Vertical differentiation in the quality of higher education when students are mobile*

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Abstract

This paper analyzes in a two-country model the impact of students' mobility on the country-specific level of higher educational quality. Individuals decide whether and where to study based on their individual ability and the implemented quality of education. We show that countries which may be asymmetric with respect to their higher education systems choose generally suboptimally differentiated levels of educational quality.

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1 Introduction

The Bologna Process aims at establishing the so-called European Higher Education Area. A common structure of higher education and more comparable degree programs will help to alleviate access of students to higher education systems across the member states of the European Union. This will increase students' mobility. At the same time, there is a tendency in many countries to attach more importance to fees as a means to finance higher education.

The aim of this paper is to analyze how governments react to higher mobility of students by determining the country-specific quality level of higher education. For this we focus on a two-period, two-stage game with two countries. At the first stage, governments choose the education quality level. At the second stage, individuals make their decisions about higher education and labor supply given the quality levels chosen by the governments. In particular, individuals decide whether (and where) to study. For this, they compare the lifetime income with higher education to the lifetime income when uneducated. This comparison differ across individuals because they differ with respect to their innate ability to benefit from higher education. Specifically, the returns to higher education depend both on the quality level of education as well as on the innate ability. Both together generate the skill-units an individual is endowed with after having acquired education. Hence the education level affects the structure of labor supply in

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the subsequent period both through two channels: directly because it affects productivity and indirectly because it modifies the incentives to become skilled.

The educational quality level is restricted to be a uniform, country-specific level of educational quality meaning that - once decided by the social planner or the government - it applies to all students of a country. The choice of this level is influenced by the government criterion and by a graduates return probability. We consider a variety of government's welfare criteria, parameterized by the relative weight on natives and foreigners. At one extreme, a government is only concerned with the natives and at the other extreme it is only concerned with the residents. The graduates return probability specifies the chances for a graduate student who has studied abroad to come back in his home country.

Ability types of students are not observed. With pure fee-financing, this assumption does not induce distortions in the individuals' choices because the private benefit and the social benefit of taking up a given educational level coincide. As a result, in a closed economy, a government that is concerned about its citizens welfare chooses the same educational level with or without observing abilities and there is no welfare loss. Instead, in the case where economies are open and students freely choose where to study, the fact that ability is not observable plays a central role. Since governments have no full control on who study and who work in their countries, the choice of the education level may be used as a way to attract or to repel individuals according to their ability. The incentives to do so depend on the government's welfare criterion -that is on the relative weight on natives and foreigners- and on the graduate return probability.

We first describe for a closed economy the decision of an individual to acquire higher education and the resulting labor supply of skilled and unskilled workers before allowing for mobility of students. For a two-country setting, we then establish that it is optimal that the two countries differentiate their quality levels as this enables individuals to split according to their abilities. The assumption that there is a single uniform educational quality that applies to all students of a country plays an important role here. This restriction cuts off the possibility that a government can achieve the first best policy which would be to provide a specific educational level for each ability type of students. It opens up also the possibility of welfare gains when the countries offer distinct levels and graduate students can freely move. Albeit strong, the restriction to one educational level reflects the observed limited variability of the quality of educational programs within a country (at least relative to the variability across countries). In reality ability types are very numerous which we proxy by assuming an infinte number of ability types and there are positive costs for each differentiated educational level. In that case, governments can only provide a limited number of specific educational levels and cannot reach the first best educational levels by matching all ability types. The restriction to one educational level is then only a simplification which does not alter the results for any limited number of educational levels.

We then study the choices of the two countries at a Nash equilibrium and compare them with the optimal levels when there is one or two levels offered. We show that at a symmetric equilibrium, the identical educational level is too high compared with the optimal one. At an asymmetric equilibrium, governments tend to be suboptimally differentiated. The reason is that governments' decisions about the educational levels are distorted by external effects resulting from students' mobility. The existence and the extent of the inefficiency depend both on the welfare criterion and the graduate return probability.

The plan is as follows. Section 2 describes the model and analyzes a closed economy. Section

3 considers two open economies, Section 4 concludes, and Section 5 gathers some proofs.

2 Closed economy

2.1 The model

The analysis is conducted in a stationary overlapping generations model in which the population is constant. The economy is kept as simple as possible. There is a single consumption good that is produced by skilled and unskilled labor through a linear technology. The good cannot be stored and there is no capital. In each period higher education has to be financed via tuition fees by the students.[1] Since students do not have any income they have to borrow money in the first period in order to finance their studies. Borrowing takes place between the individuals of one generation (not all decide to study) and possibly between generations. Credit markets are perfect and we are at the golden rule: the interest rate in the steady state without frictions is equal to the population growth rate, here zero (see Gale, 1973).

The production sector The production sector in each country uses two kinds of input:[2] labor supplied by individuals with and without higher education, L_s (skilled labor) and L_u (unskilled labor) respectively.[3] Production takes place according to a linear technology where the wage rates of the skilled workers, w_s, and the unskilled workers, w_u, are assumed to be given and constant

$$F\left(L_u, L_s\right) = w_s L_s + w_u L_u. \tag{1}$$

Production is thus completely determined by the labor supply of skilled and unskilled workers which in turn is given by the individuals' decisions to acquire higher education.

The demand for higher education Individuals are distinguished by an ability parameter, y, which reflects individually different benefits from higher education. The distribution of abilities is identical in each country and assumed for simplicity to be uniform in the range $[0, \overline{y}]$.

To be skilled, an individual must receive some education. Quality of education or the educational level, respectively, is denoted by e. The quantity of skilled labor provided by an educated worker with education level e depends on her ability y: it is given by ye. For simplicity, we assume that the amount of money spent for higher education per individual, given by $c(e)$, only depends on the educational level. Put differently, costs of education are proportional to the number of students for a given quality.[4] The cost function c is assumed to be increasing and strictly convex. Throughout the paper, to avoid corner solutions, we shall assume that marginal costs of education increase indefinitely with the level: $\lim_{e \to \infty} c'(e) = \infty$.

[1] In a companion paper (Demange, Fenge and Uebelmesser, 2008), we have shown that pure fee-financing of higher education is optimal if credit markets are perfectly competitive. Since we model higher education as a private good without externalities this is a natural outcome.

[2] We abstain here from explicitly considering capital in the production technology. Taking the effect of education on capital into account would be interesting, but it is outside the scope of the present paper.

[3] This corresponds to empirical evidence according to which mobility increases with education. See, e.g., Ehrenberg and Smith (1993).

[4] Education is thus considered here to be a private good.

If an individual decides to study, she pays the educational costs as fees during the first period, $c(e)$, and earns no wage income. In the second period, the educated worker receives a gross wage rate w_s for each unit of effective labor supply so that the wage income depends on her ability y: $w_s ye$. Thus her lifetime income is

$$w_s ye - c\left(e\right). \tag{2}$$

If the individual decides not to study she receives a wage income w_u in both periods. Hence, her lifetime income is

$$2w_u. \tag{3}$$

The individual compares both lifetime incomes and chooses the option which maximizes her income. The decision whether to study or not depends on the ability of the individual. The marginal ability type who is indifferent between both options is given by

$$y^F = \frac{2w_u + c(e)}{w_s e}. \tag{4}$$

Individuals with a lower ability, $y < y^F$, do not study and are employed as unskilled workers. Individuals with a higher ability, $y > y^F$, take up higher education in the first period and work as skilled workers in the second period.

Employment We describe here how the educational level e determines the supply of skilled and unskilled workers on the labor market.

As already mentioned, the population growth rate is assumed to be nil. In each period, employment consists of young and old unskilled workers and old skilled workers. Let an educational level e and a threshold ability level of skilled workers y^F be given. The number of unskilled workers per generation, denoted by N_u, is equal to y^F and the number of skilled workers, denoted by N_s, is equal to $\overline{y} - y^F$. The employment of unskilled labor is given by

$$L_u = 2\int_0^{y^F} 1\ dz = 2y^F = 2N_u \tag{5}$$

and the *effective* skilled labor by

$$\begin{aligned} L_s &= \int_{y^F}^{\overline{y}} zedz = e\left(\frac{\overline{y}^2 - \left(y^F\right)^2}{2}\right) = \left(\overline{y} - y^F\right) e\left(\frac{\overline{y} + y^F}{2}\right) \\ &= N_s e\left(\frac{\overline{y} + y^F}{2}\right) \end{aligned} \tag{6}$$

which is equal to the number of skilled workers multiplied by their average ability and the educational level.

2.2 Optimal allocation

Under complete information on individuals' abilities, a social planner can determine the level of education and the ability of those who study. The criterion is aggregate production net of education cost at a steady state, given by $F(L_s, L_u) - N_s c(e)$. This is the criterion that obtains in a fully fledged overlapping generations economy in which the planner treats all generations equally. In other words, we are at the golden rule with an implicit interest rate equal to the population growth rate, which is here equal to zero (see Gale, 1973).

