Household Relational Contracts, Fertility and Divorce

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Abstract

The theory of relational contracts is applied to a household model where players decide about the number of children, the form of their relationship and whether to stay together or get separated. We make precise the idea how cooperation can be enforced within a setup where players are solely driven by their self interest. Since there is a potential conflict between individually optimal and efficient decisions, self-enforcing transfers help to support cooperation. Cooperation is restricted by incentive compatibility and enforcement conditions, which state that the future prospects induced by currently costly cooperative behaviour have to be sufficiently high. Thus Becker´s claim that the Coase theorem can be applied to divorce decisions is not true if transfers necessary to support cooperation cannot be enforced. If divorce is associated with sufficiently high costs, marriage can be used as a commitment device. On the other hand, too high separation barriers make divorce threats uncredible, thus reducing punishment possibilities and incentives to cooperate.

JEL Classification: C73, D13, J12, J13, J24

1 Introduction

The casual assumption that family members always act cooperatively and necessarily get Pareto efficient outcomes has been challenged beginning with Ott (1992) and later continued by Konrad, Lommerud (2000) or Lundberg and Pollak (2003). If a household determines its resource allocation using Nash bargaining and if its members cannot commit to particular actions in the future, the resulting allocations are not necessarily Pareto efficient. But as households are engaged in a repeated game, cooperative behavior leading to Pareto efficient outcomes can even be sustained if household members are driven by self interest and not able to make binding commitments. A non-cooperative equilibrium serves as the off-equilibrium punishment supporting cooperation. Nevertheless, in those models the repeated interaction is not modelled explicitly and constraints ensuring enforcement of cooperative behaviour are rather assumed non-binding, making it sufficient to look at a one-shot game. Although not played in equilibrium, the non-cooperative outcome determines the resulting allocation, since Nash-bargaining assigns each player their outside utilities plus a fixed share of the cooperation surplus. We think that the theory of relational contracts[1] gives an appropriate tool to get over these drawbacks and helps to gain new insights of decision making within households. We develop a general fertility model within an infinitely repeated game. Since children are modelled as a public good, individual utility maximizing often leads to an inefficiently low fertility level, giving room for Pareto improvements. Divorce can potentially be used as a punishment following non-cooperative behaviour. Constraints ensuring incentive compatibility and enforceability of transfers are crucial determinants for the resulting outcomes. Furthermore, outside options do not fully determine the resource allocations, but rather place a lower bound on the payoffs players receive.

Since divorce is associated with different kinds of costs, marriage can serve as a commitment device to support voluntary transfers, a thought also captured by Rowthorn (1999) or Matouschek and Rasul (2008). But if divorce barriers are too high, they can also impede cooperation as a divorce threat becomes uncredible if it is not optimal for a player to file for divorce after non-cooperation. Furthermore, divorce on the equilibrium path reduces cooperation possibilities by decreasing future rewards of cooperation. The result that divorce reduces the enforceability of cooperation goes back to Lommerud (1989), who assumes that cooperation is driven by "voice enforcement" rather then players´ self interest within a repeated setup and is supported empirically by Lundberg and Rose (1999), where a higher divorce risk is associated with lower levels of specialization. We derive explicitly the role separation or divorce plays for the enforcement of transfers to enhance cooperation and increase fertility and reject Becker´s (1993) claim that the Coase theorem can be applied to divorce decisions. If necessary transfers to maintain a relationship cannot be enforced, an efficient relationship cannot be maintained. The main contribution of our paper is therefore including enforcement problems into household decision making without loosing insights gained by previous approaches using Nash bargaining.

2 Model Setup

A household, founded at the beginning of the game, consists of a primary and a secondary earner ($i = 1, 2$). In any period $t = 1, 2, 3, ...$, the primary earner spends his total working time (normalized to 1) on the labour market, while player 2 divides her time between working and raising children. In each of the

[1] Initially developed for labour markets and agency situations, see Bull (1987), MacLeod/Malcomson (1989) or Levin (2003).

periods 1 and 2, exactly one child can be born, causing total monetary costs c, paid by player 1. Total time needed to raise one child amounts to g. For simplicity, we assume that g only accrues in the period the child is born[2] and that $g \leq 1$.

Per period utility functions amount to $u_{it} = x_{it} + \varphi(n_t)$, $\varphi(0) = 0$, $\varphi\prime > 0$ and $\varphi\prime\prime \leq 0$, where x is a private consumption good. The wage both receive depends on previous experience. This experience is captured by the variable

$$D_{it} = d_{i1} + \rho d_{i2} + ... + \rho^{t-2} d_{it-1} \tag{1}$$

($0 < \rho < 1$ and $D_1 = 0$), where $d_{1t} = 1$ and $d_{2t} = 1 - g_{nt}$ denotes players´ working times in period t. This results in wage functions $w_i(D_{it})$, with $w_i' > 0$ and $w_i'' \leq 0$.

As experience gained later has less weight in D, any experience loss at the beginning cannot fully be substituted by later working time.

Living together, the timing withing one period is the following:

1. If not happened yet, the partners can get married or stay together unmarried

2. Player 1 gives player 2 a voluntary transfer b_t. This transfer can also adopt negative values, meaning that it is effectively paid by the secondary earner.

3. In periods 1 and 2, they decide whether to get one additional child (decision must be unanimously).

4. Both work and receive their wages; if present, player 2 also raises their child, while player 1 pays c.

5. An additional transfer $\bar{b}_t$ can be made.

6. Both players observe the realizations of continuation outside utilities for the next period, $V_{t+1} = (V_{1t+1}, V_{2t+1})$, $V_{it} \in \{V^l, V^h\}$, $V^l, V^h \in]-\infty; \infty[$, $V^l < V^h$.

