Two-sided Certification: 
The Market for Rating Agencies

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Abstract
Rating agencies tend to offer certification services to both sides of the market: to firms and investors. Conflicts of interest might arise, since firms have an incentive to ”optimize” their rating to attract favorably priced financing. We show in a theoretical model, that a credible rating agency will sell its services to both sides of the market to maximize its own profits. Furthermore we identify markets in which two-sided certification compared to no certification and one-sided certification has a strong welfare increasing effect. Hence, certification services paid by firms does not necessarily indicate distorted certification results.

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1 Introduction

The market for public debt will grow significantly in the upcoming years due to the massive recovery programs which are enacted following the current financial and economic crisis, as politicians revive keynesian theories to stimulate the economy. The question arises, how the debt burden will be repaid and how investors might value the public debt issuance. Thereby, the allocative role of efficient credit markets with its sanctioning mechanisms (Stiglitz and Weiss, 1978) are essential to maintain market discipline as proposed by Lane (1993). Financial intermediaries play a key role in the evaluation of financial products and increase the efficiency in sovereign credit markets, as it is shown in the following paper.

The importance of credit ratings grew significantly during the last decades, as the number and complexity of financial products increased. Certification providers seek to overcome the unequal distribution of information in markets to increase efficiency. Since the findings on asymmetric information by Akerlof (1970), a broad literature developed to reduce transaction costs of market participants. Therein the informed party signals (Spence, 1973) its private information to the uninformed party by e.g. building up reputation, issuing warranties or by using third parties, which are credible and certify the information. The uninformed party will build expectations on the quality of the signal and thereafter values the acquired information. Additionally, the uninformed party will try to differentiate the sellers in the market and screen the investment opportunities to identify the best option (Rothschild and Stiglitz (1976) and Wilson (1977)).

The mechanism is especially prevalent in the world of financial markets. The seller of a financial product possesses private information on the products quality, which is unobservable by other market participants, who have expectations on the quality, which might be lower than the actual quality. On the one hand, the seller demands a credible signal to maximize the profit of the product. On the other hand, the buyer seeks to reduce the informational costs to find a suitable investment object and therefore demands a rating. The literature on the rating industry is not very rich; Strausz (2005) explained the high concentration in the market and Lizzeri (1999) proofed a rather non-intuitive result, that in many circumstances a rating agency will never disclose any information but captures the entire gains from trade.

We contribute to the literature in developing a model which describes the business model of the big rating agencies, which sell their service to both sides of the market - to investors and firms. The certification industry literature so far solely focused
This paper is organized as follows: the next section briefly describes the rating market and the dominant institutions. Section 3 presents three variations of a model describing the market for credit ratings. Thereafter, section 4 discusses the policy implications and section 5 concludes.

2 The market for rating agencies

Ratings are "summary measures of assessment over the probability that a borrower will default" (Fitch, 2002). Their main purpose is the independent evaluation of the quality of an investment regarding its repayment likelihood. Typically ratings are grouped into different rating classes, which comprise a specific default probability.

The market for rating agencies is highly concentrated. The two biggest rating agencies, Moody’s and Standard & Poors, share 80 percent of the market and together with the number three, Fitch Ratings, the market share becomes 95 percent. The operating margins of the leading rating agencies is close to 50 percent and relatively stable over the last years, even in the current turmoil of the financial markets. Strausz (2005) shows that high profit margins are required to establish credibility and reputation, as rating agencies otherwise might not withstand capture. Prices therefore can even exceed static monopoly prices. A widespread argument for the high concentration of rating agencies is the rigorous accreditation procedure of rating agencies by the Securities and Exchange Commission in the US. This is only partly true, since currently 10 Nationally Recognized Statistical Rating Organizations exist. Moreover, also in other regions a high concentration is observable, e.g. in Japan two players share most of the market, namely the Japan Credit Rating Agency and Rating and Investment Information Inc..

Generally, the business model of rating agencies can be boiled down to two main pillars. On the one hand, they offer certification services and on the other hand, they offer consultancy, mainly to banks and institutional investors. The combination of the two business areas led to a discussion on conflicts of interest which might evolve, as agencies are consulting banks on the same products as they evaluate in a later stage. The key role in the current financial crisis underlines this major problem, which will only be briefly discussed in this paper.

The business model with respect to the sales model of certification services changed significantly over time. Before 1970, ratings were primarily sold to investors, who
paid for the certification service. A subscription was required to obtain information, which were thereafter private information of subscribers. After 1970, the rating agencies decided to additionally sell their services to the other side of the market, to firms or issuers. After the firm received a rating, the information was immediately public and could be observed by all market participants. In addition a large number of small rating agencies entered the market, which serve the investor’s side and sell directly to buyers on a subscription basis.

It is often argued that the current business model of rating agencies is susceptible to bribery, since firms might influence the rating procedure in order to obtain a favorable rating. Rating agencies were accused of being responsible for the financial crisis, since they offered the certification service both, investors and firms. Primarily the sale of ratings to the originator of product is under steady criticism, as conflicts of interest might evolve.