The choice of the level of education and of the minimum ability of those who study, e and y respectively, fully determines skilled and unskilled labor from (5) and (6). Hence defining

$$W(y, e) = w_s L_s + w_u L_u - N_s c(e) \tag{7}$$

where from (5) and (6) L_u and L_s are functions of e and y and N_s is a function of y alone, the objective is to maximize $\underset{e,y}{Max}\ W(y, e)$.

The impact of a marginal increase in e keeping the set of students fixed is given by

$$\begin{aligned} \frac{\partial W}{\partial e} &= w_s \frac{\partial L_s}{\partial e} + w_u \frac{\partial L_u}{\partial e} - N_s c'(e) \\ &= (\overline{y} - y)\left[w_s \frac{\overline{y} + y}{2} - c'(e)\right]. \end{aligned} \tag{8}$$

It is equal to the effect of the quality level on the production of the skilled minus the increase in costs.

The impact of a marginal increase in the minimum ability level y, keeping the education level fixed, is given by

$$\begin{aligned} \frac{\partial W}{\partial y} &= w_s \frac{\partial L_s}{\partial y} + w_u \frac{\partial L_u}{\partial y} - c(e) \frac{\partial N_s}{\partial y} \\ &= -w_s e y + 2 w_u + c(e). \end{aligned} \tag{9}$$

It is equal to the net impact on the productivity of a student of ability just equal to y from becoming skilled compared to remaining unskilled where the impact is measured at the steady state situation.

The objective function is concave in e and in y. At the optimum, assumed to be interior, the level of education and the threshold ability level are characterized by the following first-order conditions

$$w_s \frac{\overline{y} + y}{2} = c'(e) \tag{10}$$

$$w_s e y - 2 w_u - c(e) = 0 \tag{11}$$

that is, the marginal gain from a change in educational quality on the average student, $w_s \frac{\overline{y}+y}{2}$, is equal to the marginal costs, and the net gain of education for the marginal student is null.

In the sequel, we put a superscript * to indicate the values at the optimum solution for the educational levels and the threshold ability. In the following, individuals' abilities are assumed not to be observable (or contractible) by governments.

2.3 Government's decision of e

Due to informational asymmetries, the set of students cannot be chosen in the same way as an omniscient social planner does. The government chooses the level of education taking account of the individual decisions which are determined by the threshold level of ability. The welfare criterion of the government is still the aggregate production net of education cost at a steady state.

Given an educational level, the ability threshold which determines who decides to study is denoted by $y^F(e)$ (see equation (4)). Thus, the government's objective is

$$\underset{e}{Max}\, W(y^F(e), e) = w_s L_s + w_u L_u - N_s c(e) \tag{12}$$

in which skilled and unskilled labor levels are those determined by the threshold ability level

$$N_s = \overline{y} - y^F(e), L_u = 2y^F(e), L_s = N_s e\left(\frac{\overline{y} + y^F(e)}{2}\right) \tag{13}$$

The impact on welfare due to a marginal change of education is composed of two terms: an indirect one through the selection of abilities and a direct one. Formally, the marginal change in welfare that results from an increase in the educational level chosen by the government is given by

$$\frac{dW}{de} = \frac{\partial W}{\partial y}\frac{dy^F}{de} + \frac{\partial W}{\partial e} \tag{14}$$

where $\frac{dy^F}{de}$ denotes the change in the threshold ability level and thus in the selection of abilities.

The key point is that individuals' choices are not distorted in our model under full fee financing. In other words, the optimal ability associated with a given educational level coincides with that chosen by individuals. Specifcally $\frac{\partial W}{\partial y}(y^F(e), e)$ is identically null as can be seen from (4) and (9). An immediate consequence is that the optimal allocation and the maximal value for welfare can be reached even without observing abilities. By choosing the optimal level e^*, the associated optimal set of students is selected, those with ability larger than $y^* = y^F(e^*)$, and surely the government cannot do better.

3 Two open economies with mobile students

We study the same model as before - now, however, with two economies where students are mobile.[5] In particular, individuals who decide to study have no migration costs and choose the country where they attain higher education. Graduates may stay in the country where they have completed higher education or come back to their home country with some return probability, as described below. As a benchmark, we start by analyzing the choice made by an omniscient social planner who can decide on the level of education in each country and on the ability of those who study and at which level. An alternative interpretation is that the two countries cooperate in their choice of the levels of education and have complete information on abilities.

[5]This corresponds to empirical evidence according to which mobility increases with education. See, e.g., Ehrenberg and Smith (1993).

3.1 Optimal allocation

We consider the aggregate welfare over the two countries as the objective. The fact that there is a uniform educational level in each country is a constraint. This opens up the possibility of overall welfare gains if distinct educational levels are chosen in both countries and students are mobile. The omniscient social planner can choose two levels of education and the ability of those who study and at which level. Denote by e^A and e^B the educational levels (even though here an educational level is not necessarily attached to a specific country). With obvious notation overall welfare is

$$W = w_s L_s + w_u L_u - c(e^A)N_s^A - c(e^B)N_s^B. \tag{15}$$

The function is concave, hence the optimum is characterized by first-order conditions. Arguing directly, it is optimal to split individuals according to their abilities. If $e^A \geq e^B$ for instance, let y^{AB} be the minimum ability of those who are assigned to a high educational level, and y^u the minimum ability of those who are allowed to study. Individuals with an ability between y^u and y^{AB} acquire the low level of education e^B and those with an ability between y^{AB} and $\overline{y}$ acquire the high level e^A, as depicted in Figure 1.

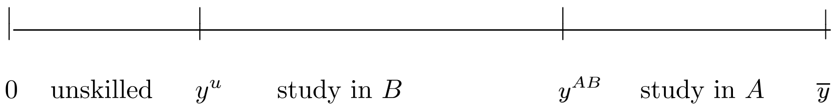

Figure 1: Threshold levels y^u and y^{AB} for $e^A > e^B$:

The choice of the levels of education and of the abilities of those who acquire a given level fully determine the number of students in each program as well as skilled and unskilled labor:

$$N_s^A = 2\left(\overline{y} - y^{AB}\right), N_s^B = 2\left(y^{AB} - y^u\right) \tag{16}$$

$$L_s = \left(\overline{y} - y^{AB}\right)(\overline{y} + y^{AB})e^A + \left(y^{AB} - y^u\right)(y^{AB} + y^u)e^B,\ L_u = 4y^u. \tag{17}$$

With similar computations as in the case of a single quality level, the optimum is characterized by the following first-order conditions

$$\begin{aligned} w_s\frac{\overline{y} + y^{AB}}{2} - c'(e^A) = 0 \text{ and } w_s\frac{y^{AB} + y^u}{2} - c'(e^B) &= 0 &(18)\\ -w_s e^A y^{AB} + c(e^A) = -w_s e^B y^{AB} + c(e^B) \text{ and } - w_s e^B y^u + c(e^B) &= -2w_u &(19)\end{aligned}$$

These conditions are easily interpreted. Conditions (18) say that the educational levels are optimal given the thresholds, that is given the set of students. The marginal gain from a change of the high educational level for the average student, $w_s\frac{\overline{y}+y^{AB}}{2}$, is equal to the marginal costs, $c'(e^A)$, and similarly for the lower level of education. Conditions (19) say that the ability of the students in each program is optimal given the education levels proposed. The net gain of top education relative to the lower level is null for the student with marginal ability y^{AB} and the net gain of the low level of education compared to remaining uneducated is null for the marginal student y^u.

We shall denote by $(\overline{e}^*, \underline{e}^*)$ these optimal levels.

3.2 Nash Equilibrium in educational levels

Consider now the situation with informational asymmetries and full mobility for students. As in a closed economy, a government chooses the level of education taking account of the individual decisions. In an open economy, individuals face more choices and their decisions are affected by the education levels chosen by both countries. Mobility thus generates a game between the two countries. Before spelling out the game and in particular the government's criterion, we analyze individuals' decisions.

3.2.1 Individual choices

We first need to consider the free choice of individuals when the two countries have chosen their education levels, e^A an e^B.

A young individual born in country I, $I = A, B$ now not only has to decide whether to study but also where to study. Since wages are constant, the lifetime income of a young who decides to study in I is $ye^I w_s - c\left(e^I\right)$. This implies that the maximum lifetime income of a y-young individual who decides to become skilled is

$$V_s(y) = \max[ye^A w_s - c\left(e^A\right), ye^B w_s - c\left(e^B\right)]$$

Similarly, the lifetime income of an unskilled worker is unchanged, given by $2w_u$ in both countries. The individual chooses to be skilled if $V_s(y) \geq 2w_u$.

In the symmetric case where educational levels are equal, $e^A = e^B = e$, individuals are indifferent between studying in either country. In that case we shall assume that they split equally (as occurs, for example, if they do not move at all).