7. The decision whether to continue the relationship is made. If both want to stay together, it is sustained for another period. In any other case, the relationship is terminated, without any option to get together again in the future.

Note that step 5 only plays a role in periods 1 and 2, since in periods where getting children is not possible, everything between steps 3. and 5. is fixed, giving no room for strategic decisions.

A termination of the relationship has the following consequences:

[2]This does not influence any qualitative results

- Both receive their outside utilities, which can adopt the values V^h or V^l, with $V^h > V^l$. These are continuation values, but for simplicity we assume that they - although not monetary - can be equally distributed over periods. V_{it+1} can either capture issues outside the relationship like potential new partners or issues within like love or caring and is not restricted to positive values. We denote Prob$\{V_{1t} = V^h \mid n\} = p_n$ and Prob$\{V_{2t} = V^h \mid n\} = q_n$. Here, we only allow for two exogenously given parameter values with probabilities dependent on the number of children present, but in principle V_i^{t+1}could be chosen more generally, for example making it continous or time dependent. As this only complicates issues without providing really new insights, we use a relatively simple characterization. Furthermore, we do not make any assumptions concering the causal impact of the number of children on probalities, although it seems likely that p_n/q_n are decreasing in n. Note that V does not include any monetary transfer between spouses.

- Furthermore the per period utility from children of the primary earner after a split amounts to $\theta u(n)$, $\theta \leq 1$. Here, we want to account for potentially different legislations determining the access of the primary earner to his children.

If the couple was married (we assume that divorce without termination and termination without divorce is not possible), a divorce has two further effects:

- At the beginning of the first period following, each has to bear exogenous divorce costs k_i (can be monetary or not).

- Player 2 receives a monetary transfer $\phi * (w_{1t} - w_{2t})$, $0 \leq \phi \leq 1/2$, from player 1 in each subsequent period. This redistribution rule is independent from the number of children, since monetary education costs have already been taken account for in the period the child was born. Making it dependent on the wage difference only at the date of divorce would not change anything qualitatively.

All other parts of the within-relationship utilities remain unchanged and are added to the values just described.

3 Household Relational Contracts (HRC)

Players have to decide about the form of their relationship, the number of children, voluntary bonus payments and whether they get divorced or stay together. We assume that they formulate a Household Relational Contract (HRC) which specifies all actions players have to take conditional on all possible histories[3]. But it is not possible to write binding contracts that are contingent on actions

[3]For a complete characterisation of HRC´s see Fahn and Rees (2009).

or outcomes[4]. Thus the actions in any period have to form a subgame perfect equilibrium. We deviate from the "standard" relations contracts literature assuming that any transfer has to be self-enforcing. Thus it is not possible to write explicit contracts on some fixed transfer which is used to distribute the surplus.

As any transfer has to be self enforcing and therefore form a subgame perfect equilibrium, any player reneging on paying a bonus part of the HRC has to be punished. We follow MacLeod, Malcomson (1989) assuming that after someone did not stick to the arrangement, any trust between the partners is lost and the relationship is soured. Thus the harshest possible punishment is used (Abreu (1986)), implying that the subgame perfect equilibrium with the lowest payoff for the player that reneged is played. In the relational contract literature, this corresponds to a termination of the relationship. In our setup, separation can also be used. But recall that separation/divorce is connected with costs and can therefore only be used if it is rational for at least one player.

Furthermore, we use the following formulations:

$U_{it}^0 = \sum\limits_{\tau=t}^{\infty} \delta^{\tau-t} u_{i\tau}^0$:= Continuation utility of player i in period t within relationship if no voluntary transfers are made in any period $\tau \geq t$.

$\hat{U}_{it} = \sum\limits_{\tau=t}^{\infty} \delta^{\tau-t} \hat{u}_{i\tau}$:= Continuation utility of player i in period t if divorce takes place in period t.

We further distinguish whether players still believe in their partner´s goodwill and are therefore willing to cooperate or not. In the first case, the game is situated in the cooperative state (C). If the relationship is soured and cooperation cannot be sustained anymore, it is in the non-cooperative state (NC). Whether we are in (C) or (NC) only depends on if promised transfers have been paid or not. All other actions can be rewarded or punished by bonus payments, therefore do not directly influence the state in which the game is situated. Summing up, (C) corresponds to being on the equilibrium path within a HRC, while (NC) describes situations off-equilibrium necessary to back up equilibrium.

The transition rule follows a simple trigger strategy. If the game is in (C) at the beginning of a period and if all promised transfers are paid, the following period remains in the cooperative state. If at least one partner reneged on a promised bonus, the game moves to (NC) and stays there forever, as any reputation regarding cooperation is destroyed. Finally, we assume that the game starts in (C).

Continuation utility of player i in period t in state (C) is denoted U_{it}^C, while U_{it}^{NC} describes this utility in the non-cooperative state.

We use state-contingent utilities because different cases are possible, on and off the equilibrium path. In (NC) for example, divorce can only be used as punishment if it is optimal for at least one player to file for divorce compared to staying together without voluntary transfers. Therefore, divorce can be used

[4] Of course, this assumptions can be questioned when it comes to the number of children, which certainly is verifiable. But it is hard to imagine that a court would enforce contracts determining payments contingent on the number of children.

as a punishment either always, never or just for some realizations of outside utilities (meaning that it would not necessarily take place in periods directly following a reneging). In the cooperative state, many different cases are possible as well. The relationship could be separated for some or all realizations of outside utilities and transfers could be paid or not. Using state-contingent utilities therefore enables us to give a general characterization of the results without having to look at all possible cases separately.

