EU-type carbon emissions trade, overlapping emissions taxes and the (re)distribution of welfare*

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Abstract

In the European Union, the emissions reductions commitments are achieved by a joint emissions trading scheme covering some part of their economies (ETS sector) and by a national emissions tax in the rest of their economies (non-ETS sector). Applicable are also emissions taxes overlapping with the trading scheme. Restricting our focus on cost-effective allocations, this paper investigates the allocative and distributive consequences of increasing the overlapping emissions tax. For quasi-linear utility functions and for a class of parametric utility and production functions it turns out that emissions tax increases are exactly offset by permit price reductions. As a consequence permit-exporting [permit importing] countries lose [win] from the increase in the emissions tax. These results are not general. By means of a numerical example we point out that export-import reversals and welfare reversals may emerge.

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Key words: emissions taxes, emissions trading, international trade

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1 Introduction

In the Kyoto Protocol the EU committed to reduce its greenhouse gas emissions by 8% in 2012 from its baseline emissions in 1990. In order to fulfill this commitment the EU has established an emissions trading scheme (ETS) in 2005 (see EU 2003) allowing for EU-wide trade in emissions permits. With respect to emissions control the economies of all member states are split into two sectors. The installations covered by the ETS, referred to as the ETS sector, include combustion installations, mineral oil refineries, coke ovens, installations producing and processing ferrous materials, mineral installations and industrial plants for the production of pulp and paper. In the rest of the economy, called the non-ETS sector (that mainly consists of private households and transportation, emissions control is the national governments' responsibility and is carried out through instruments other than emissions trading. Another peculiarity of EU emissions control is the existence of emissions or energy taxes in the ETS sectors overlapping with the ETS (Johnstone 2003, Sorrell and Sijm 2003, International Energy Agency 2007).

We aim at studying a hybrid EU-type policy in a stylized way. Under the simplifying assumption that in their non-ETS sectors national governments control emissions through a sectoral emissions tax, box 2 in Table 1 shows the mix of complementary and overlapping policy instruments that will be considered. Box 1 in Table 1 represents the limiting case in which no tax is levied in the ETS sector while box 3 can be interpreted as the state in which the rate of the tax in the ETS sector is so high as to 'dry up' the permit market by having driven its price to zero.

	Emissions control in the ETS sector via		
	ETS	ETS and sectoral tax	Sectoral tax
Emissions control in the non-ETS sector via sectoral tax	1	2	3

Table 1: EU-type emissions control in a two-sector economy

To capture that policy in a multi-country model we translate the EU commitment of emissions reductions into an upper bound, say $\bar{c}$, for total emissions in the group of countries. The EU burden sharing agreement (EU 1999) is then interpreted as a political decision to split the overall emissions cap $\bar{c}$ into national caps, c_i, one for each member state $i = 1, \ldots, n$, satisfying $\sum_j c_j = \bar{c}$. Throughout the rest of this paper we will take the partition $(c_1, \ldots, c_n)$ of the overall cap $\bar{c}$ as given. As observed above, the EU ETS covers only part of each member state's economy. Therefore, the national cap c_i needs to be further split into a cap for the ETS and the non-ETS sector. The cap for the non-ETS sector is then implemented by a sectoral emissions tax whereas the cap of the ETS sector defines the country's budget of emissions permits to be issued by the national government and allocated to the installations in its ETS sector.

It is obvious that such hybrid policy is exposed to many pitfalls of cost ineffectiveness, in particular when overlapping taxes in the ETS sectors are included in the policy mix. To the best of our knowledge, there are only two papers studying international emissions trading with *overlapping* emissions taxes. Böhringer et al. (2007) use a partial equilibrium model and Eichner and Pethig (2008) a general equilibrium model to assess policy (in)effectiveness when national governments have discretion in fixing an overlapping tax and their budget of emissions permits. Eichner and Pethig (2008) establish that cost effectiveness for the group of countries is attained of and only if there is a tax in the ETS sector (possibly zero) that is uniform across countries and a tax in the non-ETS sector that is also uniform across countries at a rate equal to the sum of the permit price and the rate of the tax in the ETS sector. The important message is that as long as the emissions tax in the ETS sectors is uniform across countries it can be fixed at different levels without compromising cost effectiveness for the group of countries.

In the present paper we will briefly reconstruct the model developed in Eichner and Pethig (2008) and restate their efficiency result. Our subsequent analysis will be based on that model. However, we will exclusively consider cost-effective policies disregarding (empirically existing) cost *in*effective policies and possible corrective actions that are the main emphasis of Eichner and Pethig (2008). Thus restriction serves to direct as clearly as possible the focus on the objective of the present paper, namely to study the *distributional* consequences of overlapping policies as described above (box 2 in Table 1).

To motivate our interest in that distributional issue we first consider only one country ($n = 1$) whose national cap is $\bar{c}$. In that country there is only one sector and an ETS overlaps with an emissions tax. The cost-effectiveness conditions for a hybrid policy would then be as described above, i.e. for each box in Table 1, one can find and characterize a cost-effective policy. The important observation in the present context is, however, that the country's welfare is the same under each type of cost-effective policy.[1]

In contrast, if we set $n > 0$, restrict our attention to a one-sector economy, fix some partition $(c_1, \ldots, c_n)$ and design the ETS as permit trading among all countries in the group, the welfare of the countries will differ, in general, for different levels of cost-effective hybrid policies. In terms of distributional impact, the policies in the boxes 1, 2 and 3 of

[1]From the large theoretical and applied literature on this issue (e.g. Shiell 2003) it is well known that this result only holds for models on a high level of abstraction (e.g. lumpsum recycling of revenues from taxing emissions or auctioning emissions permits; no need of trial and error to fix the correct tax rates etc.).

Table 1 are no longer equivalent. The intuition is that an individual country loses wealth to the benefit of all other countries or gains wealth at the expense of all other countries, when it, respectively, imports or exports permits. If starting from some initial equilibrium the emissions tax is increased the permit price reduces by the same amount. While emissions and production levels are unaffected, the value of imports and exports of permits changes which, in turn, affects the countries' welfare as follows: permit-exporting countries lose while permit-importing gain from an increase in the tax rate.

To our knowledge, this distributional issue has not been analyzed in a *two-sector* general equilibrium framework which captures the EU emissions control in a stylized way. This paper aims to determine the welfare effects of changes in the emissions tax in the ETS sector. More specifically, denote by $u_i(c, t_y)$ the welfare of country i in the (unique) competitive equilibrium of the multi-country model which exists when $c = (c_1, \ldots, c_n)$ is the vector of national emissions caps and when t_y is the rate of the emissions tax in the countries' ETS sectors.[2] We are interested in the sign of the derivative $\partial u_i(c, t_y)/\partial t_y$ whose calculation turns out to be non-trivial because it requires carrying out a full-scale comparative static analysis of the multi-country model.

Due to the model's complexity we do not succeed in fully characterizing the distributional impact of variations in t_y (from $t_y = 0$ to some high level of t_y for which the permit prices becomes zero). However, we do obtain clear-cut analytical answers for quasi-linear utility and production functions. In both cases we show that an increase in the tax rate t_y is exactly offset by a reduction in the equilibrium permit price with the consequence that all firms' production is unaffected. Yet the value of imports and exports of permits changes and analogous to the one-sector model permit-exporting countries lose while permit importing countries gain from an increase in the tax rate.

With continuous increases in the tax rate welfare changes in a strictly monotone way. However, the perfect offset of variations in the tax rate by changes in the equilibrium permit price turns out to be a special case that is typical only in models where the market of emissions permits is isolated from all other markets. In general, market interdependence effects, i.e. repercussions of tax variations in markets beyond the permit market, render imperfect the offset between the tax rate and the permit price. This is demonstrated by means of a numerical example in which one of the countries that initially exports permits eventually starts importing permits upon continuous increases in the tax rate t_y. As a consequence of that export-import reversal the country switches from welfare gains to losses. The central message of the paper is that the distributional impact of variations in the

[2] As we restrict our attention to cost-effective policies it is necessary to assume the tax rate to be uniform across countries.

emissions tax in the countries' ETS sector is significant, and since in the real world market-interdependence effects are not negligible, that impact is less clear-cut in direction than suggested by partial equilibrium analysis.

The paper is organized as follows. Section 2 investigates cost-effective emissions control in a model with one country whose economy consists of one sector. The emissions of that sector are regulated by an emissions trading scheme overlapping with an emissions tax. Next, we extend that model in section 3 by assuming that the country belongs to a group of countries which has implemented a joint emissions trading scheme. Finally, in sections 4 and 5 we study cost-effective emissions control and the incidence of the emissions tax overlapping with the ETS in a group of countries with two-sector economies. Section 6 concludes.

2 Cost-effective carbon emissions control in a single country

Consider a single country i embedded in the world economy. That economy produces the quantity y_{si} with the help of the input fossil fuel e_{yi} according to the strictly increasing and concave production function[3]

$$y_{si} = Y^i(e_{yi}). \tag{1}$$

The representative consumer derives utility from consumption y_i of the good according to the increasing and quasi-concave utility function

$$u_i = U^i(y_i). \tag{2}$$

Good Y and fossil fuel are traded on world markets at constant prices p_y and p_e, respectively. All fuel input is assumed to be imported from the rest of the world. Then the country's trade balance is given by

$$p_y(y_{si} - y_i) - p_e e_{yi} = 0. \tag{3}$$

Since CO_2 emissions are proportional to the input of fossil fuel, we simply denote both by the same symbol. National emissions are restricted to some exogenously given emissions level $\bar{c} = c_i > 0$, i.e.

$$c_i = e_{yi}. \tag{4}$$

[3]Upper case letters denote functions and subscripts attached to them indicate partial derivatives.

To characterize the efficient allocation the social planner solves the Lagrangian

$$\mathcal{L} = U^i(y_i) + \lambda_h\{p_y[Y^i(e_{yi}) - y_i] - p_e e_{yi}\} + \lambda_e(c_i - e_{yi}), \tag{5}$$

where c_i, p_e and p_y are positive constants and λ_h and λ_e are Lagrange multipliers. The first-order condition for an interior solution can be rearranged to read

$$p_y Y_e^i = p_e + \mu_e, \tag{6}$$

where $\mu_e = \lambda_e/\lambda_h$. (6) is the rule production efficiency (or cost effectiveness) and requires the marginal abatement cost to match the price of fossil fuel and the shadow price of emissions.

The next step is to investigate the national emissions control that - under conditions of perfect competition is capable to decentralize the efficient allocation. To that end the country may install an emissions trading scheme (ETS) or introduce an emissions tax at rate t_y or both. Denoting by π_e the permit price which will clear the permit market (4), the profit of the aggregate firms is given by

$$p_y Y^i(e_{yi}) - \pi_e(e_{yi} - c_i) - (t_y + p_e)e_{yi} \tag{7}$$

and the first-order condition for profit maximization reads

$$p_y Y_e^i = \pi_e + t_y + p_e. \tag{8}$$

The consumer of the country maximizes her utility subject to her budget constraint

$$z_i = p_y y_i, \tag{9}$$

where her income is given by $z_i = g_{yi}^* + t_{yi}e_{yi}$. The consumer's income stems from two sources. She is owner of the firm and gets its maximum profits g_{yi}^*. In addition she gets tax revenues which are transferred in a lumpsum way.