Assume now that education levels are distinct. We take $e^A > e^B$. The return to education increases with ability. As a result, the individuals who choose to study in A and not in B are those with high enough ability and the individuals who stay unskilled are those with low enough ability. Specifically, let y^{AB} be the type of an individual who is indifferent between studying in A and B. It is the value y defined by

$$ye^B w_s - c\left(e^B\right) = ye^A w_s - c\left(e^A\right). \tag{20}$$

Analogously, let y^u be the type of an individual who is indifferent between studying and not studying. It is defined by

$$ye^B w_s - c\left(e^B\right) = 2w_u. \tag{21}$$

These expressions are valid only if both levels are chosen such that $y^u < y^{AB} < \overline{y}$. For given educational levels, young individuals partition themselves according to their abilities as depicted in Figure 1. Comparing the choices (20) and (21) with the optimal thresholds (19) given the education levels (e^A, e^B), we find that *the optimal partition associated with given education levels coincides with that chosen by individuals.*

An immediate consequence is that the optimum can be obtained even without observing ability levels: If the optimal levels of education are implemented, it suffices to let individuals choose whether (and where) to study. These optimal levels can be thought as resulting from the decisions of a union of countries acting in a cooperative way. We consider next a Nash equilibrium and compare the levels of education to the optimal ones. We are interested in whether countries want to match their levels of education leading to a symmetric equilibrium or whether they aim at differentiating their levels - especially by increasing the level above that of the other country.

3.2.2 Return rate and Welfare criterion

We need first to define the game, and especially the criterion on which governments base their choices. With migration, population is variable within a country, and a variety of welfare criterion may be considered (see e.g. the discussion in Blackorby, Bossert and Donaldson, 2006). We shall consider here a resident welfare criterion where the government maximizes the net wage sum of all residential workers (natives and foreigners). This criterion is affected by a 'graduate return probability'. Later on we consider a native welfare criterion where the government maximizes the net wage sum of natives whereever they work. As explained below, a 'native criterion' is obtained as a special case when the return probability is taken equal to 1 (even if graduates do not need to come back).

Skilled workers are indifferent between working in either country, so that any migration decision they may take is rational. Their behavior is described as follows. Students who have studied abroad come back to work in their home country with probability π, called the return rate. Those who study in their home country do not move afterwards and remain as skilled workers in their home country. The return rate π is independent of everything else, and in particular of the ability type. The return rate determines the skilled labor force in each country as follows.

Let us assume that $e^A > e^B$. From country A and B the top ability students study in country A. Their number is $2\left(\overline{y} - y^{AB}\right)$ (as given by 16). Half of the students in A come from country B, and among those a proportion $\pi \in [0,1]$ comes back to the home country B as skilled workers. Hence only the fraction $1 - \pi/2$ of skilled workers with education level e^A works in A and the remaining fraction, $\pi/2$, works in B. Similarly, the low ability types of both countries study in country B. Their total number is $2\left(y^{AB} - y^u\right)$, and among those a fraction $\pi/2$ will work in country A and the fraction $1 - \pi/2$ in country B.

The residential welfare of a country is defined as the aggregate lifetime income of the residents. Since the cost of education is entirely borne by a student, the lifetime of a skilled worker is defined as his wage diminished by the cost of its education level. There are three types of (workers) residents: the skilled workers with education levels e^A or e^B and the unskilled workers. We consider each category in turn.

The average effective labour supply of a skilled worker with education level e^A is $\frac{\overline{y}+y^{AB}}{2}e^A$. Hence the average lifetime of a skilled worker with education level e^A is $w_s\frac{\overline{y}+y^{AB}}{2}e^A - c(e^A)$. Similarly the average effective labour supply of a skilled worker with education level e^B is

$\frac{y^{AB}+y^u}{2}e^B$, which yields an average lifetime equal to $w_s\frac{y^{AB}+y^u}{2}e^B - c(e^B)$. The average lifetime of an unskilled worker is $2w_u$.

Weighting by the number of residents in the three categories and rearranging, the residential welfare of country A is given by:

$$\begin{aligned} W^A(e^A,e^B) &= 2\left(1-\frac{\pi}{2}\right)\left(\overline{y}-y^{AB}\right)\left[w_s\frac{\overline{y}+y^{AB}}{2}e^A - c\left(e^A\right)\right] + \\ & 2\frac{\pi}{2}\left(y^{AB}-y^u\right)\left[w_s\frac{y^{AB}+y^u}{2}e^B - c(e^B)\right] + 2w_u y^u \end{aligned} \tag{22}$$

Analogously, the resident welfare criterion for country B is:

$$\begin{aligned} W^B(e^A,e^B) &= 2\frac{\pi}{2}\left(\overline{y}-y^{AB}\right)\left[w_s\frac{\overline{y}+y^{AB}}{2}e^A - c\left(e^A\right)\right] + \\ & 2\left(1-\frac{\pi}{2}\right)\left(y^{AB}-y^u\right)\left[w_s\frac{y^{AB}+y^u}{2}e^B - c(e^B)\right] + 2w_u y^u \end{aligned} \tag{23}$$

When e^A is smaller than e^B, the welfare expressions are exchanged. In the symmetric situation where both countries choose the same level e, the labor force is equally split between the countries, and welfare in each country amounts to

$$W^A(e,e) = W^B(e,e) = w_s\left(\overline{y}-y^u\right)\left(\overline{y}+y^u\right)e/2 + 2y^u w_u - c(e)(\overline{y}-y^u). \tag{24}$$

The ability composition of those who study in A or in B depends on which level is larger, but not on the return probability rate. As a result, total welfare is independent on that rate. But the return rate determines the share of this total welfare hence affects incentives. Specifically, let $TW(e^A,e^B)$ denote the total welfare associated to two education levels, e^A, e^B. We have

$$\begin{aligned} TW(e^A,e^B) &= w_s\left[2\left(\overline{y}-y^{AB}\right)\frac{\overline{y}+y^{AB}}{2}e^A + 2\left(y^{AB}-y^u\right)\frac{y^{AB}+y^u}{2}e^B\right] \\ & +4w_u y^u - 2\left(\overline{y}-y^{AB}\right)c\left(e^A\right) - 2\left(y^{AB}-y^u\right)c\left(e^B\right) \end{aligned} \tag{25}$$

The identity $TW(e^A,e^B) = W^A(e^A,e^B) + W^B(e^A,e^B)$ holds true whatever the return rate π. But the return rate determines each country's welfare hence their incentives to choose an education level. In particular, the larger the return rate is, the less the skilled labor in a country depends on the country decision. At one extreme, where the return rate is null ($\pi = 0$), each country ends up with the skilled labor force that graduated in the country. This opens up the possibility of competition for students. At the other extreme case where the return rate is one ($\pi = 1$), each country ends up with the same labor force. As we shall see in the last case, the residential criterion coincides with a 'native' welfare criterion. We analyze both cases. When graduates only partially return to their home country ($0 < \pi < 1$) the welfare criteria can be written as a combination of these extreme cases. To see this, let us simplify notation and denote by L_A be the lifetime wealth of a skilled worker with education level e^A, and similarly for L^B. From (22) we have

$$
\begin{aligned}
W^A(e^A, e^B) &= 2(1-\pi/2)L^A + 2\,(\pi/2))\,L^B + 2w_u y^u \\
&= (1-\pi)[2L^A + 2w_u y^u] + \pi[L^A + L^B + 2w_u y^u] \qquad (26)
\end{aligned}
$$

which can be written as $(1-\pi)W^A_{|\pi=0} + \pi W^A_{|\pi=1}$ since the first term in square bracket is the welfare of A for π equal to zero, and the second term the welfare of A for π equal to one.

In the following we analyze szenarios with differing return probabilities of students. As we have just shown this changes the welfare function the government uses to set the educational level. The degree of differentiation between the countries and the optimality of differentiation in the Nash equilibrium depends crucially on the return probability. We start with the extreme szenario that all students who take up higher education in a foreign country return after graduation to their home country

3.3 Graduates return to their home country ($\pi = 1$): the native welfare criterion

According to the native's principle, a government is interested in the well-being of the natives even those who have left the country, and does not care about the immigrants. When all students come back to work home, the residents and the natives coincide. As a result, for a return probability equal to one, $\pi = 1$, the resident's welfare criterion coincides with the welfare of natives. Substituting $\pi = 1$ in the welfare functions (22) and (23) yields the native welfare function which is for both countries:

$$
\begin{aligned}
W^A_{nat}\left(e^A, e^B\right) &= W^B_{nat}\left(e^A, e^B\right) = w_s \left[\left(\overline{y} - y^{AB}\right) \left(\frac{\overline{y} + y^{AB}}{2} e^A - c\left(e^A\right) \right) \right. \qquad (27) \\
&\quad \left. + \left(y^{AB} - y^u\right) \left(\frac{y^{AB} + y^u}{2} e^B - c\left(e^B\right) \right) \right] + w_u 2y^u
\end{aligned}
$$

With a native criterion and fee financed regimes, the welfare in each country is half the total welfare. This can be seen by comparing (27) with the welfare of the social planner (15):

$$
W = w_s \left[\left(\overline{y} - y^{AB}\right)(\overline{y} + y^{AB})e^A - 2c(e^A) + \left(y^{AB} - y^u\right)(y^{AB} + y^u)e^B - 2c(e^B) \right] + w_u 4y^u
$$

This observation is sufficient to infer that the incentives of both countries and the social planner are aligned. As a result, if one country chooses one of the optimal level, say the largest one $\overline{e}^*$, the other country optimal choice is the lowest $\underline{e}^*$, (and vice versa). The reason is that with a return pobability of one both countries have the same labor force whereever the workers have been educated. We summarize:

Proposition 1 *If countries maximize the welfare of their natives they choose in the Nash equilibrium the optimal differentiation of educational levels.*

3.4 Graduates stay where they were educated ($\pi = 0$)

Now we consider the other extreme szenario that all student who study abroad stay there after graduation. As it will become clear in a moment, in this case the welfare levels change in a discontinuous way when the educational levels of the two countries approach one another. We, therefore, study carefully in the following the transition from the asymmetric case to the symmetric case.

