The Role of Uncertainty and Learning for the Success of International Climate Agreements

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and

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Abstract

Despite ongoing research and regular updates on the current scientific knowledge about climate change through for instance the International Panel on Climate Change (IPCC), there remain large uncertainties about the impacts of climate change for environmental damages as well as abatement and mitigation costs. This paper studies the role of uncertainty and learning in the strategic context of the formation of international environmental agreements (IEAs). We generalize and qualify previous results obtained for instance in Na and Shin (1998), Kolstad (2007) and Kolstad and Ulph (2006, 2008) and highlight the driving forces of different outcomes. It is shown that the role of learning for the success of IEAs can be related the learning effect, strategic effect and stability effect. Compared to previous studies, we derive more positive conclusions. The veil of uncertainty has only a positive impact under special conditions but even then devices for mitigating negative outcomes are available.

Keywords: international environmental agreements, climate change, coalition formation, game theory, uncertainty, learning effect, strategic effect, stability effect.

JEL-Classification: C72, D62, D81, H41, Q20.

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0. Introduction

Climate change is one of the greatest challenges to international co-operation the world is currently facing (Stern 2006). International response to this challenge can be traced back to 1988 when the Intergovernmental Panel on Climate Change (IPCC) was founded – an international body that gathers and summarizes current world-wide scientific evidence on climate change. Successive reports of the IPCC confirmed that climate change is a serious problem caused by human activities. However, it was not until 1997 when 38 countries agreed to specific emission ceilings under the Kyoto Protocol to be meet in the "commitment period" 2008-2012. Again, it was not before 2002 when this treaty was ratified. This did not happen before several concessions had been granted to various participants and after the USA had declared to withdraw from the treaty all together.

Currently, in the light of the Stern report (Stern 2006) and the most recent IPCC report (IPCC 2007), a "Post-Kyoto" agreement is being negotiated that sets emission ceilings for the time after 2012, called the "second commitment period" (2012-2020). The aim is to reduce emissions further and to encourage participation of the major polluter, the USA, as well as the new emerging polluters China and India.

The difficulties of designing a successful climate agreement, characterized by a high participation of all major polluters and an agreement on effective emission reductions, can be related to at least *three major factors* (Finus 2008). *First*, climate change is a "public bad" with global dimension. Consequently, there are strong free-rider incentives. Despite there is a global gain from cooperation, it is even more attractive to save on abatement costs and to benefit from the abatement efforts of other countries.

Second, climate change features a large time scale due to the underlying carbon cycle in the atmosphere. Costs of emission reduction occur today whereas the benefits from slowing climate change will only be enjoyed in the future. Thus, only governments with a farsighted view find it attractive to burden their voters today for the benefit of future generations.

Third, the effect of climate change as well as the impacts of policy responses are associated with large uncertainties. On the one hand, the impact of greenhouse gas concentration on the climate, the impact of the climate on the environment and the relation between changes of environmental quality and its evaluation by people is uncertain. Also the prediction about future abatement costs is difficult as future abatement technologies, economic growth and the relation between growth and emissions are not known today (Stern 2006). For instance, President George Bush justified the withdrawal of the USA from the Kyoto Protocol with uncertainty. In a letter to Senators, dated March 13, 2001, he wrote: "I oppose the Kyoto Protocol … we must be very careful not to take actions that could harm consumers. This is especially true given the incomplete state of scientific knowledge" (quoted from Kolstad 2007, p. 69).

In this paper we focus in particular on the first and third item. That is, we analyze how uncertainty and learning affect the formation of international climate agreements in the presence of free-rider incentives. In the following section, we review the literature and relate our research. In section 2, we outline our model and describe the driving forces. Section 3 derives the solutions for the three "uncertainty cases" and three "learning scenarios" in the context of the formation of an IEA which we consider in this paper. Section 4 presents and discusses the overall results. Section 5 summarizes the main conclusions, points at the limitations of our model, put them in a wider perspective and proposes future research issues.

1. Literature Review and Topic of Paper

The difficulties of designing a successful climate agreement has been addressed in three strands in the literature where our paper belongs to the third strand.

The *first strand of literature* analyzed *optimal policy responses in the presence of uncertainty,* abstracting from free-rider incentives (e.g. Kolstad 1996a, b, Ulph and Maddison 1997 and Tol 2002, 2005). On the one hand, the possibility that better information will be available in the future leads to the suggestion that abatement activities should be delayed. For instance, if it is expected that cheaper abatement options will emerge in the future, current abatement activities may turn out to be unnecessarily costly. On the other hand, the irreversibility of greenhouse gas accumulation calls for early action in case it turns out that climate damages are more severe than expected. Moreover, current stricter environmental regulation may boost technological progress, making abatement cheaper in the future.

However, even in the simple models of Ulph (1998), Ulph and Ulph (1997) and Ulph and Maddison (1997) it is difficult to obtain general conclusions.[1] For our purposes three results are important. First, total accumulated emissions are higher in the Nash equilibrium than in the social optimum, with and without learning. Second, the value of information (i.e. in the form of higher expected global welfare) is positive in the social optimum, except for the special case of negatively correlated damages in which this value is zero. In our analysis, we call this the *learning effect*. Third, if damages are sufficiently negatively correlated, the value of information can become negative in the Nash equilibrium. We call this the *strategic effect* in our analysis.

For instance, in Ulph (1998) the strategic effect is illustrated with two countries that have the same abatement cost function and uncertainty is about environmental damages with two possible states. Ex-ante, players are symmetric as they share the expectation about environmental damages. Without learning, they are also ex-post symmetric, but with learning they are ex-post asymmetric if damages are perfectly negatively correlated (i.e. one

1 These models assume only two periods and two countries which may differ only in terms of damage costs. The abatement technology is fixed and the uncertainty is restricted to environmental damages and to two states of the world.

player will have low and one will have high damages). Without learning, this leads to symmetric Nash equilibrium emission levels which are cost-effective due to the assumption of symmetric abatement cost function. With learning, the asymmetric Nash equilibrium departs from an cost-effective allocation and hence the value of information is negative.

The *second strand of literature* analyzed *the design of international environmental agreements in the presence of free-rider incentives*, abstracting from uncertainty (see e.g. the summaries in Finus 2003, 2008 and Barrett 2003). This literature showed for instance that whenever the gains from cooperation would be large, stable treaties achieve only little. However, a departure from a first-best design can improve upon this negative conclusion. For instance, it has been shown that modest abatement targets can boost participation sufficiently to make up for downgraded abatement targets (Barrett 2002, Finus and Maus 2008 and Finus and Rundshagen 1998). Moreover, regional agreements with different abatement targets may be a pragmatic intermediate step as long as a comprehensive global treaty cannot be reached (Asheim et al. 2006 and Eyckmans and Finus 2006). Finally, asymmetries among players in terms of abatement and damage costs render cooperation difficult in the absence of transfers, but this difficulty can be mitigated through appropriate transfers (e.g. Weikard et al. 2006)

The *third strand of literature* combines both approaches, studying *the role of uncertainty and learning for the formation and stability of international environmental agreements*. The first paper is due to Na and Shin (1998) which is an extension of Ulph (1998) to three players and coalition formation. Like Ulph (1998), they find that the value of information is negative. Now, ex-ante asymmetry under full learning is bad not only due to the strategic effect but also due to the *stability effect*, as we call it in our analysis. The information about asymmetric gains from cooperation implies smaller coalitions than under the veil of uncertainty.

Later papers by Kolstad (2007), and Kolstad and Ulph (2006, 2008) introduce apart from no and full learning a third scenario which they call partial learning. In the context of a two-stage coalition formation game, partial learning means that players have to take their membership decision in the first stage under uncertainty, but will learn the true parameter values of the payoff function before taking their abatement decisions in the second stage. In contrast, under full learning players know the true parameter values before the first stage and under no learning they neither know these values before the first nor the second stage. Kolstad (2007) considers the case of perfectly positively correlated environmental damages (implying that players are ex-post symmetric not only under no and partial but also full learning). He shows that under full learning stable coalitions are larger than under no learning, though this implies lower expected welfare under full learning compared no learning, as proved in Kolstad and Ulph (2006). Thus, again the value of information is negative. For partial learning they find mixed evidence as multiple equilibria emerge in their model. Similar mixed results are obtained for the case of uncorrelated uncertainty about damages in Kolstad and Ulph (2006, 2008). An important limitation of their model is the assumption of a linear payoff function. This implies boundary solutions for equilibrium abatement, with no abatement for non-signatories and maximum abatement for signatories.

Finally, Dellink et al. (2008) analyze coalition formation based on a climate model with empirical estimates on the distribution of the parameters of the benefit and abatement cost functions for twelve world regions. They consider uncorrelated uncertainty on all benefit and cost parameters but only the scenarios of no and full learning (alternative: the scenarios of full and no learning. Just sounds better to me!). Their results show mixed evidence of the role of learning, depending on the expected value of the global benefit parameter.

