ratio. Unskilled parents' education decision is affected via the change in the current and the future wage ratio. A higher current wage ratio means higher relative education cost. Unskilled parents tend to invest less in the education of their children due to their higher relative education after technology changed. This implies a negative effect on HCA and we can expect less skilled individuals in the future. But at the same time, next period's wage ratio is also higher after occurrence of SBTC. This means becoming skilled and earning the skilled wage in the future is more beneficial and parents tend to educate their children more. The increase in next period's wage ratio causes a positive effect on HCA and we can expect more skilled individuals in the future. The education decision of skilled parents is, in contrast to the one of unskilled parents, only affected via changes in the future wage ratio. This wage ratio is higher and we have again a positive effect on HCA. In sum, changes in wage ratios cause positive and negative effects on parents' education decision. Formally, the negative and positive effects can be shown in the following derivatives holding α_t and α_{t+1} constant:

$$\frac{\partial \lambda_t^{u*}}{\partial b} = \frac{1}{1+\beta}\left[\underbrace{\frac{\beta}{n\eta}\frac{-1}{\left[\frac{w_t^s}{w_t^u}\right]^2}\frac{\partial \frac{w_t^s}{w_t^u}}{\partial b}}_{(-)} \underbrace{-\frac{-1}{\left[\frac{w_{t+1}^s}{w_{t+1}^u}-1\right]^2}\frac{\partial \frac{w_{t+1}^s}{w_{t+1}^u}}{\partial b}}_{(+)}\right] \lesseqqgtr 0 \quad \text{and} \qquad (17)$$

$$\frac{\partial \lambda_t^{s*}}{\partial b} = \underbrace{\frac{1}{1+\beta}\left[-\frac{-1}{\left[\frac{w_{t+1}^s}{w_{t+1}^u}-1\right]^2}\frac{\partial \frac{w_{t+1}^s}{w_{t+1}^u}}{\partial b}\right]}_{(+)} > 0 \qquad (18)$$

where $t \in [k, \infty]$.

So far, the net effect of changes in wages and wage ratios on HCA is unclear.

Proposition 2. *If SBTC leads to higher or lower HCA is uniquely determined by the sign of $\alpha_t \frac{\partial \lambda_t^{s*}}{\partial b} + (1-\alpha_t)\frac{\partial \lambda_t^{u*}}{\partial b}$ where current and subsequent skill formation, i.e., α_t and α_{t+1}, are hold constant.*

Proof. See appendix A.3. □

That means I can simply derive the direction of the net effect by concentrating on SBTC effects on education decisions, $\frac{\partial \lambda_t^{u*}}{\partial b}$ and $\frac{\partial \lambda_t^{s*}}{\partial b}$, without taking into account that changed education decisions also affect α_{t+1} which itself influences education decisions and HCA. Regarding the sign of $\alpha_t \frac{\partial \lambda_t^{s*}}{\partial b} + (1-\alpha_t)\frac{\partial \lambda_t^{u*}}{\partial b}$, I find that

Proposition 3. *the value of $\alpha_t \frac{\partial \lambda_t^{s*}}{\partial b} + (1-\alpha_t)\frac{\partial \lambda_t^{u*}}{\partial b}$ is negative for values of α_t close to zero and is positive for $\alpha_t \rightarrow 1$.*

Proof. See appendix A.4. □

In cases with currently few skilled adults, i.e., for low levels of α_t, the net effect is negative because the negative effect outweighs the positive effects. Parents invest on average less in the education of their children and we can expect less skilled individuals in the future after SBTC occurred. Reasons for this result are: many parents are currently unskilled and face the negative effect and the negative effect is relatively strong. Contrariwise, in cases with currently many skilled adults, i.e., for high levels of α_t, the net effect is positive since the positive effects outweigh the negative effect. Here, parents invest more in the education of their children and we can expect more skilled individuals in the future after the technology change. In these cases, the negative effect is relatively weak or even not present because current wage ratios are close to one or even equal to one.[13] Additionally, the main determinant of parents' education decision is the incentive to invest in children's education which is the wage ratio in the next period. The effect via next period's wage ratio is always positive as discussed above.