With distinct levels with e^A larger than e^B, welfare criteria are given by (22), (23) taking π equal to 0:

$$W^A = w_s \left(\overline{y} - y^{AB}\right)(\overline{y} + y^{AB})e^A + 2y^u w_u - 2c(e^A)\left(\overline{y} - y^{AB}\right) \tag{28}$$

$$W^B = w_s \left(y^{AB} - y^u\right)(y^{AB} + y^u)e^B + 2y^u w_u - 2c(e^B)(y^{AB} - y^u). \tag{29}$$

and in the symmetric situation where both countries choose the same level e welfare in each country amounts to

$$W^A = W^B = w_s\left(\overline{y} - y^u\right)(\overline{y} + y^u)e/2 + 2y^u w_u - c(e)(\overline{y} - y^u). \tag{30}$$

It is instructive to analyze the two benchmark situations where either both countries choose the single-constrained optimum e^* of a closed economy, or where they choose the optimal differentiation levels. None of these choices constitute an equilibrium.

Single-constrained optimum e^* Consider the symmetric situation in which each country chooses the single-constrained optimum e^*. Let country A contemplate changing its educational level from e^* to e^A. We show that a marginal increase in education level has a dramatic selection effect that is profitable to A. Let $e^A > e^* = e^B$. Individuals partition themselves according to the ability thresholds y^{AB} and y^u, as defined by (20) and (21). We show that the set of individuals who decide to study is unchanged, $y^u = y^*$ but those who decide to study in A are those with a large enough ability, larger than the average ability of the students. The following reasoning is illustrated in Figure 2.

We shall use that at the optimum e^* the marginal gain from a change in the educational level on the average student, $w_s\frac{\overline{y}+y^*}{2}$, is equal to the marginal cost, $c'(e^*)$. This implies that e^* is 'too high' for the marginal student y^* and 'too low' for the top ability student. Specifically, consider the lifetime income of a young individual with ability y^*, $y^*ew_s - c(e)$, as a function of e. It is concave and the derivative is negative at $e = e^*$, i.e. the educational level e^* is 'too high' for the marginal student y^*, i.e. $y^*w_s - c'(e^*) < 0$. Hence the lifetime income $y^*ew_s - c(e)$ is larger at $e = e^*$ than at $e = e^A$. Furthermore, the same argument is valid for y smaller than y^*. Hence, individuals with ability smaller than y^* remain unskilled (as skilled they would prefer to study in B, but by definition of y^* they do not benefit from that). In contrast, the educational level e^* is too small for a top ability student $\overline{y}$, i.e. $\overline{y}w_s - c'(e^*) > 0$. Using a similar argument as above, this implies that a young individual with ability $\overline{y}$ studies in A. It follows that if A increases slightly its educational level, the set of individuals who decide to study is unchanged, $y^u = y^*$.

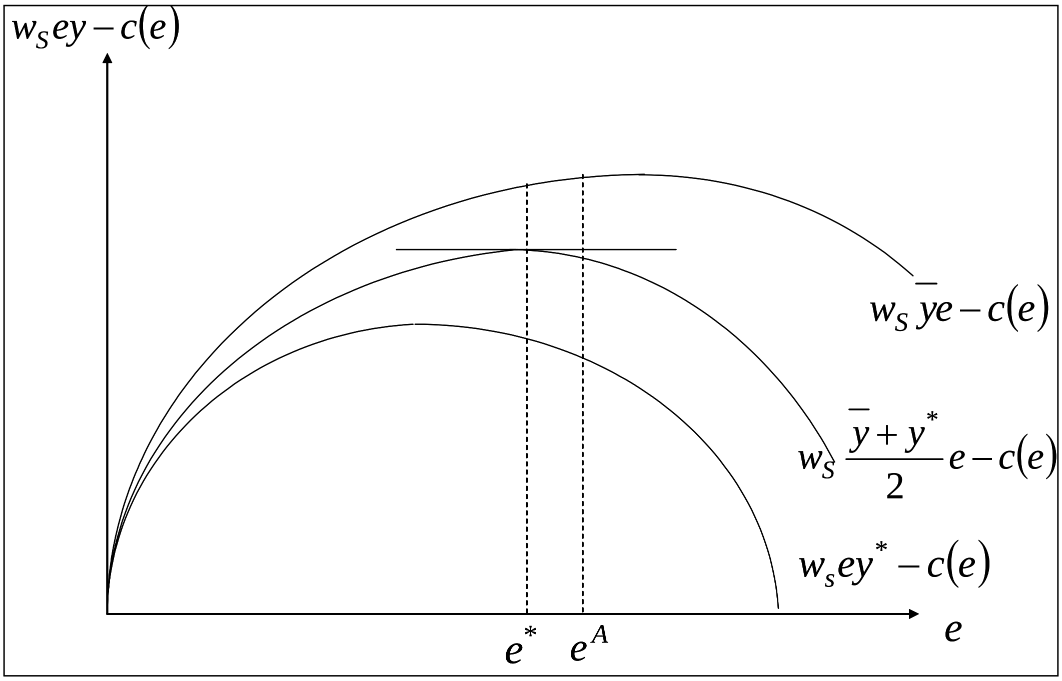

Figure 2: The sorting of ability types to countries with a single-constrained educational level.

Consider now y^{AB} the type of an individual who is indifferent between studying in A and B, as defined by (20) $ye^*w_s - c(e^*) = ye^Aw_s - c\left(e^A\right)$. Observe that by convexity of c we have that

$$y^{AB} = \frac{c\left(e^A\right) - c\left(e^*\right)}{w_s(e^A - e^*)} \geq \frac{c'(e^*)}{w_s} = \frac{\overline{y} + y^*}{2}. \tag{31}$$

where the last equation follows from the first-best conditions (see also Figure 3). Hence, country A, by providing a higher educational level than B not only attracts the best students but also deters half of the bottom students at least. This is true whatever the level e^A strictly larger than e^*. Taking the limit of y^{AB} when e^A tends to e^* gives

$$y^{lim}(e^*) = \lim_{e^A \to e^*} \frac{c\left(e^A\right) - c\left(e^*\right)}{w_s(e^A - e^*)} = \frac{c'(e^*)}{w_s} = \frac{\overline{y} + y^*}{2}. \tag{32}$$

Thus if A increases slightly its educational level, the overall set of individuals who decide to study is unchanged, given by the individuals whose ability is larger than y^*. Individuals with ability larger than the average over the students, those with ability in $(\frac{\overline{y}+y^*}{2}, \overline{y})$ study in A, and individuals with ability lower than the average, those with ability in $(y^*, \frac{\overline{y}+y^*}{2})$, study in B.

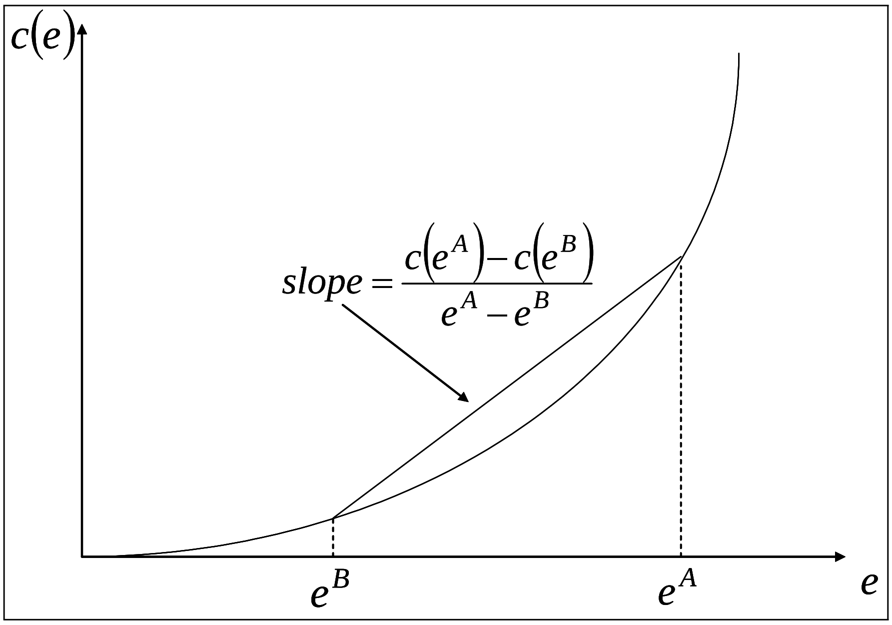

Figure 3: Threshold y^{AB} increases with educational levels e^A and e^B.