This paper is in the tradition of the theoretical papers by Na and Shin (1998), Kolstad (2007) and Kolstad and Ulph (2006, 2008). We want to relate the different aspects of uncertainty, learning and formation of an IEA in one framework and to work out their driving forces. For this we combine and generalize the different frameworks of these papers. In the tradition of Na and Shin (1998), we base our analyisis on a strictly concave payoff function. In the tradition of Kolstad (2007) and Kolstad and Ulph (2006, 2008), we consider three scenarios, no, partial and full learning, without restriction on the number of players. Like in all of these papers, we focus on uncertainty of environmental damages. This helps to isolate effects and is unanimously considered to be the most important source of uncertainty. Different from these papers, we systematize the effects of uncertainty and learning not according to whether environmental damages are positively, negatively correlated (or not correlated at all) but whether there is a) uncertainty about the level of environmental damages and b) uncertainty about the distribution of environmental damages or c) both, which we call cases 1, 2 and 3 subsequently.

2. Model Outline

2.1 Coalition Formation Game

Consider a coalition formation game in which players (i.e. countries in our context) decide in the first stage whether to join an agreement or to remain singletons. This leads to a coalition structure, $K = \{ S, 1_{n-m} \}$, which is a partition of players, with n being the total number of players, m the size of coalition S, and N the set of players, with $S \subseteq N$. Due to the simple structure of the coalition formation game, i.e. there can be at most one non-trivial coalition, coalition structure K is uniquely determined by coalition S. Typically, we will denote a member of S by i and call it a signatory and a non-member of S by j and call it a non-signatory. This game, also called cartel formation game, has been applied widespread in the literature analyzing the stability of international environmental agreements (IEAs). See Barrett (2003) and Finus (2003) for surveys.

In the second stage, players choose their economic strategies which are abatement levels q_i in our setting.[2] The payoff function is given by

(1) $$\Pi_i = B_i\left(\sum_{k=1}^{n} q_k\right) - C_i\left(q_i\right) \ , \ i \in N$$

where $B_i\left(\sum_{k=1}^{n} q_k\right)$ is country i's concave benefit function from global abatement and $C_i\left(q_i\right)$ its convex abatement cost function from individual abatement.

Following the mainstream assumption in the literature on coalition formation (e.g. Bloch 2003 and Yi 1997), we assume that the coalition derives its optimal economic strategies from maximizing the aggregate payoff of the coalition

(2) $$\max_{q^S}.\sum_{i \in S} \Pi_i(S) \Rightarrow \sum_{i \in S} B_i^{'}\left(\sum_{k=1}^{n} q_k\right) = C_i^{'}\left(q_i\right) \ \ \forall\, i \in S$$

whereas as singletons maximize their own payoff

(3) $$\max_{q_j}.\, \Pi_j(S) \Rightarrow B_j^{'}\left(\sum_{k=1}^{n} q_k\right) = C_j^{'}\left(q_j\right) \ \ \forall\, j \notin S$$

where q^S is the vector of abatement levels of those players that belong to coalition S, $B_k^{'}$ and $C_k^{'}$ are the derivatives of B_k and C_k with respect to q_k, respectively, with $k \in \{i, j\}$. The simultaneous solution of the maximization problems (2) and (3) delivers equilibrium abatement levels $q_i^*(S)$, $\forall\, i \in S$, and $q_j^*(S)$, $\forall\, j \notin S$, assuming a unique and interior

2 An alternative specification which produces equivalent results is to consider damage cost functions from global emissions and benefit functions from individual emissions. This equivalence holds as long as non-negativity constraints are observed as discussed for instance in Diamantoudi and Sartzetakis (2006) and Rubio and Ulph (2006). The assumption of a static payoff function is not only made in Kolstad (2007), Kolstad and Ulph (2006, 2008) and Na and Shin (1998), but also in many other papers on the formation of IEAs (see e.g. Finus 2003). This is certainly a strong assumption on which we comment in section 5.

solution. That is, the coalition acts de facto as a single player. Coalition S with m members and the $n-m$ singletons play a (coalitional) Nash equilibrium among each other.[3]

Substituting the equilibrium abatement levels for a given coalition S into the payoff functions delivers the payoffs of signatories, $\Pi^*_{i\in S}(S)$, and non-signatories, $\Pi^*_{j\notin S}(S)$ in the second stage of the coalition formation game.

The game is solved by backward induction. That is, stable coalitions are determined by invoking the stability concept of internal and external stability, widely used in the literature on IEAs (see the surveys e.g. in Barrett 2003 and Finus 2003):

(4) internal stability: $\Pi^*_{i\in S}(S) \geq \Pi^*_{i\notin S}(S\setminus\{i\}) \;\; \forall\, i\in S$

(5) external stability: $\Pi^*_{j\notin S}(S) > \Pi^*_{j\cup S}(S\cup\{j\}) \;\; \forall\, j\notin S$.

That is, no signatory should have an incentive to leave coalition S to become a non-signatory and no non-signatory should have an incentive to join coalition S. In order to avoid knife-edge cases, we assume that if players are indifferent between joining coalition S and remaining outside, they will join the agreement. A coalition which is internal and externally stable is called stable and denoted by S^*. Note that the coalition structure comprising only singletons is stable by definition and hence existence of an equilibrium is guaranteed.[4]

In case there is more than one stable coalition, we apply the Pareto-dominance criterion as applied in Finus and Rundshagen (2006). That is, stable coalitions which are Pareto-domi-

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3 Note that if $S=N$ (i.e. players form the grand coalition) this corresponds to the social optimum and if either $S=\{i\}$ or $S=\emptyset$ (i.e. all players play as singletons) this corresponds to the Nash equilibrium.

4 The reason is that the singleton coalition structure can be generated by $S=\emptyset$, which is internally stable by definition and externally stable as $S=\emptyset$ and $\tilde{S}=\{i\}$ generate the same coalition structure.

nated by an other stable coalition are dropped. We denote the set of Pareto-undominated stable coalitions by Ψ^*.

2.2 The Three Learning Scenarios

Following Kolstad and Ulph (2006, 2008), we distinguish three *scenarios* when players learn about the uncertain parameters: 1) full learning, 2) partial learning and 3) no learning. *Full learning* can be considered as a benchmark case in which players learn about the true parameter values before taking the membership decision in the first stage. For *partial learning* it is assumed that players decide about membership under uncertainty but know that they will learn about the true parameter values before deciding upon abatement levels in the second stage. Hence, the membership decision, i.e. internal and external stability, is based on expected payoffs, under the assumption that players will take the correct decision in the second stage. Finally, under *no learning* also the abatement decision is taken under uncertainty. That is, players derive their abatement strategies by maximizing expected payoffs. The membership decisions are also taken based on expected payoffs, though they differ from those under partial learning, given that less information is available.[5]

It is worthwhile pointing out that our assumption implies that learning takes the form of perfect learning (Kolstad and Ulph 2008). That is, if players learn about parameter values, no uncertainty remains on which we comment in section 5.

When comparing and evaluating the three scenarios in section 3, three effects are at work which we call 1) *learning effect*, 2) *strategic effect* and 3) *stability effect*. The *learning effect* is best illustrated by assuming that all players would unconditionally cooperate, i.e. $S = N$. Then, the assumption of joint welfare maximization of the coalition in the second stage implies that the social optimum is implemented. We can conclude that the aggregate payoff over all

[5] Na and Shin (1998), who do not consider partial learning, call no learning ex-ante negotiations and full learning ex-post negotiations.

players under full and partial learning is at least as high as under no learning, i.e. the learning effect is not negative. This follows simply from two facts. First, under full and partial learning, players get it “right” as they know the true parameter values in the second stage. Second, full and partial learning lead to the same outcome as we abstract from coalition formation in the first stage (and hence from stability effects). Under no learning, the choice of optimal abatement levels can only be a second-best solution.

The *strategic effect* comes into play when we depart from the assumption of full cooperation, i.e. $S \neq N$, but still abstract from stability. Again, full and partial learning are the same in the second stage but different from no learning. Given that the m coalition members and the $m-n$ non-coalition members play a (coalitional) Nash equilibrium among each other in the second stage, when choosing abatement levels, predictions are difficult. The payoff of signatories depends on the strategies of non-signatories and vice versa. For instance, the aggregate payoff depends on the average total abatement and the allocation between signatories and non-signatories. Thus, it is no longer straightforward to draw conclusions regarding the role of learning on aggregate welfare and aggregate abatement.

Finally, the *stability effect* makes predictions even more difficult. For instance, suppose that for a given coalition S, payoffs of signatories and non-signatories were higher under learning than under no learning. However, the stable coalition under learning was smaller than under no learning. Then, if this stability effect was strong enough it could compensate for the learning effect.

Taken together, all three effects have to be considered in order to draw conclusions about the role of learning for the success of international environmental agreements.

2.3 The Three Uncertainty Cases

In this section, we discuss the three uncertainty cases, systematizing the role of uncertainty on coalition formation. In order to make progress, the consideration of a particular payoff function, as well as the parameters that are uncertain and their distributions is required due to the complexity of coalition formation. In order to avoid boundary solutions, as in Kolstad (2007) and Kolstad and Ulph (2006, 2008), we consider a strictly concave payoff function which is still simple enough for an analytical analysis:[6]

$$(6) \qquad \Pi_i = b_i \sum_{k=1}^{n} q_k - c_i \frac{q_i^2}{2} \;, \; i \in N \,, \; b_i > 0 \,, \; c_i > 0$$

where b_i is a benefit parameter, $b_i \sum_{k=1}^{n} q_k$ are the benefits from global abatement, c_i is a cost parameter, and $c_i \frac{q_i^2}{2}$ are the abatement costs from individual abatement.

