International trade, factor shares in production and factor prices*

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Abstract

This paper presents a trade model with capital and labor as factors of production. The main contribution of this paper is that it considers a new type of firm heterogeneity, which is empirically relevant: firms in this paper differ with respect to their factor shares in production. Therefore, this paper addresses the following four empirical facts on globalization, firms' factor shares and factor prices: (*i*) firms within narrowly defined industries exhibit a large degree of heterogeneity in factor shares in production; (*ii*) exporters are, on average, more capital intensive than non–exporters; (*iii*) globalization decreases labor's share in national income; (*iv*) the larger the share of exporters in the industry, the larger the increase in the industry's wages due to globalization.

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1 Introduction

Recent empirical research has documented several empirical facts on firms' factor shares and how globalization affects the firms' factor shares and factor prices.

First, firms within narrowly defined industries exhibit a large degree of heterogeneity with respect to their factor shares in production (Leonardi, 2007; Alvarez and Lopez, 2005; Bernard and Jensen, 1999). Second, the exporting firms are, on average, more capital intensive than the non–exporting firms (Alvarez and Lopez, 2005; Bernard and Jensen, 1999). Third, globalization decreases labor's share in national income (Guscina, 2006; Harrison, 2002). Fourth, the larger the share of exporters in the industry, the larger the increase in the industry's wages due to globalization (Amiti and Davis, 2008).[1]

This paper presents an intra–industry trade model, which addresses these empirical facts. Firms in this model produce with two factors of production, capital and labor. Most crucially, firms in this model differ with respect to their factor shares in production, *not* with respect to total factor productivity. Furthermore, it is assumed that the countries' capital stocks are determined endogenously in terms of the Ramsey growth model.[2]

It is shown that, under quite general parameter restrictions, factor prices differ such that only the more capital intensive firms start to export after trade liberalization. The empirical relevance of these parameter restrictions is also discussed. Furthermore, it is shown that trade liberalization leads to an increase in the industry's average capital intensity in production, and, at the same time, to an increase in wages relative to capital returns. The increase in wages is larger, the larger the share of exporters in the industry.

Therefore, the main contribution of this paper is twofold. From an empirical point of view, this paper presents a theoretical setup, which can explain several empirical facts on globalization, factor shares and factor prices. From a technical point of view, this paper shows how firm heterogeneity with respect to factor shares in production can be included into an analytically tractable general equilibrium model.

The most crucial difference between this paper and the previous literature on trade with firm heterogeneity (e.g., Melitz, 2003; Bernard et al., 2003; Bernard et al., 2007; Baldwin and Robert–Nicoud, 2008; Melitz and Ottaviano, 2008) is the different definition of firm heterogeneity. All previous papers define firm heterogeneity with respect to total factor

[1] Note that the empirical study of Amiti and Davis (2007) is the first one which shows that the effect of globalization on the industry's wages crucially depends on whether the industry is export oriented or not.

[2] It is shown in the main part that the extension to a Ramsey growth setup is necessary for two reasons. First, it leads to an unambiguous relationship between the firms' factor shares and relative factor prices in equilibrium. Second and more importantly, the extension to a Ramsey growth setup leaves the model analytically tractable.

productivity and explain quite successfully why trade liberalization leads to a firm selection in favor of the more productive firms. However, just by construction, these previous papers do not focus on those empirical facts on globalization, which consider factor shares and factor prices.

One exception in the previous literature, and therefore most complementary to the present paper, is the work by Bernard et al. (2007). They combine a Melitz (2003)–setup with a Heckscher–Ohlin trade model, i.e. they assume that countries produce two goods with two factors of production. Within sectors firms behave monopolistically competitively. Furthermore, within sectors firms are *hetero*geneous with respect to total factor productivity, but *homo*geneous with respect to factor shares.

The paper by Bernard et al. (2007) provides important insights into the inter–industry and intra–industry factor relocations due to trade liberalization. By construction, the paper by Bernard et al. (2007) does not analyze how heterogeneity in factor shares interacts with globalization.

Since the present paper does not consider endowment–based comparative advantage like Bernard et al. (2007), it cannot address inter–industry relocations with trade liberalization. However, given certain parameter restrictions, the results in the present paper match all empirical facts on the interaction between heterogeneity in factor shares and globalization. Most importantly, the present paper also supports heterogeneity in factor shares *among* the exporting firms and a higher capital share of exporting firms, compared to non–exporting firms of the *same* sector.

Several assumptions of the present model are adopted from the previous literature on trade with firm heterogeneity. Intra–industry trade in the present model results from Dixit–Stiglitz preferences by households. Since households accordingly regard the varieties of different firms as imperfect substitutes, large–group monopolistic competition between firms results. Furthermore, firm heterogeneity results from the following assumption: firms are identical before market entry; only after firms entered the market and paid sunk market entry costs, they draw their technology parameter (which is the factor share parameter in the present model) from a given probability distribution.

In order to leave the model analytically tractable, several assumptions are made. The most important are the following. First, it is assumed that countries are perfectly symmetric, i.e. preference and technology parameters are identical in both countries. Second, the probability distribution, from which households draw their technology parameter after market entry, is uniform and has an identical support in both countries. Third, it is assumed that trade liberalization is bilateral and symmetric, i.e. both countries reduce their import tariffs simultaneously and by the same amount.

Finally, this paper only compares the autarky steady state with the free trade steady state. The *adjustment path* from the autarky steady state to the free trade steady state is not analyzed.

The structure of the paper is as follows. Section 2 describes the ingredients into the general equilibrium model: subsection 2.1 gives an overview of all central assumption; subsection 2.2 describes the countries' production side and subsection 2.3 the countries' demand side; subsection 2.4 describes the dynamic setup, which is based on the Ramsey growth model; subsection 2.5 describes the market entry and the firms' supply decision to the domestic market. Section 3 derives the general steady state equilibrium for the closed economy. Section 4 derives the general steady state equilibrium for the open economy and describes the reaction of all variables to trade liberalization. Section 5 illustrates the effects of globalization with the help of a numerical example. Section 6 concludes. All proofs are relegated to the appendix.

2 The model

2.1 Overview

This paper analyzes intra–industry trade between the home country H and the foreign country F. The general setup of this model draws on the previous heterogeneous firms trade literature (e.g., Melitz, 2003; Bernard et al., 2003; Bernard et al., 2007; Baldwin and Robert–Nicoud, 2008; Melitz and Ottaviano, 2008). The following assumptions on households, firm behavior, countries and market entry are adopted from this literature.

Households in each country consume a continuum of imperfectly substitutable varieties of a differentiated good Q. Like in Dixit and Stiglitz (1977), these varieties of good Q are aggregated via a CES function to give the households' utility. Firm behavior can therefore be described by large–group monopolistic competition, i.e. each firm regards the prices of all other varieties and factor prices as given.

The production side of each country only consists of this single monopolistically competitive industry. Therefore, the "industry–wide" consequences of trade liberalization are identical with the "country–wide" consequences of trade liberalization.

Firms do not know their technologies before market entry. Only after a firm has paid sunk market entry costs, it randomly draws its technology parameter from an exogenous probability distribution. Production leads to variable costs and fixed costs in each period. The combination of sunk market entry costs and the random draw of the technology parameter after market entry gives rise to firm heterogeneity with respect to technologies in equilibrium. Due to fixed production costs firms only start with production after market entry if the technology parameter they have drawn allows them to produce with sufficiently low marginal costs.

The following two features distinguish the present paper from the previous literature on trade with firm heterogeneity:

(i) Countries H and F are endowed with *two* factors of production, labor L and capital K. Firms produce their output with a CES production function and are heterogeneous with respect to the factor share parameters of the CES production function.[3] In order to concentrate on firm heterogeneity in factor shares, firms are *homo*geneous with respect to total factor productivity.

(ii) Each country's capital endowment is determined endogenously via the Ramsey growth model and therefore flexible in the long–run. Each country's labor endowment, in contrast, is constant over time.

In order to get straightforward analytical results, four assumptions are made.

First, countries H and F are symmetric in every respect. Second, the distribution from which firms randomly draw their factor share parameter after market entry is identical for both countries and uniform. Third, trade liberalization is bilateral and symmetric. Fourth, the households' utility function and the firms' production functions have an identical elasticity of substitution. Fifth, labor and capital are perfectly mobile within a country, but perfectly immobile between countries.

2.2 Production

A single firm, which is characterized by the factor share parameter ϕ, produces its unique variety of good Q with the following CES production function:[4]

$$q(\phi) = \left(\phi^{1-\alpha} \cdot l^{\alpha} + (1-\phi)^{1-\alpha} \cdot k^{\alpha}\right)^{1/\alpha}, \quad 0 \leq \phi \leq 1, \quad 0 < \alpha < 1, \tag{1}$$

where l and k denote the input of labor and capital. In this paper, firms differ with respect to ϕ.

If the prices for labor and capital are not identical, firms have different marginal production costs. It will be argued in subsection 2.5 that firms only start with production after

[3] The present setup accordingly assumes that the firms' *marginal* costs have different factor shares, while the firms' *fixed* costs have identical factor shares. However, it is straightforward to extent the present setup to heterogeneity with respect to the factor shares of fixed costs. For example, the factor shares of marginal and fixed costs could be identical for each single firm. The qualitative implications of the model would not change since the 'degree' of firm heterogeneity would just increase with this extension.

[4] Since each country's capital stock is determined endogenously via the Ramsey growth model, time matters. However, the time index t is included only when necessary.

market entry if they have drawn a ϕ from the interval $[0, \phi^*]$, with $0 < \phi^* \leq 1$. The boundary ϕ^* is endogenous and its general equilibrium value is derived in section 3 for the closed economy and in section 4 for the open economy.

The production function in equation (1) leads to the following marginal cost function:

$$c(\phi) = \left(\phi \cdot w^{1-\sigma} + (1-\phi) \cdot r^{1-\sigma}\right)^{1/(1-\sigma)}, \quad \sigma = \frac{1}{1-\alpha} > 1, \tag{2}$$

with w and r denoting the price per unit labor and capital. The parameter σ stands for the elasticity of substitution between labor and capital. If $w \neq r$, the labor share parameter ϕ influences the firms' marginal costs.

Production leads to a fixed input requirement f in each period. It is assumed that f is produced with the average technology over all active firms; this average technology is characterized by the average factor share parameters $\widetilde{\phi}$ and $1 - \widetilde{\phi}$.[5] Under the assumption of a uniform distribution of ϕ on the unit interval, $\widetilde{\phi}$ results as follows:

$$\widetilde{\phi} = \int_0^{\phi^*} \phi \cdot \mu(\phi)\, d\phi = \frac{\phi^*}{2}, \tag{3}$$

where $\mu(\phi) = \frac{1}{\phi^*}$ denotes the conditional probability density for ϕ on the subinterval $[0, \phi^*]$ of the unit interval. Total production costs TC of a single firm with labor share parameter ϕ are therefore given by:

$$TC = c(\phi) \cdot q(\phi) + c\left(\widetilde{\phi}\right) \cdot f. \tag{4}$$

2.3 Demand

The preferences of households are described by a utility function like in Dixit and Stiglitz (1977). More specifically, households aggregate the continuum of varieties of good Q according to a CES function to give the aggregate consumption good Q. Since the varieties are indexed by the labor share parameter ϕ, the aggregate consumption good Q results as:

$$Q = \left(\int_0^{\phi^*} q(\phi)^{(\sigma-1)/\sigma} \cdot \mu(\phi) \cdot N d\phi\right)^{\sigma/(\sigma-1)}, \quad \sigma > 1. \tag{5}$$

N equals to total mass of heterogeneous firms, which are distributed on the interval $[0, \phi^*]$ according to the density $\mu(\phi)$. The elasticity of substitution in consumption, which is given by σ, is assumed to be identical with the elasticity of substitution in production.

The price index P, which is dual to the CES function in equation (5) is given by:

$$P = \left(\int_0^{\phi^*} p(\phi)^{1-\sigma} \cdot \mu(\phi) \cdot N d\phi\right)^{1/(1-\sigma)}. \tag{6}$$

[5] The results of the model would not change in a qualitative sense if, alternatively, f were produced only with labor or only with capital.

Applying Shephard's Lemma, total demand for a variety with labor share parameter ϕ can be derived as:

$$q(\phi) = M_{cons} \cdot P^{\sigma-1} \cdot p(\phi)^{-\sigma}. \tag{7}$$

M_{cons} denotes that part of total factor income which households use for consumption, i.e. $M_{cons} = P \cdot Q$.[6] Aggregate expenditures for a variety with labor share parameter ϕ are accordingly given by:

$$p(\phi) \cdot q(\phi) = M_{cons} \cdot P^{\sigma-1} \cdot p(\phi)^{1-\sigma}. \tag{8}$$

Profit maximizing firms equalize marginal revenue with marginal costs of production and therefore apply the following pricing rule:

$$p(\phi) = \frac{\sigma}{\sigma-1} \cdot c(\phi) = \frac{\sigma}{\sigma-1} \cdot \left(\phi \cdot w^{1-\sigma} + (1-\phi) \cdot r^{1-\sigma}\right)^{1/(1-\sigma)}. \tag{9}$$

Substituting equation (9) into equation (6) leads to the following expression for the price index P:

$$P = \left(\int_0^{\phi^*} \frac{\sigma}{\sigma-1} \cdot \left(\phi \cdot w^{1-\sigma} + (1-\phi) \cdot r^{1-\sigma}\right) \cdot \mu(\phi) \cdot N d\phi\right)^{1/(1-\sigma)} = \left(N \cdot p\left(\widetilde{\phi}\right)^{1-\sigma}\right)^{1/(1-\sigma)}. \tag{10}$$

Furthermore, substituting equation (9) into equation (7) leads to the following demand for a single variety $q(\phi)$:

$$q(\phi) = M_{cons} \cdot P^{\sigma-1} \cdot \left(\frac{\sigma}{\sigma-1} \cdot c(\phi)\right)^{-\sigma}. \tag{11}$$

Finally, substituting equation (11) into equation (5) leads to the following expression for the aggregate consumption good Q:

$$\begin{aligned} Q &= \left(M^{(\sigma-1)/\sigma} \cdot P^{(\sigma-1)\cdot(\sigma-1)/\sigma} \cdot \int_0^{\phi^*} \frac{\sigma}{\sigma-1} \cdot \left(\phi \cdot w^{1-\sigma} + (1-\phi) \cdot r^{1-\sigma}\right) \cdot \mu(\phi) \cdot N d\phi\right)^{\sigma/(\sigma-1)} \\ &= \left(M^{(\sigma-1)/\sigma} \cdot P^{(\sigma-1)\cdot(\sigma-1)/\sigma} \cdot p\left(\widetilde{\phi}\right)^{1-\sigma} \cdot N\right)^{\sigma/(\sigma-1)}. \end{aligned} \tag{12}$$

Comparing equation (12) with equation (5) implies that the following equality holds:

$$Q = \left(\int_0^{\phi^*} q(\phi)^{(\sigma-1)/\sigma} \cdot \mu(\phi) \cdot N d\phi\right)^{\sigma/(\sigma-1)} = \left(q\left(\widetilde{\phi}\right)^{(\sigma-1)/\sigma} \cdot N\right)^{\sigma/(\sigma-1)}. \tag{13}$$

Equation (13) shows that it is immaterial for the households whether they consume the N *hetero*geneous varieties or whether they consume N *homo*geneous varieties, which are produced with the average labor share parameter $\widetilde{\phi} = \frac{\phi^*}{2}$.

[6]Note that the country's capital stock is determined endogenously via the Ramsey growth model, i.e. if M denotes *total* factor income, only part of it is used for consumption. The remainder is used for investment purposes. Subsection 2.4 describes the dynamic setup.

2.4 Dynamic setup

This paper assumes endogenous capital accumulation and intertemporal optimizing behavior of households in terms of the Ramsey growth model.

Households use the variety $q\left(\widetilde{\phi}\right)$ for investment purposes and choose their consumption and investment level each period such that lifetime utility V is maximized.[7] ρ denotes the time discount rate and u the instantaneous utility function. Including the time index t, lifetime utility of the representative household is then given by:

$$V = \sum_{t=0}^{\infty} \frac{u\left(Q_t\right)}{(1+\rho)^t}, \tag{14}$$

where Q_t is the aggregate consumption good as defined by equation (13).