In words, for e^A arbitrarily close but larger than e^*, A has the same number of students as at the initial situation, but the ability composition is increased. This results in an improvement of welfare in country A. Simple computation gives that welfare is increased by

$$w_s \left(\frac{\overline{y} - y^*}{2} \right)^2 e^*.$$

Hence, we can summarize:

Proposition 2 *If countries start with the optimal educational level in a closed economy and open up their borders students' mobility induces countries to increase the educational level above this optimal level.*

Optimal differentiation levels $\overline{e}^*, \underline{e}^*$. Consider now the optimal distinct levels, with for instance A choosing the highest level: $e^A = \overline{e}^*$ and $e^B = \underline{e}^*$. Let country A contemplate changing its educational level. The impact on welfare can be decomposed into two effects: a welfare effect on the current students and a migration effect due to a change in their incentives, which results in a change of the threshold. As we have seen, the level $e^A = \overline{e}^*$ is the optimal level for the set of students, hence the first effect is null. On the other hand, by lowering its educational level, A attracts some students from B and the migration effect can be shown to be positive. From similar arguments, at the optimal lower level $\underline{e}^*$, B has an incentive to increase quality level. In other words, at the optimal distinct education levels, countries mainly compete over the marginal students, those who are indifferent between the two countries. This generates a force towards less differentiation. These arguments are made clear by considering marginal changes in welfare.

Let us start from unequal levels, say $e^A > e^B$. As long as we consider variations in educational levels that are small enough so that the educational level in A is still higher than in B, A continues to attract the students with the highest ability. Hence, a marginal change in e^A or in e^B modifies the allocation of the students at the margin only through the modifications of the thresholds y^{AB} and y^u. A marginal change in e^A yields the marginal change in W^A

$$dW^A = 2\left(\overline{y} - y^{AB}\right)\left[w_s \frac{\overline{y} + y^{AB}}{2} - c'(e^A)\right] + 2[y^{AB}e^A w_s - c\left(e^A\right)](-\frac{\partial y^{AB}}{\partial e^A}). \tag{33}$$

The marginal change is composed of two terms. The first term is related to the efficiency gains of the current population of students in A that result from changing the educational level. The second term results from changes of that population through the modification of the threshold. It is equal to the change in the number of students, $-2\frac{\partial y^{AB}}{\partial e^A}$, multiplied by the gain per such student, $[y^{AB}e^A w_s - c\left(e^A\right)]$

Similarly, a marginal change in the lower educational level e^B yields the marginal change in W^B

$$\begin{aligned} dW^B &= 2\left(y^{AB} - y^u\right)\left[w_s \frac{y^{AB} + y^u}{2} - c'(e^B)\right] + 2[y^{AB}e^B w_s - c\left(e^B\right)]\frac{\partial y^{AB}}{\partial e^B} \\ &\quad +2[-y^u e^B w_s + w_u + c\left(e^B\right)]\frac{\partial y^u}{\partial e^B}. \end{aligned} \tag{34}$$

The terms can be interpreted similarly as terms which capture the efficiency effect and the change in the population, here both at the top and at the bottom ability levels.

When $e^A > e^B$, we have that country A prefers a lower educational level than optimal. More students study in A than in B due to a marginal decrease in e^A and the second term in (33) is always negative. For country B from the convexity of c, increasing e^B increases the threshold value y^{AB}, meaning here that the number of students in B increases, which results in a gain.[6] However at the same time, this changes the incentives to become skilled. As a result, the optimal educational values (cf. (18)) surely do not form an equilibrium. The first term - the efficiency term - is null. We have seen that the second term in (33) is negative. Hence, at the optimal levels, the country with the largest educational level is incited to decrease the quality so as to increase the number of its students. The reason is that a country takes into account the impact of its educational level on a fraction of the students only. Since the attracted students are indifferent between the two educational levels, there is no welfare loss on the aggregate: these students move from B to A with roughly the same lifetime levels which results in a transfer of welfare from B to A.

More generally, competition on the marginal students who are close to being indifferent between the two countries is a force towards less differentiation. However, as seen above, an improvement in the educational level with the lowest level also changes the incentives to become skilled, that is the value y^u, in a way that is not generally clear.

[6] Actually, $2[y^{AB}e^B w_s - c\left(e^B\right)]\frac{\partial y^{AB}}{\partial e^B}$ is exactly the opposite of $2[y^{AB}e^A w_s - c\left(e^A\right)](-\frac{\partial y^{AB}}{\partial e^A})$, meaning that there is only a transfer of welfare between the two countries at the margin, see later.

Furthermore, and more importantly, as education levels are equalized, a massive reallocation of students take place. This results in a discontinuous change in welfare levels.

In most cases, the reallocation of students leads to clear incentives for a country. For example, consider a symmetric situation (e, e). By a similar argument as used when considering the level e^*, a country that increases marginally its level attracts all students with ability larger than the value $y^{lim} = c'(e)/w_s$ (by the same computation as in (32)). Similarly, a country that decreases marginally its level attracts all students with ability lower than this value. There are overall the same number of students as at the symmetric situation (y^u changes marginally and only if a country decreases its level) but the ability composition of those who study in A or in B is affected. When the net benefit from educating high ability students, those with ability larger than y^{lim} is strictly larger than the net benefit from educating low ability students, those with ability between y^{lim} and y^u, increasing marginally the educational level above that of the other country e so as to attract the high ability students is beneficial. This was shown to be the case at the optimal single level e^*. Similarly, when the net benefit from educating low ability students is larger than that from educating high ability students, a country surely benefits from choosing its education level slightly below that of the other country. There is a (unique) level, denoted by $\hat{e}$, for which these benefits are equalized. Such a level is larger than e^* and is the only possible candidate for a symmetric equilibrium.

We may nevertheless show that under some conditions on the cost function, no differentiation occurs at equilibrium.

Proposition 3 *With a linear cost function and bounded educational levels there is a unique equilibrium in which both countries choose the maximum level.*
With a quadratic cost function, an equilibrium in pure strategies is surely symmetric. Countries choose both the same educational level $\hat{e}$.

Proof. See the Appendix. ■

In the next section an illustration is given by a quadratic cost function in which welfare W^A is represented for a fixed value of e^B and the parameters $\overline{y} = 10;\ w_s = 2;\ w_u = 1$ (insert figures). For $e^A \in [0; 10]$ and $e^B \in [0; 10]$ the restriction holds that $\overline{y} \geq y^{AB}$. For $e^A < e^B$, W^A is linear and increasing in e^A. There is a jump when e^A reaches e^B because A and B now share the students equally. There is another jump (which can be shown to be in the same direction, equal to the first one) when e^A becomes larger than e^B because the roles of A and B are exchanged with A now attracting the high ability students.

3.4.1 Illustration

In the following, we provide an example to illustrate the discontinuities of the welfare function as established above. We do not solve for the Nash-equilibrium, but we study how the welfare of country A changes for different values of e^A when country B provides the optimal minimal educational level $\underline{e}^*$. Under the constraint that country B chooses $\underline{e}^*$, this allows to determine the welfare maximizing educational level for country A and to evaluate this level relative to the optimal level $\overline{e}^*$.

More precisely, assume that the cost functions of the educational levels are quadratic: $c\left(e^A\right) = \left(e^A\right)^2$ and $c\left(e^B\right) = \left(e^B\right)^2$. For this example, we take the following parameter values as given: $\overline{y} = 10$; $w_s = 2$ and $w_u = 1$. Then, we can calculate the optimal differentiation of the educational levels by using the first-order conditions of the social planner (18):

$$\begin{aligned} \overline{e}^* &= 8.1 \\ \underline{e}^* &= 4.3 \end{aligned}$$

We have to observe that the educational levels are in the range of $e^A, e^B \in [0, 10]$ in order to satisfy the constraint that $\overline{y} \geq y^{AB}$.