Since divorce utilities depend on a player´s outside options, we write $\hat{U}_{it}(V_{it})$. Furthermore, it is also possible that cooperation utilities are contingent on outside options via transfers. As bonus payments can be used to distribute a surplus or to sustain cooperation, their values will generally depend on V_t. For a given realization of V_t we write $U_{it}^C(V_t)$, while U_{it}^C denotes expected cooperation utilities (seen from periods earlier than t, when period t outside utilities are not known yet).

All values can be defined recursively, with

$$U_{it}^C(V_t) = u_{it}^C(V_t) + \delta U_{it+1}^C \quad (2)$$

$$U_{it}^{NC}(V_t) = u_{it}^{NC}(V_t) + \delta U_{it+1}^{NC} \quad (3)$$

$$U_{it}^0 = u_{it}^0 + \delta U_{it+1}^0 \quad (4)$$

If a relationship exists in period t in state C that includes transfers, we can write $U_{2t}^C(V_t) = u_{2t}^0 + b(V_t) + \delta U_{2t+1}^C$. Writing down divorce utilities recursively does not help anything, since once the couple is divorced, the relationship cannot change to a different form anymore.

Contrary to the standard relational contracts models, the HRCs in this setup are not stationary.We have not said anything about savings until now, as players´ risk neutrality makes them irrelevant, but from now on we will implicitly assume that players could save or borrow at the interest rate δ without explicitly including it into our model.

Since the periods $t = 1, 2$ - where the couple is able to get children - are quite different from the following ones, we also treat them separately, using backwards induction and therefore start analysing periods $t \geq 3$. There, fertility decisions have already been made and efficiency gains through cooperation can only be by obtained by a continuation of the marriage. Furthermore, we first assume that the couple marries at the beginning of period 1 and later analyze how the additional option to marry at any date t influences our results.

Finally, any surplus not required for efficiency increases is distributed according to a Household Welfare Function. We do not introduce such a function here explicitly. For the use of a Household Welfare Function within a HRC framework see Fahn, Rees (2009).

4 Household Relational Contracts Taking Marriage Bbefore $t = 1$ As Given

4.1 Periods $t \geq 3$

Recall that transfers $\bar{b}_t$ are irrelevant here, why we omit them in this chapter. For any realization of outside utilities, V_1 and V_2, divorce will take place, if

$$\widehat{U}_{1t}(V_{1t}) + \widehat{U}_{2t}(V_{2t}) > U_{1t}^0 + U_{2t}^0 \tag{5}$$

Note that the optimality of divorce cannot be influenced by transfers between the partners, since those would cancel out taking utility sums. If this inequality does not hold, we call a continuation of the marriage efficient, since the total surplus of divorce is lower.

But the marriage is only continued if it is either in the interest of both or if one partner is able to compensate the other one for staying together. Assume we are at the end of period $t-1$, and both have to decide whether to file for divorce, taking their outside utilities V_t into account. If $U_{it}^0 \geq \widehat{U}_{it}(V_{it})$, $i = 1, 2$, both want to stay together anyway. If this condition only holds for one partner, a transfer needed to prevent the other to file for divorce. Then, Incentive Compatibility (IC) constraints ensure that the bonus is high enough to make staying together optimal for both, leading to[5]:

(IC 1)

$$u_{1t}^0 - b_t(V_t) + \delta U_{1t+1}^C \geq \widehat{U}_{1t}(V_{1t}) \tag{6}$$

(IC 2)

$$u_{2t}^0 + b_t(V_t) + \delta U_{2t+1}^C \geq \widehat{U}_{2t}(V_{2t}) \tag{7}$$

Note that a bonus to fulfill (IC 1) and (IC 2) can always be found if $U_{1t}^0 + U_{2t}^0 \geq \widehat{U}_{1t}(V_{1t}) + \widehat{U}_{2t}(V_{2t})$. This coincides with Becker´s (1991) claim that - applying the Coase theorem - divorce takes place if and only if total gains are higher than remaining together. But since bonus payments are costly and not legally enforceable, a player paying a transfer has to be rewarded by future prospects. As reneging on paying a promised bonus leads to a move to the non-cooperative state, the discounted utility in state (C) has to be sufficiently higher than that in (NC) to induce a player to pay a transfer. Thus any positive bonus payment is enforceable if $b_t(V_t) \leq \delta(U_{1t+1}^C - U_{1t+1}^{NC})$ and any negative if $-b_t(V_t) \leq \delta(U_{2t+1}^C - U_{2t+1}^{NC})$. As the bonus is paid before the realizations of outside utilities of period $t+1$ are known, the right hand sides of the conditions represent expected values and are identical for all realizations of V_t. Therefore, dynamic enforcement (DE) constraints have to hold for all possible bonus payments, leading to the following conditions:

(DE 1):

$$\max b_t \leq \delta(U_{1t+1}^C - U_{1t+1}^{NC}) \tag{8}$$

[5] To be fully correct, we would have to multiply both sides with δ, but omit it for simplicity

(DE 2)

$$-\min b_t \leq \delta(U^C_{2t+1} - U^{NC}_{2t+1}) \tag{9}$$

Contrary to MacLeod/Malcomson (1989) or Levin (2003), adding both (DE) constraints does not necessarily give a sufficient condition for the possible set of bonus payments. Here,

$$\max b_t - \min b_t \leq \delta(U^C_{1t+1} + U^C_{2t+1} - U^{NC}_{1t+1} - U^{NC}_{2t+1}) \tag{10}$$

is only necessary but not sufficient, implying that the total surplus cannot be distributed arbitrarily. Assume for example that $U^0_{1t} > \hat{U}_{1t}(V_{1t})$ and $\hat{U}_{2t}(V_{2t}) > U^0_{2t}$. Then, due to (8) a positive bonus is needed to maintain relationship in period 2 which means that player 1 has to receive part of the net gain from staying together in the future.