Given this theoretical background, I can turn to the analysis of SBTC effects on HCA in the short- and long-run. Results are illustrated in figure 4 where, for

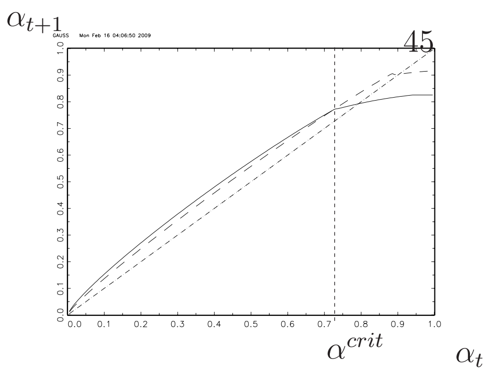

Figure 4: Impact of SBTC on HCA in the low-fertility country, $b = 4$ and $b' = 6.5$.

the case of the above introduced low-fertility country, two HCA paths with different technology levels are displayed. The solid (resp. dashed) HCA-curve in this figure depicts the accumulation path for a low (resp. high) level of the technology parameter b. The economy follows the solid HCA-curve before and switches to the dashed HCA-curve after the occurrence of SBTC.[14] Since the economy converges to a higher steady-state value of average human capital after SBTC, it faces a positive effect in the long-run. The short-run effect is ambiguous and depends on the current development stage at time k where SBTC occurs. If average human capital α_k is lower than α^{crit}, which denotes the intersection of the accumulation paths before and after SBTC occurrence, human capital growth will

[13]See the wage figure 1 for further details.
[14]SBTC is simulated by a rise in b from 4 to 6.5.

drop and consequently, the economy faces a slow-down in human capital growth in the short-run. In the case of $\alpha_k > \alpha^{crit}$ in contrast, the economy is positively affected and human capital growth accelerates in the short-run.

4.2 Comparison of SBTC Effects on HCA in Economies with Different Characteristics

In comparing SBTC effects on HCA between two economies with different characteristics, I am interested in the question if SBTC has different or same effects in developed and developing countries. On way to distinguish between developed and developing economies in my simplified model is to differentiate two economies by their fertility rate. Empirical evidence shows that parents in less developed countries tend to have on average a larger number of children than in developed countries. One other possibility to differentiate to account for differences in the education cost per child e_t. Less developed countries might be less efficient in the provision of education of the same quality than developed countries. In the following, I am concentrating on economies which differ in their fertility rate.

Starting with the comparison, I like to point out first that economies with more children per parent follow a lower HCA-curve than economies with less children per parent as displayed in figure 3. Low-fertility economies after SBTC occurrence are likely to be in a case as discussed above and displayed in figure 4. They benefit in the long-run while the short-run performance is unclear. To explore the likely SBTC effects in a high-fertility economy, the same technology change is now analysed in the case of the above introduced high-fertility country in figure 5. Contrary to the low-fertility country, the steady-state is negatively affected because the high-fertility economy converges to a lower level of human capital in the long-run after SBTC has occurred. The explanation is that the steady-state of high-fertility countries comprises low average human capital or in other words low proportion of skilled adults. I showed that HCA is negatively affected in development stages with low average human capital. Regarding the short-run effect, human capital accumulation is clearly negatively affected assuming that α_k is smaller than the steady-state level.

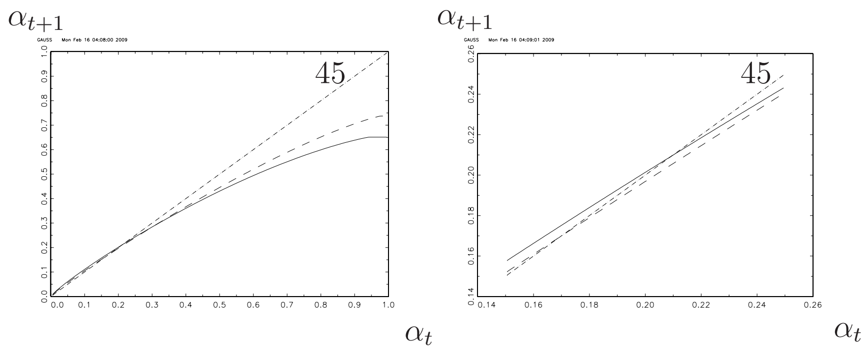

Figure 5: Impact of SBTC on HCA in the high-fertility country, $b = 4$ and $b' = 6.5$.

Comparing the results, low-fertility countries rather benefit in the long-run from SBTC and must not fear the negative effect though the growth of average human capital may slow down in the short-run. Contrary, high-fertility countries are rather harmed in their human capital accumulation in both the short- and long-run.