Depending on whether the educational level of country A is above, equal to or below the educational level of country B the welfare function of country A changes. If $e^A > \underline{e}^*$, country A will attract all the high ability students of both countries and welfare is given by

$$W^A = w_s \left(\overline{y} - y^{AB}\right)(\overline{y} + y^{AB})e^A + 2y^u w_u - 2c(e^A)\left(\overline{y} - y^{AB}\right) \tag{35}$$

If $e^A = \underline{e}^* = e$, we assume that the students split equally between both countries so that welfare is

$$W^A = w_s \left(\overline{y} - y^u\right) \frac{(\overline{y} + y^u)}{2} e + 2y^u w_u - c(e)(\overline{y} - y^u). \tag{36}$$

If $e^A < \underline{e}^*$, country A will educate all the low ability students of both countries and welfare is

$$W^A = w_s \left(y^{AB} - y^u\right)(y^{AB} + y^u)e^A + 2y^u w_u - 2c(e^A)(y^{AB} - y^u). \tag{37}$$

Hence the welfare function is piecewise defined and not continuous in e^A (see Figure 4). Which educational level country A will choose depends on the maximum welfare levels of the piecewise-defined welfare function. In the first case, $e^A > \underline{e}^*$, the welfare function is strictly concave and has a maximum at $e^A = 6.2$. The maximal welfare is $W^A\left(e^A > \underline{e}^*\right) = 537.8$. In the second case, country A chooses the same level as country B: $e^A = e^{B*} = 4.3$ and welfare is 269.5 from (36). In the last case, $e^A < e^{B*}$, welfare increases linear with e^A and the maximum is achieved by approaching the educational level $\underline{e}^*$ from below. The maximal welfare is $\lim\limits_{e^A \to e^{B*}} W^A\left(e^A < e^{B*}\right) \approx 44.05$.

In this case of a quadratic cost function, country A maximizes its welfare by choosing an educational level which is higher than the optimal level in country B, $\widehat{e}^A = 6.2 > e^{B*}$.

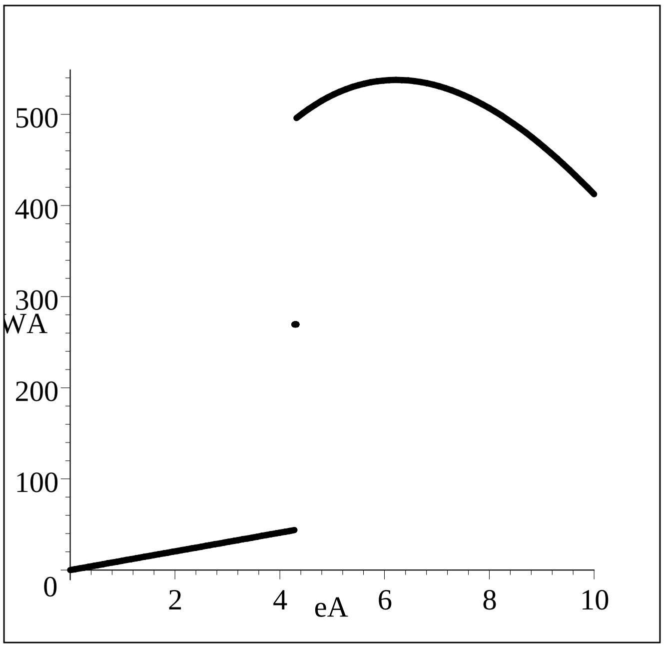

Figure 4: Welfare function of country A at $e^B = e^{B*}$

This example shows that even in a case where the choice of the educational level is restricted to $\underline{e}^*$ for country B, the level chosen by country A falls short of the optimally differentiated level, $\widehat{e}^A < \overline{e}^*$.

3.5 Graduates only partially return to their home country $(0 < \pi < 1)$

We consider first the case that countries choose different educational levels and assume by convention that $e^A > e^B$. In this case we show that for all return probabilities smaller than 1 the optimal differentiation of educational levels is never a Nash equilibrium.

From equation (26) we know that the welfare function of both countries can be written as:

$$W^A(e^A, e^B) = (1-\pi)[2L^A + 2w_u y^u] + \pi[L^A + L^B + 2w_u y^u]$$

and

$$W^B(e^A, e^B) = (1-\pi)[2L^B + 2w_u y^u] + \pi[L^A + L^B + 2w_u y^u]$$

Evaluating the first-order conditions at the optimum $(\overline{e}^*, \underline{e}^*)$ we know from section 3.3. that the derivatives of $L^A + L^B + 2w_u y^u$ with respect to e^A and e^B are equal to zero because countries with $\pi = 1$ choose the optimal differentiation. Since $\pi < 1$ some weight in the welfare function is given to residents which are not natives. Concentrating on this first part of the welfare functions:

$$(1-\pi)\left(2L^A + 2w_u y^u\right) = (1-\pi)\left(2\left(\overline{y} - y^{AB}\right)\left[w_s \frac{\overline{y} + y^{AB}}{2} e^A - c\left(e^A\right)\right] + 2w_u y^u\right) \tag{38}$$

and

$$(1-\pi)[2L^B + 2w_u y^u] = (1-\pi)\left[2\left(y^{AB}-y^u\right)\left[w_s\frac{y^{AB}+y^u}{2}e^B - c(e^B)\right] + 2w_u y^u\right] \qquad (39)$$

the first order conditions are given by (33) and (34) both multiplied by $(1-\pi)$. Now we can prove the following proposition:

Proposition 4 *Assume that $\pi < 1$ and $e^A > e^B$. Optimally differentiated educational levels do not constitute an equilibrium. Country A has an incentive to choose an educational level less than $\overline{e}^*$. If the optimal differentiation is not too large, i.e. $\underline{e}^* > \frac{\overline{e}^*}{2}$, country B has an incentive to choose an educational level higher than $\underline{e}^*$.*

Proof. Assume the countries provide the optimally differentiated educational levels $e^A = \overline{e}^*$ and $e^B = \underline{e}^*$. From condition (18) the first terms in squared brackets in (33) and 34) are zero.

The second term in (33) is negative since $[y^{AB}e^A w_s - c\left(e^A\right)] > 0$ and

$$\frac{\partial y^{AB}}{\partial e^A} = \frac{c'\left(e^A\right) - w_s y^{AB}}{w_s\left(e^A - e^B\right)} > 0$$

(see Figure 3). Hence, country A's welfare increases if e^A decreases below $\overline{e}^*$.

The second term in (34) is positive since $[y^{AB}e^B w_s - c\left(e^B\right)] > 0$ and

$$\frac{\partial y^{AB}}{\partial e^B} = \frac{w_s y^{AB} - c'\left(e^B\right)}{w_s\left(e^A - e^B\right)} > 0$$

The third term in (34) is negative since $[-y^u e^B w_s + w_u + c\left(e^B\right)] = -w_u$ by definition of y^u (see (21). And at $e^B = \underline{e}^*$ we get:

$$\left.\frac{\partial y^u}{\partial e^B}\right|_{e^B=\underline{e}^*} = \frac{c'\left(e^B\right) - w_s y^u}{w_s e^B} > 0$$

because $c'\left(e^B\right) = w_s\frac{y^{AB}+y^u}{2} > w_s y^u$. A higher e^B than $\underline{e}^*$ is welfare improving for country B if and only if

$$\left[[y^{AB}e^B w_s - c\left(e^B\right)]\frac{\partial y^{AB}}{\partial e^B} - w_u\frac{\partial y^u}{\partial e^B}\right]_{e^B=\underline{e}^*} > 0 \qquad (40)$$

We compare both terms factorwise. First we have $y^{AB}e^B w_s - c\left(e^B\right) > y^u e^B w_s - c\left(e^B\right) = 2w_u > w_u$. Second we show that $\frac{\partial y^{AB}}{\partial e^B} > \frac{\partial y^u}{\partial e^B}$ if $e^B > \frac{e^A}{2}$ at the optimal differentiation. For the following calculation we use that the optimally differentiated educational level e^B satisfies $c'\left(e^B\right) = w_s\frac{y^{AB}+y^u}{2}$:

$$\begin{aligned} & \frac{w_s y^{AB} - c'\left(e^B\right)}{w_s\left(e^A - e^B\right)} - \frac{c'\left(e^B\right) - w_s y^u}{w_s e^B} \\ = \; & \frac{1}{\left(e^A - e^B\right) w_s e^B}\left[\left(e^B - \frac{e^A}{2}\right) w_s\left(y^{AB} - y^u\right)\right] \end{aligned}$$

If $e^B > \frac{e^A}{2}$ we have $\frac{\partial y^{AB}}{\partial e^B} > \frac{\partial y^u}{\partial e^B}$ which proves the proposition. ■

The intuition for this result is that a country takes into account the impact of its educational level on foreign students who only partially return to their home country. Since the attracted students are indifferent between the two educational levels, there is no welfare loss on the aggregate. Those foreign students who stay in the country where they have been educated have at the margin the same lifetime income which results in a transfer of welfare from their home country to the country of education. The country with the higher educational level has an incentive to decrease this level in order to attract foreign students. The country with the lower edcational level has an incentive to increase this level so that the threshold ability level and the number of students increase.