Examples

The conditions derived so far give a general overview of what is possible or necessary within a relationship, including a lot of possible cases. Now, we want to see what exactly influences efficiency or enforcement problems. Thus we will look at special cases and make some simplifying assumptions. The wage function adopts the functional form $w_i(D_{it}) = D_{it}\varpi_i + \xi_i$, with ϖ_i and ξ_i exogenously given, positive and constant over time. Furthermore, we assume $\varpi_1 = \varpi_2$ and $\xi_1 = \xi_2$ and analyse impacts of different wage functions later. This simplifies the analysis, since after-divorce transfers are constant over time and independent from when the marriage is separated. $\hat{U}_{it} - U^0_{it}$ is also constant over time. When analyzing what is possible for such contracts, we differentiate between the aims of getting an efficient result and of redistributing the surplus in any way we want.

Getting an efficient result means staying married when the total gain of remaining together exceeds total divorce utility and getting divorced when it is optimal. The last case never is a problem. If it is efficient to get divorced, this will happen.

When it is efficient to stay married in a period, it can already be in the interest of both players (implying $U^0_{it} \geq \hat{U}_{it}(V_{it})$, $i = 1, 2$). Then transfers - if enforceable - are only used to redistribute the surplus. If $\hat{U}_{it}(V_{it}) > U^0_{it}$ for one player, a bonus is needed to reach an efficient outcome. In both cases, we are mainly interested in factors that influence the enforceability of transfers.

First, assume that for realisations(V^h, V^h) staying together is efficient (meaning that the same is true in all other cases as well). If $U^0_{it} > \hat{U}_{it}(V^h)$, $i = 1, 2$, no bonus payments are needed to prevent divorce, but it is also not possible to redistribute the surplus, leading to a situation without any transfers between partners. The reason is that in state (NC), no one files for divorce, since it is not rational to do so. A positive bonus is not possible because this makes player 1 worse off than no bonus at all, while the same is true for player 2 if the bonus is supposed to be negative.

Now we have $\hat{U}_{2t}(V^h) > U^0_{2t}$, meaning that a positive bonus payment is needed to maintain the relationship. The minimum amount of this transfer is determined by the (IC) constraint of player 2, which equals $U^C_{2t}(V_{1t}, V^h) \geq \widehat{U}_{2t}(V^h)$.

Inserting values and rearranging gives the minimum amount of b for 2 not filing for divorce (assume we can play an equilibrium where divorce never happens; time subscript for number of children are omitted since they are given in periods $t \geq 3$; b_t denotes expected values)

$$b_t(V_{1t}, V^h) \geq \frac{1}{1-\delta}\phi(w_{1t} - w_{2t}) - \sum_{\tau=t+1}^{\infty} \delta^{\tau-t} b_\tau + V^h - k_2 \tag{11}$$

One upper bound for the transfer is given by (IC 1)

$$b(V_{1t}, V^h) \leq \frac{1}{1-\delta}(1-\theta)u(n) + \frac{1}{1-\delta}\phi(w_{1t} - w_{2t}) - \sum_{\tau=t+1}^{\infty} \delta^{\tau-t} b_\tau - V_{1t} + k_1 \tag{12}$$

Combining both constraints gives

$$V^h - k_2 - [k_1 + \frac{1}{1-\delta}(1-\theta)u(n) - V_{1t}] \leq 0 \tag{13}$$

Equation (13) identifies the condition when continuing marriage is efficient. The first two terms describe the gain net of transfers of divorce of player 2, while the term in brackets denotes the net gain of staying together for player 1. As already mentioned above, if it is efficient to stay together, it is always possible to find a bonus that fulfills both (IC) constraints. Thus, the (DE) constraint of player 1 is decisive for the enforceability of a transfer [6]. There, it is further crucial when 2 actually files for divorce in the non-cooperative state. If $\hat{U}_{2t}(V^l) > U^0_{2t}$, she will do so in any case, yielding

$$b_t(V_{1t}, V^h) \leq \delta[U^C_{1t+1} - \widehat{U}_{1t+1}] \tag{14}$$

and

$$b_t(V_{1t}, V^h) \leq \delta[\frac{1}{1-\delta}(1-\theta)u(n) - \sum_{\tau=t+1}^{\infty} \delta^{\tau-(t+1)} b_\tau + \frac{1}{1-\delta}\phi(w_{1\tau} - w_{2\tau}) - (p_n V^h + (1-p_n)V^l) + k_1] \tag{15}$$

Adding equations (11) and (15) leads to the necessary and sufficient condition for the enforceability of a bonus necessary to maintain the relationship if $\hat{U}_{2t+1}(V^l) > U^0_{2t+1}$ and $V_{2t} = V^h$:

$$V^h - k_2 + \phi(w_{1t} - w_{2t}) - \delta[k_1 + \frac{1}{1-\delta}(1-\theta)u(n) - p_n V^h - (1-p_n)V^l] \leq 0 \tag{16}$$

[6] Actually, the stricter of the both constraints (IC 1) and (DE 1) determines player 1´s boundaries of the bonus. But if (IC 1) is stricter, this is not a problem if staying together is efficient.