We show in a theoretical model, that a credible equilibrium exists, where rating agencies sell to both sides of the market to increase their revenues and maximize their profits. Contrary to Lizzeri’s (1999) no revelation result, we show that strong incentives exist for the rating agency to credibly assess the financial product or firm.

3 Model

The model consists of four players: one informed seller, two uninformed buyers and one intermediary. The seller owns a product of quality \( q \) which we assume to be uniformly distributed on the interval \([0, 1]\). The intermediary has no value for the object while the seller has a reservation utility of \( \alpha q \) with \( \alpha \in [0, 1] \). The buyers only know the distribution of the product’s quality and therefore build expectations on the true value. The seller has no possibility to communicate the quality of his product \( q \) directly and credibly to the buyers. The intermediary however owns a technology to evaluate and communicate the product’s quality, which is perfect. If there is demand for an evaluation, by either the seller or the buyers, the intermediary can determine the quality \( q \) at no cost. In case the seller demands a rating, the intermediary can communicate the quality \( q \) credibly to the market, which is thereafter known to all buyers, hence public information. If one or both buyers demand an evaluation of the product, the information disclosed by the buyer is private information to the respective buyer.

The game of the model comprises 4 stages.
(1) The intermediary determines prices $p_s$ and $p_b$ for a rating sold to the seller and to one buyer, respectively.

(2) The seller may choose to order a rating from the intermediary for the price $p_s$. If a rating is sold, the information about the true quality $q$ will immediately become public information.

(3) The buyers decide simultaneously and (independently) whether to order a rating for the product. Buyers, who decide to order a rating, pay price $p_b$ and receive the information on the quality $q$ as their private information.

(4) The product is sold in a first-price (common value) auction. Finally, payoffs are realized.

Discussion of assumptions:
We assume that the intermediary is honest and uses a perfect information revelation technology. Furthermore, we assume that the intermediary has no competitors and can exploit her full monopoly power, which is in line with recent contributions, as e.g. by Strausz (2004), who motivates the high concentration and earning in the industry. In addition we allow the intermediary to discriminate in prices between sellers and buyers, which is plausible, as different goods are sold to both sides of the market - on seller’s side public information is revealed, while on buyer’s side private information is traded. The intermediary acts as a profit-maximizing monopolist.

The utility of the seller depends on the consumption or the sale of a single product. Depending on the market price, the seller will decide to sell the product or otherwise consume it, which constitutes an endogenous outside option in the model.\footnote{One could also think of the consumption option as a secondary market, which is not explicitly modelled.}

All buyers are symmetric ex-ante and no experience or reputation dynamics for selling good or bad quality in the market arise. The buyers bid for the product in a first-price sealed-bid auction. It seems a natural way to model the selling stage, as initial public offerings in financial markets follow this structure. Furthermore in all parts of the game the information structure is known to the buyers: in the first-price auction buyers are aware of the opponent’s information holdings. As the academic research is quite silent about picking the ”right” equilibrium in common value auctions with asymmetric informed bidders we choose a first-price auction (Larson, 2008).\footnote{We follow thereby the findings by Wilson (1967), Rob (1985), Kagel and Levin (1999), Campbell and Levin (2000) and Kim (2008). Second-price common value auctions feature multiple equilibria. Sequential bargaining with a stackelberg leader yields to similar results.}

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The market is characterized by a single parameter $\alpha \in [0, 1]$ which is known to all market participants. This parameter captures the reservation utility of the seller, which is $\alpha q$ for the quality level $q$. The buyer’s value the product of quality $q$ with $q$, hence the differences in valuation $(1-\alpha)q$ generate the possible gains from trade. By applying a market parameter we partly embed a basic adverse selection framework, where ex-ante no trade occurs.

Finally, we assume that a seller is the first to decide whether to order a rating. This is an intuitive approach as the seller initially decides whether to produce or sell a product.

3.1 The market without the certifier

It is known since Akerlof’s (1970) seminal work on adverse selection, that in markets with $\alpha > \frac{1}{2}$ trades collapse due to the asymmetrically informed parties – we will refer to this market as ‘Lemon Market’ (Figure 1). The market with $\alpha \leq \frac{1}{2}$ (combined with a first-price sealed-bid auction) has a unique equilibrium in which all goods are sold for a price of $\frac{1}{2}$ which is the expected quality for the buyers. Hence, all possible gains from trade are exploited, even without the intermediary - we refer to such a market as ‘Efficient Market’. The implications building on Akerlof’s (1970) lemon model are summarized in the following Proposition:

Proposition 1 The Market without the Intermediary

(a) In a Lemon Market the optimal bidding strategy for every buyer is to bid 0. All sellers with a positive quality parameter $q$ will not sell their good. There is no welfare generated in a Lemon Market.