Each country's labor endowment is assumed to be constant over time. Investments therefore only compensate for depreciation of capital. If δ denotes the capital depreciation rate, investments in the steady state are given by:

$$I_t = K_{t+1} - (1-\delta) \cdot K_t. \tag{15}$$

I_t denotes the amount of variety $q\left(\widetilde{\phi}\right)$ which is invested in period t, K_t and K_{t+1} denote the country's capital stocks in t and $t+1$.

Households own the production factors and lend them out to firms for production purposes. Given that households behave perfectly competitively, Baxter (1992) shows that the steady state of a Ramsey growth setup is determined by several necessary first order conditions.[8] Four of them already determine the factor price ratio $\frac{w}{r}$ in the steady state as a function of the parameters ρ, δ and σ and the endogenous average labor share parameter $\widetilde{\phi}$.[9] This is summarized by result 1.

Result 1: The factor price ratio $\frac{w}{r}$ in the steady state is determined by the parameters ρ, δ and σ and the endogenous average labor share parameter $\widetilde{\phi}$:

$$\frac{w}{r} = \left(\frac{\widetilde{\phi} \cdot (\rho+\delta)^{1-\sigma}}{(1-1/\sigma)^{1-\sigma} - (1-\widetilde{\phi}) \cdot (\rho+\delta)^{1-\sigma}}\right)^{1/(\sigma-1)}. \tag{16}$$

[7] Total demand for variety $q\left(\widetilde{\phi}\right)$ is accordingly given by $p\left(\widetilde{\phi}\right)^{-\sigma} \cdot P^{\sigma-1} \cdot M_{cons} + I$, where I denotes demand for investment purposes. Due to the *uniform* distribution of ϕ on the unit interval, the model remains analytically solvable, even if any other variety $q(\phi)$ with $\phi \neq \widetilde{\phi}$ is used for investments. However, the model is most straightforward to solve if variety $q\left(\widetilde{\phi}\right)$ is used for investments. The reason is that the relationship between $\widetilde{\phi}$ and ϕ^* is given by $\widetilde{\phi} = \frac{\phi^*}{2}$ (equation (3)); therefore, as it will be shown in subsections 3.2 and 4.1, the amount of $q\left(\widetilde{\phi}\right)$, which is produced in the steady state, is a relatively simple function of parameters.

[8] Baxter (1992), p. 738.

[9] Note that $\widetilde{\phi}$ depends on the upper bound ϕ^* of the interval $[0, \phi^*]$ (equation (3)). The general equilibrium value for ϕ^* will be derived in section 3 for the closed economy and in section 4 for the open economy.

Proof: see appendix A. □

The model remains analytically tractable due to result 1. In a *static* model, in contrast, $\frac{w}{r}$ has to be determined with the factor market equilibrium conditions, which are non–linear in $\frac{w}{r}$ in the case of CES production functions.

Result 1 has the following implications:

(*i*) $\frac{w}{r}$ is strictly larger than zero only if $\left(1-\frac{1}{\sigma}\right)^{1-\sigma} > \left(1-\widetilde{\phi}\right)\cdot(\rho+\delta)^{1-\sigma}$. Since $\sigma > 1$ and $0 < 1-\widetilde{\phi} \leq 0.5$, this condition definitely holds if $\frac{1-1/\sigma}{\rho+\delta} < 2^{1/(\sigma-1)}$. In order to avoid an equilibrium with a factor price equal to zero, the further analysis is restricted to parameter values which are in line with $\frac{1-1/\sigma}{\rho+\delta} < 2^{1/(\sigma-1)}$.

(*ii*) $\frac{w}{r}$ is *larger* than unity if $1 < \frac{1-1/\sigma}{\rho+\delta}$ and $\frac{w}{r}$ is *smaller* than unity if $1 > \frac{1-1/\sigma}{\rho+\delta}$. Empirical estimates for ρ, δ and σ do not contradict with $1 < \frac{1-1/\sigma}{\rho+\delta}$, i.e. with the case of $\frac{w}{r} > 1$:

Empirical estimates for σ are in the range from 1.5 to 10 (Johnson, 1997; Bilbiie et al., 2007; Broda and Weinstein, 2007). Since time–to–build has been estimated to be between 4 and 8 quarters (Koeva, 2000), a period t (see equation (15)) refers to a period of 4 to 8 quarters. Empirical estimates for ρ and δ are both around 0.1 (0.2) if t equals 4 (8) quarters (Backus et al., 1992; Aguiar and Gopinath, 2007).

The case of $\frac{w}{r} > 1$ finds additional empirical support since firms, which produce more capital intensively, are found to have higher growth rates, smaller failure rates (Doms et al., 1995) and higher profits (Caves, 1998). Therefore, the case of $\frac{w}{r} > 1$ also leads to a firm selection into export markets, which is documented by empirical research (see subsection 4.2).

However, note that $\frac{w}{r} > 1$ does *not* necessarily imply that the analyzed countries are *relatively* capital abundant. Since countries H and F are not compared with any third country with a different factor price ratio, countries H and F can either be relatively capital abundant or relatively labor abundant.

Due to both implications, assumption 1 is assumed to hold for the further analysis:

Assumption 1: $1 < \frac{1-1/\sigma}{\rho+\delta} < 2^{1/(\sigma-1)}$.

If the first part of assumption 1 did not hold, i.e. if $1 > \frac{1-1/\sigma}{\rho+\delta}$, the factor price ratio $\frac{w}{r}$ would be *smaller* than unity and all main results of this paper would be, in a qualitative sense, the mirror image. If the second part of assumption 1 did not hold, i.e. if $\frac{1-1/\sigma}{\rho+\delta} < 2^{1/(\sigma-1)}$, the equilibrium $\widetilde{\phi}$ would be zero (see subsection 3.2) and no firm would demand labor. Figure *A*1 in the appendix illustrates the parameter restrictions imposed by assumption 1.

Assumption 1 has the following additional implications:

(i) The per unit costs $c(\phi)$ and the price $p(\phi) = \frac{\sigma}{\sigma-1} \cdot c(\phi)$ *in*crease with an increase in the labor share parameter ϕ:

$$\frac{\partial c(\phi)}{\partial \phi} = \frac{1}{1-\sigma} \cdot c(\phi)^{\sigma} \cdot w^{1-\sigma} \cdot \left(1 - \left(\frac{w}{r}\right)^{\sigma-1}\right) > 0 \quad \text{since} \quad \sigma > 1 \quad \text{and} \quad \frac{w}{r} > 1. \tag{17}$$

Revenue $q(\phi) \cdot p(\phi)$ accordingly *in*creases with a decrease in ϕ:

$$\frac{\partial \Big(q(\phi) \cdot p(\phi)\Big)}{\partial \phi} = M_{cons} \cdot P^{\sigma-1} \cdot (1-\sigma) \cdot p(\phi)^{-\sigma} \cdot \frac{\partial p(\phi)}{\partial \phi} < 0. \tag{18}$$

(ii) The steady state factor price ratio $\frac{w}{r}$ increases with a decrease in $\widetilde{\phi}$ (see appendix B for the proof).

Finally, result 1 can be used to formulate result 2:

Result 2: The ratio of the marginal costs of any two firms with labor share parameters ϕ_1 and ϕ_2 results as:

$$\frac{c(\phi_1)}{c(\phi_2)} = \left(\frac{\frac{\phi_1}{\widetilde{\phi}} \cdot \left[(1-1/\sigma)^{1-\sigma} - (\rho+\delta)^{1-\sigma}\right] + (\rho+\delta)^{1-\sigma}}{\frac{\phi_2}{\widetilde{\phi}} \cdot \left[(1-1/\sigma)^{1-\sigma} - (\rho+\delta)^{1-\sigma}\right] + (\rho+\delta)^{1-\sigma}} \right)^{1/(1-\sigma)}. \tag{19}$$

Proof: see appendix C. □

Note that the ratio $\frac{c(\phi_1)}{c(\phi_2)}$ is positive due to assumption 1. Since both ϕ_1 and ϕ_2 are equal or smaller than ϕ^*, the ratios $\frac{\phi_1}{\widetilde{\phi}}$ and $\frac{\phi_2}{\widetilde{\phi}}$ are equal or smaller than 2 (see equation (3)). Therefore, both the numerator and the denominator on the right–hand side of equation (19) are positive due to $\frac{1-1/\sigma}{\rho+\delta} < 2^{1/(\sigma-1)}$.

2.5 Market entry and supply decision to the domestic market

An infinitely dividable number of potential entrants into the market exists. By the time of market entry the entrants do not know their labor share parameter ϕ yet, i.e. firms are completely identical by the time of market entry.[10] The market entry procedure can be divided into three steps.

First, entering the market leads to a sunk input requirement f_E, which is also produced with the average technology over all active firms.[11] The sunk market entry costs are therefore given by $c\left(\widetilde{\phi}\right) \cdot f_E$.

[10] Like in Melitz (2003) and Bernard et al. (2003), the firms' ex–ante uncertainty about their technology parameter and, hence, their per unit costs $c(\phi)$ reflects the firms' ex–ante uncertainty about, e.g., the skills of their workers.

[11] Again, the results of the model would not change in a qualitative sense if, alternatively, f_E were produced only with labor or only with capital.

Second, after market entry firms randomly draw their labor share parameter ϕ from the interval $[0,1]$ according to a uniform distribution with density $g(\phi) = 1$. The variable costs of the firm are then given by $c(\phi)$. Each firm keeps the drawn ϕ for the rest of its life.

Third, after the draw of ϕ, the firm decides whether it actually starts with production or not. Since fixed production costs $c\left(\widetilde{\phi}\right) \cdot f$ occur in each period in which the firm produces, it does not necessarily start with production after market entry. Due to $\frac{w}{r} > 1$ in the steady state, a firm actually starts with production only if its individual ϕ is equal or smaller than the threshold value ϕ^*. At ϕ^*, a firm's profits $\pi(\phi^*)$ in each period are exactly equal to zero, i.e. the following holds:

$$\pi(\phi^*) \;=\; \Big(p(\phi^*) - c(\phi^*)\Big) \cdot q(\phi^*) - c\left(\widetilde{\phi}\right) \cdot f \;=\; \frac{q(\phi^*) \cdot p(\phi^*)}{\sigma} - c\left(\widetilde{\phi}\right) \cdot f \;=\; 0. \quad (20)$$

Equation (20) will be called *zero cutoff profit* condition in the following. A firm which has drawn a labor share parameter $\phi < \phi^*$ earns positive profits $\pi(\phi) > 0$ since revenues $p(\phi) \cdot q(\phi)$ *in*creases with a decrease in the labor share parameter ϕ (equation (18)). A firm which has drawn a $\phi > \phi^*$ immediately exits the market and does not start with production since it would otherwise earn negative profits each period.

In each period, a firm may be hit by a negative technology shock with probability θ, $0 < \theta < 1$. If a firm is hit by such a shock, it has to exit the market. The negative shock guarantees that a constant amount of sunk market entry costs arises in each period of the steady state.

3 General steady state equilibrium — closed economy

In the case of two *closed* economies, the general steady state equilibrium can be derived for each economy separately. The general steady state equilibrium for either country is characterized by the following 5 conditions, which have to hold in each period of the steady state:

1. the factor price ratio in the steady state (equation (16))
2. production of each variety has to equal demand for each variety (equation (7)) at price $p(\phi) = \frac{\sigma}{\sigma-1} \cdot c(\phi)$ (equation (9))
3. the zero cutoff profit condition (equation (20))
4. a free entry condition, i.e. expected lifetime profits from market entry are equal to the sunk market entry costs
5. the factor market equilibrium conditions.

These 5 conditions can be used to solve for the following 5 variables in the steady state: the threshold labor share parameter ϕ^* and the average labor share parameter $\widetilde{\phi} = \frac{\phi^*}{2}$; supply $q\left(\widetilde{\phi}\right)$ of the firm which produces with $\widetilde{\phi}$; the factor price ratio $\frac{w}{r}$; the country's capital stock K and investment I; the mass of firms N.

Subsection 3.1 derives the free entry condition. Subsection 3.2 uses the free entry condition and the zero cutoff profit condition to derive $\widetilde{\phi}$ and $q\left(\widetilde{\phi}\right)$ in the steady state. Subsection 3.3 finally derives the factor market equilibrium conditions and solves for K and N in the steady state.

3.1 Free entry condition

Firms enter the market only if they expect ex–ante, i.e. before market entry, that the sum of discounted lifetime profits at least covers the sunk market entry costs. Market entry stops if the following free entry condition holds:

$$G\left(\phi^*\right) \cdot \sum_{t=t'}^{\infty} E\left(\pi\left(\phi\right) \Big| \phi \leq \phi^*\right) \cdot \left(\frac{1-\theta}{1+\rho}\right)^t \quad = \quad c\left(\widetilde{\phi}\right) \cdot f_E. \tag{21}$$

The left hand side of equation (21) denotes the sum of expected discounted lifetime profits and the right hand side of equation (21) denotes the sunk market entry costs.

Period t' denotes an arbitrary period in which a firm enters. $G\left(\phi^*\right)$ stands for the probability that the entrant draws a $\phi \leq \phi^*$, i.e. $G\left(\phi^*\right)$ stands for the probability for a successful market entry. Due to the uniform distribution of ϕ on $[0, 1]$ it follows that $G\left(\phi^*\right) = \phi^*$. Furthermore, $E\left(\pi\left(\phi\right) \Big| \phi \leq \phi^*\right)$ denotes the ex–ante expected profits in a single period t in the steady state, given that the firm starts with production after market entry. The term $(1-\theta)^t$ accounts for the fact that the entrant will reach period t only with probability $(1-\theta)^t$. The term $(1+\rho)^{-t}$ discounts future profits to current period values.[12]

Firms ex–ante expect that a *successful* market entry will bring them the average profits over all active firms. Therefore,

$$E\left(\pi\left(\phi\right) \Big| \phi \leq \phi^*\right) \quad = \quad \frac{1}{\sigma} \cdot \int_0^{\phi^*} q\left(\phi\right) \cdot p\left(\phi\right) \cdot \frac{g(\phi)}{G(\phi^*)} d\phi - c\left(\widetilde{\phi}\right) \cdot f, \tag{22}$$

where $\frac{g(\phi)}{G(\phi^*)} \equiv \mu(\phi)$ denotes the conditional density of ϕ, given that $\phi \leq \phi^*$. The term $\int_0^{\phi^*} q\left(\phi\right) \cdot p\left(\phi\right) \cdot \mu(\phi) d\phi$ accordingly denotes average revenues over all active firms. Considering equations (8) and (9), average revenues over all active firms can be derived as follows:

$$\int_0^{\phi^*} q(\phi) \cdot p(\phi) \cdot \mu(\phi) d\phi \quad = \quad M_{cons} \cdot P^{\sigma-1} \cdot p\left(\widetilde{\phi}\right)^{1-\sigma} \quad = \quad q\left(\widetilde{\phi}\right) \cdot p\left(\widetilde{\phi}\right). \tag{23}$$

[12] Note that neither ϕ^*, $\pi\left(\phi\right)$ nor $\widetilde{\phi}$ get a time index t since the free entry condition denotes a relationship in the steady state. Due to a constant labor endowment over time, ϕ^*, $\pi\left(\phi\right)$ and $\widetilde{\phi}$ are identical in each period of the steady state.

Therefore, the average per period profits over all active firms result as:[13]

$$\int_0^{\phi^*} \pi(\phi) \cdot \mu(\phi) d\phi \quad = \quad \frac{p(\widetilde{\phi}) \cdot q(\widetilde{\phi})}{\sigma} - c\left(\widetilde{\phi}\right) \cdot f \quad = \quad \pi\left(\widetilde{\phi}\right), \quad \text{with } \widetilde{\phi} = \frac{\phi^*}{2}. \tag{24}$$

Finally, considering $G(\phi^*) = \phi^*$ and using the formula for an infinite geometric series, the free entry condition can be simplified as follows:

$$\frac{q\left(\widetilde{\phi}\right) \cdot p\left(\widetilde{\phi}\right)}{\sigma} - c\left(\widetilde{\phi}\right) \cdot f \quad = \quad c\left(\widetilde{\phi}\right) \cdot \frac{f_E}{\phi^*} \cdot \frac{\rho + \theta}{1 + \rho}. \tag{25}$$

3.2 $\widetilde{\phi}$ and $q\left(\widetilde{\phi}\right)$ in general equilibrium

The zero cutoff profit condition (equation (20)) and the free entry condition (equation (25)) can be used to solve for the steady state values of $\widetilde{\phi}$ and $q\left(\widetilde{\phi}\right)$.