Comparing equations (13) and (16) helps to work out potential conflicts between efficiency and enforceability of remaining married and therefore show when Becker´s claim is not true. Two parts in equation (16) complicate the continuation of an efficient marriage. The mandatory transfer $\phi(w_{1t} - w_{2t})$ does not cancel out as in (13), since (15), the (DE) constraint of player 1, only incorporates future periods. Thus a sufficiently high value of ϕ makes cooperation for 2 too unattractive. Furthermore, the discounting δ can impose a problem. Player 1 is compensated by a continuation of the marriage for behaving cooperatively. As cooperation entails current costs but future rewards, low values of δ can discount cooperation gains too much to enforce a necessary transfer. Finally, high values of p_n will complicate cooperation.

If $\hat{U}_{2t}(V^l) < U^0_{2t}$ cooperation is further impeded. Then, player 2 only files for divorce in (NC) when her outside utility is high. This limits her potential to punish non-cooperative behaviour of 1, since actual punishment only occurs with probability q_n. With probability $(1-q_n)$, 2 remains within the relationship for at least one additional period. (DE 1) now becomes

$$b_t(V_{1t}, V^h) \leq \delta[U^C_{1t+1} - U^{NC}_{1t+1}] \tag{17}$$

with

$$U^C_{1t+1} - U^{NC}_{1t+1} = q_n(U^C_{1t+1} - \hat{U}_{1t+1}) + (1-q_n)(u^C_{1t+1} - u^0_{1t+1} + \delta(U^C_{1t+2} - U^{NC}_{1t+2})) \tag{18}$$

As $u^0_{1t+1} > \hat{u}_{1t+1}$[7], the maximum enforceable amount of a positive bonus is lower than in equation (15). Thus low values of q_n can also complicate cooperation.

If divorce happens on the equilibrium path, transfers to sustain an efficient relationship are even harder to enforce. This reduces the gains of future cooperation (since $U^C_{it} = \hat{U}_{it}$ in some cases) and therefore the amounts players are willing to transfer to their spouses to keep them.

Finally, we want to give an example under for when a bonus can solely be used for redistribution. Assume that $U^0_{it} > \widehat{U}_{it}(V^l)$, $i = 1, 2$, indicating that no transfer is needed to prevent divorce in (V^l, V^l). A positive bonus to redistribute the surplus is not enforceable if $U^0_{it} > \widehat{U}_{it}(V^h)$, all t, $i = 1, 2$, since divorce is not a credible threat. If $U^0_{2t+1} \leq \widehat{U}_{2t+1}(V^h)$, a positive transfer can in principle be enforced, since player 2 will file for divorce in (NC) when her outside utility is high. This leads to the following dynamic enforcement constraint of player 1 (DE 1)

$$b_t(V^l, V^l) \leq \delta[q_n(U^C_{1t+1} - \hat{U}_{1t+1}) + (1-q_n)(u^C_{1t+1} - u^0_{1t+1} + \delta(U^C_{1t+2} - U^{NC}_{1t+2}))] \tag{19}$$

Divorce in equilibrium, for example if $V_{2t+1} = V^h$, further reduces the maximum amount of b_t in (19) since this would again imply an identity of utilities in states (C) and (NC).

[7] otherwise, staying together would not be efficient in case (V^h, V^h)

Summing up, Becker´s (1991) claim is violated if efficiency cannot be sustained due to too low discounted gains of cooperation or too high actual gains of filing for divorce for one partner. This will happen if discount factors are too low, redistribution after divorce is too high or if divorce on the equilibrium path reduces the numbers of states where cooperation leads to a gain at all. Pure redistribution furthermore requires a credible divorce threat in at least one state, meaning that divorce has to happen in (NC) but not in state (C).

4.2 Relational Contracts in Periods $t = 1, 2$

In each of the first two periods, the couple has to decide whether to get an additional child, thereby taking the following game into account. If it is in the interest of both to get a child, no further redistribution is needed. But as children are a public good in our setup, the individually optimal number will in many cases be lower than the efficient one, requiring transfers between the partners to shift parts of the surplus. The credible punishment needed to enforce a transfer is still mainly constituted by divorce. Only after period 1, non-cooperation could be punishmend by the refusal of getting a child in period 2. Thus relationships that are very stable in a sense that no transfers are needed to maintain them can have less children than marriages considered more unstable.

Recall that our setup allows for a second transfer $\bar{b}_t$, paid after the fertility decision has been made. Thus we use the two different transfers b_t and $\bar{b}_t$ separately, where $\bar{b}_t$ is meant to give fertility incentives. Nevertheless, reneging on paying any of the bonus payments leads to a move to state (NC) with the consequence the relationship gets soured and no bonus is enforceable anymore.

Periods 1 and 2 are actually identical in their basic structure, with one difference: After period 1, the value of staying together versus getting divorced not only includes the parts discussed in the previous chapter, but also the option to get an additional child in period 2. Thus we will analyse both periods separately, starting with the second.

4.2.1 Period 2

Let us denote $n_1 \in \{0, 1\}$ the number of children already present after period 1 and $n_2 \in \{0, 1\}$ the number of children born in period 2. Since an existing relationship is needed to get a child, we assume that no one filed for divorce after the first period.

We just analyse fertility decisions here. Since they do not depend on current realisations of outside utilities (the decision whether to continue the relationship is made before), we adjust notation and write $U_{it}^C(n_1 + n_2)$ for $t = 1, 2$. Note that both transfers are not independent from each other.

