Tax Evasion and Collusion

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Abstract

This paper analyzes the interaction of tax evasion and collusion. We show that the possibility of evading taxes has an impact on the critical discount factor, a measure which is usually treated as a proxy for the sustainability of collusion. We establish that the effect of tax evasion via the critical discount factor on firm and industry activity depends on (i) the relation between benefits from tax evasion and legal after tax profits, and (ii) whether there is effective cartel law enforcement. By these means, we find that (i) the separability of tax evasion and firm activity may no longer hold if firms decide about forming a cartel and (ii) isolated enforcement decisions may have counter-intentional consequences.

Keywords: Tax Evasion, Activity Level, Cartel, Law Enforcement

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1 Introduction

1.1 Motivation and Main Result

"People of the same trade seldom meet together, even for merriment and diversion, but the conversation ends in a conspiracy against the public, or in some contrivance to raise prices." This quote was true in the days of Adam Smith, who stated this opinion in *An Inquiry Into the Nature and Causes of the Wealth of Nations*, and is nowadays still of undisputed actuality. The European Commission, for instance, announced on October 1st 2008 that it would fine wax producers 676 million euro for maintaining a price-fixing and market-sharing cartel between 1992 and 2005. Consequently, collusion is a very current topic and additional knowledge on circumstances facilitating collusion is highly warranted. In this contribution, we analyze how the decision to evade taxes interacts with the decision to collude.

We study the central incentive compatibility condition for stable collusion and show how the critical discount factor changes if firms can decide on whether to evade taxes. Based on the use of trigger strategies, the standard reasoning is that cartels are formed if the profits lost due to the punishment phase are sufficiently high to prevent deviation from the cartel agreement, i.e. if future payoffs are discounted only to some extent.\(^1\) The focus on the critical discount factor is thus standard in the literature. For instance, Symeonidis (2002) argues that “a standard way to examine the impact of exogenous factors on cartel sustainability [...] is to examine the comparative static properties of the critical discount factor. In particular, a change in any exogenous variable that causes the critical discount factor to increase makes collusion less likely, since collusion is then sustainable for a smaller set of deltas [discount factors].” We show that the lower bound of discount factors allowing for stable collusion may in fact depend on whether firms can evade taxes.

Interestingly, the interaction of the respective decisions is ambiguous. We indeed show that tax evasion may further or hinder cartelization. The effect depends on the way that tax evasion changes total profits of a representative firm in the different states of interest, those being the firm’s profit in the cartel, the firm’s profit if it is the only firm deviating

\(^1\)See, for instance, Chapter 6 in Tirole (1988) for a textbook representation of the model of collusion.
from the cartel agreement, and the firm’s profit in the competitive outcome. We also identify antitrust enforcement as a second factor influencing the way in which tax evasion impacts the decision to form a cartel.

We relate our analysis to two important points. First, it has been established that the decision to evade taxes leaves the firm’s activity choice undistorted in many circumstances (e.g., Yaniv 1995). In the paper at hand, the firm's activity choice is unaffected at the margin but may be changed by the different likelihood for stable collusion. If the critical discount factor with tax evasion differs from that without tax evasion, firm as well as industry output will be different depending on the possibility of evading taxes. Consequently, the firm’s activity choice and the decision whether to evade taxes are no longer separable. Second, we discuss how uncoordinated tax and cartel enforcement may have counter-intentional effects. This is done as, in the present setting, the interaction of the decisions to evade taxes and to engage in a cartel imply subtle interrelations between enforcement of tax and cartel law.

1.2 Relation to the Literature

The effect of tax evasion feasibility on the likelihood of collusion is - to the best of our knowledge - unstudied. This comes despite the fact that determinants of the critical discount factor have long been the subject of study (see, e.g., Chapter 4 in Motta 2004). These comprise, for instance, multi-market contacts and firm cost asymmetries. The already mentioned Symeonidis (2002), as a further example, studies the consequence of multiproduct firms on the critical discount factor. Further factors which may make collusion easier or more difficult to sustain are discussed in the comprehensive surveys by, e.g., Feuerstein (2005) and Kaplow and Shapiro (2007). We add a further important dimension that is worth incorporating. Possibilities for tax evasion may strengthen cartelization incentives under certain circumstances.