Using the pricing rule $p(\phi) = \frac{\sigma}{\sigma-1} \cdot c(\phi)$, the zero cutoff profit condition and the free entry condition can be written as follows:

$$\text{zero cutoff profit condition:} \qquad \frac{q(\phi^*)}{\sigma - 1} \quad = \quad \frac{c(\widetilde{\phi})}{c(\phi^*)} \cdot f \tag{26}$$

$$\text{free entry condition:} \qquad \frac{q(\widetilde{\phi})}{\sigma - 1} - f \quad = \quad \frac{f_E}{\phi^*} \cdot \frac{\rho + \theta}{1 + \rho}. \tag{27}$$

Using equation (7), the ratio $\frac{q(\phi^*)}{q(\widetilde{\phi})}$ is given by $\frac{q(\phi^*)}{q(\widetilde{\phi})} = \left(\frac{c(\widetilde{\phi})}{c(\phi^*)}\right)^{\sigma}$, which can be transformed to:

$$q(\phi^*) \quad = \quad q\left(\widetilde{\phi}\right) \cdot \left(\frac{c(\widetilde{\phi})}{c(\phi^*)}\right)^{\sigma}. \tag{28}$$

Substituting equation (28) into equation (26) and considering equation (19) with $\phi_1 = \widetilde{\phi} = \frac{\phi^*}{2}$ and $\phi_2 = \phi^*$ leads to result 3:

Result 3: $q\left(\widetilde{\phi}\right)$ in the closed economy steady state is given as follows:

$$q\left(\widetilde{\phi}\right) \quad = \quad (\sigma - 1) \cdot \left(\frac{c(\widetilde{\phi})}{c(\phi^*)}\right)^{1-\sigma} \cdot f = \frac{(\sigma - 1) \cdot (1 - 1/\sigma)^{1-\sigma}}{2 \cdot (1 - 1/\sigma)^{1-\sigma} - (\rho + \delta)^{1-\sigma}} \cdot f. \tag{29}$$

Result 3 shows that $q\left(\widetilde{\phi}\right)$ in the steady state only depends on the parameters σ, ρ, δ and f; $q\left(\widetilde{\phi}\right)$ does not depend on the steady state value of $\widetilde{\phi}$. Furthermore, $q\left(\widetilde{\phi}\right)$ is positive due to assumption 1.

[13] Note that the equality $\int_0^{\phi^*} \pi(\phi) \cdot \mu(\phi) d\phi = \pi\left(\widetilde{\phi}\right)$ crucially depends on the fact that the factor share parameters ϕ and $1 - \phi$ are to the power of unity in the marginal cost function (equation (2)). Therefore, if the marginal cost function is taken to the power of $1 - \sigma$, the resulting expression is *linear* in the factor share parameters: $c(\phi)^{1-\sigma} = \phi \cdot w^{1-\sigma} + (1 - \phi) \cdot r^{1-\sigma}$.

Substituting $q\left(\widetilde{\phi}\right)$ into equation (27), the free entry condition can be solved for the average labor share parameter $\widetilde{\phi}=\frac{\phi^*}{2}$ in the closed economy steady state. This leads to result 4:

Result 4: The average labor share parameter $\widetilde{\phi}$ in the closed economy steady state is given by:

$$\widetilde{\phi} \;=\; \frac{0.5\cdot f_E\cdot\frac{\rho+\theta}{1+\rho}}{\frac{q(\widetilde{\phi})}{\sigma-1}-f} \;=\; \frac{\left[2\cdot(1-1/\sigma)^{1-\sigma}-(\rho+\delta)^{1-\sigma}\right]\cdot(\rho+\theta)\cdot 0.5\cdot f_E}{\left[(\rho+\delta)^{1-\sigma}-(1-1/\sigma)^{1-\sigma}\right]\cdot(1+\rho)\cdot f}. \quad (30)$$

Note that $\widetilde{\phi}$ is positive due to assumption 1.

Equation (30) shows that $\widetilde{\phi}$ increases if f_E increases, while it decreases if f increases. The intuition for this reaction of $\widetilde{\phi}$ to f_E and f is as follows:

(*i*) If f_E increases, market entry ceteris paribus becomes less attractive. Therefore, less firms enter the market and competition decreases. Due to less competition, even firms with a labor share parameter $\phi>\phi^*$ can earn positive profits each period. More labor intensive firms accordingly start with production, which increases the average labor share parameter $\widetilde{\phi}$.

(*ii*) If f increases, per period profits $\pi(\phi)$ decrease. At the initial threshold labor share parameter ϕ^* per period profits become negative and only firms with a smaller labor share parameter can survive. Therefore, market entry is successful only with a smaller labor share parameter, i.e. the average labor share parameter $\widetilde{\phi}$ decreases.

3.3 Factor market equilibrium conditions

Applying Shephard's Lemma to the marginal cost function (equation (2)) and considering demand $q(\phi)$ (equation (7)) leads to the following factor market equilibrium conditions; in order to simplify further derivations, the capital depreciation rate δ is normalized to zero and investments in the steady state are equal to zero as well:[14]

$$\int_0^{\phi^*}\phi\cdot w^{-\sigma}\cdot c\,(\phi)^{\sigma}\cdot p\,(\phi)^{-\sigma}\cdot P^{\sigma-1}\cdot M\cdot\mu(\phi)\cdot N d\phi+\widetilde{\phi}\cdot w^{-\sigma}\cdot c\left(\widetilde{\phi}\right)^{\sigma}\cdot\widetilde{f}\cdot N = L \quad (31)$$

[14] Note that households demand variety $q\left(\widetilde{\phi}\right)$ only for consumption purposes if $\delta=0$. If $\delta>0$, investments in the steady state would be $I=\delta\cdot K$ (equation (15)). Therefore, if $\delta>0$, the additional labor demand for investment purposes would be $\widetilde{\phi}\cdot w^{-\sigma}\cdot c\left(\widetilde{\phi}\right)^{\sigma}\cdot\delta\cdot K$ and the additional capital demand for investment purposes would be $(1-\widetilde{\phi})\cdot r^{-\sigma}\cdot c\left(\widetilde{\phi}\right)^{\sigma}\cdot\delta\cdot K$. While the factor market equilibrium conditions remain linear in the variables K and N if $\delta>0$, solving equations (31) and (32) for K and N would be more cumbersome.

$$\int_0^{\phi^*} (1-\phi)\cdot r^{-\sigma}\cdot c(\phi)^{\sigma}\cdot p(\phi)^{-\sigma}\cdot P^{\sigma-1}\cdot M\cdot \mu(\phi)\cdot N d\phi + \left(1-\widetilde{\phi}\right)\cdot r^{-\sigma}\cdot c\left(\widetilde{\phi}\right)\cdot \widetilde{f}\cdot N = K, \quad (32)$$

with $\widetilde{f} = f + \frac{f_E}{\phi^*}\cdot\theta$. Since f and f_E arise for a single firm, the term $\widetilde{f}\cdot N$ denotes *total* fixed input requirements, which arise in a single period of the steady state.[15]

Using equations (10), (16), the pricing rule $p(\phi) = \frac{\sigma}{\sigma-1}\cdot c(\phi)$ and $\int_0^{\phi^*}\phi\cdot\mu(\phi)d\phi = \widetilde{\phi}$, the factor market equilibrium conditions can be simplified as follows:[16]

$$\widetilde{\phi}\cdot\left(\frac{\widetilde{\phi}\cdot(1-1/\sigma)^{1-\sigma}}{(1-1/\sigma)^{1-\sigma}-(1-\widetilde{\phi})\cdot\rho^{1-\sigma}}\right)^{\sigma/(1-\sigma)}\cdot\left(q\left(\widetilde{\phi}\right)+\widetilde{f}\right)\cdot N = L \quad (33)$$

$$\left(1-\widetilde{\phi}\right)\cdot\frac{(1-1/\sigma)^{\sigma}}{\rho^{\sigma}}\cdot\left(q\left(\widetilde{\phi}\right)+\widetilde{f}\right)\cdot N = K. \quad (34)$$

Equations (33) and (34) can be solved for N and K in the closed economy steady state. This is summarized by result 5:

Result 5: Solving equations (33) and (34) for N and K in the closed economy steady state leads to the following result:

$$N = \frac{\left((1-1/\sigma)^{1-\sigma}-(1-\widetilde{\phi})\cdot\rho^{1-\sigma}\right)^{\sigma/(1-\sigma)}}{\widetilde{\phi}^{1/(1-\sigma)}\cdot(1-1/\sigma)^{\sigma}\cdot\left(q(\widetilde{\phi})+\widetilde{f}\right)}\cdot L \quad (35)$$

$$K = \frac{1-\widetilde{\phi}}{\widetilde{\phi}^{1/(1-\sigma)}}\cdot\left((1-1/\sigma)^{1-\sigma}\cdot\rho^{\sigma-1}-\left(1-\widetilde{\phi}\right)\right)^{\sigma/(1-\sigma)}\cdot L. \quad (36)$$

Result 3 has shown that $q\left(\widetilde{\phi}\right)$ is a function of the parameters σ, ρ, δ, f. Result 4 has shown that $\widetilde{\phi}$ is a function of the parameters σ, ρ, δ, f, θ and f_E. Therefore, both N and K are functions of the parameters σ, ρ, δ, f, θ and f_E as well. Note that $\delta = 0$ in order to simplify the derivation of N and K.

Equation (35) shows that N depends negatively on $q\left(\widetilde{\phi}\right)$ and $\widetilde{f}$. If $q\left(\widetilde{\phi}\right)$ increases or if the fixed input requirements f or f_E increase, the equilibrium number of firms must be smaller due to resource constraints.

Furthermore, it can be shown that equation (36) implies a negative relationship between K and $\widetilde{\phi}$ (see appendix E for the proof). If the average labor share parameter $\widetilde{\phi}$ decreases,

[15] The sunk entry costs parameter f_E has to be multiplied by, first, the death rate θ since the share θ of the active firms is forced to exit the market in each period, i.e. the share θ of the active firms is replaced in each period of the steady state. Second, f_E has to be multiplied by $\frac{1}{\phi^*}$ in order to account for the fact that only the share $G(\phi^*) = \phi^*$ of the entering firms actually becomes active after market entry. Therefore, if N firms are active in a period of the steady state, $\frac{N}{\phi^*}$ firms actually had to enter the market. Since *un*successful market entry also leads to entry costs, f_E has to be multiplied by $\frac{1}{\phi^*}$.

[16] Cf. appendix D for the derivation of equations (33) and (34).

i.e. the average capital share parameter $1 - \widetilde{\phi}$ increases, country–wide capital demand by firms increases. Households accordingly invest more and the capital stock in the steady state increases.

4 General steady state equilibrium — open economy

Both countries now liberalize trade. For example, tariffs in the autarky equilibrium have been prohibitively high and are now reduced to zero.

The derivation of the general steady state equilibrium of the open economy focuses only on one country since countries are assumed to be symmetric in every respect. The general steady state equilibrium of the open economy is characterized by the following 6 conditions, which have to hold in each period of the steady state:

1. the factor price ratio in the steady state $\frac{w}{r}$ (equation (16))
2. production of each variety has to equal demand for each variety (equation (7)) at price $p(\phi) = \frac{\sigma}{\sigma-1} \cdot c(\phi)$ (equation (9))
3. the zero cutoff profit condition for the domestic market
4. the zero cutoff profit condition for the foreign market
5. the open economy free entry condition
6. the factor market equilibrium conditions.

These 6 conditions can be used to solve for the following 6 variables in the open economy steady state: the threshold labor share parameter ϕ^* and the average labor share parameter $\widetilde{\phi} = \frac{\phi^*}{2}$; supply $q\left(\widetilde{\phi}\right)$ of the firm which produces with $\widetilde{\phi}$; the factor price ratio $\frac{w}{r}$; the country's capital stock K and investment I; the mass of firms N; the share of exporting firms $\frac{N_X}{N}$.

One critical assumption which is adopted from the previous heterogeneous firms literature is that trade is costly. It is assumed that entering the foreign market leads to sunk entry costs. For simplicity, iceberg transport costs are assumed to be zero.

As in the previous literature, a firm makes its export decision *after* it has drawn its labor share parameter ϕ. Due to sunk costs for entering the foreign market, not all firms that supply to the domestic market also supply to the foreign market after trade liberalization.

Subsections 4.1 and 4.2 analyze the firms' supply decision to the domestic and the foreign market in the open economy steady state.

Subsection 4.3 analyzes how the free entry condition changes due to the additional export profits by the exporting firms. Subsection 4.4 derives the price index P in the open economy steady state. Subsection 4.5 finally uses the factor market equilibrium conditions to derive the countries' capital stocks and the equilibrium number of firms.

4.1 Supply decision to the domestic market

Trade liberalization does not change the firms' supply decision to the domestic market as described in subsection 2.5.

After market entry and the draw of the labor share parameter ϕ, firms still only start with production and supply to the domestic market if they have drawn a labor share parameter ϕ which is equal or smaller than the threshold value ϕ^*. At ϕ^*, a firm's profits from supply to the *domestic* market $\pi(\phi^*)$ are exactly equal to zero, i.e. the following holds:

$$\pi(\phi^*) \;=\; \Big(p(\phi^*) - c(\phi^*)\Big) \cdot q(\phi^*) - c\left(\widetilde{\phi}\right) \cdot f \;=\; \frac{q(\phi^*) \cdot p(\phi^*)}{\sigma} - c\left(\widetilde{\phi}\right) \cdot f \;=\; 0. \quad (37)$$

The zero cutoff profit condition for the supply to the domestic market therefore does not change with trade liberalization. Again, using the pricing rule $p(\phi) = \frac{\sigma}{\sigma-1} \cdot c(\phi)$, equation (37) can be transformed to:

$$\frac{q(\phi^*)}{\sigma - 1} \;=\; \frac{c\left(\widetilde{\phi}\right)}{c(\phi^*)} \cdot f. \quad (38)$$

Therefore, equation (28) and equation (19) with $\phi_1 = \widetilde{\phi} = \frac{\phi^*}{2}$ and $\phi_2 = \phi^*$ can be used again in order to formulate result 6:

Result 6: $q\left(\widetilde{\phi}\right)$ in the open economy steady state is given as follows:

$$q\left(\widetilde{\phi}\right) \;=\; (\sigma - 1) \cdot \left(\frac{c(\widetilde{\phi})}{c(\phi^*)}\right)^{1-\sigma} \cdot f \;=\; \frac{(\sigma - 1) \cdot (1 - 1/\sigma)^{1-\sigma}}{2 \cdot (1 - 1/\sigma)^{1-\sigma} - (\rho + \delta)^{1-\sigma}} \cdot f. \quad (39)$$

Note that $q\left(\widetilde{\phi}\right)$ in the steady state does not change with trade liberalization (see result 3). $q\left(\widetilde{\phi}\right)$ still only depends on the parameters σ, ρ, δ and f and is independent of the steady state value of $\widetilde{\phi}$.

4.2 Supply decision to the foreign market

Foreign demand for a domestic variety, which is denoted by $q_X(\phi)$, is given by:[17]

$$q_X(\phi) \;=\; M_{cons} \cdot P^{\sigma-1} \cdot p(\phi)^{-\sigma}. \quad (40)$$

[17] Since no variable export costs exist, goods prices do not differ between countries; the price $p(\phi)$ therefore does not get a subscript X, even if it refers to exports. Furthermore, M_{cons} and P do not get a country index due to symmetry between countries.