Now we turn to the analysis of symmetric educational levels. Except for a return probability of one ($\pi = 1$), there is in general a jump when the education levels become equalized. The reason comes from the students' mobility which results in a change in the skilled labor force.

Keeping the education level e^B fixed, start with a lower level for A and increase it. When e^A reaches $e = e^B$ because A and B now share the students equally. When e^A becomes larger than $e = e^B$, the roles of A and B are exchanged with A now attracting the high ability students. Hence there is another jump. We show that the result of these moves on the welfare criteria are equal, and in particular is in the same direction.

Lemma 5 *The jumps in A welfare when e^A approaches e from below or from above are equal:*

$$lim_{e^A \to <e}[W^A(e^A, e) - W^A(e, e)] = [W^A(e, e) - lim_{e^A \to >e} W^A(e^A, e)] \tag{41}$$

At a symmetric equilibrium (if any), the jumps must be null.

Proof. Let $TW(e^A, e^B)$ denote the total welfare associated to two education levels, e^A, e^B. We have

$$\begin{aligned} TW(e^A, e^B) = w_s & \left[2\left(\overline{y} - y^{AB}\right) \frac{\overline{y} + y^{AB}}{2} e^A + 2\left(y^{AB} - y^u\right) \frac{y^{AB} + y^u}{2} e^B \right] \\ & + 4w_u y^u - 2\left(\overline{y} - y^{AB}\right) c\left(e^A\right) - 2\left(y^{AB} - y^u\right) c\left(e^B\right) \end{aligned}$$

Total welfare is independent on the probability p, and is continuous. Note that $TW(e^A, e^B) = W^A(e^A, e^B) + W^B(e^A, e^B)$. Hence taking the limit (assuming the limit exist..)

$$\lim_{e^A \to <e, e^B \to >e} W^A(e^A, e^B) + W^B(e^A, e^B) = \lim_{e^A \to <e, e^B \to >e} TW(e^A, e^B) \tag{42}$$

and the limit of the RHS is equal to $TW(e, e)$ by continuity of TW.

By symmetry we have $W^A(e^A, e^B) = W^B(e^B, e^A)$. This gives

$$\lim_{e^A \to <e, e^B \to >e} W^B(e^A, e^B) = \lim_{e^A \to >e, e^B \to <e} W^A(e^A, e^B)$$

Plugging into (42)

$$\lim_{e^A \to <e, e^B \to >e} W^A(e^A, e^B) + \lim_{e^A \to >e, e^B \to <e} W^A(e^A, e^B) = TW(e, e) \tag{43}$$

By symmetry we have $2W^A(e,e) = TW(e,e)$. Taking $e^B = e$ in the above equation, the result follows.

Since the jumps are in the same direction, jumps must be null at a symmetric equilibrium: if the jump is positive at (e,e), a country benefits by proposing a level higher than e and if it is negative it benefits by proposing a smaller one. ■

Intuition. The ability of those who decide to study does not depend on the return rate. The reason is that for e^A and e^B close to e, there is roughly the same total number of students as at the symmetric situation in which both countries choose e. The total cost of education and the total welfare are the same. This means that around symmetric levels (e,e), countries are playing approximately a constant two-person game. The ability composition of those who study in A or in B depends on which level is larger. The jump in A welfare when e^A is decreased from above towards e is exactly compensated by the jump in B welfare. Exchanging the role of A and B gives the result.

Let $\hat{e}$ be the value for which the jump is null. At this level, the net benefit from educating high ability students, those with ability larger than y^{AB}, is exactly equal to the net benefit from educating low ability students, those with ability between y^{AB} and y^u. The essential insight is that this level does not depend on the return rates.

Consider (22) and (23). The difference between $W^A(e^A,e^B)$ and $W^B(e^A,e^B)$ is:

$$\begin{aligned} & W^A(e^A,e^B) - W^B(e^A,e^B) \\ = \ & 2(2p-1)\left[\left(\overline{y}-y^{AB}\right)\left[w_s\frac{\overline{y}+y^{AB}}{2}e^A - c\left(e^A\right)\right] - \left(y^{AB}-y^u\right)\left[w_s\frac{y^{AB}+y^u}{2}e^B - c(e^B)\right]\right] \end{aligned}$$

The values of the thresholds are independent of p. The value $\widehat{e}$ where there is no jump is the one where the above expression is null at $\widehat{e} = e^A = e^B$, hence is independent of p as well. The condition is

$$\left(\overline{y}-y^{lim}\right)\left[w_s\frac{\overline{y}+y^{lim}}{2}e - c\left(e\right)\right] = \left(y^{lim}-y^u\right)\left[w_s\frac{y^{lim}+y^u}{2}e - c(e)\right] \tag{44}$$

where $y^{lim} = \frac{c'(e)}{w_s}$.

For some values of return probability the symmetric equilibrium exists and the educational level $\widehat{e}$ given by (44) is independent of the values of the return probability of graduates less than 1: $0 \leq \pi < 1$. Now we can compare this symmetric equilibrium with the one obtained in the closed economy. From (11) and (10) the single constrained optimum is given by:

$$\frac{c'\left(e^*\right)}{w_s} = \frac{\overline{y}+y^*}{2}$$

Generally, the educational level in the symmetric equilibrium, $\widehat{e}$, is higher than the single-constrained optimum e^*.

3.5.1 Illustration

In our example of a quadratic cost function with parameter values $\overline{y} = 10$; $w_s = 2$ and $w_u = 1$, the educational level of country B at which the welfare function of country A is continuous is given by $\widehat{e} = 8.285$ independently of all return probabilities in the range of: $0 \le \pi < 1$. For example at $\pi = 0$ the welfare function of A is shown in Figure 5:

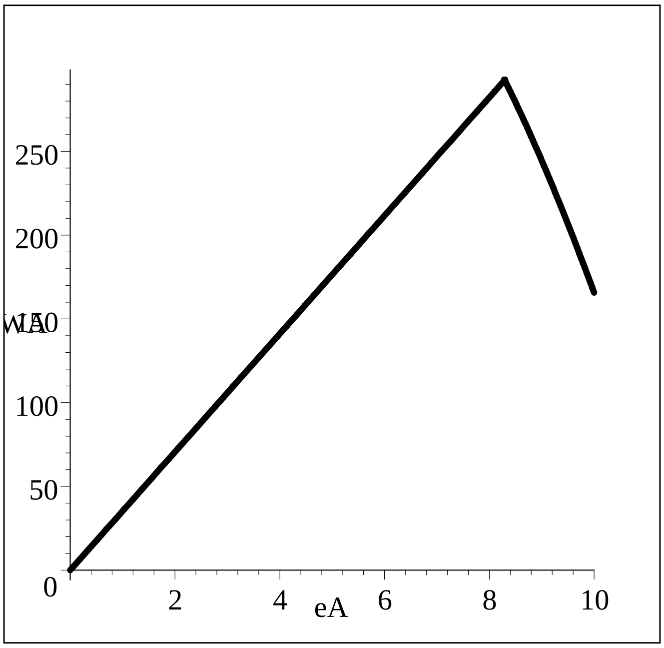

Figure 5: Welfare of country A at $e^B = 8.285$ and $\pi = 0$.

For $\pi = 2/3$:

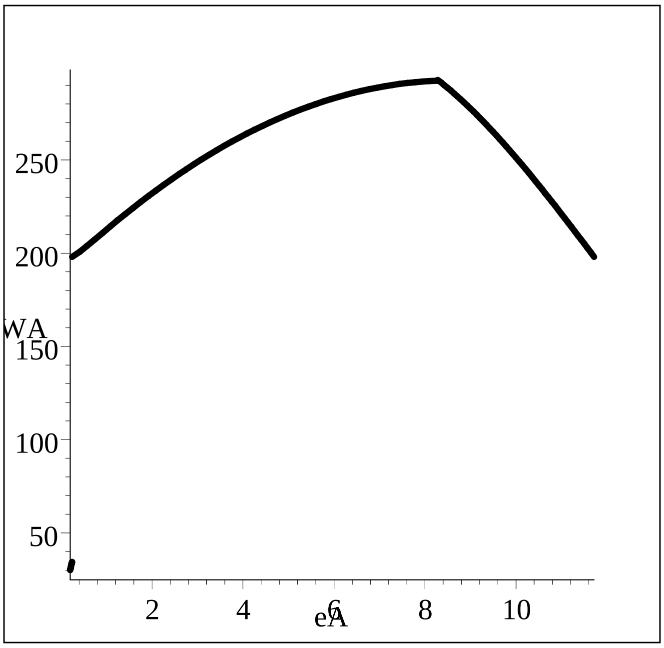

Figure 6: Welfare of country A at $e^B = 8.285$ and $\pi = 2/3$.