Comparing (5) an (8) immediately yields

Proposition 1.
The equilibrium allocation of the competitive economy (1), (2), (4), (8), (9) is cost-effective if and only if $t_y \in [0, \mu_e]$*. In addition, the permit is determined by* $\pi_e = \mu_e - t_y$*.*

According to Proposition 1 cost-effectiveness can be achieved by levying no emissions tax ($t_y = 0$). Then the standard $\bar{c}$ is implemented by an ETS which is in operation with a positive price $\pi_e = \mu_e$. In the other polar case the emissions tax rate is set such that $t_y = \mu_e$. As a consequence the international permit market exhibits a zero equilibrium price ($\pi_e = 0$) and hence can be dismissed. These equivalence between taxes and standards goes back to Baumol and Oates (1971). However, Proposition 1 provides the additional information that intermediate cost-effective solutions are captured when positive tax rates $t_y > 0$ coexist with an operating ETS ($\pi_e > 0$).

Finally, it is worth mentioning that a switch from a pure emissions tax to a pure ETS has neither allocative nor distributional effects. The latter can be seen from the consumer's exogenous income which in equilibrium turns out to be independent of π_e and t_y, formally $z_i = p_y Y^i(e_{yi}) - p_e e_{yi}$.

3 Cost-effective emissions control in a group of countries

In this section we consider again the economy of the country i presented in (1) and (2) but now assume that country i belongs to a group of n countries which as a whole committed itself to restrict its *total* emissions to some level $\bar{c} > 0$. To meet that emissions target countries take part in a *joint* emissions trading scheme with mandatory participation and may levy a national emissions tax. To install the ETS each country i is assigned a national emissions cap $c_i \geq 0$ such that $\sum_i c_i = \bar{c}$. Country i issues the amount c_i of marketable permits to be allocated to its firms. These permits can then be traded among all countries of the group. A market for permits will arise with the aggregate supply being fixed at $\sum_i c_i$ and with permit price that will clear the market

$$\sum_i c_i = \sum_i e_{yi}. \tag{10}$$

Consider now a social planner who aims to maximize the weighted sum of the utilities of all countries' representative consumers subject to (1), (2), (10) and the group's consolidated trade balance

$$\sum_j \left[p_y(y_{sj} - y_j) - p_e e_{yj}\right] = 0. \tag{11}$$

Solving the pertinent Lagrangian

$$\mathcal{L} = \sum_j \alpha_j U^j(y_j) + \lambda_e \sum_j (c_j - e_{yj}) + \lambda_h \sum_j \left\{p_y \left[Y^j(e_{yj}) - y_j\right] - p_e e_{yj}\right\}, \tag{12}$$

where the α_j for $j = 1, \ldots, n$ denote constant positive welfare weights, yields the first-order conditions

$$p_y Y_e^i = p_e + \mu_e \qquad i = 1, \ldots, n. \tag{13}$$

The allocation rule (13) requires to equalize marginal abatement costs across countries.

The behavior of aggregate firms and the consumer's budget is already presented in (8) and (9), respectively. Then we conclude

Proposition 2.
The equilibrium allocation of the competitive economy (1), (2), (8), (9), (10), is cost-effective if and only if $t_{yi} = t_y \in [0, \mu_e]$ *for all* $i = 1, \ldots, n$. *In addition, the permit price is determined by* $\pi_e = \mu_e - t_y$.

Proposition 2 is a straightforward extension of Proposition 1. In a group of countries a prerequisite for cost-effectiveness is uniformity of emissions tax rates. Then proposition 2 can be viewed as an equivalence result between emissions taxes and permits and "convex" combinations between them.

In the sequel, we analyze the allocative and redistributive effects of switching from a pure cost-effective ETS scheme to a pure cost-effective emissions tax. To that end we investigate the impacts of parametric changes in t_y on cost-effective competitive equilibria in comparative static analysis (Appendix A). The results are summarized in

Proposition 3.
The incidence of the emissions tax is given by

Δe_i	$\mathrm{d}\pi_e$	$\mathrm{d}e_{yi}$	$\mathrm{d}\Delta e_i$	$\mathrm{d}y_i$	$\mathrm{d}z_i$	$\mathrm{d}u_i$
$\mathrm{d}t_y, \Delta e_i > 0$	-1	0	0	$-$	$-$	$-$
$\mathrm{d}t_y, \Delta e_i < 0$	-1	0	0	$+$	$+$	$+$

Table 2: Tax incidence in the one-sector model; $\Delta e_i := c_i - e_{yi}$

From Proposition 3 we infer that an increase in the tax rate t_y is exactly offset by a reduction in the permit price $(\mathrm{d}\pi_e/\mathrm{d}t_y) = -1)$ with the consequence that the firms' production and input decisions are unaffected $(\mathrm{d}y_{si}/\mathrm{d}t_y = \mathrm{d}e_{yi}/\mathrm{d}t_y = 0)$. With this information the change of the consumer's budget is given by

$$\mathrm{d}z_i = \mathrm{d}_{yi} = \Delta e_i \ \mathrm{d}\pi_e, \tag{14}$$

where $\Delta e_i := c_i - e_{yi}$ is the amount of permits exported $(\Delta e_i > 0)$ or imported $(\Delta e_i < 0)$ by country i. Then (14) shows that increasing t_y reduces the permit-exporting country's budget and the consumption of good Y, and hence reduces welfare in that country. The effects are reversed in a permit-importing country.

4 Cost-effective EU-style carbon emissions control

Now we turn to a group of n countries embedded in the world economy whose economies consist of two sectors X^i and Y^i producing two consumption goods x_{si} and y_{si} with the help of fossil fuel, e_{xi} and e_{yi}, by means of the production functions

$$x_{si} = X^i(e_{xi}) \qquad \text{and} \qquad y_{si} = Y^i(e_{yi}) \tag{15}$$

that are increasing and strictly concave. The representative consumer of country i derives utility from consumption x_i and y_i of these goods according to the quasi-concave utility function

$$u_i = U^i(x_i, y_i) \tag{16}$$

that is increasing in both arguments. Good X is non-tradable, and hence domestic consumption is required to match domestic production

$$x_i = x_{si}. \tag{17}$$

Good Y and fossil fuel again are traded on world markets at constant prices p_y and p_e, respectively, and all fuel input is assumed to be imported from the rest of the world.

As in the previous section the group as a whole committed itself to restrict its total emissions to some level $\bar{c} > 0$ and each country is assigned a *national emissions cap* $c_i \geq 0$ such that $\sum_j c_j = \bar{c}$. In each country the national emissions cap needs to be split up into two *sectoral caps* c_{yi} and c_{xi} satisfying

$$c_i = c_{xi} + c_{yi}. \tag{18}$$

The sectoral caps are assumed to restrain emissions in the following way[4]

$$c_{xi} = e_{xi}, \tag{19}$$

$$\sum_i c_{yi} = \sum_i e_{yi}. \tag{20}$$

Consider now a social planner who aims to maximize the weighted sum of the utilities of all countries' representative consumers subject to (15)–(20) and the group's consolidated trade balance

$$\sum_j \left[p_y(y_{sj} - y_j) - p_e(e_{xj} + e_{yj})\right] = 0 \tag{21}$$

[4]Equation (19) is required to hold for all i and is therefore more restrictive than the constraint $\sum_j c_{xj} = \sum_j e_{xj}$. The rationale of the differential treatment of the sectors X and Y is to model in the next section the institutional setting of the European Union where the ETS covers the sectors Y of all member states only while each member state is obliged to implement the cap c_{xi} in its sector X.

vis-à-vis the rest of the world. Solving the associated Lagrangean

$$\begin{aligned}\mathcal{L} &= \sum \alpha_j\, U^j(x_j, y_j) + \sum_j \lambda_{xj}\left[X^j(e_{xj}) - x_j\right] + \sum_j \lambda_{cj}(c_j - c_{yj} - e_{xj}) \\ &+ \lambda_h \sum_j \{p_y[Y(e_{yj}) - y_j] - p_e(e_{xj} + e_{yj})\} + \lambda_e \sum_j (c_{yj} - e_{yj}), \end{aligned} \tag{22}$$

a cost-effective allocation is characterized by the marginal conditions

$$\frac{U_y^i}{U_x^i} = \frac{p_y}{\mu_{xi}} \qquad \text{for } i = 1, \dots, n, \tag{23}$$

$$\mu_{xi} X_e^i = p_y Y_e^i = p_e + \mu_e \qquad \text{for } i = 1, \dots, n, \tag{24}$$

where $\mu_{xi} := \lambda_{xi}/\lambda_h$ and $\mu_e := \lambda_e/\lambda_h$, and where λ_e, λ_h. The cost-effective allocation requires consumption efficiency (23) by equalizing the marginal rates of substitution and the price ratios across countries, and production efficiency (24) by equalizing marginal abatement costs across sectors and countries.

Having characterized the cost-effective allocation as a benchmark, we now introduce into the model (15)-(21) competitive markets for good X with price p_{xi} in all countries $i = 1, \dots, n$ along with the following emissions control policies: There is an emissions tax on good Y at rate t_{yi}, an emissions tax on good X at rate t_{xi} and the group as a whole introduces an emissions trading scheme (ETS) with mandatory participation of their sectors Y. Henceforth we will refer to sector Y as the ETS sector and to sector X as the non-ETS sector. To install the ETS, each country i issues the amount c_{yi} of marketable emissions permits and allocates them to all firms in its ETS sector. A competitive market for permits will arise with the aggregate supply being fixed at $\sum_j c_{yj}$ and with the aggregate demand $\sum_j e_{yj}$ being determined by the permit price π_e as to meet the market-clearing condition (20).