(b) For an Efficient Market the optimal bidding strategy for every buyer is to bid the expected quality $q^e = \frac{1}{2}$. Every seller will accept such a bid and the entire welfare $W_{max} = \frac{1-\alpha}{2}$ is exploited.

Proof (a) We show that the pair of bidding strategies $(0,0)$ is an equilibrium. Placing a deviating bid of $b$ leads to winning the object if the bid beats the reservation utility. The expected quality of such a good is $E[q|\alpha q \leq b]$. As $q$ is uniformly distributed the expected quality is $q^e = \frac{b}{2\alpha}$. Parameter $\alpha$ is greater than $\frac{1}{2}$ and thus $q^e < b$. It is straightforward to show that this equilibrium bids are unique and that as a consequence the market collapses.

(b) Is is easy to show that bidding the own valuation is the unique equilibrium in this symmetric first-price sealed-bid auction with common values. With $\alpha \leq \frac{1}{2}$ every
seller accepts a bid \( b = \frac{1}{2} \) as \( \frac{1}{2} \geq \alpha q \) for all \( q \in [0,1] \). All products are traded and the full welfare is exploited. q.e.d.

Note: In difference to the classical lemon model by Akerlof (1970) the selling mechanism applied here leads to a unique equilibrium in the case of the Efficient Market. The logic is shown in figure 1.

For all models discussed in the following the possible welfare in the whole market to be shared is

\[
W_{\text{max}} = \int_{0}^{1} (1 - \alpha) q dq = \frac{1 - \alpha}{2}.
\]

3.2 One-sided certification

In the following subsection we will develop the results for two different versions of the model with one-sided certification. In the first case it is only the seller is eligible to buying a rating and secondly the intermediary merely offers her services to the buyer.

3.2.1 One-sided seller-certification

The intuition for the model of only the sellers being able to order a rating is straightforward. On the last stage of the model the buyers are symmetrically informed: either both know or do not know the quality. In a Lemon Market buyers will not
bid for the object, if no rating exists, whereas in the Efficient Market they will bid their expected valuations. In either case there exists a $q \in [0, 1]$ such that all sellers with a quality above $q$ will order a rating.

**Proposition 2** Bidding behavior for the case of Seller-Certification

(a) Lemon Market: For uninformed bidders the only equilibrium bid-pair is $(0, 0)$. For informed bidders the only equilibrium is the bid-pair $(q,q)$.

(b) Efficient Market: For uninformed bidders the only equilibrium bid-pair is $(q^e \overline{q}, q^e \overline{q})$ (where $q^e$ denotes the expected quality for a seller who did not order a rating). For informed bidders the only equilibrium is the bid-pair $(q,q)$.

By publicly announcing the information on the quality parameter $q$, the seller assures the product to be traded for $q$. In a Lemon Market the product will not be traded if the seller does not order a rating, whereas trades will be observed on expectation in the Efficient Market. The profit maximizing price of the certifier allows only the best part of the firms to order a rating, which leads to a certain threshold $\overline{q}$, above which all firms will order the certification service. The remaining sellers with low quality will be traded on the Efficient market or will not be traded in the Lemon Market. The following theorem states the result for the case of one-sided certification ordered by the seller (seller-certification). $\Pi_S$ denotes the expected profits on sellers side and $\Pi_C$ denotes the expected profits for the certifier.

**Theorem 1** One-sided seller-certification

If the certifier offers his service only to the seller side, the optimal pricing strategy, the critical threshold for quality above which the seller orders a rating, expected profits and expected welfare is:

(a) Lemon Market: The optimal strategy for the certifier is a price $p_s = \frac{1-\alpha}{2}$. It follows $\overline{q} = \frac{1}{2}$, $\Pi_C = \frac{1-\alpha}{4}$, $\Pi_S = \frac{1-\alpha}{8}$ and $W = \frac{3}{8}(1 - \alpha)$.

(b) Efficient Market: In this case the optimal strategy for the certifier is a price $p_s = \frac{1}{4}$. It follows $\overline{q} = \frac{1}{2}$, $\Pi_C = \frac{1}{8}$, $\Pi_S = W_{max} - \frac{1}{8}$ and $W = W_{max}$.