Generally, the benefit as well as the cost parameters could be uncertain. However, following Kolstad (2006), Kolstad and Ulph (2008) and Na and Shin (1998), in the climate context uncertainty about the benefits from reduced damages appears to be more important than uncertainty about abatement costs. Hence, we simplify the model, by dividing payoffs by the cost parameter c_i, define the benefit-cost ratio by $\gamma_i = b_i / c_i$, and hence payoff function (6) reads:

$$(7) \qquad \Pi_i = \gamma_i \sum_{k=1}^{n} q_k - \frac{q_i^2}{2} \;, \; i \in N \,, \; \gamma_i > 0 \;.$$

Henceforth, we interpret γ_i as a benefit parameter. If this parameter is uncertain, then it is represented by the random variable Γ_i, with associated distribution f_{Γ_i}.

In order to synthesize the settings of the papers on uncertainty, learning and the formation of IEAs, we structure the three *cases of uncertainty* as displayed in Table 1.

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6 A similar payoff function has for instance been used by Botteon and Carraro (1997), Barrett (2006), Finus and Maus (2008) and Na and Shin (1998).

Table 1: Three Cases of Uncertainty

Case	Ex-ante Expectations	Ex-Post Realizations	Parameter	Uncertainty about
1	symmetric	symmetric	common	level
2	symmetric	asymmetric	individual	distribution
3	symmetric	asymmetric	common and individual	level and distribution

Case 1

Case 1 is the case considered in Kolstad (2007) and Kolstad and Ulph (2006, 2008), which the authors call systematic uncertainty. Uncertainty is about a common benefit parameter, with symmetric realization for all players, i.e. $\Gamma_i = \Gamma_k \quad \forall\, i, k \in N$. Thus, uncertainty is correlated. However, we find it more illuminating to point out that in this case uncertainty is de facto about the *level of the benefits from global abatement*. For the later analysis it is helpful to point out that this implies that the sum of marginal benefits is not known and that the variance of the sum of marginal benefits is positive.

Compared to the studies mentioned above, our case 1 seems to be more general in two respects. First, our payoff function avoids boundary solutions. This implies that optimal abatement strategies are continuous and are a function of the benefit parameter. Second, we do not assume any particular distribution for the uncertain benefit parameter.

Case 2

Case 2 is the case considered in Na and Shin (1998). Uncertainty is about the individual benefit parameter and its realization is asymmetric among players. Like in Na and Shin (1998), we consider that the random variables Γ_i, $\forall\, i \in N$, are perfectly correlated. Unlike the model of Na and Shin (1998) with three players, we consider an arbitrary number of players. Because of the larger complexity we adopt a specific distribution for parameter Γ_i, namely a uniform distribution:

(8) $$f_{\Gamma_i}(\gamma_i) = \begin{cases} \frac{1}{n} & for \quad \gamma_i = k,\ k \in N \\ 0 & otherwise \end{cases}$$

which implies expected value, $E[\Gamma_i]$, and variance, $Var[\Gamma_i]$, as follows:

(9) $$E[\Gamma_i] = \frac{n+1}{2} \text{ and } Var[\Gamma_i] = \frac{n^2-1}{12} .$$

Perfect correlation implies $\bigcup_{i=1}^{n} \Gamma_i = N$, $\Gamma_i \neq \Gamma_k$, $\forall\, i \neq k \in N$. That is, the realization of vector Γ is composed of all the elements of N, i.e. $\{1,...,n\}$. The different possible vectors only differ in the order of the elements and hence the sum of elements is always the same. In other words, the sum of marginal benefits is fixed and consequently its variance is zero.

Here, perfect correlation implies that uncertainty is purely about the *distribution of the benefits from abatement*, as the level of global benefits from abatement is constant. That is, vector Γ can be interpreted as different shares of the global benefits from abatement as modeled for instance in Dellink et al. (2008).[7] Different from Na and Shin (1998), we also consider the case of partial learning.

Case 3

Case 3 is a combination of the previous two cases. It can be interpreted as uncertainty about a common and an individual parameter. This translates in our setting into uncertainty about the *level and distribution of benefits*. This is captured by assuming the same uniform distribution as in case 2, except that all random variables, Γ_i, $i = 1,...,n$, are uncorrelated and therefore identically and independently distributed. Hence, the sum of marginal

[7] Let $\gamma_i = \lambda_i \sum_{k=1}^{n} \gamma_k = \lambda_i L$ where λ_i denotes the share of global benefits of player i, with $\sum_{k=1}^{n} \lambda_k = 1$, and $L = \sum_{k=1}^{n} \gamma_k$ the level of global benefits. Then, in case 2 $L = n(n+1)/2$, and $\lambda_i = 2j/(n(n+1))$, $j \in \{1,2,...,n\}$.

benefits is unknown and the variance of the sum of marginal benefits is positive and hence larger than in case 2 but smaller than in case 1. Different from Kolstad and Ulph (2008) our distribution allows for more than two values of the random variables.

Comparision of the Uncertainty Cases

In cases 1, 2 and 3 a common feature is that all players share the same believes about the distribution of the uncertain parameter and consequently are ex-ante symmetric. This is certainly a simplification and requires that some coordination has taken place ex-ante on which we comment in section 5.

The fundamental difference between the three cases concerns the ex-post realization of the random variables Γ_i. In case 1, regardless of the realization, the elements of vector γ are symmetric, though their sums differ. In case 2, the possible realizations of Γ_i contain asymmetric parameter values, but the sum of the elements is always the same. Case 3 is in between: possible realizations of Γ_i contain symmetric and asymmetic elements. The degree of asymmetry - measured by the variance of the elements - is positive, therefore larger than in case 1, but lower than in case 2. This will turn out to be crucial for the formation of stable coalitions in the full learning scenario. Asymmetric parameter values imply an asymmetric distribution of the gains from cooperation which makes stable cooperation difficult. This will be particularly relevant for case 2 and to a lesser extent for case 3. The fact that the sum of elements is the same for all vectors in case 2, i.e. the sum of marginal benefits is known, implies that the learning is zero as we show in section 3.

3. Solution of the Model

In this section we solve the model for the three learning scenarios and the three uncertainty cases. As pointed out in section 2.1, we solve the game backward, starting with the second stage.

3.1 Second Stage of Coalition Formation

3.1.1 Full and Partial Learning

In the full and partial learning scenarios players know the realization of the random variables Γ_k, which are γ_k. Hence, given that a coalition structure $K = \{S, 1_{(n-m)}\}$ has formed in the first stage, the optimal abatement levels of the members of coalition S and the singletons $j \notin S$ follow from the maximization procedure described in (2) and (3), based on payoff function (7), which delivers:

(10) $$q_i^*(S) = \sum_{\ell \in S} \gamma_\ell \quad \forall\, i \in S\,,\; q_j^*(S) = \gamma_j \quad \forall\, j \notin S\,.$$

Total abatement is given by:

(11) $$Q^*(S) = \sum_{i \in S} q_i^*(S) + \sum_{j \notin S} q_j^S(S) = m \sum_{i \in S} \gamma_i + \sum_{j \notin S} \gamma_j\,.$$

Using (10) and (11), equilibrium payoffs are given by:

(12) $$\Pi_{i \in S}^*(S) = \gamma_i \left(m \sum_{\ell \in S} \gamma_\ell + \sum_{j \notin S} \gamma_j \right) - \frac{1}{2} \left(\sum_{\ell \in S} \gamma_\ell \right)^2$$

(13) $$\Pi_{j \notin S}^*(S) = \gamma_j \left(m \sum_{i \in S} \gamma_i + \sum_{k \notin S} \gamma_k \right) - \frac{1}{2} (\gamma_j)^2 = \frac{1}{2} (\gamma_j)^2 + m \gamma_j \sum_{i \in S} \gamma_i + \gamma_j \sum_{k \neq j \notin S} \gamma_k$$

(14) $$\begin{aligned} \Pi^*(S) &= \sum_{i \in S} \Pi_i^*(S) + \sum_{j \notin S} \Pi_j^*(S) = \\ &\frac{1}{2} m \left(\sum_{i \in S} \gamma_i \right)^2 + \left(\sum_{i \in S} \gamma_i \right) \left(\sum_{j \notin S} \gamma_j \right) (1+m) + \left(\sum_{j \notin S} \gamma_j \right)^2 - \frac{1}{2} \sum_{j \notin S} (\gamma_j)^2 \end{aligned}$$

In order to compare the scenarios of full and partial learning with no learning in the second stage[8], we also compute expected total abatement and payoffs from (11) and (14), respec-

[8] That is, we abstract from internal and external stability of coalitions which is analyzed in the first stage of coalition formation.

tively. For a random coalition S of size m (recalling that the total number of players is n) the expected total abatement in cases 1, 2, and 3 is given by:

$$E\left[Q^{FL}\left(S,\Gamma\right)\right]=E\left[Q^{PL}\left(S,\Gamma\right)\right]=\left(m^2-m+n\right)E\left[\Gamma_k\right] \tag{15}$$

where the superscript FL stands for full learning and PL for partial learning. That is, in all three cases, expected total abatement is the same, provided we assume in case 1 the same distribution of Γ_k as in cases 2 and 3 where $E\left[\Gamma_k\right]=\left(n+1\right)/2$.