In this institutional setting the profits of the aggregate sectoral firms are[5]

$$p_{xi} X^i(e_{xi}) - (t_{xi} + p_e)e_{xi} \qquad \text{and} \qquad p_y Y^i(e_{yi}) - \pi_e(e_{yi} - c_{yi}) - (t_{yi} + p_e)e_{yi},$$

and the associated first-order conditions for profit maximization read

$$p_{xi} X_e^i(e_{xi}) = t_{xi} + p_e \qquad \text{and} \qquad p_y Y_e^i(e_{yi}) = t_{yi} + p_e + \pi_e. \tag{25}$$

The consumer of country i maximizes her utility $U^i(x_i, y_i)$ subject to her budget constraint

$$z_i = p_{xi} x_i + p_y y_i, \tag{26}$$

[5]The way profits are defined for the ETS sector implies gratis allocation of permits to that sector. Due to the high level of abstraction of the model under consideration, allocating permits via auction would leave the results unchanged.

where $z_i := g^*_{xi} + g^*_{yi} + t_{xi}e_{xi} + t_{yi}e_{yi}$ is her income consisting of the firm's maximum profits, g^*_{xi} and g^*_{yi}, and the tax revenues, $t_{xi}e_{xi} + t_{yi}e_{yi}$, recycled to the consumer in a lumpsum fashion. The first-order conditions for utility maximization yield the demand function for good X,

$$x_i = D^i(p_{xi}, z_i). \tag{27}$$

Proposition 4. *(Eichner and Pethig 2008)*[6]
The equilibrium allocation of the competitive economy (15)-(20) and (25)-(27) is cost-effective, if and only if

$$t_{xi} = t_x \quad \textit{and} \quad t_{yi} = t_y \in [0, \mu_e] \qquad \textit{for all} \quad i = 1, \ldots, n \tag{28}$$

and

$$t_x = \pi_e + t_y. \tag{29}$$

The important message of Proposition 4 is that a cost-effective allocation can be attained by means of a policy mix consisting of an ETS and an emissions tax whose rate is uniform across all sectors and countries. If (29) is satisfied, the overlap of the ETS with an emissions tax in the ETS sectors is not distortionary because for the firms in the ETS sector it is the total price of energy input and emissions, $p_e + \pi_e + t_y$, that matters. The firms' demand for energy and emissions permits depends on that total price irrespective of what its components are.

Casual evidence of carbon emissions control in the EU suggests that none of the three equalities in (28) and (29) are satisfied. There are positive tax rates t_{yi} in some member states (International Energy Agency 2007) but they tend to be low relative to the (implicit) tax rates t_{xi} in the non-ETS sectors. The average permit price $\breve{\pi}_e$ also was very low during the first trading phase 2005-2007 suggesting that $\breve{t}_x > \breve{\pi}_e + \breve{t}_y$ on EU average during the last years.

Although Proposition 4 provides straightforward guidelines for improving the cost-effectiveness of carbon emissions control in the EU, the focus of the present paper is on the distributional impacts of the hybrid EU-style policy mix. To avoid coping with distributional consequences of cost-ineffective allocations we will restrict our attention to cost-effective policies exclusively. In other words, (28) and (29) are assumed to hold throughout the rest of the paper. It follows for fixed national emissions caps, c_i, t_y (with $t_{yi} = t_y$ all i) is the sole autonomous policy instrument. In practice, employing a uniform tax rate t_y presupposes an internationally coordinated tax policy or alternatively, a supranational

[6]See also the stylized analysis of overlapping regulation in Böhringer et al. (2007).

fiscal authority fixing the tax rate t_y and requiring all governments to set the tax rate t_x in their non-ETS sectors as to satisfy (29). We will make use of the latter interpretation and refer to the supranational fiscal authority as the center.

Observing the cost-effective conditions (28) and (29) the center has some discretion in fixing the tax rate t_y. In fact, there is a range of tax rates t_y supporting cost-effective competitive equilibria with the following polar cases:

(i) Suppose the center fixes the tax rate at $t_y = 0$. The cost-effective emissions control then consists of a tax-and-cap policy in each country's domestic non-ETS sector and an international ETS covering the ETS sectors of all countries. Although in this case no overlapping regulation is employed both instruments are still linked through the cost-effective condition $t_x = \pi_e$.

(ii) Suppose the center fixes the rate at some high level, say $\bar{t}_y > 0$, such that in the resultant equilibrium total demand for permits equals total supply at price $\pi_e = 0$. The efficiency condition then is $t_x = \bar{t}_y$. A strange feature of this scenario is that in spite of $\pi_e = 0$ the market for emissions permits is still in operation. This polar case will play a benchmark role in the subsequent sections.

It follows that associated to each $t_y \in [0, \bar{t}_y]$ there is a *cost-effective* competitive equilibrium. However, we do not yet know how these equilibria differ with respect to the distribution of the countries' income and welfare. Our goal is to explore the distributive impacts of variations in t_y. These effects will be investigated by means of a comparative static analysis of our multi-country model in the subsequent section. To ease the exposition, we will omit some of the tedious calculations refering the reader to the full-scale comparative statics elaborated in the Appendix.

5 Incidence of the uniform emissions tax overlapping with the ETS

5.1 Comparative statics using general functional forms

In this section we start from an initial competitive equilibrium for some vector $c = (c_1, \ldots, c_n)$ of national emissions caps and for some $t_y \in [0, \bar{t}_y]$. We will leave the national emissions cap unchanged but will disturb the initial equilibrium by a small (exogenous) variation in t_y and determine the displacement effects characterizing the new cost-effective (!) equilibrium reached after the shock. Ultimately, we are interested in the associated redistribution of national welfare as measured by changes in the utility of the countries' representative consumers which turn out to be (Appendix B)[7]

$$\frac{\mathrm{d}u_i}{\lambda_i \mathrm{d}t_y} = t_y \left(\frac{\alpha_i \delta_i - \beta_i \gamma_i}{\gamma_i} \right) \left(\frac{\mathrm{d}\pi_e}{\mathrm{d}t_y} + 1 \right) + \left(\frac{\alpha_i t_y D_z^i + \gamma_i}{\gamma_i} \right) \Delta e_{yi} \frac{\mathrm{d}\pi_e}{\mathrm{d}t_y}, \tag{30}$$

where $\alpha_i := -\frac{X_e^i}{p_{xi} X_{ee}^i} > 0$, $\beta_i := -\left(\frac{1}{Y_{ee}^i} + \frac{1}{p_{xi} X_{ee}^i} \right) > 0$, $\delta_i := \alpha_i - \beta_i t_y D_z^i$, $\gamma_i := \alpha_i X_e^i - D_p^i - (x_i + \alpha_i t_y) D_z^i$. In addition, $\lambda_i > 0$ is the marginal welfare of income in country i and $\Delta e_{yi} := c_i - e_{xi} - e_{yi}$ is the amount of permits exported ($\Delta e_{yi} > 0$) or imported ($\Delta e_{yi} < 0$) by country i. Although it can be shown (Appendix B) that $\alpha_i \delta_i > \beta_i \gamma_i$ and that $\gamma_i > 0$ under weak restrictions (30) only yields limited information on the sign of $\mathrm{d}u_i/\mathrm{d}t_y$. We are able to infer from (30) that sign $\frac{\mathrm{d}u_i}{\mathrm{d}t_y} = -\mathrm{sign}\, \Delta e_{yi}$, if $t_y = 0$ and $\mathrm{d}\pi_e/\mathrm{d}t_y < 0$, and that

$$\frac{\mathrm{d}u_i}{\mathrm{d}t_y} < 0, \quad \text{if} \quad t_y > 0, \Delta e_{yi} > 0, \frac{\mathrm{d}\pi_e}{\mathrm{d}t_y} \in]-1, 0]$$

and if $\eta_{xz}^i := x_i D_z^i / z_i$ is sufficiently small (see Appendix B). Yet in general, the sign of $\mathrm{d}u_i/\mathrm{d}t_y$ is ambiguous for permit-exporting countries as well as for permit-importing countries. It crucially depends on the sign and magnitude of $\mathrm{d}\pi_e/\mathrm{d}t_y$ the specification of which requires to explore how the permit market responds to variations in the tax rate t_y. Since the permit market is at the core of the EU-style emissions control we will investigate the determinants of $\mathrm{d}\pi_e/\mathrm{d}t_y$ in more detail.

Observe first that in the initial equilibrium the equations (20) and (25) hold so that the clearance of the permit market can be expressed by

$$\sum_j \left[E^{xj}(p_{xj}, q_e) + E^{yj}(q_e) \right] = \sum_j c_j, \tag{31}$$

where $q_e := p_e + \pi_e + t_y$, and where $E^{xi}(\cdot)$ and $E^{yi}(\cdot)$ are sectoral demand functions for energy and permits implicitly contained in (25). If in (31) the prices $p_{x1}, \ldots, p_{xn}$ clear the national markets for good X, equation (31) determines the equilibrium permit price, π_e, for some given t_y. Differentiating (31) with respect to t_y yields, after some rearrangement of terms,

$$\frac{\mathrm{d}\pi_e}{\mathrm{d}t_y} = -1 - \frac{\sum_j \left(E_{p_{xj}}^{xj} \frac{\mathrm{d}p_{xj}}{\mathrm{d}t_y} \right)}{\sum_j \left(E_{q_e}^{xj} + E_{q_e}^{yj} \right)} = -1 + \frac{\sum_j \left(\alpha_j \frac{\mathrm{d}p_{xj}}{\mathrm{d}t_y} \right)}{\sum_j \beta_j}. \tag{32}$$

According to (32) changes in the tax rate t_y are exactly offset by opposite changes in the permit price π_e unless $\sum_j [\alpha_j (\mathrm{d}p_{xj}/\mathrm{d}t_y)] \neq 0$. This term is clearly zero in partial equilibrium models where Y is the only consumer good. However, in market economies with

[7] For convenience of notation good y is chosen as numeraire $p_y \equiv 1$).

more than one consumer good, the interdependence effects $\mathrm{d}p_{xi}/\mathrm{d}t_y$ will lead to $\mathrm{d}\pi_e/\mathrm{d}t_y \neq -1$, in general. In Appendix B these interdependence effects are calculated as

$$\frac{\mathrm{d}p_{xi}}{\mathrm{d}t_y} = \frac{\delta_i + \Delta e_{yi} D_z^i}{\gamma_i} \frac{\mathrm{d}\pi_e}{\mathrm{d}t_y} + \frac{\delta_i}{\gamma_i}. \tag{33}$$

From inserting (33) into (32) follows, after some rearrangement of terms,

$$\frac{\mathrm{d}\pi_e}{\mathrm{d}t_y} = -\frac{1}{1 + \frac{\sum_j \frac{\alpha_j D_z^j}{\gamma_j} \Delta e_{yj}}{\sum_j \frac{\alpha_j \delta_j - \beta_j \gamma_j}{\gamma_j}}}. \tag{34}$$

Not surprisingly, (34) allows for deviations of $\mathrm{d}\pi_e/\mathrm{d}t_y$ from -1 in either direction as does (32). However, closer inspection of (34) shows that progress can be made in the special case of utility functions taking on the functional form $U^i(x_i, y_i) = V^i(x_i) + y_i$ with V^i being increasing and strictly concave in x_i. For that class of so-called quasi-linear utility functions the income effect of the demand for good X is known to be zero ($D_z^i \equiv 0$) such that (34) turns into $\mathrm{d}\pi_e/\mathrm{d}t_y = -1$.

Proposition 5.
If the utility functions U^i from (16) are quasi-linear the incidence of the emissions tax is given by Table 3.