**Proof** (a) The maximization problem of the certifier is given by:

$$ \max_{p_s} \Pi(p_s) = (1 - \overline{q})p_s $$

s.t. $(1 - \alpha)q - p_s \geq 0 \quad \forall q \in [\overline{q}, 1]$ 

There is no possibility to sell the product without ordering a rating and a rated product will be traded at price $q$. From the seller’s condition we obtain $\overline{q} = \frac{p_s}{1-\alpha}$, as
the quality threshold for ordering a rating. Plugging the result into the objective function we get \( \Pi(p_s) = p_s(1 - \frac{p_s}{1 - \alpha}) \). Taking the derivative with respect to \( p_s \) yields \( p_s = \frac{1 - \alpha}{2} \), and hence, \( \bar{q} = \frac{1}{2} \) with corresponding profit \( \frac{1 - \alpha}{4} \). The profit of the seller in this segment is:

\[
\int_{\frac{1}{2}}^{1} (1 - \alpha) q dq - \frac{1 - \alpha}{4} = \frac{1 - \alpha}{8}
\]

As the lower segment is not traded a rent of \( \frac{1 - \alpha}{8} \) is lost due to the adverse selection problem. Overall welfare the realized welfare adds up to \( W = \frac{3}{8}(1 - \alpha) \).

b) The maximization problem of the certifier is given by:

\[
\max_{p_s} \Pi(p_s) = (1 - \bar{q}) p_s \\
\text{s.t. } (1 - \alpha) q - p_s \geq \frac{1}{2} \bar{q} - \alpha q, \quad \forall q \in [\bar{q}, 1]
\]

Hence, the critical quality level of the seller’s \( \bar{q} \) is determined by \( (1 - \alpha) \bar{q} - p_s = \frac{1}{2} \bar{q} - \alpha \bar{q} \). Solving the equation yields \( \bar{q} = 2p_s \). Plugging the result into the profit function we obtain \( \Pi(p_s) = p_s(1 - 2p_s) \). Maximizing w.r.t. \( p_s \) gives \( p_s = \frac{1}{4} \) and \( \bar{q} = \frac{1}{2} \) with a corresponding profit of \( \frac{1}{8} \) for the certifier. The remainder of the market is traded without a certificate at a price of \( \frac{1}{4} \) and as all products are traded in this market the profit of the seller is \( \Pi_S = \frac{1 - \alpha}{2} - \frac{1}{8} \). As in the market without a certifier \( W_{\text{max}} \) is realized. q.e.d.

### 3.2.2 One-sided buyer-certification

If the certifier solely offers the service to the buyers of the product, the objective of ordering the information is fundamentally different. The seller requires publicly announced ratings to verify the quality of his product and thereby differentiates himself from the rest of the market. In contrast, a buyer can only realize information rents if she exclusively possesses the information. In our setting, the buyers decided simultaneously and therefore can merely build expectations on the likelihood of being the only consumer of the certification service. Thus, the only symmetric equilibrium is a mixed-strategy equilibrium in which each buyer decides with a certain probability \( \omega \) to order a rating.

By assumption the buyers are aware of the distribution of the information in the
market when they enter the first-price auction. The bidding behavior for asymmetrically informed bidders in such a first-price auction results in a unique equilibrium (Weverbergh, 1979). The corresponding equilibria for both markets are shown in the following Proposition.

**Proposition 3** Bidding behavior for the case of seller-certification

(a) Lemon Market: For two uninformed bidders the only equilibrium bid-pair is $(0, 0)$.

For two informed bidders the only equilibrium is the bid-pair $(q, q)$.

If there is only one informed bidder his bidding strategy is $b = \alpha q$ and the uninformed does not bid at all. The expected payoff of a buyer given he is exclusively informed is $V_{IB} = \frac{1 - \alpha}{2}$. (refer to the appendix for further computations)

(b) Efficient Market: For two uninformed bidders the only equilibrium bid-pair is $(q_e, q_e)$ (where $q_e$ denotes the expected quality for a seller which is $\frac{1}{2}$).

For two informed bidders the only equilibrium is the bid-pair $(q, q)$.

If there is only one informed bidder his bidding strategy is $b = \frac{1}{2} q$. The uninformed mixes on the interval $[0, \frac{1}{2}]$ according to distribution function $F(b) = 2b$. The expected payoff of a buyer given he is the only one informed in this auction format is $V_{IB} = \frac{1}{6}$.

In both cases the expected profit of being uninformed is 0.

**Proof** With regard to both markets two potential values of being exclusively informed exist:

Lemon Market: The (expected) payoff of being exclusively informed is $q - \alpha q = (1 - \alpha)q$. Expressed as an ex-ante expectation, the value of being exclusively informed is $(1 - \alpha)q_e = \frac{1 - \alpha}{2}$.

Efficient Market: The expected payoff of being exclusively informed is $F(\frac{1}{2} q)(1 - \frac{1}{2})q = \frac{1}{2} q^2$. Expressed as an ex-ante expectation the value of being exclusively informed is

$$V_{IB} = \int_0^1 \frac{1}{2} q^2 dq = \frac{1}{6}.$$

q.e.d.

As the buyers randomize over the decision to order a rating using symmetric mixed strategies, they will have in expectations a profit of zero; buyers gamble for profits. From the perspective of the intermediary the most profitable case is to sell her service to both investors. The following theorem 2 captures the solution of the game:

\[3\text{Relaxing this assumption yields ....}\]
**Theorem 2** One-sided buyer-certification

If the intermediary offers her service only on the buyer side the optimal pricing strategy depending on market structure, the probability of ordering a rating by each buyer, the expected profits and the expected welfare are the following:

(a) Lemon Market: The optimal strategy for the certifier is to set a price of \( p_b = \frac{1 - \alpha}{4} \). It follows \( \omega = \frac{1}{2} \), \( \Pi_C = \frac{1 - \alpha}{4} \), \( \Pi_F = \frac{1 - \alpha}{8} \) and \( W = \frac{3}{8} (1 - \alpha) \).