However, the average equilibrium payoffs differ between the three cases and are given, respectively, by:

Case 1:

$$E\left[\Pi^{FL}_{i\in S}\left(S,\Gamma\right)\right]=E\left[\Pi^{PL}_{i\in S}\left(S,\Gamma\right)\right]=\left(\frac{m^2}{2}-m+n\right)E\left[\Gamma_i^2\right]$$

$$E\left[\Pi^{FL}_{j\notin S}\left(S,\Gamma\right)\right]=E\left[\Pi^{PL}_{j\notin S}\left(S,\Gamma\right)\right]=\left(m^2+n-m-\frac{1}{2}\right)E\left[\Gamma_j^{\,2}\right] \tag{16}$$

$$E\left[\Pi^{FL}\left(S,\Gamma\right)\right]=E\left[\Pi^{PL}\left(S,\Gamma\right)\right]=\left(m^2\left(n-\frac{m}{2}\right)+\left(n-\frac{1}{2}\right)\left(n-m\right)\right)E\left[\Gamma_k^2\right]$$

Case 2:

$$E\left[\Pi^{FL}_{i\in S}\left(S,\Gamma\right)\right]=E\left[\Pi^{PL}_{i\in S}\left(S,\Gamma\right)\right]=\frac{\left(n+1\right)\left(6n^2+\left(3m^2-5m+4\right)n+2m^2-4m\right)}{24}$$

$$E\left[\Pi^{FL}_{j\notin S}\left(S,\Gamma\right)\right]=\left[\Pi^{PL}_{j\notin S}\left(S,\Gamma\right)\right]=\frac{\left(n+1\right)\left[3n^2+\left(3m^2-3m+1\right)n+2m^2-2m-1\right]}{12} \tag{17}$$

$$E\left[\Pi^{FL}\left(S,\Gamma\right)\right]=E\left[\Pi^{PL}\left(S,\Gamma\right)\right]=$$
$$\frac{\left(n+1\right)\left(6n^3+\left(6m^2-6m+2\right)n^2+\left(-3m^3+5m^2-2m-2\right)n-2m^3+2m\right)}{24}$$

Case 3:

$$E\left[\Pi_{i\in S}^{FL}\left(S,\Gamma\right)\right]=E\left[\Pi_{i\in S}^{PL}\left(S,\Gamma\right)\right]=\frac{\left(n+1\right)\left(n\left(3m^2-5m+6\right)+3m^2-7m\right)}{24}$$

(18) $$E\left[\Pi_{j\notin S}^{FL}\left(S,\Gamma\right)\right]=E\left[\Pi_{j\notin S}^{PL}\left(S,\Gamma\right)\right]=\frac{\left(n+1\right)\left(3n^2+n\left(3m^2-3m+2\right)+3m^2-3m-2\right)}{12}$$

$$E\left[\Pi^{FL}\left(S,\Gamma\right)\right]=E\left[\Pi^{PL}\left(S,\Gamma\right)\right]=$$
$$\frac{\left(n+1\right)\left(6n^3+\left(6m^2-6m+4\right)n^2+\left(-3m^3+7m^2-4m-4\right)n-3m^3-m^2+4m\right)}{24}$$

where in (16) we leave $E\left[\Gamma_k^2\right]$ unspecified as in case 1 we do not have to assume a particular distribution of the random variables Γ_k.

3.1.2 No Learning

The no learning scenario is fundamentally different from the full and partial learning scenarios as players do not know the realization of the random variables Γ_k. Hence, players derive their equilibrium abatement levels not from (2) and (3), but maximize their expected payoffs:

(19) $$\max_{q^S}.E\left[\sum_{i\in S}\Pi_i(S)\right]=$$
$$E\left[\sum_{i\in S}\Gamma_i\sum_{k=1}^{n}q_k-\frac{1}{2}\sum_{i\in S}q_i^2\right]=\sum_{i\in S}E\left[\Gamma_i\right]\sum_{k=1}^{n}q_k-\frac{1}{2}\sum_{i\in S}q_i^2\quad\forall\, i\in S$$

(20) $$\max_{q_j}.E\left[\Pi_j\right]=E\left[\Gamma_j\sum_{k=1}^{n}q_k-\frac{q_j^2}{2}\right]=E\left[\Gamma_j\right]\sum_{k=1}^{n}q_k-\frac{q_j^2}{2}\quad\forall\, j\notin S$$

where q^S is again the vector of abatement levels of coalition S. As payoffs are linear in the random variables Γ_k certainty equivalence holds. That is, maximization of expected payoffs is equivalent to the maximization of payoffs under certainty for $\gamma_k = E\left[\Gamma_k\right]$. The simultaneous solution of (19) and (20), delivers equilibrium abatement levels:

$$(21) \qquad q_i^{**}(S) = \sum_{i \in S} E\left[\Gamma_i\right] \;\; \forall\, i \in S \;\;,\; q_j^{**}(S) = E\left[\Gamma_j\right] \;\; \forall\, j \notin S$$

where use two asterisks to indicate equilibrium abatement levels in order to stress the difference to full and partial learning. Since abatement levels in (21) are constant and independent of the realization of Γ_k, they are also expected abatement levels.

From $Q^{**}(S) = \sum_{i \in S} q_i^{**}(S) + \sum_{j \notin S} q_j^{**}(S)$, we can derive directly:

$$(22) \qquad E\left[Q^{NL}(S,\Gamma)\right] = m\left(mE\left[\Gamma_i\right]\right) + (n-m)E\left[\Gamma_j\right] = \left(m^2 - m + n\right)E\left[\Gamma_k\right]$$

where superscript NL stands for no learning, and in cases 2 and 3 $E\left[\Gamma_k\right] = (n+1)/2$. Since certain equivalence holds, the equilibrium expected payoffs of the second stage are the same as those obtained for symmetric players under certainty with $\gamma = E\left[\Gamma_k\right]$ $\forall\, k \in N$:

$$(23) \qquad E\left[\Pi_{i \in S}^{NL}(S,\Gamma)\right] = \left(\frac{m^2}{2} - m + n\right)\left(E\left[\Gamma_i\right]\right)^2$$

$$(24) \qquad E\left[\Pi_{j \notin S}^{NL}(S,\Gamma)\right] = \left(m^2 - m + n - \frac{1}{2}\right)\left(E\left[\Gamma_j\right]\right)^2$$

$$(25) \qquad \begin{aligned} E\left[\Pi^{NL}(S,\Gamma)\right] &= mE\left[\Pi_{i \in S}^{NL}(S,\Gamma)\right] + (n-m)E\left[\Pi_{j \notin S}^{NL}(S,\Gamma)\right] \\ &= \left(m^2\left(n - \frac{m}{2}\right) + \left(n - \frac{1}{2}\right)(n-m)\right)\left(E\left[\Gamma_k\right]\right)^2 \end{aligned}$$

We observe that different from full and partial learning, not only expected abatement levels but also expected payoffs are the same for all three uncertainty cases, provided we would assume the same distribution in case 1 as we do in cases 2 and 3.

3.1.3 Analysis

Despite we have not yet determined stable coalitions in the first stage for the three learning scenarios and the three uncertainty cases, it is informative to conduct a comparison. This will turn out to be helpful for the interpretation of the final results in section 4. A comparison in the second stage requires assuming that a random coalition structure $K = \{ S, 1_{n-m} \}$ of size m has formed in the first stage. This enables us to compare expected total abatement and total payoffs.

Proposition 1

Let $K = \{ S, 1_{n-m} \}$ be a random coalition structure with coalition S of size m, then the expected total abatement is the same under all three learning scenarios and is given by:

$$E\left[Q^{FL}\left(S,\Gamma\right)\right] = E\left[Q^{PL}\left(S,\Gamma\right)\right] = E\left[Q^{NL}\left(S,\Gamma\right)\right] = \left(m^2 - m + n\right) E\left[\Gamma_k\right].$$

If the distribution of Γ_k in case 1 is the same as in cases 2 and 3, then the expected total abatement is also the same.

Proof: Follows immediately from (15) and (22). **(Q.E.D.)**

From Proposition 1 it is evident that differences (similarities) between the three scenarios in terms of the final outcome of coalition formation, analyzed in section 4, must exclusively stem from different (the same) stable coalition(s). We now turn to expected total payoffs.

Proposition 2

Let $K = \{ S, 1_{n-m} \}$ be a random coalition structure with coalition S of size m, then the following relations between expected total payoffs in the three uncertainty cases hold:

Case 1: $E\left[\Pi^{FL}\left(S,\Gamma\right)\right] = E\left[\Pi^{PL}\left(S,\Gamma\right)\right] > E\left[\Pi^{NL}\left(S,\Gamma\right)\right]$

Case 2: $E\left[\Pi^{FL}\left(S,\Gamma\right)\right] = E\left[\Pi^{PL}\left(S,\Gamma\right)\right] \le E\left[\Pi^{NL}\left(S,\Gamma\right)\right]$

Case 3: $E\left[\Pi^{FL}\left(S,\Gamma\right)\right]=E\left[\Pi^{PL}\left(S,\Gamma\right)\right]>E\left[\Pi^{NL}\left(S,\Gamma\right)\right]$

with strict inequality in case 2 if $S \neq N$.