Another central aspect of the study is revisiting the result that tax evasion leaves firms’ activity choice unaffected. In the following, we comment on the literature being concerned with the relation of firm activity and tax evasion. In an early contribution, Kreutzer and Lee (1986) dismissed the neutrality of a profit tax on output if monopolists can reduce their
tax liability by overstating production costs, as this would induce them to increase output. Wang and Conant (1988), building on Kreutzer and Lee (1986), introduce a probability of detection and a penalty proportional to taxes evaded. Allowing firms to choose the level of cost overstatement, they reestablish that firms’ optimal output levels will be independent of the cost-overstatement decision.² Wang (1990) generalizes the model by Wang and Conant (1988) by accounting for the probability of detection and the penalty rate as functions of cost overstatement.³ He argues that the neutrality of the profit tax hinges on whether the probability of detection and the penalty vary with cost overstatement. However, Yaniv (1996), using the same framework, establishes that this argumentation is unfounded, i.e. profit taxes remain neutral irrespective of the monopolist’s output decision. Lee (1998) revisits the aspect of an endogenous detection probability and penalty already touched upon by Wang (1990) and Yaniv (1996). In contrast to the latter contributions, who assumed that the cost overstatement determines the probability and the penalty, Lee considers various determinants for the audit probability and the penalty rate. Allowing for tax evasion, profit taxes are neutral if, for example, the probability of detection and the penalty are functions of reported profit. Furthermore, Yaniv (1995) establishes a generic model in which firm activity is generally separable from the decision to evade taxes. This always holds true if the firm decides on the amount evaded. However, if the firm decides on the fraction of profits to evade, the separability result requires an interior optimum for the evasion decision. Two more recent contributions, however, deviate somewhat from the papers discussed so far. Panteghini (2000), using real option theory, compares two different points in time with respect to a firm’s investment, where at the later point in time uncertainty about the return on investment has been resolved. Optimal timing of investment is determined by a comparison of the net present values of immediate and postponed investment. These two alternatives may, as Panteghini demonstrates, be subject to reordering due to tax evasion. Recently, Goerke and Runkel (2006) showed in a Cournot oligopoly setting that output and evasion decisions will not be independent from each other if the number of firms is endogenous. The channel

³See also Marelli (1984) and Marelli and Martina (1988) for the implications of assumptions with respect to the detection probability in the context of indirect taxes.
through which tax evasion impacts output is an indirect one, as for a given number of firms a single firm’s optimal output does not depend on tax evasion. However, since evasion increases expected profits, more firms enter the market, with consequences for individual and industry output. Like Goerke and Runkel (2006), our paper assumes that the market is an oligopoly. In our framework, the output decision of the firm may depend on tax evasion, since the latter affects the formation of a cartel in the industry considered.

The balance of the paper is as follows. Section 2 opens with a description of the model, continues with the analysis, and closes with a discussion on the interdependence of tax and cartel law enforcement. Section 3 offers concluding remarks.

2 The Model and Analysis

2.1 The Model

The model describes the calculus of $n$ symmetric and risk-neutral firms which play a game an infinite number of times.$^4$ The game itself comprises two stages. During the first stage, firms decide whether to form a cartel which influences the choice of activity in the second stage. The firm decides on the activity level and the amount of taxes to evade during the second stage. The modeling in the first stage focuses on the incentive compatibility constraint for cartel formation, i.e. the corresponding critical discount factor. The arguments relating to the second stage follow closely the model established by Yaniv (1995), which is specific neither to a certain tax base nor to the structure of the market in which the firm is active. As is standard, there are three different modus operandi that have to be distinguished. These are: cartel (K), i.e. a cartel is formed and firms adhere to the recommendation with respect to the activity level, deviation (D), i.e. firms agree on a cartel but one firm deviates from the activity level recommendation, and competition (C), i.e. no agreement on a cartel is achieved in the first place.

Stage 2:

Firm $j$, $j = 1, \ldots, n$, faces a proportional tax rate $\theta$ on its tax base, and decides on its activity

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$^4$The assumption on the time frame may be substituted by assuming that there always is a positive probability for a renewed interaction.
level and on the statement deviation $S_j$ from the true value of the tax base $b_j$. The legal after tax profits of firm $j$ are given by $\pi_j = \pi_j(A_j, A_{-j})$, where $A_j$ is the activity level of firm $j$ and $A_{-j}$ is the vector of activity levels of all other firms.\(^5\) The statement deviation causes concealment costs $g(S_j, \pi_j)$, which increase at an increasing rate with the statement deviation, $g_s, g_{ss} > 0$.\(^6\) Regarding the dependency on $\pi_j$, it seems reasonable to assume that $g_{\pi_j} \leq 0$, i.e. higher legal after tax profits make a given statement deviation less costly. However, to obtain our results, we do not need to assume this, as will become clear shortly.