(b) Efficient Market: In this case the optimal strategy for the certifier is to set a price of \( p_b = \frac{1}{12} \). It follows \( \omega = \frac{1}{2} \), \( \Pi_C = \frac{1}{12} \), \( \Pi_F = W_{\text{max}} - \frac{1}{12} \) and \( W = W_{\text{max}} \).

**Proof** a) The maximization problem of the certifier is given by:

\[
\max_{p_b} \Pi(p_b) = \omega^2 2 p_b + 2 \omega (1 - \omega) p_b \\
\text{s.t.} \quad (1 - w) V_{IB} - p_b = 0
\]

From the buyers indifference condition we obtain \( \omega = 1 - \frac{2p_b}{1 - \alpha} \). The profit function can be simplified to \( \Pi(p_b) = 2 \omega p_b \) and by plugging in the value for \( \omega \) we get \( \Pi(p_b) = 2 p_b - 4 p_b^2 \frac{1}{1 - \alpha} \). Maximizing w.r.t. price \( p_b \) yields \( p_b = \frac{1 - \alpha}{4} \). This leads to \( \omega = \frac{1}{2} \) and thus, the certifier’s profit is \( \Pi_C = \frac{1 - \alpha}{4} \). In the case of \( \alpha > \frac{1}{2} \) sellers only realize gains if the information on their quality is known by both buyers. In \( \frac{1}{4} \) of the cases they make an expected profit of \( \frac{(1 - \alpha)}{2} \) and hence the overall firm profit is \( \frac{1 - \alpha}{8} \) and welfare accumulates to \( W = \frac{3}{8} (1 - \alpha) \).

b) The maximization problem on certifier’s side is the following:

\[
\max_{p_b} \Pi(p_b) = \omega^2 2 p_b + 2 \omega (1 - \omega) p_b \\
\text{s.t.} \quad (1 - w) V_{IB} - p_b = 0
\]

Applying the expected valuation \( V_{IB} \) of \( \frac{1}{6} \) leads with the buyers indifference condition to \( \omega = 1 - 6 p_b \). The profit function can be simplified to \( \Pi(p_b) = 2 \omega p_b \) and by plugging in the value for \( \omega \) we obtain \( \Pi(p_b) = 2 p_b - 12 p_b^2 \). Maximizing this w.r.t. \( p_b \) yields \( p_b = \frac{1}{12} \), \( \omega = \frac{1}{2} \) and therefore \( \Pi_C = \frac{1}{12} \). Knowing that in such a market all projects are realized it turns out that \( \Pi_F = \frac{1 - \alpha}{2} - \frac{1}{12} \). q.e.d.

### 3.3 Two-sided certification

The model of two-sided certification combines both models already discussed. The different objectives of information revelation are stressed: a high-quality seller will try to disclosed his quality for both buyers, and the buyers will try to be exclusively informed. In the Lemon Market and the Efficient Market a seller can hope that
both buyers will order a rating. In this case he shifts his certification costs \( p_s \) to the buyers and thereby maximizes his rents. The certifier may profit from the fact, that he can sequentially segment the market. A high-quality segment \( (q \in [\bar{q}, 1]) \) where firms order the rating and a lower-quality segment in which the buyers order with a mixed-strategy probability. It is obvious that a buyer will never demand a certificate if the seller already ordered, as the certification process publicly reveals the true quality. In case of an efficient market there is still a market for uncertified goods in which bidding depends on the quality threshold \( \bar{q} \) above which a seller orders a certificate on her own.

Insert graph on market segmentation?

As in the one-sided buyer-certification there are three possible information outcomes in the bidding stage in both markets. Hereby, the distribution of the quality in the remaining market \( q \in [0, \bar{q}] \) is determined endogenously. The bidding behavior for either case is summarized in the following Proposition:

**Proposition 4** Bidding behavior for two-sided certification

(a) Lemon Market:

(a1) If both buyers are informed about the quality \( q \) the unique bidding equilibrium is \((q, q)\).

(a2) If none of the buyers knows quality the unique bidding equilibrium is \((0, 0)\).

(a3) If only one investor is informed about quality \( q \) the unique bidding equilibrium requires the informed buyer to bid \( \alpha q \) and the uninformed one does not bid at all. The expected payoff of a buyer given he is exclusively informed in this auction format is \( V_{IB} = (1 - \alpha)\frac{q^2}{2} \).

(b) Efficient Market:

(b1) If both buyers are informed about the quality \( q \) the unique bidding equilibrium is \((q, q)\).