	$\mathrm{d}\pi_e$	$\mathrm{d}p_{xi}$	$\mathrm{d}e_{yi}$	$\mathrm{d}e_{xi}$	$\mathrm{d}\Delta e_{yi}$	$\mathrm{d}x_i$	$\mathrm{d}y_i$	$\mathrm{d}z_i$	$\mathrm{d}u_i$
$\mathrm{d}t_y, \Delta e_{yi} > 0$	-1	0	0	0	0	0	$-$	$-$	$-$
$\mathrm{d}t_y, \Delta e_{yi} < 0$	-1	0	0	0	0	0	$+$	$+$	$+$

Table 3: Tax incidence in the general model in case of quasi-linear utility functions

It is easy to see that, with $D_z^i = 0$ (all i) the ETS and the overlapping (uniform) emissions tax are perfect substitutes in the sense that the total factor price, $q_e = p_e + \pi_e + t_y$, is unaffected by variations in t_y. Since $D_z^i = 0$ and $\mathrm{d}\pi_e/\mathrm{d}t_y$ also eliminate spillovers between the permit market and the national markets for good X ($\mathrm{d}p_{xi}/\mathrm{d}t_y$ becomes zero in (32)), the demand for permit remains unchanged in all sectors and hence the permit market is unaffected. However, the distributional incidence of tax shifts are pronounced: An increase in the tax rate t_y benefits permit-importing countries but reduces the welfare of permit-exporting countries. Since $\mathrm{d}e_{xi} = \mathrm{d}e_{yi} = 0$, $\mathrm{d}(\Delta e_{yi})/\mathrm{d}t_{yi} = 0$ follows, i.e. a country's permit export or import does not depend on the level of t_y.

Zero income elasticity of demand for good X appears to be a restrictive and unrealistic assumption. As pointed out above it eliminates market interdependence effects and thus "isolates" the permit market which can then be studied as in a partial equilibrium model. To gain further insights in the tax incidence without assuming zero income effects we will resort to parametric functional forms of the Cobb-Douglas type.

5.2 Comparative statics using parametric functional forms

We now parametrize the model by introducing the following constant-exponent functions

$$U^i(x_i, y_i) = x_i^{h_i} y_i^{1-h_i}, \quad X^i(e_{xi}) = e_{xi}^{a_i}, \quad Y^i(e_{yi}) = e_{yi}^{b_i}. \tag{35}$$

With these parametric functions the impact of variations in the tax rate t_y on the permit price, π_e, can be shown (Appendix C) to be

$$\frac{\mathrm{d}p_{xi}}{\mathrm{d}t_y} = \kappa_i \Delta e_{yi} \frac{\mathrm{d}\pi_e}{\mathrm{d}t_y} + \mu_i \left(1 + \frac{\mathrm{d}\pi_e}{\mathrm{d}t_y}\right), \tag{36}$$

where $\kappa_i := \frac{(1-a_i)p_{xi}a_i}{e_{xi}\left[(p_e+\pi_e)a_i + \bar{h}_i q_e\right]} > 0$, $\bar{h}_i := \frac{(1-h_i)}{h_i} > 0$, $\mu_i := \rho_i p_{xi}\left[\frac{\bar{h}_i q_e + a_i(p_e+\pi_e)}{a_i q_e} - \frac{(1-a_i)}{e_{xi}}\right] \left(\gtreqless\right) 0$ and $\rho_j := \frac{a_i}{(p_e+\pi_e)a_i + \bar{h}_i q_e} > 0$. According to (36), $\mathrm{d}p_x/\mathrm{d}t_y \neq 0$ is non-zero in general, and this is true even in case of $\mathrm{d}\pi_e/\mathrm{d}t_y = -1$.[8] More specifically, for $\mathrm{d}\pi_e/\mathrm{d}t_y = -1$ it follows from (36) that $\mathrm{d}p_{xi}/\mathrm{d}t_y > 0$ for permit-importing countries and $\mathrm{d}p_{xi}/\mathrm{d}t_y < 0$ for permit-exporting countries. If $\mathrm{d}\pi_e/\mathrm{d}t_y \neq -1$, $\mathrm{d}p_{xi}/\mathrm{d}t_y$ may be positive or negative.

Inserting (36) into (32) does not render the result more informative. However, taking another route of comparative static calculations (Appendix C) we find

$$\frac{\mathrm{d}\pi_e}{\mathrm{d}t_y} = -\frac{1}{1 - \frac{\sum_j \rho_j \Delta e_{yj}}{\sum_j \rho_j \sigma_j}}, \tag{37}$$

where $\sigma_i := \frac{\bar{h}_i e_{xi}}{a_i} + \frac{(\bar{h}_i + a_i)e_{yi}}{(1-b_i)a_i} > 0$. As in the model of Section 5.1, the change in the countries' welfare, $\mathrm{d}u_i/\mathrm{d}t_y$, is given by (30). $\mathrm{d}u_i/\mathrm{d}t_y$ remains ambiguous in sign although γ_i is now unequivocally positive. However, closer inspection of (37) reveals that $\mathrm{d}\pi_e/\mathrm{d}t_y = -1$, if

$$a_i = a \quad \text{and} \quad h_i = h \quad \text{for all}\, i. \tag{38}$$

Note that (38) does not render all countries identical. They may still differ with respect to their production functions for good Y ($b_i \neq b_j$) and their national caps ($c_i \neq c_j$) such that net exports and imports of permits will be non-zero, in general.

A remarkable consequence of the assumption (38) is that in contrast to the special case $D_z^i = 0$, all i, of Section 4.1 variations in t_y now do affect the market for good X: Combining (36) and (38) shows that a tax hike $\mathrm{d}t_y > 0$ will raise [lower] the equilibrium price p_{xi} if country i imports [exports] permits. Moreover, we know from comparing (32) and (37) that if (38) holds the opposite price changes of permit exporting and importing countries are symmetric in the sense that $\sum_j \left[\alpha_j(\mathrm{d}p_{xj}/\mathrm{d}t_y)\right] = 0$.

[8] Recall that in the previous section $D_z^i = 0$ for all i implied $\mathrm{d}\pi_e/\mathrm{d}t_y = -1$ as well as $\mathrm{d}p_{xi}/\mathrm{d}t_y = 0$. Note also that with the Cobb-Douglas utility function the income effect on the demand for both goods is always positive.

Since q_e remains unchanged the demand for energy inputs of the ETS sectors does not change either. On the other hand, the increase [reduction] in the price p_{xi} induced by $\mathrm{d}t_y > 0$ reduces [increases] the demand for energy inputs in the non-ETS sector of permit exporting [importing] countries such that exports as well as imports rise. Since it can be shown that

$$\lim_{\Delta e_{yi} \to 0} \frac{\mathrm{d}\Delta e_{yi}}{\mathrm{d}t_y} = 0,$$

we conclude that the subsets of permit exporting and importing countries are independent of the level of t_y. The comparative statics carried out in Appendix C yield

Proposition 6.
If the functions X^i, Y^i and U^i from (15) and (16) are specified by (35) and if (38) holds the incidence of the emissions tax is given by Table 4.

	$\mathrm{d}\pi_e$	$\mathrm{d}p_{xi}$	$\mathrm{d}e_{yi}$	$\mathrm{d}e_{xi}$	$\mathrm{d}\Delta e_{yi}$	$\mathrm{d}x_i$	$\mathrm{d}y_i$	$\mathrm{d}z_i$	$\mathrm{d}u_i$
$\mathrm{d}t_y, \Delta e_{yi} > 0$	-1	$-$	0	$-$	$+$	$-$	$-$	$-$	$-$
$\mathrm{d}t_y, \Delta e_{yi} < 0$	-1	$+$	0	$+$	$-$	$+$	$+$	$+$	$+$

Table 4: Tax incidence in the parametric model, when technologies of good X and preferences are the same across countries

Comparing Table 4 with Table 3 reveals that in both cases we observe $\mathrm{d}\pi_e/\mathrm{d}t_y = -1$ and the qualitative changes in $\mathrm{d}e_{yi}$, $\mathrm{d}y_i$, $\mathrm{d}z_i$ and $\mathrm{d}u_i$ are the same. However, while in case of $D_z^i \equiv 0$ (Table 3) both endogenous markets, i.e. the market for good X and the permit market, remain unaffected, the parametric model satisfying (38) exhibits repercussions in both markets. As an implication, the cost-effective split of the national emissions caps into two sectoral caps depends on the level of the tax rate t_y in the parametric model satisfying (38) while it is unaffected by t_y if $D_z^i = 0$.

Apart from these differences, the restrictions imposed on the model in the Propositions 5 and 6 have an important property in common: They imply that the derivatives of all endogenous variables with respect to t_y are either zero or unconditionally positive or negative. In other words, there are functions $v = v(t_y)$ for all endogenous variables $v = \pi_e, p_{xi}, e_{xi}, e_{yi}, \Delta e_{yi}, x_i, y_i, z_i, u_i$ which are monotone or strictly monotone in the tax rate t_y. For that reason the comparative statics analysis does not only yield 'local information' for marginal variations in the tax rate but provides 'global information' about the properties of the functions $v = v(t_y)$. The most relevant properties are highlighted in

Proposition 7.
Denote by $\Delta e_{yi}(t_y)$ and $u_i(t_y)$ country i's permit trade balance and welfare, respectively,

when the center has fixed the tax rate at $t_y \in [0, \bar{t}_y]$ and suppose the functional forms are as specified in Proposition 5 or Proposition 6.

(a) If country i exports [imports] permits for some $t_y \in [0, \bar{t}_y]$, it exports [imports] permits for all $t_y \in [0, \bar{t}_y]$.

(b) Permit-exporting [permit-importing] countries lose [gain] whenever the tax rate t_y is raised such that

$$u_i(0) > u_i(\bar{t}_y), \text{ if } \Delta e_{yi} > 0 \text{ and } u_i(0) < u_i(\bar{t}_y), \text{ if } \Delta e_{yi} < 0. \tag{39}$$

We conclude that under the conditions of Proposition 7 the distributional consequences of variations in the tax rate t_y are unambiguous. Unfortunately this feature does not hold in general, i.e. if $D_z^i \neq 0$ or if (38) does not hold, we cannot draw conclusions from marginal information provided by the comparative-static analysis on the global properties of the functions $v(t_y)$. In the next Section we will therefore resort to numerical analysis of our parametric model aiming at additional global information on the functions $v(t_y)$ in a case where (38) is not satisfied. Particular attention will be placed on whether and how $\mathrm{d}\pi_e/\mathrm{d}t_y$ deviates from minus one, whether non-marginal variations of t_y may lead to export-import reversals and what the associated changes in the distribution of national welfare are.

5.3 Non-monotone changes in welfare: a numerical example

In this section we consider a parametric model of Section 5.2 for which condition (38) is not satisfied. To make progress we consider a three-country model in which the parameters take on the values $a_1 = 0.2$, $a_2 = 0.6$, $a_3 = 0.9$, $b_1 = b_2 = b_3 = 0.5$, $c_1 = 0.605$, $c_2 = 0.6$, $c_3 = 1.3$, $h_1 = h_2 = h_3 = 0.5$, $p_e = 0.2$. For this model we then compute the equilibrium allocation as a function of the tax rate t_y with the help of the tool Mathematica (Appendix C) establishing

Proposition 8.
If in the parametric model of Section 4.2 the tax rate t_y is successively raised, some permit-exporting country may eventually import permits such that its national income and welfare first decline but then increase.