The firm is audited with exogenous probability $p$. In the event of an audit, the firm will have to pay evaded taxes and a penalty which, following Yitzaki (1974), is a multiple of evaded taxes according to the penalty rate $\tau - 1 > 0$. Consequently, firm $j$’s expected profits are given by

$$\Pi_j = \pi_j(A_j, A_{-j}) + S_j \theta - pS_j \theta \tau - g(S_j, \pi_j(A_j, A_{-j})) \quad (1)$$

The firm decides on activity and the statement deviation in Stage 2. In the following, we will characterize the optimal activity choice and thereby establish that the irrelevance result referred to in the introduction holds in our framework also at the margin.

If the firm decides to deviate from a cartel agreement or if the modus operandi is competition, the firms’ activity choice is guided by

$$\frac{d\Pi_j}{dA_j} = \pi_{jA_j} (1 - g_{\pi}) = 0 \quad (2)$$

For $g_{\pi} \neq 1$, we obtain $\pi_{A_j} = 0$, which holds for both ‘deviation’ and ‘competition’. Consequently, activity choices are not directly affected by tax evasion for these modus operandi.

In the case of ‘cartel’, firms maximize joint expected profits with regard to activity. Summing equation (1) over all $n$ symmetric firms the corresponding first-order condition for $A_j$ is given by

$$\frac{d}{dA_j} \left( \sum_{i=1}^{n} \Pi_i \right) = \pi_{jA_j} (1 - g_{\pi}) + \sum_{i \neq j} \pi_{iA_j} (1 - g_{\pi}) = 0 \quad (3)$$

Given symmetric firms and $g_{\pi} \neq 1$, the last equation boils down to $\pi_{jA_j} = -\sum_{i \neq j} \pi_{iA_j}$. Therefore, the optimal activity level of firm $j$ is again independent of the decision to evade

\(^{5}\)The tax rate also determines legal after tax profits but this dependence is suppressed for convenience.

\(^{6}\)The introduction of concealment costs is analogous to the treatment in, e.g. Virmani (1989) or Cremer and Gahvari (1992).
taxes. Consequently, the activity choice $A_j$ is independent of the statement deviation $S_j$ for every modus operandi.

We will establish that tax evasion can impact the firm and industry output by affecting the modus operandi which results. For this purpose, we reformulate legal after tax profits in the following way: $\pi_j = \pi_j(A_j, A_{-j}) = \pi_j(\alpha)$, where $\alpha$ is an index representing different activity vectors. We scale $\alpha$ in a way so that $\frac{\partial \pi_j}{\partial \alpha} = 1$. This will prove to be quite convenient for the analysis to follow. The three different modus operandi of interest, K, D, and C, are three different realizations of $\alpha$, where $\pi_j(\alpha^D) > \pi_j(\alpha^K) > \pi_j(\alpha^C)$.

To end the description of the second stage, we have to investigate the decision on tax evasion. The profit-maximizing choice of the statement deviation $S_j^*$ satisfies

$$\frac{d\Pi_j}{dS_j^*} = \theta(1 - p\tau) - g_S(S_j^*, \pi_j(\alpha)) = 0$$

and changes with $\alpha$ according to

$$\frac{dS_j^*}{d\alpha} = -\frac{g_S}{g_S^2}$$

Thus, $sgn\{\frac{dS_j^*}{d\alpha}\} = sgn\{-g_S\}$. So, if marginal concealment costs due to an increase in the statement deviation decrease with the level of legal after tax profits, the statement deviation increases with $\alpha$ and therefore legal after tax profit $\pi_j$.

The expected profit function can be stated as

$$\Pi_j(\alpha) = \pi_j(\alpha) + S_j^*\theta - pS_j^*\theta\tau - g(S_j^*, \pi_j(\alpha))$$

and changes with $\alpha$ (and therefore $\pi_j$) according to

$$\frac{d\Pi_j(\alpha)}{d\alpha} = 1 - g_S(S_j^*, \pi_j(\alpha))$$

$$\frac{d^2\Pi_j(\alpha)}{d\alpha^2} = \frac{1}{g_S^2} [g_S^2 - g_{\pi\pi}g_S]$$

The way in which the expected profit function changes with $\alpha$ (and therefore $\pi_j$) will be central in the analysis to follow.

This concludes our discussion of the individual calculus for the second stage. Next, we analyze the decision of whether to form a cartel, particularly the stability of such an agreement which is critically determined by the discount factor applied to future payoffs.
Stage 1:

Collusive agreements are inherently unstable in a static setting. However, stability may obtain in a multi-period setting because punishment for deviant behavior may allow for collusion in equilibrium. The size of the discount factor $\delta$ with which future payments are discounted is of critical importance in this regard. We consider the following standard trigger strategy: (i) start cooperatively, (ii) maintain cooperative behavior if no firm deviated in the past, and (iii) revert to the competitive outcome forever should any other firm have deviated from the collusive agreement. Consequently, we assume for simplicity that past behavior is perfectly observable. As an additional impediment to collusion, we consider that collusive behavior may be sanctionable. Indeed, many countries outlaw collusion. However, in some countries, competition law exists primarily in the books and is not enforced (Gal 2008). In principle, competition authorities command over the detection probability $q \geq 0$ and the fine imposed $F > 0$ in the event of collusion being detected. In the following, we distinguish between the case of effective, $q > 0$, and ineffective, $q = 0$, cartel law enforcement. In line with most of the literature, we assume that the fine is independent of the number of periods in which collusion occurs (see, e.g., Motta and Polo 2003). Furthermore, we investigate two different scenarios: first, the fine is supposed to be a constant amount independent of other variables in the model (cf., e.g., Motta and Polo 2003), second, the fine is assumed to be proportional to the additional profits due to collusion. The latter consideration is in line with the actual legal treatment in most jurisdictions. Finally, with respect to tax evasion and cartel formation we suppose that the detection of one type of transgression does not imply the uncovering of the other type. This seems a reasonable presumption because evidence looked for during investigations is of a unique nature and respective procedures are undertaken by different authorities.

Given that the other firms stick to collusion, three different values are of importance for the representative firm. For firm $j$, the expected present value of the cartel, $V^K_j$, is given by

$$V^K_j = \Pi_j(\alpha^K) + \delta\{q(V^C_j - F) + (1 - q)V^K_j\}$$  \hspace{1cm} (9)
where $\Pi_j(\alpha^K)$ represents the expected per-period profit if the cartel is the applying modus and $V_j^C$ denotes the present value of the competitive outcome for the firm. At the end of each period, the cartel will be uncovered with probability $q$. Not only will the competitive modus be installed by competition authorities but firms also owe the fine $F$ to competition authorities. With the residual probability, the cartel remains undetected. The value of the competitive outcome for firm $j$ is given by

$$V_j^C = \Pi_j(\alpha^C) + \delta V_j^C \tag{10}$$

The last value of relevance when deliberating whether to stick to the collusive agreement is the gain from deviation. For firm $j$, the present value that follows from deviation is given by

$$V_j^D = \Pi_j(\alpha^D) + \delta V_j^C \tag{11}$$

We solve the system (9) – (11) explicitly to obtain

$$V_j^K = \frac{\Pi_j(\alpha^K) + \delta q \left[ \frac{\Pi_j(\alpha^C)}{1-\delta} - F \right]}{1 - \delta (1 - q)} \tag{12}$$

$$V_j^C = \frac{\Pi_j(\alpha^C)}{1 - \delta} \tag{13}$$

$$V_j^D = \Pi_j(\alpha^D) + \frac{\delta \Pi_j(\alpha^C)}{1 - \delta} \tag{14}$$

We are now in the position to state the central incentive compatibility condition. For the firm at hand, it must be better to stick to the agreement than to deviate, i.e. $V_j^K \geq V_j^D$ needs to hold for the cartel to be stable. The critical discount factor $\delta_T$ makes the firm indifferent between the alternative courses of action. The subscript $T$ indicates that $\delta_T$ results as critical discount factor in the case with tax evasion. Consequently,

$$\frac{\Pi_j(\alpha^K) + \delta_T q \left[ \frac{\Pi_j(\alpha^C)}{1-\delta_T} - F \right]}{1 - \delta_T (1 - q)} = \Pi_j(\alpha^D) + \frac{\delta_T \Pi_j(\alpha^C)}{1 - \delta_T} \tag{15}$$

Collusion will prevail as long as firms apply a high weight to future payoffs, i.e. $\delta \geq \delta_T$, because the punishment from deviation is not discounted too heavily. For $\delta < \delta_T$ the cartel is not stable because each firm would rather choose to deviate from collusion.
The corresponding value for the critical discount factor \( \bar{\delta}_N \) in the case without tax evasion is obtained from

\[
\frac{\pi_j(\alpha^K) + \bar{\delta}_N q \left[ \frac{\pi_j(\alpha^C) - F}{1 - \bar{\delta}_N} \right]}{1 - \bar{\delta}_N (1 - q)} = \frac{\bar{\delta}_N \pi_j(\alpha^C)}{1 - \bar{\delta}_N}
\]

(16)

To summarize, the higher the critical value for the discount factor is, the less likely it is that collusion will occur. Whether tax evasion favors collusion can therefore be evaluated by looking at the corresponding change in the critical value for the discount factor, i.e. a comparison of \( \bar{\delta}_N \) and \( \bar{\delta}_T \). The likelihood of collusion would be increased by tax evasion if \( \bar{\delta}_T < \bar{\delta}_N \) were to hold and vice versa.