(b2) If none of the buyers knows quality the unique bidding equilibrium is \((\frac{1}{2} \bar{q}, \frac{1}{2} \bar{q})\).

(b3) If only one investor is informed about quality \( q \) the unique bidding equilibrium requires the informed buyer to bid \( \frac{1}{\bar{q}} q \). The uninformed mixes on the interval \([0, \frac{1}{2} \bar{q}]\) according to the distribution function \( G(b) = \frac{2b}{\bar{q}} \). The expected payoff of a buyer given he is exclusively informed in this auction format is \( V_{IB} = \frac{1}{6} \bar{q} \).
In both cases the expected profit when being uninformed is zero.

Proof Referring to the considerations of the auction format discussed above, the bidding equilibria are derived from the expected payoffs for the exclusively informed buyer. For non-certified seller the quality is distributed according to \( \bar{F}(q) = \frac{q}{\bar{q}} \).

(a) The probability of winning the auction for the informed bidder is 1. The quality remaining un-certified in the market after the seller-certification is \( \bar{F}(q) \). In expectation the informed buyer wins an object of quality \( \bar{q} \) for a bid of \( \alpha \bar{q}^2 \), and hence realizes a profit.

(b) Let the object in the auction be of a quality \( q \). With a bid of \( \frac{1}{2}q \) the informed buyer wins with a probability of \( G(\frac{1}{2}q) = \frac{q}{\bar{q}} \). If he wins his payoff is \( q - \frac{1}{2}q = \frac{1}{2}q \). Thus, expected payoff ex-ante is determined by:

\[
V_{IB} = \bar{q} \int_0 ^{\bar{q}} \frac{q}{\bar{q}} \frac{1}{2} q dq = \bar{q} \int_0 ^{\bar{q}} \frac{1}{4} \frac{1}{3} q^3 dq = \frac{1}{6} \bar{q}.
\]

From the analysis it is obvious that the expected payoff of being uninformed is zero. q.e.d.

Putting this all together leads us to the optimal behavior of the monopolistic certifier, a seller of product with quality \( q \) and ex-ante uninformed investors.

Theorem 3 Two-sided certification

If the intermediary offers his service on both sides of the market, the optimal pricing strategy depending on market structure, the probability of each buyer ordering a rating, the expected profits and the expected welfare are the following:

(a) Lemon Market: The optimal strategy for the certifier is to set prices of \( p_s = \frac{16}{27}(1-\alpha) \) and \( p_b = \frac{2}{9}(1-\alpha) \). It follows \( \bar{q} = \frac{2}{3}, \omega = \frac{1}{3}, \Pi_C = \frac{8}{27}(1-\alpha), \Pi_S = (1-\alpha) \frac{17}{162} \) and \( W = (1-\alpha) \frac{65}{162} \).

(b) Efficient Market: The optimal strategy for the certifier is to set prices of \( p_s = \frac{3}{2}(5\sqrt{5} - 11) \) and \( p_b = \frac{1}{4}(7 - 3\sqrt{5}) \). It follows \( \bar{q} = \frac{141 - 63\sqrt{5}}{36 - 16\sqrt{5}}, \Pi_C = \frac{3}{4}(5\sqrt{5} - 11), \Pi_F = \frac{1-\alpha}{2} - \Pi_C \) and \( W = W_{\text{max}} \).

Proof (a) The maximization problem of the certifier is given by:

\[
\max_{p_s, p_b} \Pi(p_b) = (1-\bar{q})p_s + \bar{q}[\omega^22p_b + 2\omega(1-\omega)p_b] \\
\text{s.t. } (1-\alpha)q - p_s \geq \omega^2(1-\alpha)q \quad \forall q \in [\bar{q}, 1] \\
\text{s.t. } (1-w)(1-\alpha)q^2 - p_b = 0
\]
Note, that the profit function can be simplified to $p_s + \bar{q}[2\omega p_b - p_s]$. In equilibrium $\bar{q}$ is determined by the seller’s condition with equality. Using the equation we obtain 

$$(1 - \omega^2)(1 - \alpha)\bar{q} = p_s \Leftrightarrow (1 - \omega)(1 - \alpha)\bar{q} = \frac{p_s}{1 - \omega}.$$ 