Since both countries are assumed to be symmetric in every respect, the following holds:

$$q_X(\phi) \quad = \quad M_{cons} \cdot P^{\sigma-1} \cdot p(\phi)^{-\sigma} \quad = \quad q(\phi), \tag{41}$$

where $q(\phi)$ denotes *domestic* supply of a firm. Aggregate sales of an *exporting* firm after trade liberalization therefore amount to:

$$q(\phi) + q_X(\phi) \quad = \quad 2 \cdot M_{cons} \cdot P^{\sigma-1} \cdot p(\phi)^{-\sigma}. \tag{42}$$

However, entering the foreign market leads to a sunk input requirement f_{EX}, which is also produced with the average factor share parameters $\widetilde{\phi}$ and $1-\widetilde{\phi}$. The sunk entry costs into the foreign market are therefore given by $c\left(\widetilde{\phi}\right) \cdot f_{EX}$. Considering the time discount rate ρ, the death probability θ and the formula for an infinite geometric series, the per period equivalent of the sunk entry costs into the foreign market are given by: $c\left(\widetilde{\phi}\right) \cdot f_X = c\left(\widetilde{\phi}\right) \cdot f_{EX} \cdot \frac{\rho+\theta}{1+\rho}$. Note that a firm is indifferent between paying $c\left(\widetilde{\phi}\right) \cdot f_{EX}$ once or $c\left(\widetilde{\phi}\right) \cdot f_X$ in each period of its expected remaining lifetime.

Since sunk entry costs into the foreign market exist and since $\frac{w}{r} > 1$ in the steady state, only firms with a sufficiently small labor share parameter ϕ actually export. The additional threshold value for the labor share parameter is denoted by ϕ_X^*. At ϕ_X^*, a firms profits from serving the foreign market $\pi_X(\phi)$ are exactly equal to zero, i.e. the following holds:

$$\pi_X(\phi_X^*) \quad = \quad \Big(p(\phi_X^*) - c(\phi_X^*)\Big) \cdot q(\phi_X^*) - c\left(\widetilde{\phi}\right) \cdot f_X \quad = \quad \frac{q(\phi_X^*) \cdot p(\phi_X^*)}{\sigma} - c\left(\widetilde{\phi}\right) \cdot f_X \; = \; 0. \tag{43}$$

Equation (43) is the zero cutoff profit condition for the supply to the foreign market. If ϕ_X^* is strictly smaller than ϕ^*, some firms serve the domestic market, but do not export. Therefore, $\phi_X^* < \phi^*$ holds if

$$\frac{q(\phi^*) \cdot p(\phi^*)}{\sigma} \quad = \quad c\left(\widetilde{\phi}\right) \cdot f, \tag{44}$$

but, at the same time,

$$\frac{q(\phi^*) \cdot p(\phi^*)}{\sigma} \quad < \quad c\left(\widetilde{\phi}\right) \cdot f_X. \tag{45}$$

Note that equation (44) is the zero cutoff profit condition for the supply to the domestic market (equation (20)). Equations (44) and (45) indicate that firms with the labor share parameter ϕ^* can afford to supply to the domestic market, but they cannot afford to supply to the foreign market.

Dividing equations (44) and (45) by each other results in:

$$\frac{q(\phi^*) \cdot p(\phi^*)}{q(\phi_X^*) \cdot p(\phi_X^*)} \quad = \quad \frac{f}{f_X}. \tag{46}$$

Equation (46) can be used to solve for the ratio $\frac{\phi_X^*}{\phi^*}$, which leads to result 7:

Result 7: The ratio of threshold labor share parameters $\frac{\phi_X^*}{\phi^*}$ is given by:

$$\frac{\phi_X^*}{\phi^*} = \frac{\left[(1-1/\sigma)^{1-\sigma} - 0.5\cdot(\rho+\delta)^{1-\sigma}\right]\cdot\frac{f_X}{f} - 0.5\cdot(\rho+\delta)^{1-\sigma}}{(1-1/\sigma)^{1-\sigma} - (\rho+\delta)^{1-\sigma}}; \qquad (47)$$

$\frac{\phi_X^*}{\phi^*}$ is smaller than unity if $f_X > f$.

Proof: see appendix F. □

Since ϕ is uniformly distributed on $[0,1]$, the ratio $\frac{\phi_X^*}{\phi^*}$ equals

(i) the ex–ante probability that an entering firm becomes an exporter;

(ii) the share of exporting firms in the open economy.

Furthermore, $\frac{\phi_X^*}{\phi^*}$ does not depend on the equilibrium average labor share parameter $\widetilde{\phi}$.

Result 8 shows that the partial derivative of $\frac{\phi_X^*}{\phi^*}$ with respect to f_X is negative, i.e. the share of exporting firm decreases the higher the sunk costs for entering the foreign market:

Result 8: The partial derivative of $\frac{\phi_X^*}{\phi^*}$ with respect to f_X is given by:

$$\frac{\partial\left(\phi_X^*/\phi^*\right)}{\partial f_X} = \frac{\left[(1-1/\sigma)^{1-\sigma} - 0.5\cdot(\rho+\delta)^{1-\sigma}\right]\cdot\frac{1}{f}}{(1-1/\sigma)^{1-\sigma} - (\rho+\delta)^{1-\sigma}} < 0 \quad \text{since} \qquad (48)$$

$\left(1-\frac{1}{\sigma}\right)^{1-\sigma} < (\rho+\delta)^{1-\sigma}$ and $(1-\frac{1}{\sigma})^{1-\sigma} > 0.5\cdot(\rho+\delta)^{1-\sigma}$ due to assumption 1.

Finally, the zero cutoff profit condition for the foreign market (equation (43)) can be transformed to:

$$\frac{q\left(\phi_X^*\right)}{\sigma-1} = \frac{c(\widetilde{\phi})}{c(\phi_X^*)}\cdot f_X. \qquad (49)$$

Since $\widetilde{\phi}_X = \int_0^{\phi_X^*}\phi\cdot\frac{g(\phi)}{G(\phi_X^*)}d\phi = \frac{\phi_X^*}{2}$ denotes the average labor share parameter over all exporting firms, equation (49) can be used to formulate result 9:

Result 9: $q\left(\widetilde{\phi}_X\right)$ in the open economy steady state is given by:

$$q\left(\widetilde{\phi}_X\right) = (\sigma-1)\cdot\frac{c\left(\widetilde{\phi}\right)}{c\left(\phi_X^*\right)}\cdot\left(\frac{c\left(\phi_X^*\right)}{c\left(\widetilde{\phi}_X\right)}\right)^{\sigma}\cdot f_X; \qquad (50)$$

$q\left(\widetilde{\phi}_X\right)$ is already determined by the parameters σ, ρ, δ, f and f_X and is independent of the steady state values of $\widetilde{\phi}$ and $\widetilde{\phi}_X$.

Proof: see appendix G. □

Finally, note that $q\left(\widetilde{\phi}_X\right)$ equals $q\left(\widetilde{\phi}\right)$ (result 3) if $f = f_X$. The reason is that $\phi_X^* = \phi^*$ if $f = f_X$, i.e. each firm also exports if $f = f_X$.

4.3 Free entry condition — open economy

The free entry condition for the closed economy (equation (21)) has to be extended by the ex–ante expected export profits. The free entry condition for the open economy is therefore given by:

$$G(\phi^*) \cdot \sum_{t=t'}^{\infty} E\left(\pi(\phi) \middle| \phi \le \phi^*\right) \cdot \left(\frac{1-\theta}{1+\rho}\right)^t + G(\phi_X^*) \cdot \sum_{t=t'}^{\infty} E\left(\pi_X(\phi) \middle| \phi \le \phi_X^*\right) \cdot \left(\frac{1-\theta}{1+\rho}\right)^t = c\left(\widetilde{\phi}\right) \cdot f_E, \tag{51}$$

where $\pi_X(\phi)$ denotes the export profits of a firm, which produces with the labor share parameter ϕ. Since both countries are completely symmetric, $\pi_X(\phi) = \pi(\phi)$.

$G(\phi_X^*)$ stands for the probability that the entrant draws a $\phi \le \phi_X^*$, i.e. $G(\phi_X^*)$ stands for the probability that the entrant actually exports after market entry. Due to the uniform distribution of ϕ on $[0, 1]$, it follows $G(\phi_X^*) = \phi_X^*$. Furthermore, $E\left(\pi_X(\phi) \middle| \phi \le \phi_X^*\right)$ stands for the ex–ante expected export profits in a single period t in the steady state, given that the firm exports after market entry.

Firms ex–ante expect that they will have the average profits over all exporting firms, given that they have drawn a $\phi \le \phi_X^*$. Therefore, the ex–ante expected export profits result as:[18]

$$E\left(\pi(\phi) \middle| \phi \le \phi_X^*\right) = \frac{q(\widetilde{\phi}_X) \cdot p(\widetilde{\phi}_X)}{\sigma} - c\left(\widetilde{\phi}\right) \cdot f_X, \qquad \text{with } \widetilde{\phi}_X = \tfrac{\phi_X^*}{2}. \tag{52}$$

Finally, considering $G(\phi^*) = \phi^*$, $G(\phi_X^*) = \phi_X^*$, the pricing rule $p(\phi) = \frac{\sigma}{\sigma-1} \cdot c(\phi)$ and the formula for an infinite geometric series, the free entry condition for the open economy can be simplified as follows:

$$\frac{q(\widetilde{\phi})}{\sigma-1} - f + \Upsilon = \frac{f_E}{\phi^*} \cdot \frac{\rho+\theta}{1+\rho}, \tag{53}$$
$$\text{with} \quad \Upsilon = \frac{\phi_X^*}{\phi^*} \cdot \frac{c(\widetilde{\phi}_X)}{c(\widetilde{\phi})} \cdot \frac{q(\widetilde{\phi}_X)}{\sigma-1} - \frac{\phi_X^*}{\phi^*} \cdot f_X.$$

Compared to the free entry condition for the closed economy (equation (27)), the term Υ adds to the left hand–side. Equation (53) can be solved for the average labor share parameter $\widetilde{\phi} = \frac{\phi^*}{2}$ in the open economy. This is summarized by result 10:

[18] The ex–ante expected *export* profits are derived the same way as the ex–ante expected profits from supply to the *domestic* market, which are given by $E\left(\pi(\phi) \middle| \phi \le \phi^*\right)$; see equations (22), (23) and (24).

Result 10: The average labor share parameter in the steady state of the open economy is given by:

$$\widetilde{\phi} \;=\; \frac{\phi^*}{2} = \frac{0.5 \cdot f_E \cdot \frac{\rho+\theta}{1+\rho}}{\frac{q(\widetilde{\phi})}{\sigma-1} - f + \Upsilon}, \quad \text{with } \Upsilon \;=\; \frac{\phi_X^*}{\phi^*} \cdot \left(\frac{c(\widetilde{\phi}_X)}{c(\widetilde{\phi})} \cdot \frac{q(\widetilde{\phi}_X)}{\sigma - 1} - f_X \right); \qquad (54)$$

$\widetilde{\phi}$ is already determined by the parameters σ, ρ, δ, f, f_X and f_E; furthermore, $\widetilde{\phi}$ in the steady state of the *open* economy is smaller than $\widetilde{\phi}$ in the steady state of the *closed* economy (result 4) since Υ is positive.

Proof: see appendix H. □

Result 10 has the following implications:

(*i*) The industry becomes more capital intensive with trade liberalization since, first, $\widetilde{\phi}$ decreases with trade liberalization and, second, all firms with a capital share parameter of $1 - \phi_X^*$ or larger also produce for exports in the open economy.

(*ii*) The decrease in $\widetilde{\phi}$ is larger, the larger the share of exporting firms, i.e. the larger $\frac{\phi_X^*}{\phi^*}$.

(*iii*) Wages increase relative to capital returns with trade liberalization (see appendix B for the proof).

(*iv*) The increase in $\frac{w}{r}$ is larger, the larger the share of exporting firms, i.e. the larger $\frac{\phi_X^*}{\phi^*}$.

4.4 Price index in the open economy

Since foreign firms now supply to the domestic market, the price index P has to be adjusted. In the open economy the price index is given as follows:[19]

$$\begin{aligned} P &= \left(\int_0^{\phi^*} p(\phi)^{1-\sigma} \cdot \mu(\phi) \cdot N d\phi + \int_0^{\phi_X^*} p(\phi)^{1-\sigma} \cdot \xi(\phi) \cdot N \cdot \frac{\phi_X^*}{\phi^*} d\phi \right)^{1/(1-\sigma)} \\ &= N^{1/(1-\sigma)} \cdot \frac{\sigma}{\sigma - 1} \cdot \left(p\left(\widetilde{\phi}\right) + p\left(\widetilde{\phi}_X\right) \cdot \frac{\phi_X^*}{\phi^*} \right)^{1/(1-\sigma)}, \end{aligned} \qquad (55)$$

where $\xi(\phi) = \frac{1}{\phi_X^*}$ denotes the conditional probability density for ϕ on the subinterval $[0, \phi_X^*]$ of the unit interval. $\widetilde{\phi}$ is equal to $\frac{\phi^*}{2}$ and $\widetilde{\phi}_X$ is equal to $\frac{\phi_X^*}{2}$.

Note that $N \cdot \frac{\phi_X^*}{\phi^*}$ stands for the mass of *foreign* firms supplying to the domestic market. These foreign firms have their ϕ from the interval $[0, \phi_X^*]$.

[19] Cf. also equation (10) for the derivation of P.

Dividing and multiplying the right–hand side of equation (55) by $\left(1+\frac{\phi_X^*}{\phi^*}\right)^{1/(1-\sigma)}$ and simplification leads to the following price index P for the open economy:[20]

$$
\begin{aligned}
P &= N^{1/(1-\sigma)} \cdot \frac{\sigma}{\sigma-1} \cdot \left(p\left(\widetilde{\phi}\right) \cdot \frac{1}{1+\frac{\phi_X^*}{\phi^*}} + p\left(\widetilde{\phi}_X\right) \cdot \frac{\frac{\phi_X^*}{\phi^*}}{1+\frac{\phi_X^*}{\phi^*}} \right)^{1/(1-\sigma)} \cdot \left(1+\frac{\phi_X^*}{\phi^*}\right)^{1/(1-\sigma)} \\
&= N^{1/(1-\sigma)} \cdot p\left(\widetilde{\widetilde{\phi}}\right) \cdot \left(1+\frac{\phi_X^*}{\phi^*}\right)^{1/(1-\sigma)}, \quad \text{with } \widetilde{\widetilde{\phi}} = \frac{\phi^*}{2} \cdot \frac{1+\left(\frac{\phi_X^*}{\phi^*}\right)^2}{1+\frac{\phi_X^*}{\phi^*}}. \qquad (56)
\end{aligned}
$$

Since $\frac{\phi_X^*}{\phi^*} \leq 1$, the ratio $\frac{1+\left(\frac{\phi_X^*}{\phi^*}\right)^2}{1+\frac{\phi_X^*}{\phi^*}}$ is equal or smaller than unity. Therefore, $\widetilde{\widetilde{\phi}}$ is equal or smaller than $\widetilde{\phi} = \frac{\phi^*}{2}$.