2.2 The Analysis

We are interested in the interaction of the decision on tax evasion and the decision on forming a cartel. Profits are affected by the opportunity to evade taxes in that \( \Pi_j(\alpha) \geq \pi_j(\alpha) \). What we will show is that it is of critical importance for the impact of tax evasion on cartel stability in which way the term \( \frac{\Pi_j(\alpha) - \pi_j(\alpha)}{\pi_j(\alpha)} \) changes with the level of legal after tax profits.

The analysis distinguishes the cases in which the fine is constant and the case in which the fine is proportional to the increase in profits resulting from collusion. We start with the former case.

2.2.1 Constant Fine

Case (i) \( \frac{d\Pi_j(\alpha)}{d\alpha} > 0 = \frac{d^2\Pi_j(\alpha)}{d\alpha^2} \):

Suppose that the total expected profit \( \Pi_j(\alpha) \) varies proportionally with \( \alpha \) and (therefore legal after tax profits \( \pi_j \)). Referring to (7) and (8), this obtains if concealment costs can be represented, for instance, by a linear homogeneous function, \( g(S_j, \pi_j(\alpha)) = k \left( \frac{\pi_j(\alpha)}{S_j} \right) S_j \), where \( k' < 0 < k'' \). Concealment costs per unit of the tax base evaded increase at a decreasing rate with the proportion of profits to deviation. Higher legal after tax profits make a given deviation less costly but marginal costs of deviating are constant for a given

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7The curvature of the expected profit function \( \Pi_j(\alpha) \) depends on the curvature of the concealment cost function \( g(S_j^*, \pi_j(\alpha)) \), see equations (7) and (8).
proportion of legal after tax profits and taxes evaded. In this case, making use of \( \frac{d\pi_j(\alpha)}{d\alpha} = 1 \), we can employ \( \Pi_j(\alpha) = a\pi_j(\alpha), \ a > 1, \) to restate (15) as

\[
a\pi_j(\alpha^K) + \bar{\delta}_T q \left[ \frac{\pi_j(\alpha^C)}{1-\delta_T} - F \right] \frac{1}{1-\bar{\delta}_T(1-q)} = a\pi_j(\alpha^D) + \frac{\bar{\delta}_T a\pi_j(\alpha^C)}{1-\bar{\delta}_T}
\]

(17)

To relate this incentive compatibility condition to (16), the corresponding equation in the absence of tax evasion, we divide (17) by \( a \) to reach

\[
\frac{\pi_j(\alpha^K)}{1-\bar{\delta}_T(1-q)} = \pi_j(\alpha^D) + \frac{\bar{\delta}_T a\pi_j(\alpha^C)}{1-\bar{\delta}_T}
\]

(18)

Isolating the critical discount factor, we obtain

\[
\bar{\delta}_T = \frac{\pi_j(\alpha^D) - \pi_j(\alpha^K)}{(1-q)(\pi_j(\alpha^D) - \pi_j(\alpha^C)) - qF/a}
\]

(19)

**Proposition 1** Given \( \frac{d\Pi_j(\alpha)}{d\alpha} > 0 = \frac{d^2\Pi_j(\alpha)}{d\alpha^2} \) and a constant fine \( F \), tax evasion (i) leaves incentives for collusion unchanged \( (\bar{\delta}_T = \bar{\delta}_N) \) in the absence of effective cartel enforcement \( (q = 0) \), and (ii) allows for more collusion in equilibrium \( (\bar{\delta}_T < \bar{\delta}_N) \) if cartel law enforcement is effective \( (q > 0) \).

**Proof.** The first claim follows from the fact that without effective cartel enforcement, (16) and (18) are identical. The second claim follows from the fact that \( \frac{d\bar{\delta}_T}{d\alpha} < 0 \) follows from equation (19). □

If total expected profits increase linearly with legal after tax profits, we obtain an up-scaling of all \( \pi_j(\alpha) \) by a given factor. If cartel enforcement is not adapted to this fact, tax evasion eases the condition on the discount factor which needs to be fulfilled for collusion to occur because in this case profits out of a cartel increase relative to profits from deviation. However, in the absence of effective cartel law enforcement, all relevant terms are increased by the same proportion, leaving incentives to cartelize unchanged.

Referring to the activity choice of the individual firm and industry output we can therefore assert that this choice is not affected by the possibility to evade taxes if there is no effective cartel law enforcement. With effective cartel law enforcement and a constant fine, firm and
industry activity falls whenever \( \delta \in (\bar{\delta}_T, \bar{\delta}_N) \) due to tax evasion softening the incentive compatibility constraint for cartel stability. In that case, the separability of tax evasion and firm activity does no longer hold.