Proof: First we note that there is no difference between full and partial learning in the second stage. Second, in case 1, we use (16) and (25) to show, after some basic manipulation, that $E\left[\Pi^{PL}\left(S,\Gamma\right)\right]-E\left[\Pi^{NL}\left(S,\Gamma\right)\right]=Var\left[\Gamma_k\right]$, where $Var\left[\Gamma_k\right]>0$ by assumption. Third, in case 2, we use (17) and (25) and find $E\left[\Pi^{NL}\left(S,\Gamma\right)\right]-E\left[\Pi^{PL}\left(S,\Gamma\right)\right]=\frac{(n+1)\left(n+m^2-1\right)(n-m)}{24}$ which is strictly positive for $n>m$, implying $S\neq N$, and zero if $n=m$, implying $S=N$, for all $m,n\in N$ and $m\leq n$. Fourth, in case 3, we use (18) and (25) and find $E\left[\Pi^{PL}\left(S,\Gamma\right)\right]-E\left[\Pi^{NL}\left(S,\Gamma\right)\right]=\frac{\left(n^2-1\right)\left(n+m^2-m\right)}{24}$ which is strictly positive for all $m,n\in N$ and $m\leq n$. **(Q.E.D.)**

The intuition behind Proposition 2 is the following. In cases 1 and 3, under no learning, neither the individual marginal benefits from global abatement nor their sum is known, which is an important item in the maximization problem of the coalition as well as the singletons. Under full and partial learning players get it right as they choose their optimal abatement vector for each realization of the random variables Γ_k. In contrast, under no learning, players get it only right on average, as their optimal strategies are based on expected marginal benefits. This is the learning effect mentioned in section 2.2.

Case 2 is fundamentally different. First, in the social optimum, uncertainty is no longer a disadvantage. Recall that the first order conditions are given by “sum of marginal benefits is equal to individual marginal abatement costs”. In our model, marginal abatement costs are anyway known and in case 2, due to the assumption of perfect correlation of the random variables Γ_k, the sum of marginal benefits is also known. This explains why aggregate payoffs in the social optimum, i.e. $S = N$, are equal in all three scenarios. In other words, the learning effect is zero.

Second, for any $S \neq N$, the strategic effect comes into play. This has no impact on expected total benefits as expected total abatement is the same for all three scenarios (see Proposition 1). However, it has an impact on expected total abatement costs. Under full and partial learning, for each realization of the random variables Γ_k, signatories and non-signatories will choose a different abatement level. More important, the difference between signatories' and non-signatories abatement levels will vary substantially. Since all players have the same abatement cost function, this implies a cost-ineffective allocation of abatement burdens from a global point of view. Under no learning, signatories and non-signatories choose strategies which are optimal on average. Though also here signatories and non-signatories have different optimal strategies, on average - compared to the case of full and partial learning - differences are smaller. Hence, in this context, the strategic effect is a cost-effectiveness effect.

3.2 First Stage of Coalition Formation

In this section, we determine stable coalitions based on the equilibrium abatement strategies of the second stage. Different from the second stage, in the first stage we have to analyze each learning scenario separately. The reason is that different from full learning, under partial learning players do not know the realization of the random variable in the first stage and hence have to base their decisions on expected payoffs.

3.2.1 Full Learning

In order to determine equilibrium coalitions in the three uncertainty cases, we first establish a general condition for stability.

Proposition 3

Consider a coalition structure $K=\left\{S,1_{(n-m)}\right\}$ *where* S *is composed of* m *members with parameters* $\gamma_{i_1}\le\gamma_{i_2}\le\ldots\le\gamma_{i_m}$ *, and there are* $n-m$ *singletons, with parameters* $\gamma_{j_1}\le\gamma_{j_2}\le\ldots\le\gamma_{j_{n-m}}$ *. This coalition structure is stable if and only if:*

$$\begin{cases}\forall\gamma_i, i\in N \quad if \quad m=1\\ \dfrac{\gamma_{i_2}}{\gamma_{i_1}}\le\sqrt{2} \quad if \quad m=2\ \wedge\ n=2\\ \dfrac{\gamma_{i_2}}{\gamma_{i_1}}\le\sqrt{2}\ \wedge\ \gamma_{j_{n-2}}<\dfrac{\gamma_{i_1}+\gamma_{i_2}}{2} \quad if \quad m=2\ \wedge\ n\ge 3\\ \gamma_{i_1}=\gamma_{i_2}=\gamma_{i_3} \quad if \quad m=3\ \wedge\ n=3\\ \gamma_{i_1}=\gamma_{i_2}=\gamma_{i_3}\ \wedge 2\gamma_{j_{n-m}}<3\gamma_{i_1} \quad if \quad m=3\ \wedge\ n\ge 4\\ \varnothing \quad if \quad m\ge 4\end{cases}$$

Proof: See Appendix 1. **(Q.E.D.)**

The upshot of Proposition 3 is that the singleton coalition is stable by definition (see section 2.1) and that the maximum size of a stable coalition is three. This maximum coalition size will only emerge provided there are at least three players with the same benefit parameter γ_i. By applying Proposition 3 to cases 1, 2 and 3, we establish the following results.

Lemma 1

In uncertainty case 1 and the full learning scenario, the set of stable coalitions Ψ^{*FL} *and the expected equilibrium coalition size are given, respectively, by:*

$$\Psi^{*FL}=\begin{cases}\{i,j\} & if \quad n=2\\ \{i,j,k\} & if \quad n\ge 3\end{cases}, \forall i,j,k\in N \qquad\qquad E\left[m^{*FL}\right]=\begin{cases}2 & if \quad n=2\\ 3 & if \quad n\ge 3\end{cases}.$$

Proof: Follows immediately from Proposition 3 by inserting $\gamma_{i_1} = \gamma_{i_2} = \ldots = \gamma_{i_n}$, and noting that any non-trivial coalition that is internally stable Pareto-dominates the singleton coalition. (**Q.E.D.**)

Note that $E\left[m^{*FL}\right]$ has to be interpreted as the average coalition size, given the different possible realizations of the random variables Γ_k. This also applies to the subsequent lemmas.

Lemma 2

In uncertainty case 2 and the full learning scenario, the set of stable coalitions Ψ^{*FL} *and the expected equilibrium coalition size are given, respectively, by:*

$$\Psi^{*FL} = \begin{cases} \{i\} & if \quad n \leq 3 \\ \{n-1,n\} & if \quad n \geq 4 \end{cases}, \forall i \in N \qquad\qquad E\left[m^{*FL}\right] = \begin{cases} 1 & if \quad n \leq 3 \\ 2 & if \quad n \geq 4 \end{cases}$$

where $\{n-1,n\}$ *refers to the players with the highest value of* γ_i *for a given realization of the vector* γ.

Proof: In case 2, each realization of the random variables Γ_k imply a permutation of the vector $\gamma = \{1,\ldots,n\}$ where $\gamma_i = i$, $i \in \{1,\ldots,n\}$. Hence, it follows immediately from Proposition 3 that no coalition of three players can be stable. Consider now two-player coalitions. We first show that $\{n-1,n\}$ is the only non-trivial coalition that is externally stable. We proceed in three steps. First, player n must belong to any externally stable coalition. By contradiction, suppose there is an externally stable coalition $S = \{i_1, i_2\}$ and player n does not belong to it. Then, external stability requires $n < \frac{\gamma_{i_1} + \gamma_{i_2}}{2}$, which never holds as the maximum value of the right hand side of the inequality is $n - 3/2$, when $\gamma_{i_1} = n-1$ and $\gamma_{i_2} = n-2$. Second, suppose $n \in S$ but $n-1 \notin S$ and let $S = \{n-k, n\}$,

$k \in \{2,...,n-2\}$. Then coalition S is externally stable if and only if $n-1 < \frac{n-k+n}{2}$ which simplifies to $k < 2$ and hence is not admissible. Third, let $S = \{n-1, n\}$, then external stability requires $n-2 < \frac{n-1+n}{2}$ which is true.

Finally, we analyze the internal stability of $S = \{n-1, n\}$ which requires $\frac{n}{n-1} \le \sqrt{2}$ or equivalently $n \ge \frac{\sqrt{2}}{\sqrt{2}-1} \simeq 3.41$. Hence, $S = \{n-1, n\}$ is not stable for $n \le 3$, in which case the singleton coalition is stable ($m^* = 1$), and is internally stable for $n \ge 4$, in which case it Pareto-dominates the singleton coalition and therefore $m^* = 2$. **(Q.E.D.)**

Lemma 3

In uncertainty case 3 and the full learning scenario, the expected equilibrium coalition size is given by

$$E\left[m^{*FL}\right] < 3 \quad \forall \gamma .$$

Proof: We note that in case 3 there is a positive probability that the realization of the random variables Γ_k implies a permutation of the vector $\gamma = \{1,...,n\}$, i.e. $\gamma_i = i$, $i \in \{1,...,n\}$. As shown under Lemma 2, then the only stable coalition structure is the one formed by singletons $(m^* = 1)$ for $n \le 3$, and $S = \{n-1, n\}$, implying $m^* = 2$ for $n \ge 4$. Finally, from Proposition 3 the largest possible stable coalition is $m^* = 3$ and hence $E\left[m^{*FL}\right] < 3 \quad \forall \gamma$ follows. **(Q.E.D.)**

In contrast to Lemma 1 and 2, Lemma 3 is rather general and does not provide information about Ψ^{*FL}. The reason is that no closed form solution exists as the number of possible vectors γ in case 3 is very large and increases more than proportionally with the number of players. For instance, already for $n = 3$ there are 27 different possible vectors

γ. For each of these vectors, stability has to be tested using Proposition 3. For practical purposes, and the evaluation in section 4, stable coalitions are determined with an algorithm programmed with the software Matlab.