4.5 Factor market equilibrium conditions in the open economy

Applying Shephard's Lemma to the marginal cost function (equation (2)) and considering demand $q(\phi)$ (equation (7)) leads to the following factor market equilibrium conditions; again, the capital depreciation rate δ is normalized to zero in order to simplify further derivations (see also footnote 14):

$$
\begin{aligned}
&\frac{M}{P^{1-\sigma}} \cdot N \cdot \left(\int_0^{\phi^*} \phi \cdot w^{-\sigma} \cdot \left(\frac{c(\phi)}{p(\phi)}\right)^{\sigma} \cdot \mu(\phi) d\phi + \int_0^{\phi_X^*} \phi \cdot w^{-\sigma} \cdot \left(\frac{c(\phi)}{p(\phi)}\right)^{\sigma} \cdot \xi(\phi) \cdot \frac{\phi_X^*}{\phi^*} d\phi \right) \\
&\qquad + \widetilde{\phi} \cdot w^{-\sigma} \cdot c\left(\widetilde{\phi}\right)^{\sigma} \cdot N \cdot \left(\widetilde{f} + f_X \cdot \frac{\phi_X^*}{\phi^*}\right) = L \qquad (57)
\end{aligned}
$$

$$
\begin{aligned}
&\frac{M}{P^{1-\sigma}} \cdot N \cdot \left(\int_0^{\phi^*} (1-\phi) \cdot r^{-\sigma} \cdot \left(\frac{c(\phi)}{p(\phi)}\right)^{\sigma} \cdot \mu(\phi) d\phi + \int_0^{\phi_X^*} (1-\phi) \cdot r^{-\sigma} \cdot \left(\frac{c(\phi)}{p(\phi)}\right)^{\sigma} \cdot \xi(\phi) \cdot \frac{\phi_X^*}{\phi^*} d\phi \right) \\
&\qquad + \left(1-\widetilde{\phi}\right) \cdot r^{-\sigma} \cdot c\left(\widetilde{\phi}\right)^{\sigma} \cdot N \cdot \left(\widetilde{f} + f_X \cdot \frac{\phi_X^*}{\phi^*}\right) = K. \qquad (58)
\end{aligned}
$$

Considering equations (56) and (16), the pricing rule $p(\phi) = \frac{\sigma}{\sigma-1} \cdot c(\phi)$ and the definition of $\widetilde{\phi}$ and $\widetilde{\phi}_X$ leads to the following simplification of the factor market equilibrium conditions:[21]

$$
\frac{\widetilde{\phi}^{1/(1-\sigma)} \cdot \left(1-\frac{1}{\sigma}\right)^{\sigma}}{\left(\left(1-\frac{1}{\sigma}\right)^{1-\sigma} - \left(1-\widetilde{\phi}\right) \cdot \rho^{1-\sigma}\right)^{\sigma/(1-\sigma)}} \cdot \left(q\left(\widetilde{\phi}\right) \cdot \Delta_1 + \widetilde{f} + f_X \cdot \frac{\phi_X^*}{\phi^*} \right) \cdot N = L \qquad (59)
$$

$$
\left(1-\widetilde{\phi}\right) \cdot \frac{(1-1/\sigma)^{\sigma}}{\rho^{\sigma}} \cdot \left(q\left(\widetilde{\phi}\right) \cdot \Delta_2 + \widetilde{f} + f_X \cdot \frac{\phi_X^*}{\phi^*} \right) \cdot N = K, \qquad (60)
$$

$$
\text{with} \quad \Delta_1 = \frac{\widetilde{\widetilde{\phi}}}{\widetilde{\phi}} \cdot \left(\frac{c(\widetilde{\phi})}{c(\widetilde{\widetilde{\phi}})}\right)^{1-\sigma} \quad \text{and} \quad \Delta_2 = \frac{1-\widetilde{\widetilde{\phi}}}{1-\widetilde{\phi}} \cdot \left(\frac{c(\widetilde{\phi})}{c(\widetilde{\widetilde{\phi}})}\right)^{1-\sigma}.
$$

[20] Cf. appendix I for the derivation of equation (56).

[21] Cf. appendix J for the derivation of equations (59) and (60).

Equations (59) and (60) can be solved for N and K in the open economy steady state. This is summarized by result 11:

Result 11: Solving equations (59) and (60) for N and K in the open economy steady state leads to the following result:

$$N = \frac{\left((1-1/\sigma)^{1-\sigma} - \left(1-\widetilde{\phi}\right)\cdot \rho^{1-\sigma}\right)^{\sigma/(1-\sigma)}}{\widetilde{\phi}^{1/(1-\sigma)} \cdot (1-1/\sigma)^{\sigma} \cdot \left(q\left(\widetilde{\phi}\right)\cdot \Delta_1 + \widetilde{f} + f_X \cdot \frac{\phi_X^*}{\phi^*}\right)} \cdot L \tag{61}$$

$$K = \frac{1-\widetilde{\phi}}{\widetilde{\phi}^{1/(1-\sigma)}} \cdot \left(\left(1-\frac{1}{\sigma}\right)^{1-\sigma} \cdot \rho^{\sigma-1} - 1 + \widetilde{\phi}\right)^{\sigma/(1-\sigma)} \cdot \frac{q\left(\widetilde{\phi}\right)\cdot \Delta_2 + \widetilde{f} + f_X \cdot \frac{\phi_X^*}{\phi^*}}{q\left(\widetilde{\phi}\right)\cdot \Delta_1 + \widetilde{f} + f_X \cdot \frac{\phi_X^*}{\phi^*}} \cdot L. \tag{62}$$

The capital stock in the open economy is unambiguously larger than the capital stock in the closed economy (equation (36)). The reason for this is twofold: first, the average labor share parameter $\widetilde{\phi}$ decreases with trade liberalization (result (10)), i.e. the threshold labor share parameter ϕ^* decreases and firms have to be more capital intensive in order to survive after market entry. Second, the more capital intensive firms produce more in the open economy since they also export. Therefore, the industry's average capital intensity increases with trade liberalization. Investments and the steady state capital stock accordingly increase as well.

An analytical comparison between the number of firms in the open economy and in the closed economy (equation (35)) is not possible. Two opposing effects on the number of firms are present. On the one hand, exports lead to additional fixed export costs, which ceteris paribus *de*creases the number of firms. On the other hand, the industry's capital stock increases with trade liberalization, which ceteris paribus *in*creases the number of firms.

The effect of trade liberalization on the industry's capital stock is summarized by result 12:

Result 12: The industry's capital stock K increases with trade liberalization.

Proof: see appendix K. □

5 Autarky versus open economy steady state — a numerical example

The model is parameterized and solved numerically. These numerical solutions provide a visual comparison between the autarky and the open economy steady state.

All basic assumptions of the analytical model are taken over to the numerical model. Most importantly, ϕ is *uniformly* distributed on $[0, 1]$, all parameter values are in line with assumption 1 and countries are perfectly symmetric.

The parameter values are chosen as follows: $\sigma = 1.5$, $\rho = 0.2$, $\delta = 0$, $f = 1$, $f_E = 0.7$, $L = 1$ and $\theta = 0.5$. To study the impact of trade liberalization, the fixed export costs f_X decrease from $f_X \approx 1.821$ to $f_X = 1$. At $f_X \approx 1.821$ no firm exports, i.e. $\frac{\phi_X^*}{\phi^*} = 0$; at $f_X = 1$ all firms export, i.e. $\frac{\phi_X^*}{\phi^*} = 1$. The superscript *aut* in figures 1a and 1b refers to variables in the autarky situation and the superscript *ft* refers to variables in the free trade situation.

The parameter values do not necessarily relate to empirical estimates. However, as long as the parameters are in line with assumption 1, the adjustments to exposure to trade are robust with respect to the specific numerical values. Figures 1a and 1b display the values for the autarky and the free trade steady state.

Figure 1a shows that the decrease of f_X increases the share of exporting firms $\frac{\phi_X^*}{\phi^*}$ linearly from 0 to 1. The average labor share parameter $\widetilde{\phi}$ decreases with trade liberalization, i.e. the average capital share parameter $1 - \widetilde{\phi}$ accordingly increases with trade liberalization. The adjustment of $\widetilde{\phi}$ and $1 - \widetilde{\phi}$ with trade liberalization is larger, the larger the share of exporting firms.

Furthermore, figure 1a shows that the average labor share parameter over the non–exporting firms, which is denoted by $\widetilde{\phi}_N$, is always larger than the average labor share parameter over the exporting firms $\widetilde{\phi}_X$.[22] Therefore, the exporting firms are, on average, more capital intensive than the non–exporting firms.

Finally, figure 1a shows that the marginal costs $c(\widetilde{\phi})$ decrease with trade liberalization.

Figure 1b shows that the decrease of f_X increases the industry's capital stock K. Due to this positive endowment effect, the number of firms N increases with trade liberalization.

Finally, figure 1b shows that the relative labor returns $\frac{w}{r}$ increase with trade liberalization. The increase in $\frac{w}{r}$ with trade liberalization is larger, the smaller f_X, i.e. the larger the share of exporting firms $\frac{\phi_X^*}{\phi^*}$.

6 Conclusions

This paper presents a model with intra–industry trade, capital and labor as factors of production and firm heterogeneity with respect to the factor shares in production. All previous papers in the field, in contrast, consider firm heterogeneity with respect to total factor productivity.

This paper therefore considers the stylized fact that firms within narrowly defined industries exhibit a large degree of heterogeneity with respect to their factor shares in production.

First, this paper illustrates how firm heterogeneity in factor shares can be incorporated

[22]The average labor share parameter $\widetilde{\phi}_N$ is calculated as $\widetilde{\phi}_N = \frac{\phi_X^* + \phi^*}{2}$.

into an analytically tractable model. Second, this paper illustrates how firm heterogeneity in factor shares and trade liberalization interact.

The theoretical results of this paper reproduce several empirical facts on the firm and industry–level adjustments to trade liberalization. It is shown that, for quite general parameter restrictions, the exporting firms are, on average, more capital intensive than the non–exporting firms. Furthermore, trade liberalization increases capital's share in national income and increases wages relative to capital returns.

Closest to the present paper is the work by Bernard et al. (2007). These authors keep the assumption of firm heterogeneity with respect to total factor productivity, but they extend Melitz (2003) to two sectors and Heckscher–Ohlin trade.

In future research the present setup could be combined with the setup of Bernard et al. (2007). In such a setup, firms would have different factor shares both *within* sectors and *between* sectors. It could be analyzed whether the net effect of inter–industry and intra–industry factor relocations on a country's capital stock is positive or negative. This question is especially important for relatively labor–rich developing countries, which trade with relatively capital–rich developed countries.

Appendix

A Proof of result 1

Baxter (1992), p. 738, shows that relative factor prices in the steady state of a Ramsey growth model are determined by four necessary first order conditions for a competitive general equilibrium. Extending the setup of Baxter (1992) by monopolistic competition between firms and including time indices, these necessary first order conditions result as follows:

$$r_t + (1-\delta)\cdot p\left(\widetilde{\phi}_t\right) \quad = \quad p_t^K, \tag{63}$$

$$r_t = p\left(\widetilde{\phi}_t\right)\cdot\left(1-\frac{1}{\sigma}\right)\cdot\left(\widetilde{\phi}_t^{1-\alpha}\cdot l_t^\alpha + \left(1-\widetilde{\phi}_t\right)^{1-\alpha}\cdot k_t^\alpha\right)^{(1-\alpha)/\alpha}\cdot\left(1-\widetilde{\phi}_t\right)^{1-\alpha}\cdot k_t^{\alpha-1} \tag{64}$$

$$w_t = p\left(\widetilde{\phi}_t\right)\cdot\left(1-\frac{1}{\sigma}\right)\cdot\left(\widetilde{\phi}_t^{1-\alpha}\cdot l_t^\alpha + \left(1-\widetilde{\phi}_t\right)^{1-\alpha}\cdot k_t^\alpha\right)^{(1-\alpha)/\alpha}\cdot\widetilde{\phi}_t^{1-\alpha}\cdot l_t^{\alpha-1} \tag{65}$$

$$\frac{p_{t+1}^K}{p\left(\widetilde{\phi}_t\right)} \quad = \quad (1+\rho), \tag{66}$$

where p_t^K denotes the price per unit capital in period t, w_t the price per unit labor in period t and r_t the capital rental rate in period t.

Equation (63) is a zero profit condition for the households capital lending behavior. Households realize zero profits from lending capital out to firms if the price per unit capital in period t equals the capital returns in period t, plus what is left from the unit of capital in $t+1$; since one unit of the average good in t leads to one unit of capital in $t+1$, the remaining $1-\delta$ units of capital in $t+1$ are evaluated with $p\left(\widetilde{\phi}_t\right)$.

Equations (64) and (65) imply that equilibrium factor prices equal the value of the marginal product for each factor. Equation (66) denotes the Euler equation.

The time index is removed now for a steady state analysis. Equations (66) and (64) can be substituted into equation (63) to give:

$$\left(1-\frac{1}{\sigma}\right)\cdot\left(\widetilde{\phi}^{1-\alpha}\cdot l^\alpha + \left(1-\widetilde{\phi}\right)^{1-\alpha}\cdot k^\alpha\right)^{(1-\alpha)/\alpha}\cdot\left(1-\widetilde{\phi}\right)^{1-\alpha}\cdot k^{\alpha-1} + 1 - \delta \quad = \quad 1+\rho. \tag{67}$$

Manipulation of equation (67) leads to:

$$\frac{l}{k} \quad = \quad \left(\frac{\left(\frac{\rho+\delta}{(1-1/\sigma)\cdot\left(1-\widetilde{\phi}\right)^{1-\alpha}}\right)^{\alpha/(1-\alpha)} - \left(1-\widetilde{\phi}\right)^{1-\alpha}}{\widetilde{\phi}^{1-\alpha}}\right)^{1/\alpha}. \tag{68}$$

Furthermore, equation (65) can be transformed to:

$$\frac{w}{p\left(\widetilde{\phi}\right)\cdot(1-1/\sigma)} \quad = \quad \widetilde{\phi}^{1-\alpha}\cdot\left(\widetilde{\phi}^{1-\alpha} + \left(1-\widetilde{\phi}\right)^{1-\alpha}\cdot\left(\frac{k}{l}\right)^\alpha\right)^{(1-\alpha)/\alpha}. \tag{69}$$

Substituting equation (68) into equation (69) leads to the following:

$$\frac{w}{p\left(\widetilde{\phi}\right)\cdot(1-1/\sigma)} \quad = \quad \left(\frac{\widetilde{\phi}\cdot(\rho+\delta)^{\alpha/(1-\alpha)}}{(\rho+\delta)^{\alpha/(1-\alpha)} - \left(1-\widetilde{\phi}\right)\cdot(1-1/\sigma)^{\alpha/(1-\alpha)}}\right)^{(1-\alpha)/\alpha}. \tag{70}$$

Furthermore, from equations (63) and (66) it follows that

$$\frac{r}{p\left(\widetilde{\phi}\right)} \quad = \quad \rho+\delta \qquad \text{and} \qquad \frac{r}{p\left(\widetilde{\phi}\right)\cdot(1-1/\sigma)} \quad = \quad \frac{\rho+\delta}{1-1/\sigma}. \tag{71}$$

Dividing equations (70) and (71) by each other results in:

$$\frac{w\Big/\left(p\left(\widetilde{\phi}\right)\cdot(1-1/\sigma)\right)}{r\Big/\left(p\left(\widetilde{\phi}\right)\cdot(1-1/\sigma)\right)} \quad = \quad \left(\frac{\widetilde{\phi}\cdot(\rho+\delta)^{\alpha/(1-\alpha)}}{(\rho+\delta)^{\alpha/(1-\alpha)} - \left(1-\widetilde{\phi}\right)\cdot(1-1/\sigma)^{\alpha/(1-\alpha)}}\right)^{(1-\alpha)/\alpha}\cdot\frac{1-1/\sigma}{\rho+\delta}$$

$$= \left(\frac{\widetilde{\phi} \cdot (1 - 1/\sigma)^{\alpha/(1-\alpha)}}{(\rho+\delta)^{\alpha/(1-\alpha)} - \left(1 - \widetilde{\phi}\right) \cdot (1 - 1/\sigma)^{\alpha/(1-\alpha)}} \right)^{(1-\alpha)/\alpha} . \tag{72}$$

Considering $\sigma = \frac{1}{1-\alpha}$ or $\frac{\sigma-1}{\sigma} = \alpha$ leads to the following expression for $\frac{w}{r}$:

$$\frac{w}{r} = \left(\frac{\widetilde{\phi} \cdot (\rho+\delta)^{1-\sigma}}{(1 - 1/\sigma)^{1-\sigma} - \left(1 - \widetilde{\phi}\right) \cdot (\rho+\delta)^{1-\sigma}} \right)^{1/(\sigma-1)} . \tag{73}$$

The ratio $\frac{w}{r}$ therefore only depends on the parameters σ, ρ, δ and the endogenous labor share parameter $\widetilde{\phi}$.