**Case (ii)** \( \frac{d\Pi_j(\alpha)}{d\alpha}, \frac{d^2\Pi_j(\alpha)}{d\alpha^2} > 0 \): Suppose that the increase in expected profit \( \Pi_j(\alpha) \) increases at an increasing rate with \( \alpha \) (and therefore legal after tax profits \( \pi_j \)). Referring to (7) and (8), this obtains, for instance, if concealment costs can be stated as \( g(S_j, \pi_j(\alpha)) = k\left(\frac{\pi_j(\alpha)}{S_j}\right) \), with \( k' < 0 < k'' \). In other words, total concealment costs decrease at a decreasing rate with the proportion of profits to deviation. Stated alternatively, concealment costs only depend on the statement deviation relative to legal after tax profits.

Replicating the argument above, we first consider the case without effective cartel law enforcement \( (q = 0) \). From (16), we deduce that

\[
\pi_j(\alpha^K) = (1 - \bar{\delta}_N)\pi_j(\alpha^D) + \bar{\delta}_N\pi_j(\alpha^C)
\]

(20)

Consequently, \( \pi_j(\alpha^K) \) is a convex combination of \( \pi_j(\alpha^D) \) and \( \pi_j(\alpha^C) \) with the use of \( \bar{\delta}_N \) as weight.\(^8\) This observation helps to prove

**Proposition 2** Given \( \frac{d\Pi_j(\alpha)}{d\alpha}, \frac{d^2\Pi_j(\alpha)}{d\alpha^2} > 0 \), tax evasion erodes collusion incentives \( (\bar{\delta}_T > \bar{\delta}_N) \) in the absence of effective cartel enforcement \( (q = 0) \).

**Proof.** The case considered ensures that \( \Pi_j(\alpha) \) is strictly convex in \( \alpha \) while \( \pi_j(\alpha) \) is linear in \( \alpha \). From Jensen’s inequality, it follows for a strictly convex function \( \Pi_j(\alpha) \) that

\[
\Pi_j(\alpha^K) = \Pi_j((1 - \bar{\delta}_N)\alpha^D + \bar{\delta}_N\alpha^C) < (1 - \bar{\delta}_N)\Pi_j(\alpha^D) + \bar{\delta}_N\Pi_j(\alpha^C)
\]

(21)

Therefore, to reestablish equality of payoffs from collusion versus deviation, the discount factor needs to increase. Note, that \( \Pi_j(\alpha^K) \) is a fixed value not affected by the required increase in the discount factor. Consequently, it holds in the case of tax evasion that

\[
\Pi_j(\alpha^K) = (1 - \bar{\delta}_T)\Pi_j(\alpha^D) + \bar{\delta}_T\Pi_j(\alpha^C)
\]

(22)

\(^8\)As \( \frac{d\pi_j(\alpha)}{d\alpha} = 1 \), this directly translates into \( \alpha^K \) being a convex combination of \( \alpha^D \) and \( \alpha^C \) with corresponding weights \( (1 - \delta_N) \) and \( \delta_N \), respectively.
with $\delta_T > \delta_N$. ■

In Figure 1, we depict the legal after tax profit levels and the corresponding expected profit levels $\Pi_j(\alpha)$. The possibility to evade always implies a higher profit level which is represented by the fact that the function of expected profits lies above the 45 degree line. The transformation of $\pi_j(\alpha^K)$ is clearly inferior to the linear combination of the transformations of $\pi_j(\alpha^D)$ and $\pi_j(\alpha^C)$ if the weight $\delta_N$ is maintained. Therefore, the discount factor equating payoffs from the options collusion and deviation needs to be higher if tax evasion is possible, which gives a higher weight to $\Pi_j(\alpha^C)$. Consequently, in these circumstances tax evasion erodes incentives for collusion, because collusion will not take place for $\delta \in (\delta_N, \delta_T)$.

![Figure 1: Expected Profits and Legal After Tax Profits, Case (ii), $q = 0$](image)

Next, we consider the case of effective cartel law enforcement ($q > 0$). On the one hand, the effect flowing from the strict convexity of $\Pi_j(\alpha)$ remains. On the other hand, it needs to be recognized that the monotone transformation is not applied to the expected fine due
to the cartel law violation. To illustrate this, we rearrange (16) to

\[ \pi_j(\alpha^K) - \bar{\delta}_NqF = [1 - \bar{\delta}_N(1 - q)] \pi_j(\alpha^D) + \bar{\delta}_N(1 - q)\pi_j(\alpha^C) \]

(23)

and contrast it with

\[ \Pi_j(\alpha^K) - \bar{\delta}_TqF = [1 - \bar{\delta}_T(1 - q)] \Pi_j(\alpha^D) + \bar{\delta}_T(1 - q)\Pi_j(\alpha^C) \]

(24)

**Proposition 3** Given \( \frac{d\Pi_j(\alpha)}{d\alpha} > 0 \) and a constant fine \( F \), tax evasion changes collusion incentives ambiguously in the presence of effective cartel law enforcement.