3.2.2 Partial Learning

Under partial learning, players base their decision of membership on expected payoffs as derived in section 3.1.1. We find:

Proposition 4

Under the partial learning scenario, in uncertainty cases 1, 2 and 3, the set of stable coalitions Ψ^{*PL} *and the expected equilibrium coalition size are given, respectively, by:*

$$\Psi^{*PL} = \begin{cases} \{i,j\} & if \quad n=2 \\ \{i,j,k\} & if \quad n \geq 3 \end{cases} \quad \forall i,j,k \in N \qquad\qquad E\left[m^{*PL}\right] = \begin{cases} 2 & if \quad n=2 \\ 3 & if \quad n \geq 3 \end{cases}.$$

Proof: We test internal and external stability, using the definitions provided in (4) and (5). In case 1, we use the expected payoffs provided in (16), in case 2 those provided in (17), and in case 3 those provided in (18). The results are obtained through basic, though cumbersome algebra. Again, the singleton coalition, though stable by definition, is Pareto-dominated by larger stable coalitions. (Q.E.D.)

Thus, in the partial learning scenario, due to the assumption of ex-ante symmetry, players are symmetric when deciding upon membership. This leads to larger coalitions than under full learning in cases 2 and 3. Note that the expected equilibrium coalitions size is equal to the actual coalition size as the equilibrium is independent of the realization of the random variables Γ_k.

3.2.3 No Learning

Under no learning, players base their decision of membership on expected payoffs as derived in section 3.1.2. We find:

Proposition 5

Under the no learning scenario, in uncertainty cases 1, 2 and 3, the set of stable coalitions Ψ^{*NL} *and the expected equilibrium coalition size are given, respectively, by:*

$$\Psi^{*NL} = \begin{cases} \{i,j\} & if \quad n = 2 \\ \{i,j,k\} & if \quad n \geq 3 \end{cases} \quad \forall i,j,k \in N \qquad\qquad E\left[m^{*NL}\right] = \begin{cases} 2 & if \quad n = 2 \\ 3 & if \quad n \geq 3 \end{cases}.$$

Proof: We note that the expected payoffs for the no learning scenario are the same in the uncertainty cases 1, 2 and 3 and are given by (23) and (24). We test internal and external stability, using the definitions provided in (4) and (5). Then simple algebra delivers the result above. Again, singleton coalitions are ruled out because of being Pareto-inferior.

(Q.E.D.)

In our model, the partial and no learning scenarios produce exactly the same stable coalitions, as in both scenarios players are symmetric when deciding upon membership. Also, the equilibrium coalition is independent of the realization of the random variables and hence expected and actual equilibrium coalition coincide.

4. Results and Discussion

4.1 Case 1: Uncertainty about the Level of Benefits

In case 1, players are ex-ante and ex-post symmetric. Hence, under the three learning scenarios, the stable coalitions are those of maximum size which is three (two) players if $n \geq 3$ $(n = 2)$. Consequently, total abatement is the same under all scenarios as spelled out in Proposition 1. More important, from Proposition 2 we can directly conclude that expected total payoffs under full and partial learning are the same, and higher than under no learning. The difference can be solely attributed to the learning effect.

Proposition 6

In uncertainty case 1, under the full, partial, and no learning scenarios, expected equilibrium global abatement and global payoffs are ranked as follows:

$$1)\quad E\left[Q^{*FL}\left(\Psi *^{PL},\Gamma\right)\right]=E\left[Q^{*PL}\left(\Psi^{*PL},\Gamma\right)\right]=E\left[Q^{*NL}\left(\Psi^{*NL},\Gamma\right)\right] \qquad \forall\, n\geq 2$$

$$2)\quad E\left[\Pi^{*FL}\left(\Psi *^{PL},\Gamma\right)\right]=E\left[\Pi^{*PL}\left(\Psi^{*PL},\Gamma\right)\right]>E\left[\Pi^{*NL}\left(\Psi^{*NL},\Gamma\right)\right]$$

Proof: Follows immediately from Proposition 1; Proposition 2, Case 1; Lemma 1 and Propositions 4 and 5. **(Q.E.D.)**

Comparing our result with Kolstad (2007), Kolstad and Ulph (2006, 2008), it appears that we derive completely opposite conclusions. Our results indicate that if there is only uncertainty about the level of the benefits from abatement, learning is good. They find that though full learning leads to larger stable coalitions than no learning, expected total payoffs are smaller. For partial learning they find multiple equilibria suggesting that the most likely equilibrium leads to lower membership and lower expected aggregate payoffs than full and no learning. As their and our results are derived from a different payoff function, an evaluation is difficult. Nevertheless, we would like to make two comments.

First, their model has the interesting feature that the equilibrium number of signatories differs between the three learning scenarios. That is, the equilibrium number of signatories depends on the common benefit parameter, with a difference between its realization and its expected value. Second their results, as ours, depend on a particular payoff function. Their payoff function is linear and this implies boundary solutions for equilibrium abatement levels which do not depend on the benefit parameter. This may explain their counter-intuitive result that under no learning smaller coalitions are stable but expected total payoffs are larger than under full learning. (In standard models with symmetric players and no uncertainty, total payoffs increase with the size of coalitions. See, e.g. Finus and Rundshagen

2003). Put differently, in their model the learning effect is negative, which appears to be difficult to justify.

4.2 Case 2: Uncertainty about the Distribution of Benefits

In case 2, players are ex-ante symmetric but ex-post asymmetric. For the partial and no learning scenarios, ex-post asymmetry has no effect as players base their decision about membership on expected payoffs. In our model this means that under partial and no learning the stable coalitions with maximum size form which comprise three players if $n \geq 3$ and two players if $n = 2$. Thus, a comparison between both learning scenarios depends only on the relative strength of the learning and the strategic effect. As pointed out in Proposition 2, case 2, the strategic effect (which de facto is a cost-effectiveness effect in our model) is stronger than the learning effect and this explains why no learning performs better in terms of expected total payoffs, though both are equal with respect to expected total abatement.

The effects under the full learning scenario are fundamentally different. As players learn that they are ex-post asymmetric, only small coalitions are stable. For $n < 4$ no non-trivial coalition is stable and for $n \geq 4$ only a coalition of two players, formed by those with the highest marginal benefit parameter, is stable. These smaller coalitions compared to partial learning, explain why full learning is inferior to partial learning in terms of expected total abatement and total payoffs. This is the stability effect mentioned in section 2.2.

Proposition 7

In case 2, under the full, partial and no learning scenarios, expected equilibrium global abatement and global payoffs are ranked as follows:

1) $E\left[Q^{*NL}\left(\Psi^{*PL},\Gamma\right)\right]=E\left[Q^{*PL}\left(\Psi^{*PL},\Gamma\right)\right]>E\left[Q^{*FL}\left(\Psi^{*FL},\Gamma\right)\right]\ \forall\, n\geq 2$

2) $$\begin{cases} E\left[\Pi^{*NL}\left(\Psi^{*NL},\Gamma\right)\right]=E\left[\Pi^{*PL}\left(\Psi^{*PL},\Gamma\right)\right]>E\left[\Pi^{*FL}\left(\Psi^{*FL},\Gamma\right)\right] & if \quad n=2\vee n=3 \\ E\left[\Pi^{*NL}\left(\Psi^{*NL},\Gamma\right)\right]>E\left[\Pi^{*PL}\left(\Psi^{*PL},\Gamma\right)\right]>E\left[\Pi^{*FL}\left(\Psi^{*FL},\Gamma\right)\right] & if \quad n\geq 4 \end{cases}$$

Proof: See Appendix 2. **(Q.E.D.)**

Thus, we generalize the finding of Na and Shin (1998) to more than three players, namely if there is uncertainty only with respect to the distribution of the benefits from global abatement, learning has a negative impact for the outcome of coalition formation. The intuition is simple: they veil of uncertainty has a positive impact for cooperation if players would learn that the gains from cooperation are unequally distributed. Moreover, additionally to Na and Shin (1998), we also show that partial learning leads to lower expected payoffs than no learning due to the strategic effect.

In order to appreciate these results, two comments are in order. First, as mentioned in section 2.2, the superiority of no learning could be weakened if players had different abatement cost functions and/or if there were uncertainty also about the abatement cost parameter. Second, as the inferiority of full learning is due to the asymmetric distribution of the gains from cooperation, we expect that transfers could improve upon the outcome.

4.3 Case 3: Uncertainty about the Level and Distribution of Benefits

Like in case 2, in case 3 players are ex-ante symmetric but ex-post asymmetric. The average degree of asymmetry is positive, therefore larger than in case 1, but smaller than in case 2. This is because case 3 combines the level effect of case 1 and the distribution effect of case 2. Not surprisingly, this improves upon the relative performance of full learning compared to case 2, but weakens it compared to case 1.