Referring to the buyer’s indifference condition and using $q^e_q = \frac{1}{2}\bar{q}$ we see that $\frac{1}{2}p_{\bar{q} + \omega} = p_b \Leftrightarrow \omega = \frac{p_s}{2p_b} - 1$. To receive a function $\bar{q}(p_s, p_b)$ we use $1 - \omega^2 = \frac{p_s}{p_b}(1 - \frac{p_s}{4p_b})$ in the equation stated above, which yields $\bar{q} = \frac{4p_s^3}{(1 - \alpha)(4p_b - p_s)}$. Plugging the two expressions for $\omega$ and $\bar{q}$ into the objective function we obtain the profit of the intermediary as:

$$\Pi(p_s, p_b) = p_s - \frac{8p_s^3}{(1 - \alpha)(4p_b - p_s)}.$$

Maximizing the profit function w.r.t. $p_s$ and $p_b$ we finally obtain $p_s = \frac{16}{27}(1 - \alpha)$ and $p_b = \frac{2}{9}(1 - \alpha)$. The derived functions imply $\bar{q} = \frac{2}{3}$ and $\omega = \frac{1}{3}$. The profit for the certifier is $\Pi_C = \frac{8}{9}(1 - \alpha)$. In the market with quality below $\bar{q}$ a share of $1 - \left(\frac{2}{3}\right)^2 = \frac{5}{9}$ is traded. Hence, the overall welfare adds up to:

$$W = \frac{5}{9} \int_0^{\frac{2}{3}} (1 - \alpha)qdq + \int_{\frac{2}{3}}^1 (1 - \alpha)qdq = (1 - \alpha)\frac{65}{162}.$$

As the buyers do not make any profit in equilibrium the firm profit is determined by $\Pi_F = W - \Pi_C = (1 - \alpha)\frac{17}{162}$.

(b) To formulate the maximization problem of the intermediary the expected winning bid for an object of quality $q$ sold in a first-price auction with asymmetrically informed bidders $E[b_{\text{win}}|q]$ is required. With a probability of $\frac{q}{\bar{q}}$ the informed bidder wins with a bid of $\frac{1}{2}q$. With a probability of $1 - \frac{q}{\bar{q}}$ the uninformed wins with an expected bid of $\frac{1}{4}q + \frac{1}{2}\bar{q} = \frac{1}{4}(q + \bar{q})$. Thus,

$$E[b_{\text{win}}|q] = \frac{q}{\bar{q}} \cdot \frac{1}{2}q + (1 - \frac{q}{\bar{q}}) \cdot \frac{1}{4}(q + \bar{q}) = \frac{1}{4}\bar{q} + \frac{q^2}{4\bar{q}}.$$

As the left hand side of the seller’s constraint is increasing faster than the right hand side, we formulate the maximization problem for the intermediary operating on both sides as the following:

$$\max_{p_s, p_b} \Pi(p_s, p_b) = (1 - \bar{q})p_s + \bar{q}[\omega^22p_b + 2\omega(1 - \omega)p_b]$$

s.t. 

$$\begin{align*}
(1 - \alpha)\bar{q} - p_s &= \omega^2(1 - \alpha)q + 2\omega(1 - \omega)(\frac{1}{2} - \alpha)\bar{q} + (1 - \omega)^2(\frac{1}{2} - \alpha)\bar{q} \\
(1 - w)\frac{1}{2}q^e_q - p_b &= 0
\end{align*}$$
This is due to \( E[b_{\text{win}} | \bar{q} = \frac{1}{4} \bar{q} + \frac{q^2}{4\bar{q}} = \frac{1}{2} \bar{q}] \). The three parts of the seller’s condition correspond to the profits by being rated by both investors, rated by only one investor, and not being rated at all. In the latter case, there will be trade without any information in the remainder of the Efficient Market.

The seller’s condition can be reformulated as \((1 - \omega^2)(1 - \alpha)\bar{q} - p_s = (1 - \omega^2)(\frac{1}{2} - \alpha)\bar{q}\). Solving the constraints for \(\omega\) and \(\bar{q}\) we obtain \(\omega = 1 - \frac{6p_b}{q}\) and \(\bar{q} = \frac{2p_s}{1 - \omega^2}\). Replacing \(\omega\) gives \(\bar{q} = \frac{18p^2}{6p_b - p_s}\) and by plugging the result into the profit function of the certifier we get the reduced maximization problem:

\[
\max_{p_s, p_b} \Pi(p_s, p_b) = p_s - 6p_b^2 \frac{6p_b + p_s}{6p_b - p_s}
\]

s.t. \(0 \leq \omega, \bar{q} \leq 1\).

In reduced form version of the model one explicitly has to assure that the constraints for the values of \(\omega\) and \(\bar{q}\) are fulfilled. Using the expressions for the two parameters derived above the constraints are equivalent to

\(3p_b \leq p_s \leq 6p_b - 18p^2_b\).

Solving this problem gives the parameters stated in the Theorem. q.e.d.

### 4 Results

The model reveals some striking results. Without a certification service two market outcomes arise: in one market the costs of asymmetric information does not hinder investors and firms from exchanging their products. The reservation utility of the best firm is lower than the expected quality of all firms by the investors and consequently, all products are traded in the market; an 'efficient market' arises.

Contrarily, in a market in the sense of Akerlof (1970), the asymmetric information problem leads to the collapse of the entire market. No trades will be observed in this 'lemon market'.