Assume that getting an additional child in period 2 is efficient, indicated by

$$U_{12}^C(n_1 + 1) + U_{22}^C(n_1 + 1) \geq U_{12}^C(n_1) + U_{22}^C(n_1) \tag{20}$$

Then, the couple chooses $n_2 = 1$, if there exist bonus payments $b_2(V_2)$, $\bar{b}_2(n_1)$ and $\bar{b}_2(n_1 + 1)$ such that

(IC)

$$U_{i2}^{C}(n_1+1) \geq U_{i2}^{C}(n_1) \tag{21}$$

(DE)

$$\bar{b}_2(n_1+1) \leq \delta(U_{13}^{C}(n_1+1) - U_{13}^{NC}(n_1+1)) \tag{22}$$

$$-\bar{b}_2(n_1+1) \leq \delta(U_{23}^{C}(n_1+1) - U_{23}^{NC}(n_1+1)) \tag{23}$$

$$\bar{b}_2(n_1) \leq \delta(U_{13}^{C}(n_1) - U_{13}^{NC}(n_1)) \tag{24}$$

$$-\bar{b}_2(n_1) \leq \delta(U_{23}^{C}(n_1) - U_{23}^{NC}(n_1)) \tag{25}$$

Conditions (22) and (24) are only relevant for positive positive values and therefore just affect player 1, while (23) and (25) cover negative transfer player 2 has to pay.

Additionally, conditions (6) - (9) ensure incentive compatibility and enforceability of b_2. There, we just have to take into account that reneging on paying b_2 can also have an impact on n_2 by a switch to state (NC). Thus the set of potential punishments becomes larger compared to later periods. Furthermore, an adequate redistribution via b_2 can increase the number of situations when $n_2 = 1$ is possible. If only one partner is able to make a credible divorce threat, b_2 can help to construct a contract leading to more cooperation.

To get a better intuition, assume that $n_2 = 1$ is efficient. Furthermore, we have $U_{2t}^{0}(n_1+1) > \hat{U}_{2t}(n_1+1, V^h)$, all t, indicating that player 2 will never file for divorce in (NC). Thus $\bar{b}_2(n_1+1)$ cannot be positive. If it is not in the individual interest of player 2 to get a child, this is only possible if sufficient incentives can be given using a negative $\bar{b}_2(n_1)$. This requires a credible divorce threat by player 1. The transfer b_t can be used to redistribute surplus ex ante. The constraints this bonus faces are less strict than in previous periods, since player 1 can further be punished by $n_2 = 0$. Nevertheless, if $U_{it}^{0}(n_1+1) > \hat{U}_{it}(n_1+1, V^h)$, all t, $i = 1, 2$, no transfer $\bar{b}_2$ is enforceable.

Altogether, it is again crucial whether divorce can be used as a punishment threat or not. Any situation where this is not possible - because either divorce is not optimal or divorce is part of the equilibrium - makes bonus payments $\bar{b}_2$ harder to enforce.

4.2.2 Period 1

Assume that $n_1 = 1$ is efficient, indicated by

$$U_{11}^{C}(1) + U_{21}^{C}(1) \geq U_{11}^{C}(0) + U_{21}^{C}(0) \tag{26}$$

Then, the couple chooses $n_1 = 1$ if there exist bonus payments $b_1(V_1)$, $\bar{b}_1(1)$ and $\bar{b}_1(0)$ in line with conditions (8), (9) and

(IC)

$$U_{i1}^{C}(1) \geq U_{i1}^{C}(0) \tag{27}$$

(DE)

$$\bar{b}_1(1) \leq \delta(U_{12}^{C}(1) - U_{12}^{NC}(1)) \tag{28}$$

$$-\bar{b}_1(1) \leq \delta(U_{22}^{C}(1) - U_{22}^{NC}(1)) \quad (29)$$

$$\bar{b}_1(0) \leq \delta(U_{12}^{C}(0) - U_{12}^{NC}(0)) \quad (30)$$

$$-\bar{b}_1(0) \leq \delta(U_{22}^{C}(0) - U_{22}^{NC}(0)) \quad (31)$$

(28) and (30) correspond to positive, (29) and (31) to negative values of $\bar{b}_1$.

Equations (6) and (7), the IC constraints with respect to b_1, are not necessary here, since by assumption the relationship exists at the beginning of period 1.

4.3 Comparative Statics

The results derived so far are quite general, allowing the model to capture a lot of different cases. Now, we want to take a closer and more detailed look at the impact different parameters can have. Each fertility or divorce decision is made because it is efficient or because - if the decision is not efficient - necessary transfers to support the efficient outcome are not enforceable. Thus we will always take both aspects into account, efficiency and enforceability, which is of special importance when it comes to policy recommendations.

Since our model is quite general, the following analysis concentrates on some special cases. Nevertheless, we are convinced that these are sufficient to present the most relevant aspects of our setup.

We first give a summary of important conditions for fertility and divorce decisions and will refer to them in the following to give our arguments more transparency.

Conditions Determining Efficiency

In period 2, getting a child with $n_1 = 0$ when the couple stays together in the future in all cases is optimal, if:

$$\frac{2}{1-\delta}u(1) - c - g[(\varpi_2 + \xi_2) + \frac{\delta}{1-\delta}\varpi_2\rho] \geq 0 \quad (32)$$

In period 1, getting a child (versus getting no child at all) when the couple stays together in the future in all cases is optimal, if:

$$\frac{2}{1-\delta}u(1) - c - g[\xi_2 + \frac{\delta}{1-\delta}\varpi_2] \geq 0 \quad (33)$$

In period 2, getting a child with $n_1 = 0$ when divorce takes place in period 3 with probability 1 is optimal for player 1, if:

$$u(1) + \frac{\delta}{1-\delta}\theta u(1) - c - \sum_{\tau=t+1}^{\infty} \delta^{\tau-t}\phi(w_{1\tau} - w_{2\tau}) \geq 0 \quad (34)$$

It is optimal for player 2, if:

$$\frac{1}{1-\delta}u(1) - gw_{21} - \frac{\delta}{1-\delta}\varpi_2\rho g + \sum_{\tau=t+1}^{\infty}\delta^{\tau-t}\phi(w_{1\tau} - w_{2\tau}) \geq 0 \quad (35)$$