B Factor price ratio $\frac{w}{r}$ and the average labor share parameter $\widetilde{\phi}$

The partial derivative of the factor price ratio $\frac{w}{r}$ with respect to the average labor share parameter $\widetilde{\phi}$ is given by:

$$\frac{\partial (w/r))}{\partial \widetilde{\phi}} = \frac{1}{\sigma - 1} \cdot \left(\frac{w}{r}\right)^{2-\sigma} \cdot \frac{(\rho+\delta)^{1-\sigma} \cdot \left[(1 - 1/\sigma)^{1-\sigma} - (\rho+\delta)^{1-\sigma}\right]}{\left((1 - 1/\sigma)^{1-\sigma} - \left(1 - \widetilde{\phi}\right) \cdot (\rho+\delta)^{1-\sigma}\right)^2} . \tag{74}$$

The partial derivative $\frac{\partial(w/r))}{\partial \widetilde{\phi}}$ is negative since the squared bracket in the numerator is negative due to assumption 1.

C Proof of result 2

The ratio of the marginal costs $c(\phi_1)$ and $c(\phi_2)$ is given as follows:

$$\begin{aligned} \frac{c(\phi_1)}{c(\phi_2)} &= \left(\frac{\phi_1 \cdot w^{1-\sigma} + (1-\phi_1) \cdot r^{1-\sigma}}{\phi_2 \cdot w^{1-\sigma} + (1-\phi_2) \cdot r^{1-\sigma}} \right)^{1/(1-\sigma)} \\ &= \left(\frac{\phi_1 + (1-\phi_1) \cdot \left(\frac{r}{w}\right)^{1-\sigma}}{\phi_2 + (1-\phi_2) \cdot \left(\frac{r}{w}\right)^{1-\sigma}} \right)^{1/(1-\sigma)} . \end{aligned} \tag{75}$$

Substituting $\frac{w}{r}$ from equation (16) into equation (75) leads to:

$$\frac{c(\phi_1)}{c(\phi_2)} = \left(\frac{\phi_1 + (1-\phi_1) \cdot \frac{\widetilde{\phi} \cdot (\rho+\delta)^{1-\sigma}}{(1-1/\sigma)^{1-\sigma} - \left(1-\widetilde{\phi}\right) \cdot (\rho+\delta)^{1-\sigma}}}{\phi_2 + (1-\phi_2) \cdot \frac{\widetilde{\phi} \cdot (\rho+\delta)^{1-\sigma}}{(1-1/\sigma)^{1-\sigma} - \left(1-\widetilde{\phi}\right) \cdot (\rho+\delta)^{1-\sigma}}} \right)^{1/(1-\sigma)} . \tag{76}$$

Simplification leads to:

$$\begin{aligned} \frac{c(\phi_1)}{c(\phi_2)} &= \left(\frac{\phi_1 \cdot (1 - 1/\sigma)^{1-\sigma} - \phi_1 \cdot (\rho+\delta)^{1-\sigma} + \widetilde{\phi} \cdot (\rho+\delta)^{1-\sigma}}{\phi_2 \cdot (1 - 1/\sigma)^{1-\sigma} - \phi_2 \cdot (\rho+\delta)^{1-\sigma} + \widetilde{\phi} \cdot (\rho+\delta)^{1-\sigma}} \right)^{1/(1-\sigma)} \\ &= \left(\frac{\frac{\phi_1}{\widetilde{\phi}} \cdot \left[(1 - 1/\sigma)^{1-\sigma} - (\rho+\delta)^{1-\sigma}\right] + (\rho+\delta)^{1-\sigma}}{\frac{\phi_2}{\widetilde{\phi}} \cdot \left[(1 - 1/\sigma)^{1-\sigma} - (\rho+\delta)^{1-\sigma}\right] + (\rho+\delta)^{1-\sigma}} \right)^{1/(1-\sigma)} . \end{aligned} \tag{77}$$

D Factor market equilibrium conditions in the closed economy

The factor market equilibrium conditions are given by:

$$\int_0^{\phi^*} \phi \cdot w^{-\sigma} \cdot c(\phi)^{\sigma} \cdot p(\phi)^{-\sigma} \cdot P^{\sigma-1} \cdot M \cdot \mu(\phi) \cdot N d\phi + \widetilde{\phi} \cdot w^{-\sigma} \cdot c\left(\widetilde{\phi}\right)^{\sigma} \cdot \widetilde{f} \cdot N = L \tag{78}$$

$$\int_0^{\phi^*} (1-\phi) \cdot r^{-\sigma} \cdot c(\phi)^{\sigma} \cdot p(\phi)^{-\sigma} \cdot P^{\sigma-1} \cdot M \cdot \mu(\phi) \cdot N d\phi + \left(1 - \widetilde{\phi}\right) \cdot r^{-\sigma} \cdot c\left(\widetilde{\phi}\right)^{\sigma} \cdot \widetilde{f} \cdot N = K. \tag{79}$$

Using $P = N^{1/(1-\sigma)} \cdot p\left(\widetilde{\phi}\right)$ (equation (10)), the factor market equilibrium conditions can be transformed to:

$$\int_0^{\phi^*} \phi \cdot w^{-\sigma} \cdot c(\phi)^{\sigma} \cdot p(\phi)^{-\sigma} \cdot p\left(\widetilde{\phi}\right)^{\sigma-1} \cdot M \cdot \mu(\phi) d\phi + \widetilde{\phi} \cdot w^{-\sigma} \cdot c\left(\widetilde{\phi}\right)^{\sigma} \cdot \widetilde{f} \cdot N = L \quad (80)$$

$$\int_0^{\phi^*} (1-\phi) \cdot r^{-\sigma} \cdot c(\phi)^{\sigma} \cdot p(\phi)^{-\sigma} \cdot p\left(\widetilde{\phi}\right)^{\sigma-1} \cdot M \cdot \mu(\phi) d\phi + \left(1-\widetilde{\phi}\right) \cdot r^{-\sigma} \cdot c\left(\widetilde{\phi}\right)^{\sigma} \cdot \widetilde{f} \cdot N = K. \quad (81)$$

Using the pricing rule $p(\phi) = \frac{\sigma}{\sigma-1} \cdot c(\phi)$, i.e. $\frac{c(\phi)}{p(\phi)} = \frac{\sigma-1}{\sigma}$, the factor market equilibrium conditions can be simplified to:

$$\int_0^{\phi^*} \phi \cdot w^{-\sigma} \cdot c\left(\widetilde{\phi}\right)^{\sigma} \cdot p\left(\widetilde{\phi}\right)^{-1} \cdot M \cdot \mu(\phi) d\phi + \widetilde{\phi} \cdot w^{-\sigma} \cdot c\left(\widetilde{\phi}\right)^{\sigma} \cdot \widetilde{f} \cdot N = L \quad (82)$$

$$\int_0^{\phi^*} (1-\phi) \cdot r^{-\sigma} \cdot c\left(\widetilde{\phi}\right)^{\sigma} \cdot p\left(\widetilde{\phi}\right)^{-1} \cdot M \cdot \mu(\phi) d\phi + \left(1-\widetilde{\phi}\right) \cdot r^{-\sigma} \cdot c\left(\widetilde{\phi}\right)^{\sigma} \cdot \widetilde{f} \cdot N = K. \quad (83)$$

Considering $\int_0^{\phi^*} \phi \cdot \mu(\phi) d\phi = \widetilde{\phi}$, $\int_0^{\phi^*} (1-\phi) \cdot \mu(\phi) d\phi = \left(1-\widetilde{\phi}\right)$ and the expression for $c\left(\widetilde{\phi}\right)$ (equation (2)) leads to the following simplification of the factor market equilibrium conditions:

$$\widetilde{\phi} \cdot \left(\widetilde{\phi} + \left(1-\widetilde{\phi}\right) \cdot \left(\frac{r}{w}\right)^{1-\sigma}\right)^{\sigma/(1-\sigma)} \cdot \left(\frac{M}{p(\widetilde{\phi})} + \widetilde{f} \cdot N\right) = L \quad (84)$$

$$\left(1-\widetilde{\phi}\right) \cdot \left(\widetilde{\phi} \cdot \left(\frac{w}{r}\right)^{1-\sigma} + \left(1-\widetilde{\phi}\right)\right)^{\sigma/(1-\sigma)} \cdot \left(\frac{M}{p(\widetilde{\phi})} + \widetilde{f} \cdot N\right) = K. \quad (85)$$

Combining equations (7) and (10) implies $N \cdot q\left(\widetilde{\phi}\right) = \frac{M_{cons}}{p\left(\widetilde{\phi}\right)}$. Substituting this result into equations (84) and (85) leads to:

$$\widetilde{\phi} \cdot \left(\widetilde{\phi} + \left(1-\widetilde{\phi}\right) \cdot \left(\frac{r}{w}\right)^{1-\sigma}\right)^{\sigma/(1-\sigma)} \cdot \left(q\left(\widetilde{\phi}\right) + \widetilde{f}\right) \cdot N = L \quad (86)$$

$$\left(1-\widetilde{\phi}\right) \cdot \left(\widetilde{\phi} \cdot \left(\frac{w}{r}\right)^{1-\sigma} + \left(1-\widetilde{\phi}\right)\right)^{\sigma/(1-\sigma)} \cdot \left(q\left(\widetilde{\phi}\right) + \widetilde{f}\right) \cdot N = K. \quad (87)$$

Finally, substituting the factor price ratio $\frac{w}{r}$ in the steady state (equation (16)) into equations (86) and (87) and simplification leads to equations (33) and (34) in the main text.

E $\widetilde{\phi}$ and the steady state capital stock K

The capital stock in the closed economy steady state is given by:

$$K = \underbrace{\frac{1-\widetilde{\phi}}{\widetilde{\phi}^{1/(1-\sigma)}} \cdot \left(\left(1-\frac{1}{\sigma}\right)^{1-\sigma} \cdot \rho^{\sigma-1} - 1 + \widetilde{\phi}\right)^{\sigma/(1-\sigma)}}_{\equiv \Phi} \cdot L. \quad (88)$$

It can be shown that term Φ *in*creases with a decrease in $\widetilde{\phi}$. The partial derivative of Φ with respect to $\widetilde{\phi}$ results as follows:

$$\frac{\partial \Phi}{\partial \widetilde{\phi}} = \frac{\left(\frac{\widetilde{\phi}^{\frac{2-\sigma}{\sigma-1}}}{\sigma-1} - \frac{\sigma \cdot \widetilde{\phi}^{\frac{1}{\sigma-1}}}{\sigma-1}\right) \cdot \left(\left(\frac{1-\frac{1}{\sigma}}{\rho}\right)^{1-\sigma} - 1 + \widetilde{\phi}\right)^{\frac{\sigma}{\sigma-1}} - \left(\widetilde{\phi}^{\frac{1}{\sigma-1}} - \widetilde{\phi}^{\frac{\sigma}{\sigma-1}}\right) \cdot \frac{\sigma}{\sigma-1} \cdot \left(\left(\frac{1-\frac{1}{\sigma}}{\rho}\right)^{1-\sigma} - 1 + \widetilde{\phi}\right)^{\frac{1}{\sigma-1}}}{\left(\left(\frac{1-\frac{1}{\sigma}}{\rho}\right)^{1-\sigma} - 1 + \widetilde{\phi}\right)^{\frac{2\cdot\sigma}{\sigma-1}}}. \quad (89)$$

Since $\left(\frac{1-\frac{1}{\sigma}}{\rho}\right)^{1-\sigma} - 1 + \widetilde{\phi}$ is positive due to assumption 1 (note that $\delta = 0$ for the derivation of K), the denominator of the partial derivative $\frac{\partial \Phi}{\partial \widetilde{\phi}}$ is positive anyway. The sign of the partial derivative $\frac{\partial \Phi}{\partial \widetilde{\phi}}$ therefore depends on the sign of the numerator.

The sign of the numerator of the partial derivative $\frac{\partial \Phi}{\partial \widetilde{\phi}}$ can be determined as follows:

$$\left(\frac{\widetilde{\phi}^{\frac{2-\sigma}{\sigma-1}}}{\sigma-1}-\frac{\sigma\cdot\widetilde{\phi}^{\frac{1}{\sigma-1}}}{\sigma-1}\right)\cdot\left(\left(\frac{1-\frac{1}{\sigma}}{\rho}\right)^{1-\sigma}-1+\widetilde{\phi}\right)^{\frac{\sigma}{\sigma-1}}-\frac{\left(\widetilde{\phi}^{\frac{1}{\sigma-1}}-\widetilde{\phi}^{\frac{\sigma}{\sigma-1}}\right)\cdot\sigma}{\sigma-1}\cdot\left(\left(\frac{1-\frac{1}{\sigma}}{\rho}\right)^{1-\sigma}-1+\widetilde{\phi}\right)^{\frac{1}{\sigma-1}} \lessgtr 0$$

$$\Leftrightarrow \left(\widetilde{\phi}^{\frac{2-\sigma}{\sigma-1}}-\sigma\cdot\widetilde{\phi}^{\frac{1}{\sigma-1}}\right)\cdot\left(\left(\frac{1-\frac{1}{\sigma}}{\rho}\right)^{1-\sigma}-1+\widetilde{\phi}\right)-\left(\widetilde{\phi}^{\frac{1}{\sigma-1}}-\widetilde{\phi}^{\frac{\sigma}{\sigma-1}}\right)\cdot\sigma \lessgtr 0$$

$$\Leftrightarrow \left(\widetilde{\phi}^{-1}-\sigma\right)\cdot\left(\left(\frac{1-\frac{1}{\sigma}}{\rho}\right)^{1-\sigma}-1+\widetilde{\phi}\right)-\left(1-\widetilde{\phi}\right)\cdot\sigma \lessgtr 0$$

$$\Leftrightarrow \left(1-\sigma\cdot\widetilde{\phi}\right)\cdot\left(\left(\frac{1-\frac{1}{\sigma}}{\rho}\right)^{1-\sigma}-1+\widetilde{\phi}\right)-\left(\widetilde{\phi}-\widetilde{\phi}^{2}\right)\cdot\sigma \lessgtr 0$$

$$\Leftrightarrow \left(\frac{1-\frac{1}{\sigma}}{\rho}\right)^{1-\sigma}-1+\widetilde{\phi}-\sigma\cdot\widetilde{\phi}\cdot\left(\frac{1-\frac{1}{\sigma}}{\rho}\right)^{1-\sigma} \lessgtr 0$$

$$\Leftrightarrow \left(1-\sigma\cdot\widetilde{\phi}\right)\cdot\left(\frac{1-\frac{1}{\sigma}}{\rho}\right)^{1-\sigma}-1+\widetilde{\phi} \lessgtr 0. \tag{90}$$

It can be shown that $\lambda \equiv \left(1-\sigma\cdot\widetilde{\phi}\right)\cdot\left(\frac{1-\frac{1}{\sigma}}{\rho}\right)^{1-\sigma}-1+\widetilde{\phi}<0$, i.e. the numerator of the partial derivative $\frac{\partial\Phi}{\partial\widetilde{\phi}}$ is negative:

first, $\lambda<0$ holds at $\widetilde{\phi}=0$ and $\widetilde{\phi}=1$:

$$\widetilde{\phi}=0: \qquad \lambda=\left(\frac{1-\frac{1}{\sigma}}{\rho}\right)^{1-\sigma}-1<0 \quad \text{due to assumption A1;}$$

$$\widetilde{\phi}=1: \qquad \lambda=(1-\sigma)\cdot\left(\frac{1-\frac{1}{\sigma}}{\rho}\right)^{1-\sigma}<0 \quad \text{since } \sigma>1. \tag{91}$$

Second, it can be shown that the partial derivative of λ with respect to $\widetilde{\phi}$ is independent of the level of $\widetilde{\phi}$:

$$\frac{\partial\lambda}{\partial\widetilde{\phi}} = -\sigma\cdot\left(\frac{1-\frac{1}{\sigma}}{\rho}\right)^{1-\sigma}+1. \tag{92}$$

Therefore, λ is also negative for any value between 0 and 1. The partial derivative $\frac{\partial\Phi}{\partial\widetilde{\phi}}$ is therefore negative, i.e. the term Φ *in*crease with a decrease in $\widetilde{\phi}$.