Proposition 8

In case 3, under the full, partial, and no learning scenarios, expected equilibrium global abatement and global payoffs are ranked as follows:

1) $$\begin{cases} E\left[Q^{*PL}\left(\Psi^{*PL},\Gamma\right)\right] = E\left[Q^{*NL}\left(\Psi^{*NL},\Gamma\right)\right] > E\left[Q^{*FL}\left(\Psi^{*FL},\Gamma\right)\right] & if \quad n < 29 \\ E\left[Q^{*PL}\left(\Psi^{*PL},\Gamma\right)\right] = E\left[Q^{*NL}\left(\Psi^{*NL},\Gamma\right)\right] < E\left[Q^{*FL}\left(\Psi^{*FL},\Gamma\right)\right] & if \quad n \geq 29 \end{cases}$$

2) $$\begin{cases} E\left[\Pi^{*PL}\left(\Psi^{*PL},\Gamma\right)\right] > E\left[\Pi^{*NL}\left(\Psi^{*NL},\Gamma\right)\right] > E\left[\Pi^{*FL}\left(\Psi^{*FL},\Gamma\right)\right] & if \quad n < 29 \\ E\left[\Pi^{*PL}\left(\Psi^{*PL},\Gamma\right)\right] > E\left[\Pi^{*FL}\left(\Psi^{*FL},\Gamma\right)\right] > E\left[\Pi^{*NL}\left(\Psi^{*NL},\Gamma\right)\right] & if \quad 29 \leq n < 32 \\ E\left[\Pi^{*FL}\left(\Psi^{*FL},\Gamma\right)\right] > E\left[\Pi^{*PL}\left(\Psi^{*PL},\Gamma\right)\right] > E\left[\Pi^{*NL}\left(\Psi^{*NL},\Gamma\right)\right] & if \quad n \geq 32 \end{cases}$$

Proof: See Appendix 3. **(Q.E.D.)**

The relation between partial and no learning follows directly from Proposition 1 with respect to expected total abatement, and from Proposition 2, case 3 with respect to expected total payoffs, as under both scenarios the equilibrium coalition is the same. Hence, differences are due to the learning effect.

In the case of full learning, the equilibrium coalition depends on the realization of the random variables Γ_k. In some cases, only the singleton coalition, in some a two player coalition and in some a three player coalition is stable. The average coalition size, $E\left[m^{*FL}\right]$, increases with the number of players, n, but is strictly lower than 3. For instance, it is 1.5 for $n = 2$, 2.15 for $n = 10$, and 2.51 for $n = 50$. Thus, the difference between $E\left[m^{*NL}\right] = E\left[m^{*PL}\right]$ (which is 3 for $n \geq 3$) and $E\left[m^{*FL}\right]$ decreases with n. That is, the stability effect when comparing full learning with partial and no learning decreases with. Additionally, it is important to notice that under full learning the two and three player stable coalitions are not random, as in the other learning scenarios, but only those that satisfy Proposition 3. In particular, all stable two-player coalitions must include the player with the highest marginal benefit parameter, γ_i. This leads to higher expected total abatement and total payoff than a random two player coalition. It is for this reason that for $n \geq 29$ full learning generates higher expected total abatement than partial and no learning. This in turn implies that for $n \geq 29$ the expected total payoff under full learning is higher than under no learning In other words, the learning effect dominates the stability effect above this threshold. When comparing full learning with partial learning, the threshold for expected total payoffs is $n = 32$. As there is no difference between the two scenarios, in terms of the second stage of coalition formation, the difference is only due to the stability effect. Since $E\left[m^{*FL}\right] < E\left[m^{*PL}\right]$ for all $n \geq 2$, this stresses that not only the size but also the identity of coalition members matters.

Taken together, whereas in case 1 learning has a positive effect, in case 2 this is reversed, and in case 3 the evidence is mixed. This is not surprising as case 3 is a combination of cases 1 and 2. Clearly, in case 3, partial learning is better than no learning, but the relative performance of full learning depends on the number of players. Above a certain number of players, full learning performs better than no learning and even partial learning.

5. Summary and Conclusions

This paper addressed the role of uncertainty and learning for the formation of international environmental agreements (IEAs). In a two-stage coalition formation game, countries decide first whether to participate in an IEA and then on their abatement levels. We considered three learning scenarios: full, partial and no learning. That is, the uncertainty about the parameters of the benefit functions from global abatement (in the form of reduced damages) is resolved before stage 1, between stage 1 and 2, or never resolved, respectively. We also considered three uncertainty cases, namely uncertainty on 1) the level of the benefits from global abatement, 2) the distribution of the benefits and 3) both.

The results showed that learning has a positive effect on expected aggregate payoffs if the there is uncertainty on the level of the benefits of climate policy but this is reversed if the there is uncertainty on the distribution of these benefits. If both effects are at work, at least partial learning is always better than no learning. Thus, overall, our paper draws far more positive conclusions about the role of learning than previous studies.

We argued that the driving forces of our results can be related to three effects. First there is the learning effect, which is (strictly) positive in our cases 1 and 3, but zero in case 2. Hence, expected aggregate payoffs can never be lower under full and partial learning that under no learning if all countries participate in an IEA. However, due to free-rider incentives, the grand coalition may not be a stable outcome. Then the stability effect (which is (strictly) negative in our cases 2 and 3, but zero in case 1) may lead to smaller coalitions under full learning than under partial and no learning. Moreover, the strategic effect, resulting from the strategic interplay between signatories and non-signatories when choosing their equilibrium abatement levels (which is (strictly) negative in our cases 2 and 3, but zero in case 1) may improve upon the aggregate outcome under no learning compared full and partial learning.

Despite we generalized some aspects of the models by Kolstad (2007), Kolstad and Ulph (2006, 2008) and Na and Shin (1998), our model also shares some of their limitations. First, we assumed a static payoff function which does not capture the nature of stock pollutants relevant in the context of greenhouse gases. Moreover, learning takes the form of perfect learning. Richer models could allow for a dynamic payoff structure, the test for stable IEAs along the entire time path (e.g. Rubio and Ulph 2006 and Ulph 2004), and imperfect learning, e.g. in the form of a Bayesian update of beliefs (e.g. Karp and Zhang 2000 and Kelly and Kolstad 1999). No doubt, analytical solutions will be difficult to obtain for these extensions. Second, all players share the same believes about the distribution of the uncertain parameters. This simplification requires shared knowledge of current scientific evidence and/or some form of coordination ex-ante, provided for instance through the IPCC. Giving up this restrictive assumption would certainly be interesting and should not be too complicated to model. Third, we did not consider uncertainty about the abatement cost parameters with possible asymmetric realizations. Moreover, we did not consider possible transfers in order to compensate for a possible asymmetric distribution of the gains from cooperation (e.g. Fujita 2004). For both extensions we expect that the impact of learning improves, particularly in cases 2 and 3. Fourth, we could give up the assumption of risk neutrality and study the role of risk preference on the formation of IEAs (e.g. Bramoullé and Treich 2006 and Broucher and Bramoullé 2007). In the case of ex-post asymmetry and not perfectly correlated parameters this opens the possibility to explore the role of insurance contracts and other financial instruments for fostering international cooperation on climate change.

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Appendix 1: Proof of Proposition 3

Inserting equilibrium payoffs of signatories and non-signatories as provided in (12) and (13), respectively, into the definition of internal and external stability in (4) and (5), respectively, we derive:

(I)

$$\begin{cases} \gamma_i\left(m\sum_{\ell\in S}\gamma_\ell+\sum_{j\notin S}\gamma_j\right)-\frac{1}{2}\left(\sum_{\ell\in S}\gamma_\ell\right)^2\geq\gamma_i\left((m-1)\sum_{\ell\in S\setminus\{i\}}\gamma_\ell+\sum_{j\notin S\setminus\{i\}}\gamma_j\right)-\frac{1}{2}(\gamma_i)^2,\forall i\in N \\ \gamma_j\left(m\sum_{i\in S}\gamma_i+\sum_{k\notin S}\gamma_k\right)-\frac{1}{2}(\gamma_j)^2>\gamma_j\left((m+1)\sum_{i\in S\cup\{j\}}\gamma_i+\sum_{k\notin S\cup\{j\}}\gamma_k\right)-\frac{1}{2}\left(\sum_{i\in S\cup\{j\}}\gamma_i\right)^2,\forall j\notin S \end{cases}$$

where this system can be simplified to:

(II)
$$\begin{cases} \gamma_i\geq\frac{\sum_{\ell\in S\setminus\{i\}}\gamma_\ell}{\sqrt{2(m-1)}}, \quad \forall i\in S \\ \gamma_j<\frac{\sum_{i\in S}\gamma_i}{\sqrt{2m}}, \quad \forall j\notin S \end{cases}$$

Since the smallest γ_i in coalition S is γ_{i_1} and the largest γ_j among non-signatories is $\gamma_{j_{n-m}}$, the n conditions of the system in (II) can be reduced to only two sufficient conditions:

(III)
$$\begin{cases} \gamma_{i_1}\geq\frac{\sum_{\ell\in S\setminus\{i\}}\gamma_\ell}{\sqrt{2(m-1)}} \\ \gamma_{j_{n-m}}<\frac{\sum_{i\in S}\gamma_i}{\sqrt{2m}} \end{cases}$$

Moreover, summing the first set of conditions in (II) over all $i\in S$ gives a necessary condition for internal stability:

(IV) $$\sum_{i\in S}\gamma_i \geq \frac{\sum_{i\in S}\sum_{\ell\in S\setminus\{i\}}\gamma_\ell}{\sqrt{2(m-1)}} \Leftrightarrow \sqrt{2(m-1)}\sum_{i\in S}\gamma_i \geq (m-1)\sum_{i\in S}\gamma_i \Leftrightarrow (m-1)(m-3)\leq 0 \ .$$

From (IV) we can conclude that only coalitions of size $m\in\{1,2,3\}$ can be internally stable and hence stable. Thus, it remains to analyze each of these possible values for m.