Remaining married is efficient (in periods $t \geq 3$), if:

$$V_{2t} - k_2 - [k_1 + \frac{1}{1-\delta}(1-\theta)u(n) - V_{1t}] \leq 0 \quad (36)$$

Conditions Determining Enforceability

Minimum condition for the enforcement of a positive bonus payment if $\varpi_1 = \varpi_2$ and $n_1 = 0$, $n_2 = 1$:

$$\frac{1}{1-\delta}\rho g\phi + V^h - k_2 \geq 0 \quad (37)$$

If $\hat{U}_{2t}(V^l) > U_{2t}^0$ and staying together is efficient in all cases, a positive bonus to maintain marriage is enforceable, if

$$V_{2t} - k_2 + \phi(w_{1t} - w_{2t}) - \delta[k_1 + \frac{1}{1-\delta}(1-\theta)u(n) - p_n V^h - (1-p_n)V^l] \leq 0 \quad (38)$$

Mandatory After-Divorce Transfer ϕ While the after-divorce transfer has no effect on the efficiency of the number of children, it can have a positive or negative impact on the enforceability of transfers. A higher ϕ increases the incentives for 2 to file for divorce in state (NC), helping to make a positive transfer possible per se (condition (37)). But it gets harder to enforce a given positive bonus, indicated by (38). The reason is that - given she would file for divorce in (NC) - 2 has to be compensated for remaining within the marriage in the respective period, while 1 is rewarded for cooperative behaviour just one period later. Therefore, setting ϕ too high could lead to a situation where player 2 files for divorce conditional on no bonus paid but that player 1 is not willing to pay a bonus high enough to maintain the marriage. Divorce on the equilibrium path decreases the freedom to distribute the surplus of a child since utilities in states (C) and (NC) coincide. Kraft and Stebler (2006) also get the result that the probability of divorce increases with higher after-divorce transfers. Furthermore, a higher ϕ increases player 2´s individual utility of getting an additional child if divorce happens on the equilibrium path (again assuming that player 1´s wage is higher than the one of player 2) and decreases the utility of player 1 (see conditions (34) and (35)). Summing up, higher after-divorce transfers naturally increase 2´s utility share. But if they become too high and thus reduce transfer enforcement, the total pie that can be distributed shrinks.

Divorce Costs k_i Divorce costs k_i have an impact on the enforceability of transfers and whether the couple remains married or not. As higher divorce costs decrease the enforceability of transfers, they have a negative effect on fertility on first sight. But if they lead to a smaller probability of divorce on the equilibrium path, they can also have a positive effect if redistribution is possible within marriage. Therefore, they potentially serve as a kind of commitment device. This idea is partly captured by Rowthorn (1999) and Matouschek and Rasul (2008) and leads them to the conclusion that divorce costs can be "too low". In our model, the same can be true. But if credible divorce threats are not feasible anymore, divorce costs have become too high. Generally, Becker´s (1993) view that divorce laws are of no importance is not valid here, since they have an impact on enforceability. The empirical results - increase in divorce rates in the short run and decrease in the long - as reported by Friedberg (1998) and Wolfers (2006), could be explained within our model. After the introduction of laws decreasing divorce costs, resulting marriages will be separated with higher probality. But since lower costs give less commitment possibilities, fewer potential couples will actually get married - and those who do will divorce less frequently.

Player 1´s Access to Children after Divorce θ A smaller value of θ generally destroys surplus. If it leads to a situation where divorce is not efficient anymore, the couple could stay together, although getting divorced would be efficient with a higher θ. But - like k_i - it can take the role of a commitment device when it comes to the redistribution of the surplus of getting a child. When the probability of divorce is decreased and redistribution is generally possible, the freedom of redistribution becomes higher. If we assume that generally a positive bonus is required more frequently, a lower θ is even a better tool to increase commitment possibilities than higher divorce costs, since an increased k_2 makes divorce threats of player 2 less credible (condition (37)). A further difference from divorce costs results from the fact that the commitment induced by θ is not fully exogenous, since the utility difference of children between marriage and divorce of 1 increases with their number. Thus a lower θ can induce the couple to get more children to make promised transfers of player 1 more credible.

Human Capital Accumulation ρ The experience parameter ρ has an impact on efficiency of number of children and timing of childbirth. Comparing equation (32) to equation (33) shows that a smaller ρ decreases the human capital loss if the couple gets a child in period 2 instead in period 1.

Monetary Costs c and Time Costs g The costs of getting children naturally directly influence the respective efficiency. Higher direct or opportunity costs will lead to a smaller number of children (for example see (32)). While direct monetary costs only accrue ones, opportunity costs due to the time required to raise children enter twice, once via the direct time loss and once via the human capital loss. The optimal timing is not influenced by g or c, which can be seen by comparing conditions (32) and (33). Finally, equation (37) shows how time costs g influence enforceability of transfers. This happens in a similar way as with ϕ, as a higher g can increase the wage difference between partners.

Of special interest in a setup like ours where the time requirent g is given exogenously is a potential substitutability between monetary and time costs. Assume that the couple could reduce g by buying in child care to decrease the problem of 2´s human capital loss. Price and availability of bought-in child care would become crucial determinants of the optimal number of children. Thus a government considering policies to increase fertility has to analyse carefully whether direct monetary grants (reducing c) or better and cheaper access to child care facilities (reducing g) is superior. Assume that the government wants to spend an amount S to support families and that the price of reducing g by one unit equals ν (and further assume that the couple has already chosen the optimal and feasible amount of substitution which is included in the parameter values c and g). Giving the money directly to the couple results in effective monetary costs $c - S$, while using it for better child care facilities leads to effective time costs $g - \frac{S}{\nu}$. Using the example where in period 2, the decision whether to get an additional child is made, no child is present and the couple remains together afterwards, reducing g is superior with respect to efficiency if $\nu \leq (\varpi_2 + \xi_2) + \frac{\delta}{1-\delta}\varpi_2\rho$. If the wages of nannies are lower than those of the secondary earner, it is generally better to reduce g. But the level of g can further have an impact on the enforceability of transfers and the probability of divorce (via after-divorce transfers), which has to be taken into account as well.