F Proof of result 7

Considering $q(\phi)=M_{cons}\cdot P^{\sigma-1}\cdot p(\phi)^{-\sigma}$ (equation (7)), the pricing rule $p(\phi)=\frac{\sigma}{\sigma-1}\cdot c(\phi)$ and $c(\phi)$ from equation (2), equation (46) can be transformed to:

$$\frac{\phi^*+(1-\phi^*)\cdot\left(\frac{r}{w}\right)^{1-\sigma}}{\phi_X^*+(1-\phi_X^*)\cdot\left(\frac{r}{w}\right)^{1-\sigma}} = \frac{f}{f_X}. \tag{93}$$

Substituting $\frac{r}{w}$ (equation (16)) into equation (93), considering $\widetilde{\phi}=\frac{\phi^*}{2}$ and simplification leads to:

$$\frac{(1-1/\sigma)^{1-\sigma}-0.5\cdot(\rho+\delta)^{1-\sigma}}{\frac{\phi_X^*}{\phi^*}\cdot\left[(1-1/\sigma)^{1-\sigma}-(\rho+\delta)^{1-\sigma}\right]+0.5\cdot(\rho+\delta)^{1-\sigma}} = \frac{f}{f_X}. \tag{94}$$

Equation (94) can be solved for the ratio of the threshold labor share parameters $\frac{\phi_X^*}{\phi^*}$:

G Proof of result 9

This appendix argues that $q\left(\widetilde{\phi}_X\right)=(\sigma-1)\cdot\frac{c(\widetilde{\phi})}{c(\phi_X^*)}\cdot\left(\frac{c(\phi_X^*)}{c(\widetilde{\phi}_X)}\right)^{\sigma}\cdot f_X$ only depends on the parameters σ, ρ, δ, f and f_X and is independent of the steady state values of $\widetilde{\phi}$ and $\widetilde{\phi}_X$.

First, equation (19) implies that the ratio $\frac{c(\widetilde{\phi})}{c(\phi_X^*)}$ depends on σ, ρ, δ and $\frac{\widetilde{\phi}}{\phi_X^*}$; the ratio $\frac{\widetilde{\phi}}{\phi_X^*}$ is equal to $0.5 \cdot \frac{\phi^*}{\phi_X^*}$. Result 7 has shown that $\frac{\phi^*}{\phi_X^*}$ only depends on the parameters σ, ρ, δ, f and f_X.

Second, equation (19) implies that the ratio $\frac{c(\phi_X^*)}{c(\widetilde{\phi}_X)}$ depends on σ, ρ, δ and $\frac{\phi_X^*}{\widetilde{\phi}_X}$. Due to the uniform distribution of ϕ on the unit interval, the ratio $\frac{\phi_X^*}{\widetilde{\phi}_X}$ equals 2.

H Proof of result 10

This appendix argues that the term $\Upsilon = \frac{\phi_X^*}{\phi^*} \cdot \left(\frac{c(\widetilde{\phi}_X)}{c(\widetilde{\phi})} \cdot \frac{q(\widetilde{\phi}_X)}{\sigma-1} - f_X \right)$ is positive.

First, the zero cutoff profit condition for the supply to the foreign market (equation (43)) can be divided by $c\left(\widetilde{\phi}\right)$ to give:

$$\frac{p(\phi_X^*)}{c(\widetilde{\phi})} \cdot \frac{q(\phi_X^*)}{\sigma} - f_X \quad = \quad \frac{c(\phi_X^*)}{c(\widetilde{\phi})} \cdot \frac{q(\phi_X^*)}{\sigma - 1} - f_X \quad = \quad 0. \tag{95}$$

Second, since $\widetilde{\phi}_X = \frac{\phi_X^*}{2}$, equation (18) implies $\frac{\sigma}{\sigma-1} \cdot c\left(\widetilde{\phi}_X\right) \cdot q\left(\widetilde{\phi}_X\right) > \frac{\sigma}{\sigma-1} \cdot c\left(\phi_X^*\right) \cdot q\left(\phi_X^*\right)$. Therefore, if $\frac{c(\phi_X^*)}{c(\widetilde{\phi})} \cdot \frac{q(\phi_X^*)}{\sigma-1} - f_X$ exactly equals zero, $\frac{c(\widetilde{\phi}_X)}{c(\widetilde{\phi})} \cdot \frac{q(\widetilde{\phi}_X)}{\sigma-1} - f_X$ is strictly larger than zero.

I Price index P in the open economy

The price index P in the open economy is given by:

$$\begin{aligned} P \quad &= \quad \left(\int_0^{\phi^*} p(\phi)^{1-\sigma} \cdot \mu(\phi) \cdot N d\phi + \int_0^{\phi_X^*} p(\phi)^{1-\sigma} \cdot \xi(\phi) \cdot N \cdot \frac{\phi_X^*}{\phi^*} d\phi \right)^{1/(1-\sigma)} \\ &= N^{1/(1-\sigma)} \cdot \frac{\sigma}{\sigma - 1} \cdot \left(\int_0^{\phi^*} (\phi \cdot w^{1-\sigma} + (1-\phi) \cdot r^{1-\sigma}) \cdot \mu(\phi) d\phi + \int_0^{\phi_X^*} (\phi \cdot w^{1-\sigma} + (1-\phi) \cdot r^{1-\sigma}) \cdot \xi(\phi) \cdot \frac{\phi_X^*}{\phi^*} d\phi \right)^{1/(1-\sigma)} \\ &= N^{1/(1-\sigma)} \cdot \frac{\sigma}{\sigma - 1} \cdot \left(\widetilde{\phi} \cdot w^{1-\sigma} + \left(1 - \widetilde{\phi}\right) \cdot r^{1-\sigma} + \frac{\phi_X^*}{\phi^*} \cdot \left(\widetilde{\phi}_X \cdot w^{1-\sigma} + \left(1 - \widetilde{\phi}_X\right) \cdot r^{1-\sigma} \right) \right)^{1/(1-\sigma)} \end{aligned} \tag{96}$$

Dividing and multiplying the right–hand side of equation (96) by $\left(1 + \frac{\phi_X^*}{\phi^*}\right)^{1/(1-\sigma)}$, the price index P in the open economy results as follows:

$$\begin{aligned} P \quad = \quad & N^{1/(1-\sigma)} \cdot \frac{\sigma}{\sigma - 1} \\ & \cdot \left[\left(\frac{\widetilde{\phi}}{1 + \frac{\phi_X^*}{\phi^*}} + \frac{\widetilde{\phi}_X \cdot \frac{\phi_X^*}{\phi^*}}{1 + \frac{\phi_X^*}{\phi^*}} \right) \cdot w^{1-\sigma} + \left(1 - \frac{\widetilde{\phi}}{1 + \frac{\phi_X^*}{\phi^*}} - \frac{\widetilde{\phi}_X \cdot \frac{\phi_X^*}{\phi^*}}{1 + \frac{\phi_X^*}{\phi^*}} \right) \cdot r^{1-\sigma} \right]^{1/(1-\sigma)} \cdot \left(1 + \frac{\phi_X^*}{\phi^*}\right)^{1/(1-\sigma)}. \end{aligned} \tag{97}$$

Considering $\frac{\widetilde{\phi}_X}{\widetilde{\phi}} = \frac{\phi_X^*/2}{\phi^*/2} = \frac{\phi_X^*}{\phi^*}$ leads to the following simplification of the price index P:

$$\begin{aligned} P \quad &= \quad N^{1/(1-\sigma)} \cdot \frac{\sigma}{\sigma - 1} \cdot \left[w^{1-\sigma} \cdot \widetilde{\widetilde{\phi}} + r^{1-\sigma} \cdot \left(1 - \widetilde{\widetilde{\phi}}\right) \right]^{1/(1-\sigma)} \cdot \left(1 + \frac{\phi_X^*}{\phi^*}\right)^{1/(1-\sigma)} \\ &= \quad N^{1/(1-\sigma)} \cdot p\left(\widetilde{\widetilde{\phi}}\right) \cdot \left(1 + \frac{\phi_X^*}{\phi^*}\right)^{1/(1-\sigma)}, \qquad \text{with} \quad \widetilde{\widetilde{\phi}} = \frac{\phi^*}{2} \cdot \frac{1 + \left(\frac{\phi_X^*}{\phi^*}\right)^2}{1 + \frac{\phi_X^*}{\phi^*}}. \end{aligned} \tag{98}$$

J Factor market equilibrium conditions in the open economy

Considering $\left(\frac{c(\phi)}{p(\phi)}\right)^{\sigma} = \left(\frac{\sigma-1}{\sigma}\right)^{\sigma}$ leads to the following simplification of equations (57) and (58):

$$P^{\sigma-1} \cdot M \cdot \left(\frac{\sigma-1}{\sigma}\right)^{\sigma} \cdot N \cdot w^{-\sigma} \cdot \left(\int_0^{\phi^*} \phi \cdot \mu(\phi) d\phi + \int_0^{\phi_X^*} \phi \cdot \xi(\phi) \cdot \frac{\phi_X^*}{\phi^*} d\phi\right) + \widetilde{\phi} \cdot w^{-\sigma} \cdot c\left(\widetilde{\phi}\right)^{\sigma} \cdot N \cdot \left(\widetilde{f} + f_X \cdot \frac{\phi_X^*}{\phi^*}\right) = L \quad (99)$$

$$P^{\sigma-1} \cdot M \cdot \left(\frac{\sigma-1}{\sigma}\right)^{\sigma} \cdot N \cdot r^{-\sigma} \cdot \left(\int_0^{\phi^*} (1-\phi) \cdot \mu(\phi) d\phi + \int_0^{\phi_X^*} (1-\phi) \cdot \xi(\phi) \cdot \frac{\phi_X^*}{\phi^*} d\phi\right) + \left(1-\widetilde{\phi}\right) \cdot r^{-\sigma} \cdot c\left(\widetilde{\phi}\right)^{\sigma} \cdot N \cdot \left(\widetilde{f} + f_X \cdot \frac{\phi_X^*}{\phi^*}\right) = K. \quad (100)$$

The term $\int_0^{\phi^*} \phi \cdot \mu(\phi) d\phi + \int_0^{\phi_X^*} \phi \cdot \xi(\phi) \cdot \frac{\phi_X^*}{\phi^*} d\phi$ results as follows:

$$\int_0^{\phi^*} \phi \cdot \mu(\phi) d\phi + \int_0^{\phi_X^*} \phi \cdot \xi(\phi) \cdot \frac{\phi_X^*}{\phi^*} d\phi = \left(1 + \frac{\phi_X^*}{\phi^*}\right) \cdot \left(\frac{\frac{\phi^*}{2}}{1 + \frac{\phi_X^*}{\phi^*}} + \frac{\frac{\phi_X^*}{2} \cdot \frac{\phi_X^*}{\phi^*}}{1 + \frac{\phi_X^*}{\phi^*}}\right) = \left(1 + \frac{\phi_X^*}{\phi^*}\right) \cdot \widetilde{\widetilde{\phi}}, \quad (101)$$

with $\widetilde{\widetilde{\phi}} = \frac{\phi^*}{2} \cdot \frac{1+\left(\frac{\phi_X^*}{\phi^*}\right)^2}{1+\frac{\phi_X^*}{\phi^*}} = \widetilde{\phi} \cdot \frac{1+\left(\frac{\phi_X^*}{\phi^*}\right)^2}{1+\frac{\phi_X^*}{\phi^*}}$. Therefore, the factor market equilibrium conditions can be further simplified to:

$$P^{\sigma-1} \cdot M \cdot \left(\frac{\sigma-1}{\sigma}\right)^{\sigma} \cdot N \cdot w^{-\sigma} \cdot \left(1 + \frac{\phi_X^*}{\phi^*}\right) \cdot \widetilde{\widetilde{\phi}} + \widetilde{\phi} \cdot w^{-\sigma} \cdot c\left(\widetilde{\phi}\right)^{\sigma} \cdot N \cdot \left(\widetilde{f} + f_X \cdot \frac{\phi_X^*}{\phi^*}\right) = L \quad (102)$$

$$P^{\sigma-1} \cdot M \cdot \left(\frac{\sigma-1}{\sigma}\right)^{\sigma} \cdot N \cdot r^{-\sigma} \cdot \left(1 + \frac{\phi_X^*}{\phi^*}\right) \cdot \left(1 - \widetilde{\widetilde{\phi}}\right) + \left(1-\widetilde{\phi}\right) \cdot r^{-\sigma} \cdot c\left(\widetilde{\phi}\right)^{\sigma} \cdot N \cdot \left(\widetilde{f} + f_X \cdot \frac{\phi_X^*}{\phi^*}\right) = K. \quad (103)$$

Substituting $P^{\sigma-1} = N^{-1} \cdot p\left(\widetilde{\widetilde{\phi}}\right)^{\sigma-1} \cdot \left(1 + \frac{\phi_X^*}{\phi^*}\right)^{-1}$ into equations (102) and (103) and considering $\left(\frac{\sigma-1}{\sigma}\right)^{\sigma} = c\left(\widetilde{\widetilde{\phi}}\right)^{\sigma} \cdot p\left(\widetilde{\widetilde{\phi}}\right)^{-\sigma}$ leads to the following simplification:

$$w^{-\sigma} \cdot \widetilde{\widetilde{\phi}} \cdot c\left(\widetilde{\widetilde{\phi}}\right)^{\sigma} \cdot \frac{M}{p\left(\widetilde{\widetilde{\phi}}\right)} + w^{-\sigma} \cdot \widetilde{\phi} \cdot c\left(\widetilde{\phi}\right)^{\sigma} \cdot N \cdot \left(\widetilde{f} + f_X \cdot \frac{\phi_X^*}{\phi^*}\right) = L \quad (104)$$

$$r^{-\sigma} \cdot \left(1 - \widetilde{\widetilde{\phi}}\right) \cdot c\left(\widetilde{\widetilde{\phi}}\right)^{\sigma} \cdot \frac{M}{p\left(\widetilde{\widetilde{\phi}}\right)} + r^{-\sigma} \cdot \left(1-\widetilde{\phi}\right) \cdot c\left(\widetilde{\phi}\right)^{\sigma} \cdot N \cdot \left(\widetilde{f} + f_X \cdot \frac{\phi_X^*}{\phi^*}\right) = K. \quad (105)$$

Factoring out the terms $w^{-\sigma} \cdot \widetilde{\phi} \cdot c\left(\widetilde{\phi}\right)^{\sigma}$ and $r^{-\sigma} \cdot \left(1-\widetilde{\phi}\right) \cdot c\left(\widetilde{\phi}\right)^{\sigma}$ results in:

$$w^{-\sigma} \cdot \widetilde{\phi} \cdot c\left(\widetilde{\phi}\right)^{\sigma} \cdot \left(\frac{M}{p\left(\widetilde{\widetilde{\phi}}\right)} \cdot \Delta_1 \cdot \left(\frac{c\left(\widetilde{\phi}\right)}{c\left(\widetilde{\widetilde{\phi}}\right)}\right)^{-1} + N \cdot \left(\widetilde{f} + f_X \cdot \frac{\phi_X^*}{\phi^*}\right)\right) = L \quad (106)$$

$$r^{-\sigma} \cdot \left(1-\widetilde{\phi}\right) \cdot c\left(\widetilde{\phi}\right)^{\sigma} \cdot \left(\frac{M}{p\left(\widetilde{\widetilde{\phi}}\right)} \cdot \Delta_2 \cdot \left(\frac{c\left(\widetilde{\phi}\right)}{c\left(\widetilde{\widetilde{\phi}}\right)}\right)^{-1} + N \cdot \left(\widetilde{f} + f_X \cdot \frac{\phi_X^*}{\phi^*}\right)\right) = K, \quad (107)$$

$$\text{with} \quad \Delta_1 = \frac{\widetilde{\widetilde{\phi}}}{\widetilde{\phi}} \cdot \left(\frac{c\left(\widetilde{\phi}\right)}{c\left(\widetilde{\widetilde{\phi}}\right)}\right)^{1-\sigma} \quad \text{and} \quad \Delta_2 = \frac{1-\widetilde{\widetilde{\phi}}}{1-\widetilde{\phi}} \cdot \left(\frac{c\left(\widetilde{\phi}\right)}{c\left(\widetilde{\widetilde{\phi}}\right)}\right)^{1-\sigma}.$$

It can be shown that both Δ_1 and Δ_2 only depend on the parameters ρ, δ, σ, f and f_X (note that $\delta = 0$ for the derivation of the factor market equilibrium conditions). First, the ratios $\frac{\tilde{\tilde{\phi}}}{\tilde{\phi}}$, $\frac{1-\tilde{\tilde{\phi}}}{1-\tilde{\phi}}$ and $\frac{c\left(\tilde{\tilde{\phi}}\right)}{c\left(\tilde{\phi}\right)}$ result as follows:

$$
\begin{aligned}
(i) \qquad & \frac{\tilde{\tilde{\phi}}}{\tilde{\phi}} &=& \frac{1+\left(\frac{\phi_X^*}{\phi^*}\right)^2}{1+\frac{\phi_X^*}{\phi^*}}, \\
(ii) \qquad & \frac{1-\tilde{\tilde{\phi}}}{1-\tilde{\phi}} &=& \frac{\phi_X^*}{\phi^*} \quad \text{and} \\
(iii) \qquad & \frac{c\left(\tilde{\tilde{\phi}}\right)}{c\left(\tilde{\phi}\right)} &=& \left(\frac{\frac{\tilde{\tilde{\phi}}}{\tilde{\phi}} \cdot \left(\left(1-\frac{1}{\sigma}\right)^{1-\sigma} - (\rho+\delta)^{1-\sigma}\right) + (\rho+\delta)^{1-\sigma}}{\left(1-\frac{1}{\sigma}\right)^{1-\sigma}}\right)^{1/(1-\sigma)}
\end{aligned}
$$

Second, result 7 has shown that $\frac{\phi_X^*}{\phi^*}$ only depends on the parameters ρ, δ, σ, f and f_X. The ratios $\frac{\tilde{\tilde{\phi}}}{\tilde{\phi}}$, $\frac{1-\tilde{\tilde{\phi}}}{1-\tilde{\phi}}$ and $\frac{c\left(\tilde{\tilde{\phi}}\right)}{c\left(\tilde{\phi}\right)}$ therefore only depend on these parameters as well.