We first note that $m=1$ implies the singleton coalition structure which is stable by definition (see section 2.1). However, if a coalition of size $m=2$ or $m=3$ is stable, we can drop the equilibrium coalition $m=1$ by invoking the Pareto-dominance criterion.

Suppose $m=2$. For $n=2$ internal stability requires $\frac{\gamma_{i_2}}{\gamma_{i_1}}\leq\sqrt{2}$ and externally stability holds as no non-signatory is left. For $n>2$ the condition for external stability has to be added: $\gamma_{j_{n-2}} < \frac{\gamma_{i_1}+\gamma_{i_2}}{2}$.

Suppose $m=3$. Then internally stability requires $2\gamma_{i_1}\geq\gamma_{i_2}+\gamma_{i_3}$ which holds if and only if $\gamma_{i_1}=\gamma_{i_2}=\gamma_{i_3}$. Thus, for $n=3$, $\gamma_{i_1}=\gamma_{i_2}=\gamma_{i_3}$ is sufficient as external stability always holds. For $n>3$ we require additionally for external stability that $\gamma_{j_{n-3}} < \frac{3\gamma_{i_1}}{2} \Leftrightarrow 2\gamma_{j_{n-3}} < 3\gamma_{i_1}$, which completes the proof. **(Q.E.D.)**

Appendix 2: Proof of Proposition 7

The equality of expected total abatement between the partial and the no learning scenarios follows directly from Propositions 1, 4 and 5. For expected total payoffs, the relations follow directly from Proposition 2, case 2, and Propositions 4 and 5. Hence, we have established:

$$E\left[Q^{*NL}\left(\Psi *^{NL},\Gamma\right)\right]=E\left[Q^{*PL}\left(\Psi^{*PL},\Gamma\right)\right]\ \forall\, n\geq 2$$

$$E\left[\Pi^{*NL}\left(\Psi *^{NL},\Gamma\right)\right]=E\left[\Pi^{*PL}\left(\Psi^{*PL},\Gamma\right)\right]\ \textit{if}\ n=2\vee n=3$$

$$E\left[\Pi^{*NL}\left(\Psi *^{NL},\Gamma\right)\right]>E\left[\Pi^{*PL}\left(\Psi^{*PL},\Gamma\right)\right]\ \textit{if}\ n\geq 4$$

In order to establish $E\left[Q^{*PL}\left(\Psi^{*PL},\Gamma\right)\right]>E\left[Q^{*FL}\left(\Psi^{*FL},\Gamma\right)\right]$, we compute $E\left[Q^{*PL}\left(\Psi^{*PL},\Gamma\right)\right]$ from (15) and Proposition 4, and $E\left[Q^{*FL}\left(\Psi^{*FL},\Gamma\right)\right]$ from (11) and Lemma 2:

(V)

$$E\left[Q^{*PL}\left(\Psi *^{PL},\Gamma\right)\right]=\begin{cases}6 \quad if \quad n=2\\ \dfrac{(n+6)(n+1)}{2} \quad if \quad n\geq 3\end{cases}$$

$$E\left[Q^{*FL}\left(\Psi^{*FL},\Gamma\right)\right]=\begin{cases}\dfrac{n(n+1)}{2} \quad if \quad n\leq 3\\ \dfrac{n^2+5n-2}{2} \quad if \quad n\geq 4\end{cases}$$

where the relation above follows from simple algebra.

Finally, $E\left[\Pi^{*PL}\left(\Psi^{*PL},\Gamma\right)\right]>E\left[\Pi^{*FL}\left(\Psi^{*FL},\Gamma\right)\right]$ is established by computing $E\left[\Pi^{*PL}\left(\Psi^{*PL},\Gamma\right)\right]$ from (17) and Proposition 4, and $E\left[\Pi^{*FL}\left(\Psi^{*FL},\Gamma\right)\right]$ from (14) and Lemma 2. We find:

(VI)

$$E\left[\Pi^{*PL}\left(\Psi^{*PL},\Gamma\right)\right]=\begin{cases}9 \quad if \quad n=2\\ \dfrac{(n+1)\left(3n^3+19n^2-22n-24\right)}{12} \quad if \quad n\geq 3\end{cases}$$

$$E\left[\Pi^{*FL}\left(\Psi^{*FL},\Gamma\right)\right]=\begin{cases}\dfrac{n(n+1)\left(3n^2+n-1\right)}{12} \quad if \quad n\leq 3\\ \dfrac{3n^4+16n^3-30n^2+29n-6}{12} \quad if \quad n\geq 4\end{cases}$$

where the relation above follows from simple algebra.

Appendix 3: Proof of Proposition 8

The relation between partial and no learning follows directly from Propositions 1, 4 and 5 with respect to expected total abatement, and from Proposition 2, case 3, and Propositions 4 and 5 with respect to expected total payoffs.

$E\left[Q^{*PL}\left(\Psi^{*PL},\Gamma\right)\right]=E\left[Q^{*NL}\left(\Psi^{*NL},\Gamma\right)\right]$ is provided in (V). $E\left[\Pi^{*NL}\left(\Psi^{*NL},\Gamma\right)\right]$ is computed using (25) and Proposition 5 and is given by:

(VII) $$E\left[\Pi^{*NL}\left(\Psi^{*NL},\Gamma\right)\right]=\begin{cases}9 & if \quad n=2\\ \dfrac{\left(2n^2+11n-24\right)\left(n+1\right)^2}{8} & if \quad n\geq 3\end{cases}$$

$E\left[Q^{*PL}\left(\Psi^{*PL},\Gamma\right)\right]$ is computed from (18) and Proposition 4 and is given by:

(VIII) $$E\left[Q^{*PL}\left(\Psi^{*PL},\Gamma\right)\right]=\begin{cases}\dfrac{19}{2} & if \quad n=2\\ \dfrac{\left(n+1\right)\left(6n^3+40n^2-34n-78\right)}{24} & if \quad n\geq 3\end{cases}$$

$E\left[\Pi^{*FL}\left(\Psi^{*FL},\Gamma\right)\right]$ and $E\left[Q^{*FL}\left(\Psi^{*FL},\Gamma\right)\right]$ have to be computed for each n for which we use an algorithm programmed with the software package Matlab. The following table shows the results.

Table 1A: Expected Total Abatement and Coalition Size under the Three Scenarios*

n	$E\left[Q^{*FL}\left(\Psi^{*FL},\Gamma\right)\right]$	$E\left[Q^{*PL}\left(\Psi^{*PL},\Gamma\right)\right]=E\left[Q^{*NL}\left(\Psi^{*NL},\Gamma\right)\right]$	$E\left[m^{*FL}\right]$	$E\left[m^{*PL}\right]=E\left[m^{*NL}\right]$
2	4.50	6	1.50	2
3	9.11	18	1.56	3
…	…	…	…	…
10	79.17	88	2.15	3
…	…	…	…	…
20	266.62	273	2.29	3
…	…	…	…	…
25	399.58	403	2.34	3
…	…	…	…	…
28	491.61	493	2.36	3
29	525.63	525	2.37	3
30	558.93	558	2.38	3
31	593.95	592	2.39	3
32	629.88	627	2.39	3
33	666.92	663	2.40	3
…				
50	1451.81	1428	2.51	3

Table 2A: Expected Total Payoffs under the Three Scenarios*

n	$E\left[\Pi^{*FL}\left(\Psi^{*FL},\Gamma\right)\right]$	$E\left[\Pi^{*PL}\left(\Psi^{*PL},\Gamma\right)\right]$	$E\left[\Pi^{*NL}\left(\Psi^{*NL},\Gamma\right)\right]$
2	8.25	9.50	9.00
3	39.00	57.00	54.00
…	…	…	…
10	3,926.03	4,391.75	4,325.75
…	…	…	…
20	53,939.96	55,336.75	54,904.50
…	…	…	…
25	125,904.52	127,640.50	126,834.50
…	…	…	…
28	194,115.99	195,800.75	194,691.50
29	223,241.43	223,637.50	222,412.50
30	253,701.00	254,331.75	252,983.25
31	287,955.39	288,072.00	286,592.00
32	325,589.06	325,052.75	323,433.00
33	366,471.58	365,474.50	363,706.50
…	…	…	…
50	1,828,220.11	1,802,471.75	1,796,640.75

* The values are exact for $n \leq 10$ and are estimated through a Monte Carlo simulation method for higher number of players. A total of 10,000 simulations were undertaken which guarantees low margin of errors for the estimates.