5 Summary

This paper analyses the effect of skill-biased technological change (SBTC) on human capital accumulation (HCA) where human capital accumulation over time is described by the level of average human capital. This level is uniquely determined by the proportion of skilled adults, denoted by α_t, assuming that skill levels are constant.

A numerical example is used to identify the accumulation process over time because an explicit analytical solution is not achievable. The numerical example shows that the accumulation path and the long-run equilibrium (steady-state) for average human capital are uniquely defined for given parameters. Consequently, countries with different characteristics expressed by different parameter values, say differences in fertility rates, follow different HCA paths and converge to different steady-states. Consistent with empirical findings, the model predicts that countries with higher fertility rates converge to steady-states with lower proportion of skilled adults and hence with less average human capital and income levels.

The analysis of SBTC effects on HCA is conducted in two steps. Firstly, the analysis focuses on SBTC effects on HCA in general and secondly, a comparative analysis is made between low-fertility (rich) and high-fertility (poor) countries.

Regarding the first analysis, SBTC causes two effects: a negative effect relative to education investment cost and a positive effect relative to incentive to invest in education. Due to its skill-biased character, SBTC raises the wage ratio in current and subsequent periods. These increased wage ratios imply on the one hand, for current and future periods, increasing relative education cost for unskilled parents, in other words a negative effect on unskilled parents' education decisions. On the other hand, larger wage gaps increase, for current and future periods, incentive to invest in education. These incentive effects have a positive impact on all parents' education decisions. Though two forces in different directions are caused by SBTC, the net effect can be identified. The net effect on HCA is negative in situations where only few adults are skilled, i.e., for low values of α_t, because many parents are unskilled and face the negative cost effect and the cost effect is the stronger and determining factor for the education decision of unskilled parents. In contrary, HCA is positively affected in situations with many skilled adults, i.e., for high values of α_t, since relative education cost for all parents is equalised and is unaffected by SBTC. The only active force in this case is then the positive incentive effect.

Regarding the comparison between high- and low-fertility countries, the paper shows that low-fertility countries are rather positively affected in the long-run by SBTC comparing to high-fertility countries. Low-fertility countries' steady-state comprises high proportion of skilled adults and consequently, the positive incentive effect dominates. As for high-fertility countries, their steady-state comprises lower proportion of skilled adults and hence, stronger negative cost effect which dominates the positive incentive effect. In the short-run, low-fertility countries may face temporarily a slow down in human capital accumulation depending on the current skill formation. High-fertility countries are always negatively affected in the short-run.

This paper analyses the effect of skill-biased technology change on the human capital accumulation process. The effect stems from the impact of SBTC on current and future wages and consequently individuals' education decision. However, there are other factors that can affect wages and HCA in the same way as SBTC can. Globalisation, for instance, is one of the most obvious factors. Its main implications are (i) indirect effects via international competition in goods market which influence domestic wages and (ii) direct effects via competition in international labour markets. Therefore, it is also important to take into account assumptions on open economy as an extension of the current model which is only a closed economy framework.

A Appendices

A.1 Description of the HCA process

In this appendix, the HCA process is described in detail. Firstly, I divide the accumulation path into segments and secondly, analyse parents' education decision in every segment separately.

The accumulation path of an economy with certain characteristics can be divided into up to three segments. The segments are shown in figure 2 and are labelled in the following as: (i) left segment for α_t between 0 and $\widetilde{\alpha}$; (ii) middle segment for α_t between $\widetilde{\alpha}$ and $\widehat{\alpha}$; and (iii) right segment for α_t between $\widehat{\alpha}$ and 1.[15] The presence of these segments depends on parameter values and will be analysed at the end of this subsection.

In what follows, education decision of each parent's type are analysed for each segment separately. For these analyses, current and next period's wages are important. To derive later wage ratios, which are main determinants for education decisions, skilled and unskilled wages are displayed for all three segments in figure 6.

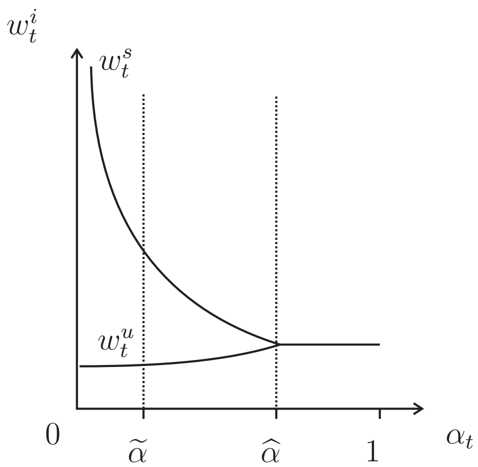

Figure 6: Skilled and unskilled wages in each segment.