Wage Levels w_i The impact of wages on fertility is a widely analyzed issue in economics, with a main focus on the fact that fertility declined over time with womens´ education levels and wages. Higher wage levels increase the opportunity costs of getting children and therefore reduce their optimal number. If the return on experience of player 2[8] (higher ϖ_2) increases, the negative effect on fertility becomes stronger, leading to results in line with the literature. Due to the quasilinearity of preferences, the wage of the primary earner does not have any effect on fertility in our setup. Furthermore, we can give an explanation why fertility decreased with female income and labor market particiation over time but is higher in countries where a larger share of women is working. In countries like France, Norway or Denmark with relatively high fertility more children are in childcare than in Germany, Italy or Austra[9]. As monetary benefits given to couples with children are quite high in Germany or Austria, it seems that their governments should better have used the money to provide more and cheaper access to child care facilities and ensure better substitutability of costs c and g.

Not only the level of 2´s income, but also the difference to player 1´s is important. Assume that $\varpi_1 > \varpi_2$, for example caused by different education

[8]since 70-90% of secondary earners in households in North America and Europe are female, see Immervoll et. al. (2009).

[9]For the exact correlation see Bradshaw and Finch (2002).

decisions. This increases the wage difference over time and therefore also a transfer the primary has to pay to the secondary earner after divorce. Thus a divorce threat of 2 becomes more credible. The narrowing gap between wages of men and women can then have a negative effect on fertility. Furthermore, education decisions of women can be affected. High levels of ϕ make a higher wage difference more attractive for secondary earners. Thus they could be tempted to choose inefficiently low education levels.

Outside Utilities and Probalities p_n and q_n Many (especially young) mothers left by their partners often stay singles, since meeting someone new is more difficult when a child is present. Therefore, it seems reasonable to assume $q_0 > q_1 \geq q_2$. In our setup, a lower probability q_n can make it more difficult to enforce a positive transfer to maintain the marriage if 2´s divorce threat is only credible for V^h (see equation (18)). Probabilities to find a new partner could further depend on the sex ratio (ratio of men/women within a cohort). Except having an impact on the distribution of gains from marriage (see Becker (1993) or Chiappori (2002)), it can influence the enforceability of transfers. A higher sex ratio should then increase q_n which can help to enforce transfer to stabilize marriages and therefore increase fertility, whereas the opposite is true for a higher p_n. This result is supported empirically for Malaysian couples by Rasul (2002), where a higher competition for females leads to higher fertility levels and greater possibilities for re-marriage to 35-44 Malay males (higher p_n) tends to significantly reduce equilibrium fertility levels.

4.4 Endogenous Marriage Decision

If we do not assume that the couple gets married before period 1, the partners can make the decision themselves at the beginning of each period. Recall that a marriage can only be withdrawn by divorce, which includes a total separation. Compared to a split of a cohabiting couple, divorce causes costs k_i and leads to the transfer $\phi(w_{1t} - w_{2t})$ that has to be paid by player 1 in each subsequent period. Everything else remains equal, both receive their outside utilities V_{it} and the per period utility player 1 derives from children decreases to $\theta u(n)$. Since the decision to get married has to be made unanimously, both have to profit from it. But marriage makes separation more difficult causing only additional costs for at least one side and is therefore associated with some efficiency loss. Thus the couple will only be willing to get married, if this efficiency loss is more than offset by other gains. If we are in a period $t \geq 3$, the couple is together but not married and staying together is efficient for all realizations of players´ outside utilities, it is not always possible to enforce a transfer that prevents a separation. Since a marriage increases the barriers to a separation, it can simply make enforcement problem disappear if it becomes optimal to stay together for both partners.

But we are more interested in how marriage effects the enforceability of transfers. It is reduced if costs are so high or transfers so low that divorce is not a credible punishment. But if discounted after-divorce transfers are higher than divorce costs k_2, a divorce threat by player 2 can become more credible. Although player 1 is worth off after a separation, he might nevertheless be willing to get married if a better enforceability of transfers makes player 2 more willing to get children. Stevenson and Wolfers (2007) support this result empirically since the declining wage gap in the USA lead to less marriages. Finally, by reducing the probability for separation in equilibrium and therefore increasing the utility differences between states (C) and (NC), a marriage can further lead to a better enforceability of transfers.

Since marriage potentially serves as a commitment device, changing divorce laws can have unintended consequences for existing ones. If the secondary earner is only willing to get children and therefore reducing her human capital trusting that her outside option is relatively high in the future, transfer or cost reductions can make her worse off. Using a similar argument, Rowthorn (1999) proposes that changes in divorce laws should never affect already existing marriages.

5 Conclusions

We use the concept of relational contracts to analyse the importance of the enforceability of voluntary within-household transfers. Although our model is quite general, we did not capture all issues that are of potential interest. Couples can decide whether to marry or just cohabit, but the matching process is not modelled. Furthermore, child welfare and the quality of education is not included. Finally, using further household production than only child care can increase punishment possibilities within a relationship and therefore increase the set of possible tools to provide incentives. Including these issues could lead to further interesting results, leaving potential for future research.

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