This conclusion follows from the fact that for small expected fines, the effect flowing from the strictly convex expected profit function dominates. But this does not need to hold if the expected fine is significant. Figure 2 illustrates the special case where the critical discount factor is unaffected by the possibility of tax evasion because the two opposing effects just cancel out. In the example, a higher [lower] fine would be associated with \( \bar{\delta}_T < \bar{\delta}_N \) [\( \bar{\delta}_T > \bar{\delta}_N \)] and accordingly the possibility to evade taxes may restrict [expand] firm activity due to tax evasion whenever \( \delta \in (\bar{\delta}_T, \bar{\delta}_N) \) [\( (\bar{\delta}_N, \bar{\delta}_T) \)].

(iii) Case \( \frac{d\Pi_j(\alpha)}{d\alpha} > 0 > \frac{d^2\Pi_j(\alpha)}{d\alpha^2} \):

Suppose that the increase in expected profit \( \Pi_j(\alpha) \) increases at a decreasing rate with \( \alpha \) (and therefore legal after tax profits \( \pi_j \)). Referring to (7) and (8), this obtains if \( g_{\pi\pi}^2 - g_{SSg_{\pi\pi}} < 0 \) and requires \( g_{\pi\pi} \) to be sufficiently large and positive. For instance, for \( g_{\pi} < 0 \) this would imply that the benefit due to higher legal after tax profits regarding concealment costs is diminishing. From the discussion for the case in which the total expected profit function is strictly convex, it follows that tax evasion will in this case motivate collusion with or without effective cartel law enforcement because now \( \Pi_j(\alpha) \) is strictly concave.

**Proposition 4** Given \( \frac{d\Pi_j(\alpha)}{d\alpha} > 0 > \frac{d^2\Pi_j(\alpha)}{d\alpha^2} \) and a constant fine \( F \), tax evasion furthers collusion incentives (\( \bar{\delta}_T < \bar{\delta}_N \)) in the absence and in the presence of effective cartel enforcement (\( q \geq 0 \)).

**Proof.** The first part of the proof relating to the case without effective cartel law enforcement is dispensable since it runs analogously to the case for \( \Pi_j(\alpha) \) being strictly convex and again
uses Jensen’s inequality. In the case of effective cartel law enforcement, the effect from the strict concavity of $\Pi_j(\alpha)$ is only reinforced by the fact that there is no transformation applied to the expected fine.

In this case the possibility of tax evasion always softens the incentive compatibility constraint for cartelization. Thus, if firms use a discount factor from the interval $(\bar{\delta}_T, \bar{\delta}_N)$, the possibility of tax evasion results in reduced activity levels.

### 2.2.2 Fine Proportional to Profit Gain Due to Collusion

In reality sanctions levied on transgressors by antitrust authorities are often in relation to the profit gain resulting from being a cartel member.\footnote{See, e.g., the German competition law (Gesetz gegen Wettbewerbsbeschränkungen), §81.} Therefore, it is of interest to highlight the
implications for the interaction of tax evasion and collusion for this type of fine. Accordingly, we assume the fine to equal 
\[ F = \beta(\Pi_j(\alpha^K) - \Pi_j(\alpha^C)) \] in the case of [no] tax evasion, \( \beta > 0 \).

**Proposition 5** Assume the fine for collusion to be proportional to the profit gain due to collusion. Then incentives for collusion

(i) are unchanged irrespective of the presence of effective cartel law enforcement for \( \frac{d\Pi_j(\alpha)}{d\alpha} > 0 = \frac{d^2\Pi_j(\alpha)}{d\alpha^2} \),

(ii) are eroded irrespective of the presence of effective cartel law enforcement for \( \frac{d\Pi_j(\alpha)}{d\alpha}, \frac{d^2\Pi_j(\alpha)}{d\alpha^2} > 0 \),

(iii) are furthered irrespective of the presence of effective cartel law enforcement for \( \frac{d\Pi_j(\alpha)}{d\alpha} > 0 > \frac{d^2\Pi_j(\alpha)}{d\alpha^2} \).

**Proof.** (i) follows from equation (18) for the sanction specified above as the linear transformation is applied to all terms.