For $\pi = 0.9$:

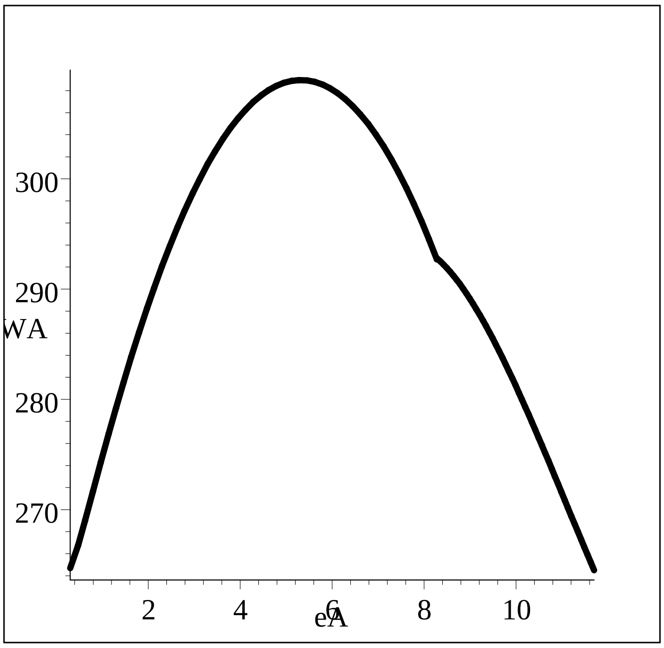

Figure 7: Welfare of country A at $e^B = 8.285$ and $\pi = 0.9$.

Only in the first two cases of $\pi = 0$ and $\pi = 2/3$ the symmetric equilibrim exists at $\widehat{e}$. This shows that the return probability π must be small enough to guarantee the existence of the symmetric equilibrium.

The single-constrained optimum assumes in our example the value $e^* = 6.765$. Hence, if the symmetric equilibrium exists the educational level is higher than in the single-constrained optimum.

4 Conclusion

Mobility of students distorts the governmental choice of quality levels of higher education. Even if countries have the option to choose differing levels of education in order to match differing ability levels of individuals the differentiation will not generally achieve the optimum. The only exception is the case where all graduates after studying in a foreign country return to their home country. In terms of welfare this is equivalent to a welfare objective where govenrments take only account of their natives. Furthermore, we have shown that under certain conditions as to the cost function, countries will choose identical educational levels in a Nash equilibrium. Competition thus clearly leads to suboptimal differentiation.

The reason is that both countries compete to attract students. In particular, at the optimal differentiation levels, the country with the largest education level has an incentive to lower its education level to attract the best students of the other country, which is harmful for its own students. Similarly the country with the lowest education level has an incentive to raise its education level.

This analysis focuses on the choice of the educational quality in a setting where education is fee-financed and students and graduates are mobile . It might be worthwhile to relax the assumption of pure fee-financing. In a further step we analyze the case of (partial) tax-financing of higher education beside fee-financing where mobile graduates affect the tax base.

5 Appendix: Proof of Proposition 3

Proof. Let us determine the best response of a country, say A, to the educational level chosen by the other country. Given $e^B = e$, consider $W^A(e^A, e)$ as a function of e^A. We need to distinguish three cases depending on e^A being smaller than, equal to, or larger than e. Also we want both countries to have students, which requires $y^u < y^{AB} < \overline{y}$. The first inequality holds true if $\frac{c'(e)}{w_s} > \frac{2w_u + c(e))}{w_s e}$ (cf. (20) and (21)), i.e. for $c(e) = e^2$ if $e^2 > 2w_u$.

1) As long as $e^A < e^B = e$, country A attracts students with low ability, i.e., between $y^u(e^A)$ and y^{AB}. The derivative is (cf. (34) where the roles of A and B are exchanged and where we

use $[-y^u e^A w_s + w_u + c(e^A)] = -w_u)$

$$\begin{aligned}\frac{\partial W^A}{\partial e^A} &= 2\left(y^{AB} - y^u\right)\left[w_s \frac{y^{AB} + y^u}{2} - c'(e^A)\right] \\ &\quad +2[y^{AB} e^A w_s - c\left(e^A\right)]\frac{\partial y^{AB}}{\partial e^A} - w_u c\left(e^A\right)\frac{\partial y^u}{\partial e^A} \\ \text{for } e^A &< e^B.\end{aligned} \tag{45}$$

Check that with a quadratic function, the derivative is linear in e^A and increasing: A prefers to be as close as possible to e in this zone.Therefore if the country prefers to be the one with the lowest level its best response is 'almost' to match the other country's level e. Furthermore, this implies that there is no asymmetric equilibrium since the country with the lower educational level increases its welfare by increasing its level.

2) Consider the zone with $e^A > e$. We have

$$\begin{aligned}\frac{\partial W^A}{\partial e^A} &= 2\left(\overline{y} - y^{AB}\right)\left[w_s \frac{\overline{y} + y^{AB}}{2} - c'(e^A)\right] - 2[y^{AB} e^A w_s - c\left(e^A\right)]\frac{\partial y^{AB}}{\partial e^A} \\ \text{for } e^A &> e^B.\end{aligned} \tag{46}$$

The derivative is null at an 'interior' best response, one that is indeed above e. Check that the best response is decreasing with e (write $\frac{\partial W^A}{\partial e^A} = 0$ and impose the solution to be larger than e). The minimum ability level of those who decide to study y^u depends only on the minimum educational level. Thus it is continuous and stays constant for e^A larger than e.

3) We need to examine carefully the behavior when e^A is close to e because of discontinuities. Take the limit of y^{AB} when e^A tends to e, denoted by y^{lim}. The limit is

$$y^{lim} = \lim_{e^A \to e} \frac{c\left(e^A\right) - c\left(e\right)}{w_s(e^A - e)} = \frac{c'(e)}{w_s}. \tag{47}$$

At the symmetric situation where $e^A = e^B = e$, we have

$$N_s = (\overline{y} - y^u), L_s = (\overline{y} - y^u)\left(\overline{y} + y^u\right)e/2, L_u = 2y^u.$$

Taking the limit when e^A tends to e from above gives

$$N_s = 2(\overline{y} - y^{lim}), L_s = \left(\overline{y} - y^{\text{lim}}\right)\left(\overline{y} + y^{\text{lim}}\right)e, L_u = 2y^u$$

and from below

$$N_s = 2(y^{lim} - y^u), L_s = \left(y^{\text{lim}} - y^u\right)\left(y^{\text{lim}} + y^u\right)e, L_u = 2y^u.$$

This gives the value for the jump when e^A is raised above e

$$lim_{e^A \to > e} W^A(e^A, e) - W^A(e, e) = w_s e[\left(\overline{y}^2 - (y^{lim})^2\right) - \frac{1}{2}(\overline{y}^2 - (y^u)^2)] - c(e)(\overline{y} - 2y^{lim} + y^u). \tag{48}$$

Observe that

$$lim_{e^A \to > e} W^A(e^A, e) + lim_{e^A \to < e} W^A(e^A, e) = 2W^A(e, e).$$

In words, for e^A close to e, there are overall the same number of students as at the symmetric situation in which both countries choose e. Hence the total cost of education and the total welfare are the same, as given by the last equation. However, the ability composition of those who study in A or in B is affected. This means that around symmetric levels (e, e), countries are playing approximately a constant two-person game. The jump when e^A is decreased from above towards e, $lim_{e^A \to > e} W^A(e^A, e) - W^A(e, e)$ is equal to that when e^A is increased from below to e $W^A(e^A, e) - lim_{e^A \to < e} W^A(e^A, e)$.

Let $\hat{e}$ be the value for which these jumps are null. At this level, the net benefit from educating high ability students, those with ability larger than y^{AB}, is exactly equal to the net benefit from educating low ability students, those with ability between y^{AB} and y^u.

Consider a value e larger than $\hat{e}$. No country benefits by improving the educational level. Each benefits from choosing an educational level just below the other one. Similarly, for a value e smaller than $\hat{e}$, a country benefits by improving the educational level above e. We are left with $(\hat{e}, \hat{e})$ as the only possibility for an equilibrium in pure strategies.

We determine conditions under which $(\hat{e}, \hat{e})$ is indeed an equilibrium. Take $e^B = \hat{e}$, and consider the welfare of A, for example (by symmetry the same argument works for B). A's welfare is continuous at $\hat{e}$. Furthermore it increases for $e^A < \hat{e}$. Hence if A's welfare decreases for $e^A > \hat{e}$, $\hat{e}$ is indeed a best response to $e^B = \hat{e}$ and $(\hat{e}, \hat{e})$ is an equilibrium. Recall that a country's welfare is concave when it has the largest education level. Therefore, A's welfare decreases for $e > \hat{e}$ if and only if the 'right' derivative $\lim_{e^A \to > e} \frac{\partial W^A}{\partial e^A}$ is negative (since, in that case, the concavity of W^A for $e > \hat{e}$ implies that $\frac{\partial W^A}{\partial e^A}$ is negative for $e > \hat{e}$ hence A's welfare decreases). ■

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