Finally, combining equations (7) and (10) implies $q\left(\tilde{\phi}\right) \cdot N = \frac{M}{p\left(\tilde{\phi}\right)}$, i.e. $q\left(\tilde{\phi}\right) \cdot N \cdot \frac{c\left(\tilde{\phi}\right)}{c\left(\tilde{\tilde{\phi}}\right)} = \frac{M}{p\left(\tilde{\phi}\right)} \cdot \frac{p\left(\tilde{\phi}\right)}{p\left(\tilde{\tilde{\phi}}\right)}$. Substituting this into equations (106) and (107) leads to:

$$
w^{-\sigma} \cdot \tilde{\phi} \cdot c\left(\tilde{\phi}\right)^{\sigma} \cdot \left(q\left(\tilde{\phi}\right) \cdot N \cdot \Delta_1 + N \cdot \left(\tilde{f} + f_X \cdot \frac{\phi_X^*}{\phi^*}\right)\right) = L \tag{108}
$$

$$
r^{-\sigma} \cdot \left(1-\tilde{\phi}\right) \cdot c\left(\tilde{\phi}\right)^{\sigma} \cdot \left(q\left(\tilde{\phi}\right) \cdot N \cdot \Delta_2 + N \cdot \left(\tilde{f} + f_X \cdot \frac{\phi_X^*}{\phi^*}\right)\right) = K. \tag{109}
$$

Substituting $c(\phi)$ (equation (2)) into equations (108) and (109) and considering $\frac{w}{r}$ in the steady state (equation (16)) finally leads to:

$$
\tilde{\phi} \cdot \left(\frac{\tilde{\phi} \cdot (1-1/\sigma)^{1-\sigma}}{(1-1/\sigma)^{1-\sigma} - \left(1-\tilde{\phi}\right) \cdot \rho^{1-\sigma}}\right)^{\sigma/(1-\sigma)} \cdot \left(q\left(\tilde{\phi}\right) \cdot \Delta_1 + \tilde{f} + f_X \cdot \frac{\phi_X^*}{\phi^*}\right) \cdot N = L \tag{110}
$$

$$
\left(1-\tilde{\phi}\right) \cdot \frac{(1-1/\sigma)^{\sigma}}{\rho^{\sigma}} \cdot \left(q\left(\tilde{\phi}\right) \cdot \Delta_2 + \tilde{f} + f_X \cdot \frac{\phi_X^*}{\phi^*}\right) \cdot N = K. \tag{111}
$$

K Proof of result 12

The capital stock in the open economy steady state is given by:

$$
K = \underbrace{\frac{1-\tilde{\phi}}{\tilde{\phi}^{1/(1-\sigma)}} \cdot \left(\left(1-\frac{1}{\sigma}\right)^{1-\sigma} \cdot \rho^{\sigma-1} - 1 + \tilde{\phi}\right)^{\sigma/(1-\sigma)}}_{\equiv \Phi} \cdot \underbrace{\frac{q\left(\tilde{\phi}\right) \cdot \Delta_2 + \tilde{f} + f_X \cdot \frac{\phi_X^*}{\phi^*}}{q\left(\tilde{\phi}\right) \cdot \Delta_1 + \tilde{f} + f_X \cdot \frac{\phi_X^*}{\phi^*}}}_{\equiv \Psi} \cdot L. \tag{112}
$$

The equation for the capital stock in the open economy differs from the equation for the capital stock in the closed economy in two respects: the term Φ differs between the closed and the open economy steady state due to the decrease of $\tilde{\phi}$ with trade liberalization; furthermore, the term Ψ adds to the right–hand side.

Appendix E has shown that the term Φ *in*creases with a decrease in $\tilde{\phi}$.

Furthermore, it can be shown that the term Ψ is strictly larger than unity as long as $\frac{\phi_X^*}{\phi^*} > 0$, i.e. as long as some firms export. The only differences between the numerator and the denominator of the term Ψ are the terms Δ_1 and Δ_2. The terms Δ_1 and Δ_2 are given as follows:

$$
\Delta_1 = \frac{\tilde{\tilde{\phi}}}{\tilde{\phi}} \cdot \left(\frac{c\left(\tilde{\phi}\right)}{c\left(\tilde{\tilde{\phi}}\right)}\right)^{1-\sigma}, \qquad \Delta_2 = \frac{1-\tilde{\tilde{\phi}}}{1-\tilde{\phi}} \cdot \left(\frac{c\left(\tilde{\phi}\right)}{c\left(\tilde{\tilde{\phi}}\right)}\right)^{1-\sigma}.
$$

The only difference between Δ_1 and Δ_2 are the ratios $\frac{\widetilde{\widetilde{\phi}}}{\widetilde{\phi}}$ and $\frac{1-\widetilde{\widetilde{\phi}}}{1-\widetilde{\phi}}$. Since $\widetilde{\widetilde{\phi}}$ equals $\widetilde{\phi} \cdot \frac{1+\left(\frac{\phi_X^*}{\phi^*}\right)^2}{1+\frac{\phi_X^*}{\phi^*}}$ and since $0 < \frac{\phi_X^*}{\phi^*} \leq 1$ in the open economy, it follows $\widetilde{\widetilde{\phi}} < \widetilde{\phi}$. Therefore, $\Delta_2 > \Delta_1$ and, finally, $\Psi > 1$.

The capital stock K therefore increase with trade liberalization since the term Φ increase with the decrease in $\widetilde{\phi}$ and since the additional term Ψ is larger than unity.

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figure 1a

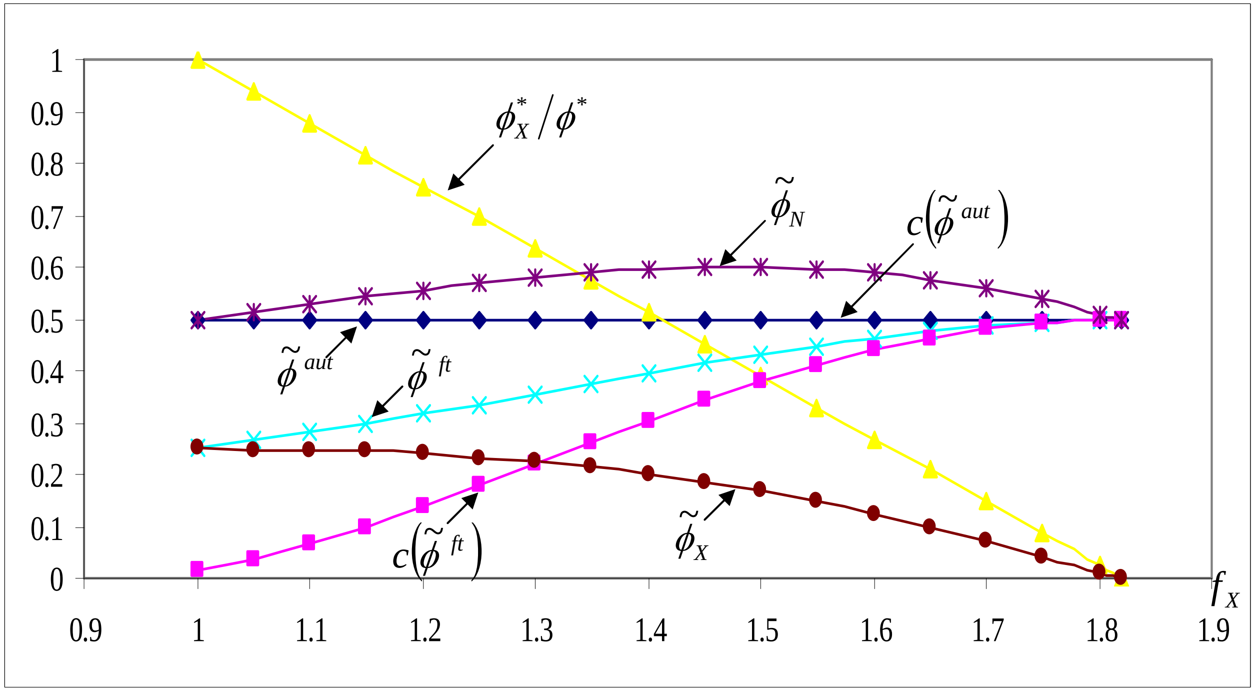


figure 1b

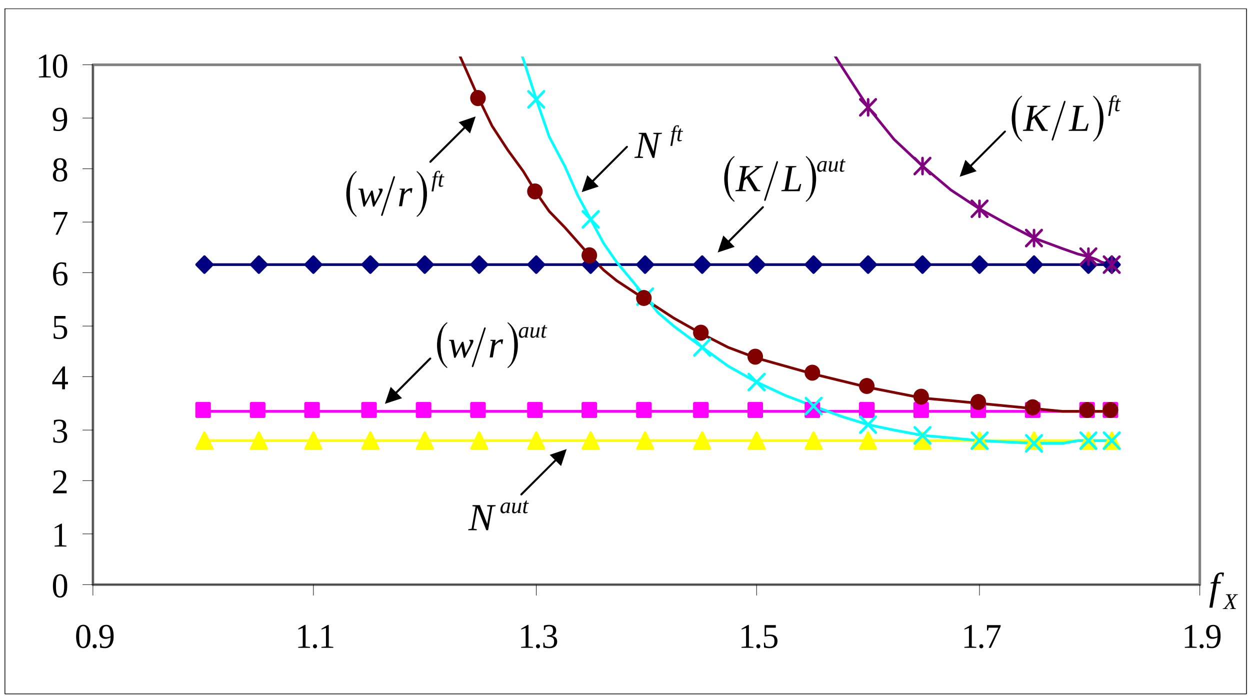
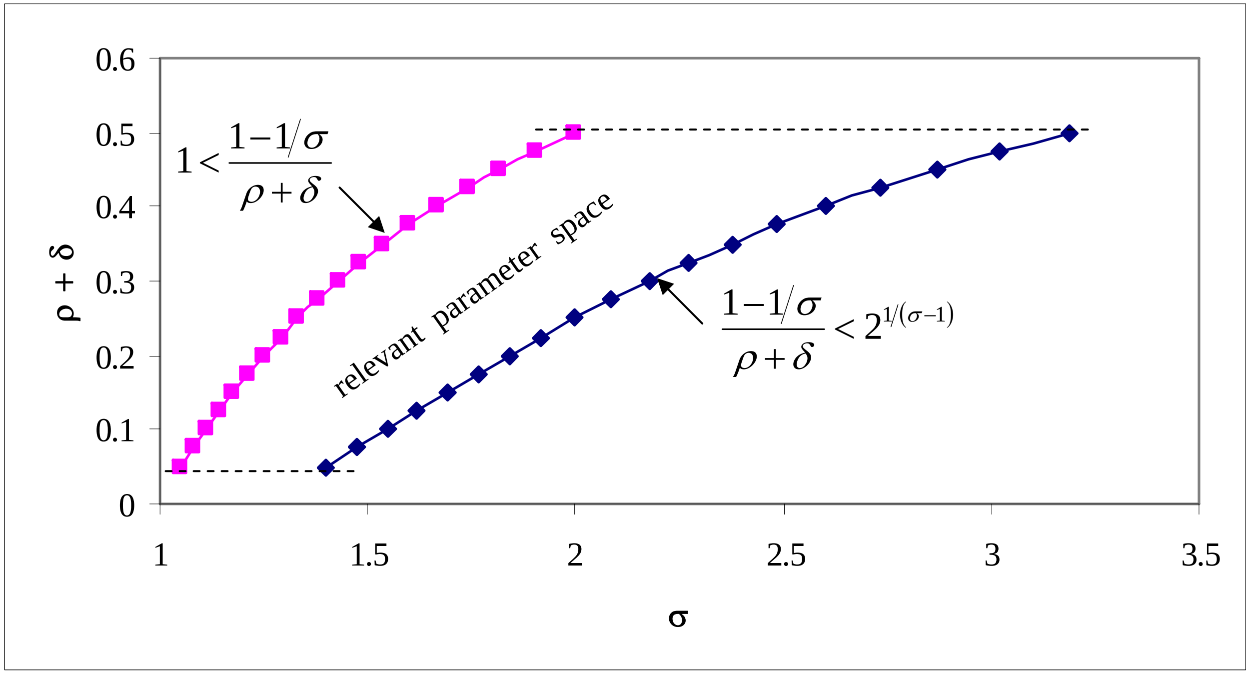


figure A1



0.6
0.5
0.4
0.3
0.2
0.1
0
ρ + δ
1
1.5
2
2.5
3
3.5
σ
$1 < \frac{1-1/\sigma}{\rho+\delta}$
relevant parameter space
$\frac{1-1/\sigma}{\rho+\delta} < 2^{1/(\sigma-1)}$