We start with the analysis of education decisions in the left segment. With

[15]Note that $\widetilde{\alpha}$ depends on the fertility rate n and is depicted in figure 3 for both the high- and the low-fertility country. Contrary, the value of $\widehat{\alpha}$ does not depend on n and is hence the same for high- and low-fertility countries.

respect to wages in this segment, the proportion of skilled adults is low which implies high wages for skilled parents and low wages for unskilled ones. Consequently, current and next period's wage ratios are high for all levels of α_t and α_{t+1} within this segment. The high current wage ratio implies high relative education cost for unskilled parents since this cost depend on both current skilled and unskilled wage. The high wage ratio in the next period implies high incentives to invest in education for all parents because the difference between skilled and unskilled wage is large.

This wage formation in the left segment induces the following education decisions. Skilled parents invest in education of all their children. This corner solution in skilled parents' education decision is the constituting and major property of the left segment. It distinguishes the left segment from the middle segment where skilled parents do not invest in education of all children. Reasons for $\lambda_t^{s*} = 1$ are the high incentive to invest in education and low enough relative education cost for skilled parents which are given by η. If η is not low enough, skilled parents do not invest in all children's education and thus, the left segment does not exist. Regarding the education decision of unskilled parents, though they face high relative education cost due to the high current wage ratio, unskilled parents invest in education at least of a small proportion of their children. If unskilled parents do not invest in education of any child, next period's skill formation equals to the current composition between skilled and unskilled adults because all children of skilled parents receive education. Consequently, high wage inequality arises in the next period which stimulates unskilled parents to invest in education of a small proportion of their children today.

Moreover, we can identify the main determinant of education decisions and the shape of the HCA-curve in this segment. Although the investment incentive is high for all parents, overall investments are relatively low, as can be deducted from the low levels of α_{t+1} because the currently large proportion of unskilled parents is restricted by their high relative education cost. The relative education cost of unskilled parents is therefore the most important determinant of HCA in this segment. With increasing α_t, wages of both skill groups converge which implies declining relative education cost for unskilled parents. This allows unskilled parents to invest more in education of their children and explains why the slope of the HCA-curve is larger than one for very small α_t. However, since the speed of wage convergence decreases with α_t, as can be seen in the wage figure 6, the slope is declining and thus, the accumulation path is concave.

Turning to the middle segment, wages are more equal here than in the left segment. This implies, via smaller current and next period's wage ratios than in the left segment, lower relative education cost for unskilled parents and lower incentive to invest for all parents. Regarding the education decision of skilled parents, they do not invest in education of all their children anymore though their relative education cost are unchanged. The reason is the lower investment incentive comparing to the situation in the left segment. Investment behaviour of unskilled parents is not changed compared qualitatively; they still invest in education of some of their children. Therefore, the HCA-curve has a downward kink at $\widetilde{\alpha}$ due to less education investments of skilled parents.

In the right segment, income levels and hence relative education cost are equalised for both types of parents. Regarding education decisions, suppose the case of η being low enough, which means that every parent has enough wealth to invest in education of all his children.[16] But this decision cannot be optimal since all adults in the next period would be skilled. Some of them would work as unskilled workers which leads to equalisation between skilled and unskilled wage in the next period, i.e., the readjustment process takes place. Due to equalised wages in the next period after the readjustment process, there is no gain of education investments. Consequently, parents invest in education of a proportion of their children only such that marginal cost equals to marginal gain. Subsequently, wage inequality in the next period is positive which stimulates current education investments. In the right segment, the incentive side is therefore the most important determinant of HCA.

Note that the accumulation path is given by a horizontal line in the right segment because education decisions do not change for different levels of the current skill formation. This stems from the fact that the optimal input factor ratio and hence current wages are constant within this segment. Same wage levels for all values of α_t imply, on the one hand, same education cost and, on the other hand, same next period's wages and hence the same investment incentive. Consequently,

[16] Note that parameter η determines in the right segment relative education cost of all parents.

education decisions are similar for all current skill formations in the right segment.