The qualitative properties of the response of the entire equilibrium allocation to successive increases in the tax rate are summarized in Table 5, and the Figures 1, 2 and 3 provide additional illustration of some of the particularly interesting functions.

	$\pi_e(t_y)$	$p_{xi}(t_y)$	$e_{yi}(t_y)$	$e_{xi}(t_y)$	$\Delta e_{yi}(t_y)$	$x_i(t_y)$	$y_i(t_y)$ $z_i(t_y)$ $u_i(t_y)$
country 1 $\Delta e_{y1} \gtrless 0$	DECR	U-SH CONV	INCR & CONV	INCR CONV	DECR CONC	INCR CONV	U-SH & CONV
country 2 $\Delta e_{y2} < 0$		INCR CONV			DECR CONV		INCR & CONV
country 3 $\Delta e_{y3} > 0$		DECR CONC		DECR CONC	INCR CONV	DECR CONC	DECR & CONV

(DECR= monotone decreasing, U-SH= u-shaped, INCR= monotone increasing, CONV= strictly convex, CONC= strictly concave)

Table 5: Equilibrium quantities and prices as functions of the tax rate t_y: numerical example

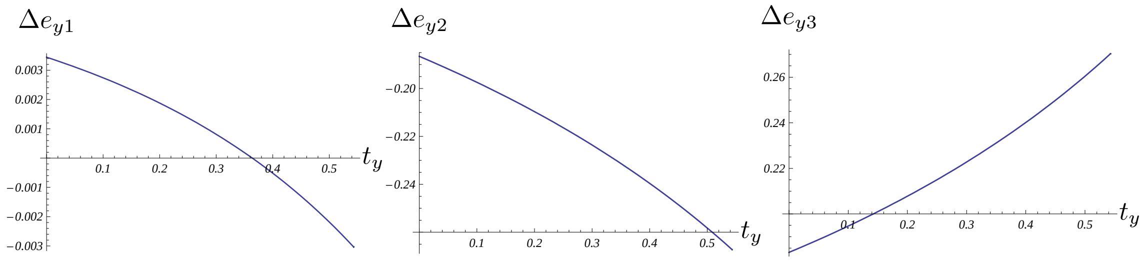

Figure 1: Exports and imports of permits

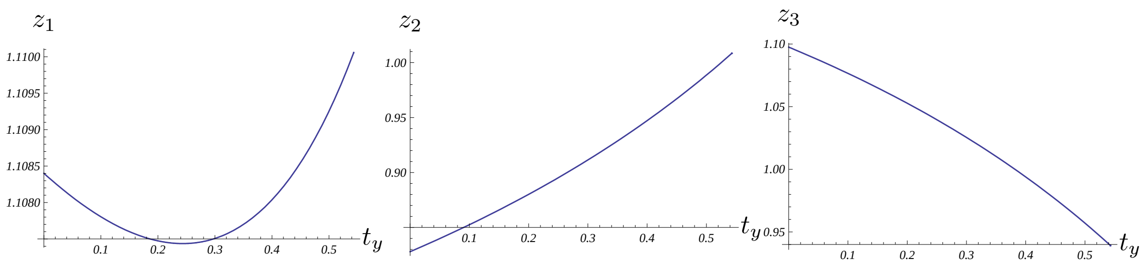

Figure 2: National incomes

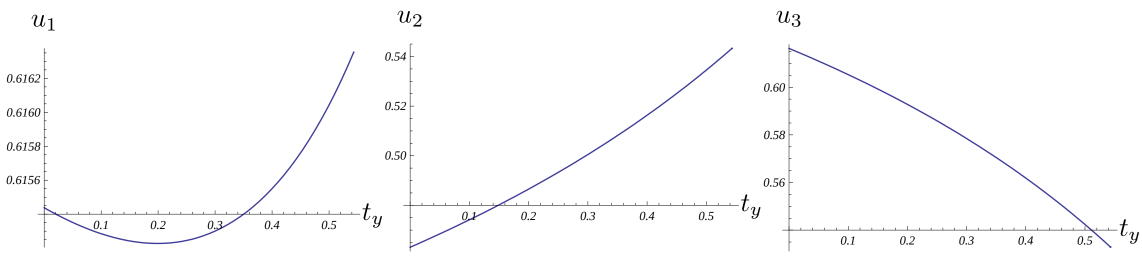

Figure 3: National welfare

A few comments on the results are in order. Although the graph of the function $\pi_e(t_y)$ (see Appendix D) is clearly negatively sloped it exhibits little curvature, if any. Nonetheless we safely conclude that $\mathrm{d}\pi_e/\mathrm{d}t_y < -1$ because $e_{yi}(t_y) = \left(\frac{p_e+\pi_e+t_y}{b_i}\right)^{\frac{1}{b_i-1}}$ is obviously strictly increasing in t_y for i= 1, 2, 3 and hence

$$\frac{\mathrm{d}e_{yi}}{\mathrm{d}t_y} = -\frac{e_{yi}}{(1-b_i)q_e}\left(\frac{\mathrm{d}\pi_e}{\mathrm{d}t_y}+1\right).$$

Country 2 imports and country 3 exports permits, and as in the model of Table 3 the levels of imports and exports rise with increasing tax rate (Figure 2). Corresponding to these changes the national income and the welfare in country 2 increase with the tax rate while national income and welfare shrink in country 3 (Figures 2 and 3).

The striking feature of the numerical example under consideration highlighted in Proposition 8 are the effects of successive tax increases on the allocation in country 1. As shown in the left panel of Figure 1 country 1 first exports permits but becomes an importer of permits when the tax continues to increase. Along with that reversal from exports to imports the price of good X, the national income and national welfare of country 1 are u-shaped functions of the tax rate (left panels of Figures 2 and 3).

The observation that export-import reversals are feasible and with them non-monotone welfare changes makes it difficult to assess correctly the impact of tax hikes on the international distribution of welfare.

6 Concluding remarks

In a stylized way, our paper adresses *distributional* consequences of a hybrid regime of CO_2 emissions control designed to capture some basic features of the regime applied in the EU since 2005. Characteristic of the EU regime is an EU-wide international ETS that coexists with national complementary and overlapping national emissions taxes. Restrictly our attention to cost-effective competitive equilibria we show that an increase in the emissions tax rate is exactly offset by a reduction in the equilibrium permit price for quasi-linear utility functions and for a class of parameteric utility and production functions. Since the reduction of the permit price lowers [raises] a permit-exporting [importing] country's budget, permit-exporting [importing] countries lose [gain] from the increase in the emissions tax. However, these results are not general, because emissions tax changes may cause effects in markets beyond the permit market. With the help of a numerical example we show that an initially permit-exporting country may switch to a permit-importing country when the emission tax is successively increased. Hence, export-import reversals are feasible such that initially welfare losing countries may turn to welfare gaining countries and vice versa.

Finally, a remark is order with respect to the partition of the national emissions caps. Throughout the paper we assumed that the national emissions caps are fixed. It is well known, of course, that changes in the national emissions cap also affect the distribution of the countries' welfare, because national emissions caps are valuable national assets similar as endowments with productive factors. Since both overlapping emissions taxes and national caps are the main determinants of national welfare the investigation of their interplay is an important and interesting task for future research.

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Appendix A: Comparative statics of the one-sector model

For given prices p_e and p_y and for given emissions tax rate t_y the cost-effective competitive equilibrium is determined by

$$\sum_j c_{yj} = \sum_j e_{yj}, \tag{A1}$$

$$z_i = p_y y_{si} + \pi_e \Delta e_i - p_e e_{yi}, \qquad i = 1, \ldots, n, \tag{A2}$$

$$y_{si} = Y^i(e_{yi}), \qquad i = 1, \ldots, n, \tag{A3}$$

$$z_i = p_y y_i, \qquad i = 1, \ldots, n, \tag{A4}$$

$$Y^i_e = p_e + \pi_e + t_y, \qquad i = 1, \ldots, n, \tag{A5}$$

where $\Delta e_{yi} := c_{yi} - e_{yi}$ is the amount of permits exported or imported by country i. In the following we carry out a comparative static analysis to specify the impact of exogenous variations in the uniform tax rate t_y. Total differentiation of (A1) - (A5) yields

$$0 = \sum_j \mathrm{d}e_{yj}, \tag{A6}$$

$$\mathrm{d}z_i = p_y \mathrm{d}y_{si} + \mathrm{d}\pi_e \Delta e_i - (p_e + \pi_e)\mathrm{d}e_{yi}, \tag{A7}$$

$$\mathrm{d}y_{si} = Y^i_e \mathrm{d}e_{yi}, \tag{A8}$$

$$\mathrm{d}z_i = p_y \mathrm{d}y_i, \tag{A9}$$

$$Y^i_{ee}\mathrm{d}e_{yi} = \mathrm{d}\pi_e + \mathrm{d}t_y. \tag{A10}$$

Inserting (A10) into (A6) we obtain

$$(\mathrm{d}\pi_e + dt_y)\sum_j \frac{1}{Y^i_{ee}} = 0 \quad \Longleftrightarrow \quad \frac{\mathrm{d}\pi_e}{\mathrm{d}t_y} = -1. \tag{A11}$$

We make use of (A11) in (A10) to obtain

$$\frac{\mathrm{d}e_{yi}}{\mathrm{d}t_y} = 0 \tag{A12}$$

and from (A8) we infer

$$\frac{\mathrm{d}y_{si}}{\mathrm{d}t_y} = 0. \tag{A13}$$

Next, we take advantage of (A11), (A12) and (A13) to rewrite (A7) as

$$\frac{\mathrm{d}z_i}{\mathrm{d}t_y} = -\Delta e_i. \tag{A14}$$

Finally, we insert into (A14) which we in turn insert into $\mathrm{d}u_i = U_y^i \mathrm{d}y_i$ to get

$$\frac{\mathrm{d}y_i}{\mathrm{d}t_y} = -\Delta e_i, \tag{A15}$$
$$\frac{\mathrm{d}u_i}{\mathrm{d}t_y} = -U_y^i \Delta e_i. \tag{A16}$$