In a lemon market a financial intermediary can partly overcome the asymmetric information problem, as a high proportion of potential trades is realized. In the efficient market, the total welfare is not affected by the introduction of the intermediary. The intermediary in this case receives a high fraction of the rents generated by the market. In an efficient market our results show that a profit-maximizing certifier prefers to operate on seller’s side, if she has to decided to offer the services to merely one side of the market, while she is indifferent in a lemon market. Figure 2
depicts the shares of all parties involved in the market if the certification service is merely offered to the seller’s side. In the lemon market not the entire welfare can be realized, but a substantial proportion of 75 percent. In both markets, the certifier extracts a high amount of the rents, which rise up to 50 percent of potential welfare in the lemon market. Firms gain in the lemon market by hiring the intermediary, as they extract 25 percent of potential welfare, which could not be realized in an alternative way. In an efficient market the intermediary does not increase welfare and the seller will be unwilling to share the potential rents with the intermediary in the market (ex-ante).

If the intermediary decides to merely sell to the investor’s side, its revenues will shrink from $\frac{1}{8}$ to $\frac{1}{12}$. It is important to notice, that the traded products differ between both sales schemes: if the seller orders a rating, the best half of the firms will demand a certificate, whereas if buyers order ratings, they cannot differentiate between good and bad firms and therefore will select randomly.

Comparing the outcomes of one-sided certification with the model in which the intermediary sells its services firstly to the sellers and, if they reject the offer, secondly to the buyers, the welfare in the lemons market increases even further. With two-sided certification, about 70 percent of all products are traded in equilibrium, including the third with highest quality. The welfare loss is down to about 20 percent

Figure 2: Profit shares with one-sided certification sold to sellers.
compared to 100 percent in the case without certification.

The profit for the intermediary is highest in the market with two-sided certification, which is not as intuitive as expected at first sight. By offering the certification service on the secondary market the intermediary faces a negative second-order effect from sellers hoping to be rated by two buyers. Through the rating offer to the seller in the lemon market, the seller with high quality levels $q$ will except the offer (firms above $\bar{q}$) and the remainder might then be evaluated on a random basis by the investors. Appendix 1 gives a summary report on the equilibrium values of the main variables in the model.

The shares of welfare in the one-sided and the two-sided model are depicted in figure 3. The graph shows the slight increase of the intermediaries’ share on welfare in the efficient market. Compared to the 50% jump in profits from offering ratings to the firm’s side instead of operating on investor’s side solely the increase in profits if offering the certification service on both sides in an efficient market is only about 8%.

Figure 3: Profit shares with two-sided certification compared to one-sided.

For the lemon market it shows the increased share of the firms’ profits which is the reduction in lost welfare compared to a one-sided market. The welfare loss due to asymmetric information reduces drastically if it is a two-sided certification intermediary operating in the market.
5 Conclusion

The rating industry is highly concentrated and offers services to both sides of the market. In principle, ratings seek to reduce transaction costs and market inefficiencies which accrue due to information asymmetries of market participants. However, the financing of the intermediaries was under steady criticism, as conflict of interest might arise and the market power could be exploited. The main criticism hereby was the payment of the rating service by the seller side. Therefore we analyze the sales mechanism of financial intermediaries and determine its influence on welfare generation.

Introducing a financial intermediary, which offers its services solely to one side of the market enables trades in a market in the Akerlof sense, which we call the lemon market. Depending on the sales scheme, either selling the certification service merely to the buyer or to the seller, the profit shares of the parties vary. The certifier can maximize its profit by selling to the sellers side.

Furthermore, we show theoretically, that the profit maximizing strategy for the rating agencies is to sell to both sides of the market, as it is done since the 1970s. Furthermore the economic welfare in financial markets increases, as more projects, which are adequately priced, are promoted.

The policy implication of the results of our model is an indirect one. It is not necessarily the case that observing intermediaries being paid by the issuers indicates a cooperation of the two parties or even beautifying the default probability or whatever the quality parameter may be. In a functioning market which fits the world of our model we would rather expect that intermediaries have a tendency to offer their services mainly on seller’s side. A policy implication could be to foster the presence of intermediaries in inefficient markets as the lemon markets as they are able to solve the inefficiency due to asymmetrically distributed information to a certain degree and even increase welfare. It is less clear whether an efficient market needs a certifier. Taking into account that the certifying procedure incorporates some economic costs it may reduce welfare to have such intermediaries.

In recent times it seemed to be a straightforward argument that the financial crisis was provoked by unjustified good ratings arising from the conflicts of interest between certifier and the firms. This paper does not claim that this was not the case. It is rather that one has to look closer at a particular market, the ratings within this market and the outcome at the end of the day whether ex-post wrong ratings were disturbed on purpose or not. To look at the agent who paid the certificate is
definitely not a sufficient reason for this argument.
References


Larson, N. (2009), ”Private value perturbartions and informational advantage in common value auctions”, Games and Economic Behavior, in press.


## Appendix

Table 1: Comparing equilibrium outcomes of different model settings

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