All three segments exist when the following parameter restrictions are fulfilled. The left segment, for instance, disappears if relative education cost for skilled parents is too high, i.e., if $\eta > \overline{\eta} = \frac{\beta}{1+\beta}\frac{1}{n}$. The middle segment would disappear if and only if relative education cost parameter η is less or equal to zero, which is ruled out by assumption, i.e., $\eta \in]0, 1[$. Therefore, the middle segment is always present. The right segment is present if the value of $\widehat{\alpha}$, derived as $[b^{\frac{1}{1-\gamma}}]/[1 + b^{\frac{1}{1-\gamma}}]$, is smaller than one. It is smaller than one when $b > 0$ which is fulfilled by assumption. Hence, the right segment is also always present.

A.2 Proof of proposition 1

Proposition 1. *For every combination of α_t, b, β, γ, η and n, there exist unique solutions for the maximisation problem of skilled and unskilled parents, i.e., for λ_t^{s*} and λ_t^{u*}. Consequently, the skill formation in the next period α_{t+1} is uniquely determined.*

Proof. Optimum education decisions depend on current and future wages as shown in equation (9). Considering that wages depend on the skill formation in the respective period, it follows that optimum education decisions depend on α_t and α_{t+1}. Furthermore, since wages depend on the technology level, education decisions depend also on parameter b. Accounting for these dependencies, the dynamic equation of α_t can be rewritten as

$$\alpha_{t+1} = \alpha_t \lambda_t^s(\alpha_t, \alpha_{t+1}, b) + (1 - \alpha_t)\lambda_t^u(\alpha_t, \alpha_{t+1}, b). \tag{19}$$

Variable α_{t+1} on the right-hand side of equation (19) determines next period's wages and is expected by parents at time t. Variable α_{t+1} on the left-hand side instead is the actual outcome of current education decisions. Due to rational expectation and certainty in the model, i.e., perfect foresight, the expected value must be equal to the actual value. Therefore, next period's skill formation α_{t+1} is only implicitly defined by equation (19).

For the following proof, both sides of equation (19) are considered as separate functions which depend on α_{t+1}. We are interested in situations where both functions are equalised. With regard to the right-hand side, the reaction of optimal education decisions to a change in the expected future skill formation derives as

$$\frac{\partial \lambda_t^{i*}}{\partial \alpha_{t+1}} = \begin{cases} \frac{-1}{1+\beta} \frac{\frac{\partial w_{t+1}^u}{\partial \alpha_{t+1}} w_{t+1}^s - \frac{\partial w_{t+1}^s}{\partial \alpha_{t+1}} w_{t+1}^u}{[w_{t+1}^s - w_{t+1}^u]^2} < 0 & \text{for } \alpha_{t+1} < \widehat{\alpha} \text{ and } 0 < \lambda_t^{i*} < 1, \\ \\ 0 & \text{else} \end{cases} \tag{20}$$

where $\frac{\partial w_{t+1}^u}{\partial \alpha_{t+1}} > 0$ and $\frac{\partial w_{t+1}^s}{\partial \alpha_{t+1}} < 0$ for $\alpha_{t+1} < \widehat{\alpha}$ and $\frac{\partial w_{t+1}^u}{\partial \alpha_{t+1}} = \frac{\partial w_{t+1}^s}{\partial \alpha_{t+1}} = 0$ else. Since the derivative in equation (20) is non-positive, the right-hand side is therefore monotonic decreasing in α_{t+1}. Intuitively, a higher proportion of skilled adults decreases the wage inequality and hence lowers the investment incentive. Parents therefore invest less in education of their children, and consequently, the right-hand side is negatively affected by a rise in α_{t+1}. Contrary, the left-hand side depends positively on and is strictly monotonic increasing in α_{t+1}.

Since the right-hand side is strictly monotonic increasing and the left-hand side is monotonic decreasing in α_{t+1}, there exist only one unique solution for α_{t+1} fulfilling equation (19). Figure 7 represents graphically the qualitative properties

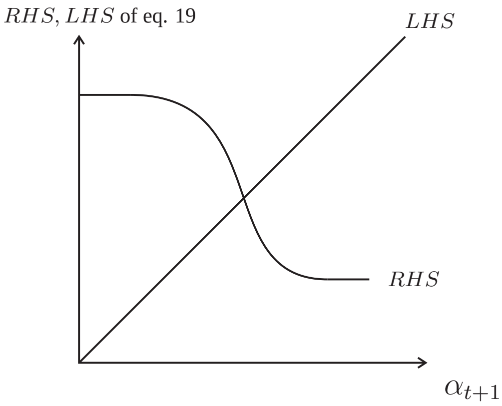

Figure 7: Unique graphical solution for α_{t+1}.