Appendix B: Comparative statics of the general model

The efficient competitive equilibrium of the multi-country economy is completely described by the following equations:

$$\sum_j c_j = \sum_j (e_{xj} + e_{yj}), \tag{B1}$$
$$x_{si} = x_i, \qquad i = 1, \ldots, n, \tag{B2}$$
$$x_{si} = X^i(e_{xi}), \qquad i = 1, \ldots, n, \tag{B3}$$
$$x_i = D^i(p_{xi}, z_i), \qquad i = 1, \ldots, n, \tag{B4}$$
$$z_i = p_{xi}x_{is} + y_{is} - p_e(e_{xi} + e_{yi}) + \pi_e(c_i - e_{xi} - e_{yi}), \quad i = 1, \ldots, n \tag{B5}$$
$$y_{si} = Y^i(e_{yi}), \qquad i = 1, \ldots, n, \tag{B6}$$
$$z_i = p_{xi}x_i + y_i, \qquad i = 1, \ldots, n, \tag{B7}$$
$$p_{xi}X_e^i(e_{xi}) = p_e + t_x, \qquad i = 1, \ldots, n, \tag{B8}$$
$$Y_e^i = p_e + \pi_e + t_y, \qquad i = 1, \ldots, n, \tag{B9}$$
$$t_x = \pi_e + t_y, \tag{B10}$$

where without loss of generality good Y is chosen as numeraire ($p_y \equiv 1$). The variables determined by (B1) - (B10) are $e_{xi}, e_{yi}, x_{si}, x_i, p_{xi}, z_i, y_{si}, y_i$ for $i = 1, \ldots, n$, π_e and t_x. The tax rate t_y treated here as an exogenous parameter. It is convenient to compress the system of equations (B1) - (B10) as follows:

$$\sum_j c_j = \sum_j (e_{xj} + e_{yj}), \tag{B11}$$
$$X^i(e_{xi}) = D^i(p_{xi}, z_i), \tag{B12}$$
$$z_i = p_{xi}X^i(e_{xi}) + Y^i(e_{yi}) - p_e(e_{xi} + e_{yi}) + \pi_e \Delta e_{yi}, \tag{B13}$$
$$p_{xi}X_e^i(e_{xi}) = Y_e^i(e_{yi}), \tag{B14}$$
$$Y_e^i(e_{yi}) = p_e + \pi_e + t_y, \tag{B15}$$
$$y_i = Y^i(e_{yi}) - p_e(e_{xi} + e_{yi}) + \pi_e \Delta e_{yi}, \tag{B16}$$

where $\Delta e_{yi} := c_i - e_{xi} - e_{yi}$ in (B13) is the amount of permits exported or imported by country i. The equations (B11) - (B15) serve to determine π_e and e_{xi}, e_{yi}, p_{xi} and z_i for $i = 1, \ldots, n$. Equation (B16) represents the current account balances, and it determines y_i after e_{xi}, e_{yi} and π_e are solved via (B11) - (B15). Our aim is to perform a comparative static analysis to specify the impact on the economy of exogenous variations in the uniform tax rate t_y. To that end (B11) through (B15) are now totally differentiated.

$$\sum_j \mathrm{d}e_{xj} + \sum_j \mathrm{d}e_{yj} = 0, \tag{B17}$$

$$X_e^i \mathrm{d}e_{xi} - D_p^i \mathrm{d}p_{xi} - D_z^i \mathrm{d}z_i = 0, \tag{B18}$$

$$\mathrm{d}z_i = x_i \mathrm{d}p_{xi} + t_y(\mathrm{d}e_{xi} + \mathrm{d}e_{yi}) + \Delta e_{yi} \mathrm{d}\pi_e, \tag{B19}$$

$$X_e^i \mathrm{d}p_{xi} + p_{xi} X_{ee}^i \mathrm{d}e_{xi} - Y_{ee}^i \mathrm{d}e_{yi} = 0 \tag{B20}$$

$$Y_{ee}^i \mathrm{d}e_{yi} - \mathrm{d}\pi_e - \mathrm{d}t_y = 0 \tag{B21}$$

To obtain (B19) we have differentiated (B13),

$$\mathrm{d}z_i = x_{si}\mathrm{d}p_{xi} + p_{xi}X_e^i \mathrm{d}e_{xi} + Y_e^i \mathrm{d}e_{yi} - p_e(\mathrm{d}e_{xi} + \mathrm{d}e_{yi}) + \Delta e_{yi}\mathrm{d}\pi_e - \pi_e(\mathrm{d}e_{xi} + \mathrm{d}e_{yi}),$$

and then made use of (B14) and (B15).

Next we consider $\mathrm{d}e_{yi} = \frac{1}{Y_{ee}^i}(\mathrm{d}\pi_e + \mathrm{d}t_y)$ from (B21) in (B20) to obtain

$$\mathrm{d}e_{xi} \quad = \quad \frac{1}{p_{xi}X_{ee}^i}(\mathrm{d}\pi_e + \mathrm{d}t_y) - \frac{X_e^i}{p_{xi}X_{ee}^i}\mathrm{d}p_{xi}. \tag{B22}$$

Summation of $\mathrm{d}e_{xi}$ from (B22) and $\mathrm{d}e_{yi}$ from (B21) yields

$$\mathrm{d}e_{xi} + \mathrm{d}e_{yi} = \alpha_i \mathrm{d}p_{xi} - \beta_i(\mathrm{d}\pi_e + \mathrm{d}t_y), \tag{B23}$$

where $\alpha_i := -\frac{X_e^i}{p_{xi}X_{ee}^i} > 0$ and $\beta_i := -\left(\frac{1}{Y_{ee}^i} + \frac{1}{p_{xi}X_{ee}^i}\right) > 0$. Inserting (B23) in (B17) gives

$$\frac{\sum_j \alpha_j \mathrm{d}p_{xj}}{\sum_j \beta_j} - \mathrm{d}\pi_e = \mathrm{d}t_y \tag{B24}$$

We take advantage of (B23) again to turn (B19) into

$$\mathrm{d}z_i = (x_i + \alpha_i t_y)\mathrm{d}p_{xi} - \beta_i t_y(\mathrm{d}\pi_e + \mathrm{d}t_y) + \Delta e_{yi}\mathrm{d}\pi_e. \tag{B25}$$

We make use of (B22) and (B25) to transform (B18) into

$$\mathrm{d}p_{xi} = \frac{\Delta_i(\mathrm{d}\pi_e + \mathrm{d}t_y)}{\gamma_i} + \frac{D_z^i \Delta e_{yi}\mathrm{d}\pi_e}{\gamma_i}, \tag{B26}$$

where $\delta_i := \alpha_i - \beta_i t_y D_z^i$ and $\gamma_i := \alpha_i X_e^i - D_p^i - (x_i + \alpha_i t_y)D_z^i$.
We insert (B26) into (B24) to obtain, after some rearrangement of terms,

$$\mathrm{d}\pi_e \sum_j \frac{\alpha_j D_z^j \Delta e_{yj}}{\gamma_j} + (\mathrm{d}\pi_e + \mathrm{d}t_y)\sum_j \left[\frac{\alpha_j \delta_j}{\gamma_j} - \beta_j\right] = 0 \tag{B27}$$

which in turn can be rewritten as

$$\frac{\mathrm{d}\pi_e}{\mathrm{d}t_y} = -\frac{1}{1 + \frac{\sum_j \frac{\alpha_j D_z^j}{\gamma_j}\Delta e_{yj}}{\sum_j \frac{\alpha_j \delta_j - \beta_j \gamma_j}{\gamma_j}}}. \tag{B28}$$

Next, we differentiate the utility function (16) to get $\mathrm{d}u_i = U_x^i \mathrm{d}x_i + U_y^i \mathrm{d}y_i$ and use $\frac{U_x^i}{p_{xi}} = \frac{U_y^i}{p_y} = \lambda_i$ to obtain

$$\frac{\mathrm{d}u_i}{\lambda_i} = p_{xi}\mathrm{d}x_i + \mathrm{d}y_i, \tag{B29}$$

where λ_i is the marginal utility of income (i.e. the Lagrange multiplier assigned to the consumers budget constraint). From (B3), (B8) and (B10) we infer

$$\mathrm{d}x_i = X_e^i \mathrm{d}e_{xi} = \frac{p_e + \pi_e + t_y}{p_{xi}} \mathrm{d}e_{xi}. \tag{B30}$$

From (B16) we obtain with the help of (B15)

$$\mathrm{d}y_i = t_y \mathrm{d}e_{yi} - (p_e + \pi_e)\mathrm{d}e_{xi} + \Delta e_{yi}\mathrm{d}\pi_e. \tag{B31}$$

Inserting (B30) and (B31) in (B29) gives

$$\frac{\mathrm{d}u_i}{\lambda_i} = (p_e + \pi_e + t_y)\mathrm{d}e_{xi} + t_y \mathrm{d}e_{yi} - (p_e + \pi_e)\mathrm{d}e_{xi} + \Delta e_{yi}\mathrm{d}\pi_e = t_y(\mathrm{d}e_{xi} + \mathrm{d}e_{yi}) + \Delta e_{yi}\mathrm{d}\pi_e. \tag{B32}$$

or, equivalently,

$$\frac{\mathrm{d}u_i}{\lambda_i \mathrm{d}t_y} = t_y \frac{\mathrm{d}e_{xi} + \mathrm{d}e_{yi}}{\mathrm{d}t_y} + \Delta e_{yi}\frac{\mathrm{d}\pi_e}{\mathrm{d}t_y}. \tag{B33}$$

From (B23) it follows that

$$\frac{\mathrm{d}e_{xi} + \mathrm{d}e_{yi}}{\mathrm{d}t_y} = \alpha_i \frac{\mathrm{d}p_{xi}}{\mathrm{d}t_y} - \beta_i \left(\frac{\mathrm{d}\pi_e}{\mathrm{d}t_y} + 1\right). \tag{B34}$$

(B26) yields

$$\frac{\mathrm{d}p_{xi}}{\mathrm{d}t_y} = \frac{\delta_i}{\gamma_i}\left(\frac{\mathrm{d}\pi_e}{\mathrm{d}t_y} + 1\right) + \frac{D_z^i \Delta e_{yi}}{\gamma_i}\frac{\mathrm{d}\pi_e}{\mathrm{d}t_y}. \tag{B35}$$

Making use of (B35) in (B34) yields

$$\frac{\mathrm{d}e_{xi} + \mathrm{d}e_{yi}}{\mathrm{d}t_y} = \left(\frac{\alpha_i \delta_i}{\gamma_i} - \beta_i\right)\left(\frac{\mathrm{d}\pi_e}{\mathrm{d}t_y} + 1\right) + \frac{\alpha_i D_z^i \Delta e_{yi}}{\gamma_i}\frac{\mathrm{d}\pi_e}{\mathrm{d}t_y}. \tag{B36}$$

We take advantage of (B36) to turn (B33) into

$$\frac{\mathrm{d}u_i}{\lambda_i \mathrm{d}t_y} = t_y \left(\frac{\alpha_i \delta_i - \beta_i \gamma_i}{\gamma_i}\right)\left(\frac{\mathrm{d}\pi_e}{\mathrm{d}t_y} + 1\right) + \left(\frac{\alpha_i t_y D_z^i + \gamma_i}{\gamma_i}\right)\Delta e_{yi}\frac{\mathrm{d}\pi_e}{\mathrm{d}t_y}. \tag{B37}$$

Lemma 1.

$$\eta_{xz}^i < \frac{p_e + \pi_e + t_y}{t_y} \cdot \frac{z_i}{p_{xi}x_i} \qquad \Longrightarrow \qquad \gamma_i > 0. \tag{B38}$$