of both functions and depicts both right-hand side (denoted by RHS) and left-hand side (denoted by LHS) of equation (19) for a given value of α_t, i.e., for given current wages, and given parameter values. □

A.3 Proof of proposition 2

Proposition 2. *If SBTC leads to higher or lower HCA is uniquely determined by the sign of* $\alpha_t \frac{\partial \lambda_t^{s*}}{\partial b} + (1-\alpha_t)\frac{\partial \lambda_t^{u*}}{\partial b}$ *where current and subsequent skill formation, i.e.,* α_t *and* α_{t+1}*, are hold constant.*

Proof. The implicit dynamic equation for α_t can be derived as

$$\alpha_{t+1} = \alpha_t \lambda_t^{s*}(\alpha_t, \alpha_{t+1}, b) + (1-\alpha_t)\lambda_t^{u*}(\alpha_t, \alpha_{t+1}, b). \tag{21}$$

Employing the implicit function theorem, we can identify what drives the effect of technology change on α_{t+1}. The implicit derivative is derived as

$$\frac{d\alpha_{t+1}}{db} = -\frac{\alpha_t \frac{\partial \lambda_t^{s*}}{\partial b} + (1-\alpha_t)\frac{\partial \lambda_t^{u*}}{\partial b}}{\alpha_t \frac{\partial \lambda_t^{s*}}{\partial \alpha_{t+1}} + (1-\alpha_t)\frac{\partial \lambda_t^{u*}}{\partial \alpha_{t+1}} - 1}. \tag{22}$$

Since a higher proportion of skilled adults in the next period implies a decreasing wage ratio, i.e., lower wage inequality, the investment incentive is lower. Parents thus choose to invest in education of a smaller number of children. Consequently, the denominator in derivative (22) is strictly negative. The overall sign of the derivative is then uniquely determined by the numerator and the changes in optimum education decisions holding α_{t+1} constant. $\square$

A.4 Proof of proposition 3

Proposition 3. *The value of* $\alpha_t \frac{\partial \lambda_t^{s*}}{\partial b} + (1-\alpha_t)\frac{\partial \lambda_t^{u*}}{\partial b}$ *is negative for values of* α_t *close to zero and is positive for* $\alpha_t \to 1$.

Proof. Current and next period's wage ratio influence current education decisions.

Their reaction to the technology change can be derived as

$$\frac{\partial \frac{w_t^s}{w_t^u}}{\partial b} = \begin{cases} \left(\frac{1-\alpha_t}{\alpha_t}\right)^{1-\gamma} & \text{for } \alpha_t < \widehat{\alpha}, \\ \\ 0 & \text{else;} \end{cases}$$

$$\frac{\partial \frac{w_{t+1}^s}{w_{t+1}^u}}{\partial b} = \left(\frac{1-\alpha_{t+1}}{\alpha_{t+1}}\right)^{1-\gamma}$$

respectively.

The following findings can be inferred from these derivatives. For the case of $\alpha_t \rightarrow 1$, on the one hand, the current wage ratio does not change because wages are equalised for values of α_t between $\widehat{\alpha}$ and 1 and is always equal to one. Consequently, the negative cost effect is invalid. On the other hand, since α_{t+1} is strictly smaller than one for $\alpha_t \in [\widehat{\alpha}, 1]$ and is hold constant in this analysis, next period's wage ratio always rises. That means that the positive incentive effect is active. As a result, SBTC affects via wages the education decisions and the HCA process positively.

Regarding the case of α_t being close to zero, note that $\alpha_{t+1} \rightarrow 0$ when $\alpha_t \rightarrow 0$ as shown graphically in figure 3. Therefore, the change in current and next period's wage ratios are going to infinity for $\alpha_t \rightarrow 0$. Concentrating on small values of α_t close to zero, we find that the change in current wage ratio is larger than that in next period's wage ratio, i.e., $\frac{\partial w_t^s/w_t^u}{\partial b} > \frac{\partial w_{t+1}^s/w_{t+1}^u}{\partial b}$. The reason is that α_{t+1} is always larger than α_t for small positive values of α_t. Note also that a small value of α_t implies high weight on the education decision of unskilled parents and only low weight on that one of skilled parents. As a result, the negative cost effect dominates the positive incentive effect in this case, and human capital accumulation is overall negatively affected. $\square$

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