Proof. Observe that

$$\gamma_i := \alpha_i X_e^i - D_p^i - (x_i + \alpha_i t_y) D_z^i = \alpha_i X_e^i - \frac{p_{xi} D_p^i}{x_i} \cdot \frac{x_i}{p_{xi}} - (x_i + \alpha_i t_y) \frac{z_i D_z^i}{x_i} \cdot \frac{x_i}{z_i}. \quad \text{(B39)}$$

Making use of the definitions $\eta_{xp}^i := \frac{p_{xi} D_p^i}{x_i}$ and $\eta_{xz}^i := \frac{z_i D_z^i}{x_i}$ (B39) turns into

$$\gamma_i = \frac{x_i}{p_{xi}} \left[\frac{\alpha_i p_{xi}}{x_i} X_e^i - \eta_{xp}^i - \left(\frac{p_{xi} x_i}{z_i} + \frac{\alpha t_y p_{xi}}{z_i} \right) \eta_{xz}^i \right]. \quad \text{(B40)}$$

With the help of the Slutzky equation (in elasticity notation), formally $\eta_{xp}^i = \eta_{xp}^{ci} - \frac{p_{xi} x_i}{z_i} \eta_{xz}^i$ with $\eta_{xp}^{ci} := \frac{\partial x_i}{\partial p_{xi}} \cdot \frac{p_{xi}}{x_i} \Big|_{u=const.} < 0$, (B40) can be rearranged to

$$\gamma = \frac{x_i}{p_{xi}} \left[\alpha_i p_{xi} \left(\frac{X_e^i}{x_i} - \frac{t_y}{z_i} \eta_{xz}^i \right) - \eta_{xp}^{ci} \right]. \quad \text{(B41)}$$

Finally, we consider (B7) to obtain

$$\frac{X_e^i}{x_i} - \frac{t_y}{z_i} \eta_{xz}^i = \frac{p_e + \pi_e + t_y}{p_{xi} x_i} - \frac{t_y}{z_i} \eta_{xz}^i = \frac{t_y}{p_{xi} x_i} \left(\frac{p_e + \pi_e + t_y}{t_y} - \frac{p_{xi} x_i}{z_i} \eta_{xz}^i \right). \quad \text{(B42)}$$

Lemma 2. *The term $\alpha_i \delta_i - \beta_i \gamma_i$ is negative.*

Proof. Observe that

$$\begin{aligned} \alpha_i \delta_i - \beta_i \gamma_i &= \alpha_i \left(\alpha_i - \beta_i t_y D_z^i \right) - \beta_i \left[\alpha_i X_e^i - D_p^i - (x_i + \alpha_i t_y) D_z^i \right] \\ &= \alpha_i \left(\alpha_i - \beta_i X_e^i \right) + \beta_i \left(D_p^i + x_i D_z^i \right). \end{aligned} \quad \text{(B43)}$$

Making use of the definitions of β_i, the elasticities η_{xp}^i and η_{xz}^i and making use of the Sluzky equation we obtain

$$\alpha_i \delta_i - \beta_i \gamma_i = \alpha_i \frac{X_e^i}{Y_{ee}^i} + \beta_i \frac{x_i}{p_{xi}} \left(\eta_{xp}^i + \frac{p_{xi} x_i}{z_i} \eta_{xz}^i \right) = \alpha_i \frac{X_e^i}{Y_{ee}^i} + \beta_i \frac{x_i}{p_{xi}} \eta_{xp}^{ci}. \quad \text{(B44)}$$

Comparative statics for quasi-linear utility functions (Table 3)

While $\mathrm{d}\pi_e$ and $\mathrm{d}u_i$ follows from setting $D_z^i = 0$ in (34) and (30), $\mathrm{d}p_{xi}$, $\mathrm{d}e_{yi}$, $\mathrm{d}e_{xi}$, $\mathrm{d}x_i$, $\mathrm{d}y_i$, $\mathrm{d}z_i$ follows from (B26), $\mathrm{d}e_{yi} = \frac{\mathrm{d}\pi_e + \mathrm{d}t_y}{Y_{ee}^i}$, (B22), (B30), (B31) and (B25), respectively.

Appendix C: Comparative statics of the parametric model

For the parametric functional forms $U^i(x_i, y_i) = x_i^{h_i} y_i^{1-h_i}$, $X^i(e_{xi}) = e_{xi}^{a_i}$, $Y^i(e_{yi}) = e_{yi}^{b_i}$ the efficient competitive equilibrium is determined by

$$\sum_j c_j = \sum_j (e_{xj} + e_{yj}), \tag{C1}$$

$$e_{xi}^{a_i} = \frac{h_i z_i}{p_{xi}}, \tag{C2}$$

$$z_i = p_{xi} e_{xi}^{a_i} + e_{yi}^{b_i} - (q_e - t_y)(e_{xi} + e_{yi}) + \pi_e c_i, \tag{C3}$$

$$p_{xi} a_i e_{xi}^{a_i - 1} = b_i e_{yi}^{b_i - 1} = q_e, \tag{C4}$$

$$b_i e_{yi}^{b_i - 1} = q_e, \tag{C5}$$

$$y_i = e_{yi}^{b_i} - (q_e - t_y)(e_{xi} + e_{yi}) + \pi_e c_i, \tag{C6}$$

where $q_e := p_e + \pi_e + t_y$. Note that (C2), (C3) and (C6) imply

$$y_i = (1 - h_i) z_i. \tag{C7}$$

Next we rearrange the system of equations (C1)-(C5). We make use of (C2) and (C4) to get

$$z_i = \frac{e_{xi} q_e}{a_i h_i}. \tag{C8}$$

From (C4) and (C5) we obtain

$$p_{xi} e_{xi}^{a_i} = \frac{e_{xi} q_e}{a_i}. \tag{C9}$$

We rearrange (C5) to

$$e_{yi}^{b_i} = \frac{e_{yi} q_e}{b_i}. \tag{C10}$$

We make use of (C8)-(C10) in (C3) to get

$$\left[\frac{(1 - h_i) + a_i h_i}{a_i h_i} q_e - t_y\right] e_{xi} - \left[\frac{1 - b_i}{b_i} q_e + t_y\right] e_{yi} - \pi_e c_i = 0. \tag{C11}$$

Total differentiation of (C1), (C5), (C6) and (C11) yields

$$\sum_j (\mathrm{d}e_{xj} + \mathrm{d}e_{yj}) = 0, \tag{C12}$$

$$\mathrm{d}e_{yi} = -\frac{e_{yi}}{(1 - b_i) q_e} (\mathrm{d}\pi_e + dt_y), \tag{C13}$$

$$\mathrm{d}z_i = \frac{\mathrm{d}y_i}{1 - h_i}, \tag{C14}$$

$$\begin{aligned}\left[\frac{(1 - h_i) + a_i h_i}{a_i h_i} q_e - t_y\right] \mathrm{d}e_{xi} - \left(\frac{1 - b_i}{b_i} q_e + t_y\right) \mathrm{d}e_{yi} \quad - \quad \mathrm{d}\pi_e c_i \\ + e_{xi}\left[\frac{1 - h_i + a_i h_i}{a_i h_i} \mathrm{d}\pi_e + \frac{1 - h_i}{a_i h_i} \mathrm{d}t_y\right] \quad - \quad e_{yi}\left[\frac{1 - b_i}{b_i} \mathrm{d}\pi_e + \frac{\mathrm{d}t_y}{b_i}\right] = 0.\end{aligned} \tag{C15}$$

(C15) can be rearranged to

$$\frac{\mathrm{d}e_{xi}}{\rho_i} = \Delta e_{yi}\mathrm{d}\pi_e - \left(\frac{1-b_i}{b_i}q_e + t_y\right)\mathrm{d}e_{yi} + (\mathrm{d}\pi_e + \mathrm{d}t_y)\left(\frac{\bar{h}_i e_{xi}}{a_i} - \frac{e_{yi}}{b_i}\right), \tag{C16}$$

where $\bar{h}_i := \frac{1-h_i}{h_i}$ and $\rho_i := \frac{a_i}{\bar{h}_i q_e + a_i(p_e+\pi_e)} > 0$. Inserting $\mathrm{d}e_{yi}$ from (C13) in (C16) yields after some rearrangement of terms

$$\frac{\mathrm{d}e_{xi}}{\rho_i} = \Delta e_{yi}\mathrm{d}\pi_e - (\mathrm{d}\pi_e + \mathrm{d}t_y)\left[\frac{\bar{h}_i e_{xi}}{a_i} + \frac{t_y e_{yi}}{(1-b_i)q_e}\right]. \tag{C17}$$

In view of (C13) and (C17), the sum of $\mathrm{d}e_{xi}$ and $\mathrm{d}e_{yi}$ is equal to

$$\mathrm{d}e_{xi} + \mathrm{d}e_{yi} = -\sigma_i\rho_i(\mathrm{d}\pi_e + \mathrm{d}t_y) + \Delta e_{yi}\rho_i\mathrm{d}\pi_e, \tag{C18}$$

where $\sigma_i := \frac{\bar{h}_i e_{xi}}{a_i} + \frac{(\bar{h}_i + a_i)e_{yi}}{(1-b_i)a_i} > 0$. Next, we insert (C18) in (C12) to obtain

$$\mathrm{d}\pi_e \sum_j \rho_j(\sigma_j - \Delta e_{yj}) = -\mathrm{d}t_y \sum_j \rho_j\sigma_j \quad \Longleftrightarrow \quad \frac{\mathrm{d}\pi_e}{\mathrm{d}t_y} = -\frac{1}{1-\sum_j \frac{\rho_j \Delta e_{yj}}{\rho_j\sigma_j}}. \tag{C19}$$

Totally differentiating (C4) yields

$$\mathrm{d}p_{xi} = \frac{(1-a_i)p_{xi}}{e_{xi}}\mathrm{d}e_{xi} + \frac{p_{xi}}{q_e}(\mathrm{d}\pi_e + \mathrm{d}t_y), \tag{C20}$$

which can be rearranged with the help of (C17) to

$$\mathrm{d}p_{xi} = \frac{(1-a_i)p_{xi}\rho_i}{e_{xi}}\Delta e_{yi}\mathrm{d}\pi_e + \mu_i(\mathrm{d}\pi_e + \mathrm{d}t_y), \tag{C21}$$

where $\mu_i := \rho_i p_{xi}\left[\frac{\bar{h}_i q_e + a_i(p_e+\pi_e)}{a_i q_e} - \frac{(1-a_i)}{e_{xi}}\right]$. $\mathrm{d}x_i$, $\mathrm{d}y_i$, $\mathrm{d}z_i$ and $\mathrm{d}u_i$ have been calculated in (B30), (B31), (C14) and (B33). We make use of (C13), (C17) and (C18) to transform (B30), (B31), (C14) and (B33) into

$$\begin{aligned}
\mathrm{d}x_i &= \frac{q_e\rho_i}{p_{xi}}\Delta e_{yi}\mathrm{d}\pi_e - (\mathrm{d}\pi_e + \mathrm{d}t_y)\frac{q_e\rho_i}{p_{xi}}\left[\frac{\bar{h}_i e_{xi}}{a_i} + \frac{t_y e_{yi}}{(1-b_i)q_e}\right], && \text{(C22)}\\
\mathrm{d}y_i &= -(p_e+\pi_e)\rho_i\Delta e_{yi}\mathrm{d}\pi_e - (\mathrm{d}\pi_e + \mathrm{d}t_y)\rho_i\left[\frac{h_i e_{xi}}{a_i} + \frac{t_y}{(1-b_i)q_e}\left(e_{yi} + \frac{1}{\rho_i}\right)\right], && \text{(C23)}\\
\mathrm{d}z_i &= -\frac{(p_e+\pi_e)\rho_i\Delta e_{yi}}{(1-h_i)}\mathrm{d}\pi_e - \frac{(\mathrm{d}\pi_e + \mathrm{d}t_y)\rho_i}{(1-h_i)}\left[\frac{h_i e_{xi}}{a_i} + \frac{t_y}{(1-b_i)q_e}\left(e_{yi} + \frac{1}{\rho_i}\right)\right], && \text{(C24)}\\
\frac{\mathrm{d}u_i}{\lambda_i} &= -t_y\sigma_i\rho_i(\mathrm{d}\pi_e + \mathrm{d}t_y) + (t_y\rho_i + 1)\Delta e_{yi}\mathrm{d}\pi_e. && \text{(C25)}
\end{aligned}$$

The signs in Table 4 follow from setting $\mathrm{d}\pi_e = -\mathrm{d}t_y$ in (C13), (C17), (C21)-(C25).

Appendix D: Numerical example (only for the referees)

In this Appendix C we show how we solved the equilibrium equation system in order to simulate the graphs. Remember that for the special functional forms the multi-country equilibrium is determined by (C1)-(C6). In the following, we transform the equations (C1)-(C6). First, we eliminate the variables p_{xi} and z_i through substitution. Invoking $z_i = \frac{e_x^{a_i} p_{xi}}{\sigma_i}$ from (C2) in (C3) we get

$$p_{xi} e_{xi}^{a_i} \frac{(1-\sigma_i)}{\sigma_i} = e_{yi}^{b_i} - (q_e - t_y)(e_{xi} + e_{yi}) + \pi_e c_i. \tag{D1}$$

Next, we invoke $p_{xi} = \frac{q_e}{a_i e_{xi}^{a_i - 1}}$, which follows from (C4) and (C5), in (D1) to obtain

$$\begin{aligned} e_{xi}\left[q_e \frac{(1-\sigma_i)}{a_i\sigma_i} + (q_e - t_y)\right] &= e_{yi}^{b_i} - (q_e - t_y)e_{yi} + \pi_e c_i \\ \Longleftrightarrow e_{xi} + e_{yi} &= \frac{e_{yi}^{b_i} - (q_e - t_y)e_{yi} + \pi_e c_i}{q_e \frac{(1-\sigma_i)}{a_i\sigma_i} + (q_e - t_y)} + e_{yi} \\ &= \frac{e_{yi}^{b_i} + \frac{q_e(1-\sigma_i)}{a_i\sigma_i} e_{yi} + \pi_e . c_i}{q_e \frac{(1-\sigma_i)}{a_i\sigma_i} + (q_e - t_y)} \end{aligned} \tag{D2}$$

The equations (C1), (C5) and (D2) now determine the equilibrium values of e_{xi}, e_{yi} for all i and π_e. Next, we insert $e_{yi} = \left(\frac{q_e}{b_i}\right)^{\frac{1}{b_i - 1}}$ from (C5) in (D2) which yields.

$$e_{xi} + e_{yi} = \frac{\left(\frac{q_e}{b_i}\right)^{\frac{b_i}{b_i-1}} + \frac{q_e(1-\sigma_i)}{a_i\sigma_i}\left(\frac{q_e}{b_i}\right)^{\frac{1}{b_i-1}} + \pi_e c_i}{q_e \frac{(1-\sigma_i)}{a_i\sigma_i} + (q_e - t_y)}, \tag{D3}$$

which in turn is inserted into (C1) to obtain

$$\sum_i \frac{\left(\frac{q_e}{b_i}\right)^{\frac{b_i}{1-b_i}} + \frac{q_e(1-\sigma_i)}{a_i\sigma_i}\left(\frac{q_e}{b_i}\right)^{\frac{1}{1-b_i}} + \pi_e c_i}{q_e \frac{(1-\sigma_i)}{a_i\sigma_i} + (q_e - t_y)} = \sum_i c_i. \tag{D4}$$

Equation (D4) implicitly determines π_e as a function of t_y, formally $\pi_e = \pi_e(t_y)$. Then we can compute $e_{yi}(t_y)$ from (C5), $e_{xi}(t_y)$ from (C4), $\Delta e_{yi}(t_y)$ from (D3) , $x_i(t_y)$ from $x_i = X^i(e_{xi})$, $y_i(t_y)$ from (C6), $z_i(t_y)$ from (C7) and $u_i(t_y)$ from $u_i = x_i^{h_i} y_i^{1-h_i}$.

The missing graphs of the numerical example:

	$i=1$		$i=2$		$i=3$	
	$t_y=0$	$\pi_e=0$ $\bar{t}_y$	$t_y=0$	$\pi_e=0$ $\bar{t}_y$	$t_y=0$	$\pi_e=0$ $\bar{t}_y$
p_{xi}	0.811	0.811	0.590	0.771	0.149	0.970
e_{xi}	0.149	0.150	0.334	0.409	0.664	0.572
e_{yi}	0.452	0.458	0.452	0.458	0.452	0.458
Δe_{yi}	0.003	-0.003	-0.186	-0.267	0.318	0.270
x_i	0.683	0.684	0.518	0.585	0.692	0.605
y_i	0.554	0.555	0.414	0.504	0.549	0.470
z_i	1.108	1.110	0.828	1.009	1.097	0.939
u_i	0.615	0.533	1.097	0.543	0.616	0.811

Table 6: Numerical values for $t_y = 0$, $\pi_e = 0$

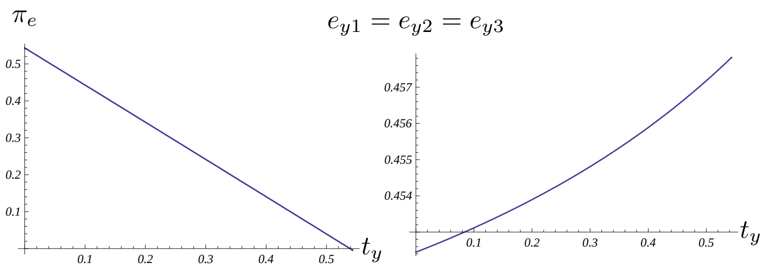

Figure 4: The permit price, the emissions in sector Y

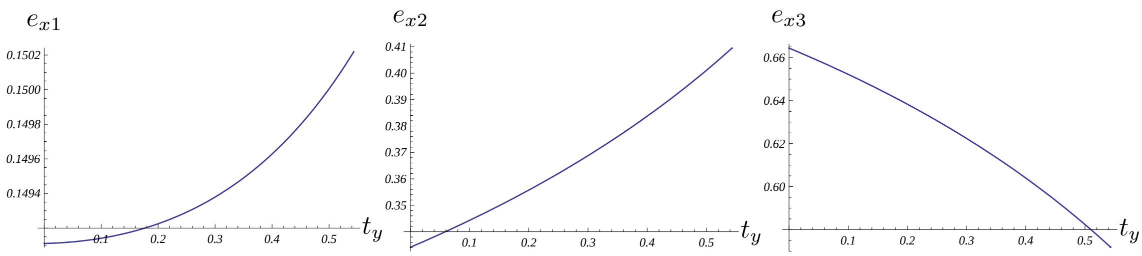

Figure 5: The emissions in sector X

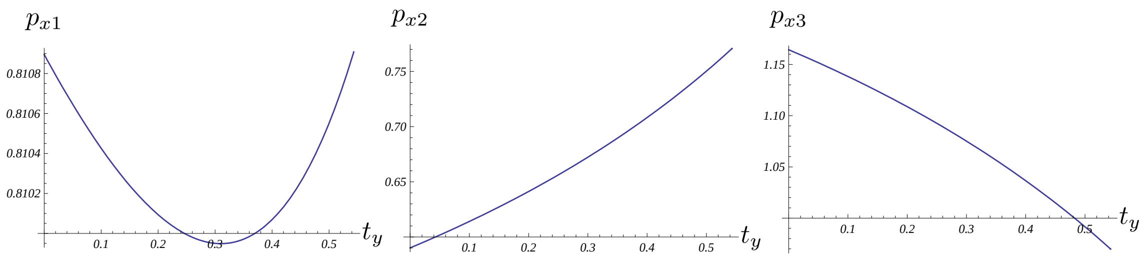

Figure 6: The domestic price of good X

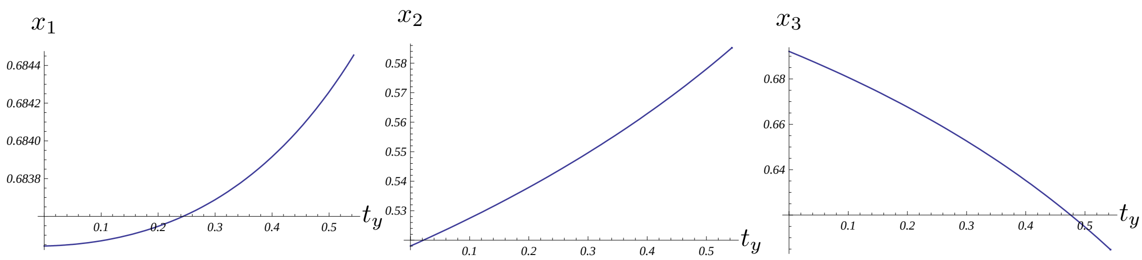

Figure 7: The consumption of good X

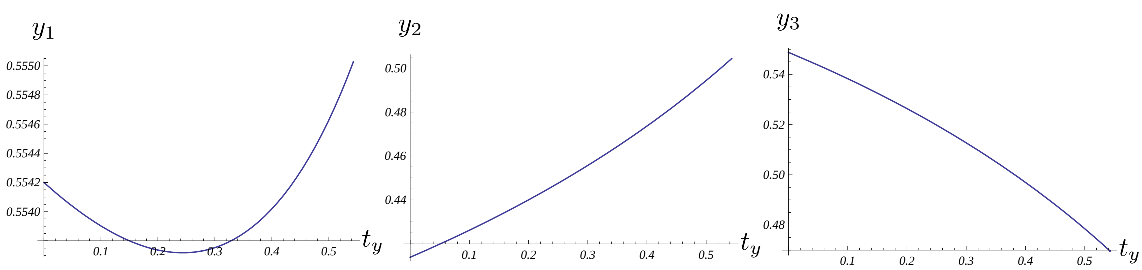

Figure 8: The consumption of good Y