To prove (ii), note that we can rearrange equation (16) to

\[ \pi_j(\alpha^K) = \frac{1 - \delta_N(1-q)}{1 - \delta_Nq\beta} \pi_j(\alpha^D) + \frac{\delta_N(1-q-q\beta)}{1 - \delta_Nq\beta} \pi_j(\alpha^C) \] (25)

Applying \( \frac{d\pi_j(\alpha)}{d\alpha} = 1 \) and the transformation \( \Pi_j(\alpha) \), we get from Jensen’s inequality

\[ \Pi_j(\alpha^K) < \frac{1 - \delta_N(1-q)}{1 - \delta_Nq\beta} \Pi_j(\alpha^D) + \frac{\delta_N(1-q-q\beta)}{1 - \delta_Nq\beta} \Pi_j(\alpha^C) \] (26)

Therefore, \( \bar{\delta}_T > \bar{\delta}_N \) holds independent of the existence of effective cartel law enforcement.

To prove (iii) use the same reasoning as for (ii) with the exception that \( \Pi_j(\alpha) \) is now strictly concave and therefore \( \bar{\delta}_T < \bar{\delta}_N \) results.

The analysis in Sections 2.2.1 and 2.2.2 has established that depending on the curvature of the expected profit function as well as the effectiveness of cartel law enforcement, the possibility to evade taxes may increase, decrease or leave unchanged incentives to collude. From this follows: first, tax evasion and output choice may no longer be separable if firms can form a cartel, and, second, tax evasion may be an important determinant of the circumstances that facilitate collusion. The fact that the interdependence of respective decisions may have implications for enforcement is an issue to which we now turn.
2.3 Discussion: Interdependence of Enforcement

The analysis considers two different offenses: firms may break cartel and/or tax law. To enforce the law, the state maintains a detection probability and a sanction for both transgressions. The interest of the paper resides in the interaction of firms’ decision on tax evasion and collusion. From the fact that we have established an interdependence between respective decisions follows that the impact of enforcement will also be more manifold than is standard. Consequently, it is of interest how a change in enforcement directed at one kind of offense impacts the prevalence of the other kind of offense.

First, consider that the detection probability for collusion, \( q \), is increased. This policy change makes stable collusion less likely. Firms evade more taxes in the modus collusion than in the state competition if it holds that \( \frac{dS_j^c}{d\alpha} > 0 \), which is true for \( gS\pi < 0 \). Accordingly, a higher detection probability makes collusion less attractive an option and will decrease the total level of tax evasion via the lower likelihood of collusion (if \( gS\pi < 0 \)). Therefore, the deterrence of collusion and tax evasion can simultaneously be improved if the probability for detecting cartels is raised.

Second, consider an increase in the probability of a tax audit, \( p \). Given the modus operandi, this certainly will make firms declare more in Stage 2. The impact on collusion is however less clear. To gain more insight, we restate the incentive-compatibility constraint in the case of tax evasion (15) as

\[
\Delta = \frac{\Pi_j(\alpha^K) + \delta_T q \left[ \frac{\Pi_j(\alpha^C)}{1-\delta_T} - F \right]}{1 - \delta_T(1 - q)} - \Pi_j(\alpha^D) - \frac{\delta_T \Pi_j(\alpha^C)}{1 - \delta_T} = 0
\]  

(27)

to state that

\[
\frac{d\delta_T}{dp} = -\frac{\Delta_p}{\Delta_{\delta_T}}
\]

(28)

where \( \Delta_{\delta_T} > 0 \) as the advantage from collusion is increasing in the discount factor and

\[
\Delta_p = \frac{\theta_T}{1 - \delta_T(1 - q)} \left[ -S_j^K + (1 - \delta_T(1 - q))S_j^D + \delta_T(1 - q)S_j^C \right]
\]

(29)

Consequently, \( \frac{d\delta_T}{dp} < (>) 0 \) if \( \Delta_p > (<) 0 \) where the latter holds if the optimal statement deviation is a strictly convex (concave) function of \( \alpha \) (and therefore legal after tax profits).
This implies that a stricter tax enforcement may decrease the critical discount factor, i.e. may increase the likelihood of collusion. With regard to changes in the detection probability, we have already referred to the fact that $\frac{dS^*}{d\alpha} > 0$ if $g_{S^*} < 0$. Therefore, an increase in the audit probability will be successful with regard to curtailing tax evasion given a certain modus operandi, but may result in collusion becoming stable and thereby in the end may even call for more tax evasion due to the higher profit levels firms realize in the cartel outcome.

3 Conclusion

This paper analyzes the interaction of tax evasion and collusion. Tax evasion in most circumstances affects the payoffs from sticking to a cartel agreement, deviating and those in a competitive environment to different degrees. Therefore tax evasion impacts the range of discount factors compatible with stable collusion. Consequently, we add to the literature by highlighting an hitherto neglected factor that affects the likelihood of collusion in a given industry.

Moreover, the finding on the interdependence of the decisions on tax evasion and collusion proves that the activity choice of firms may no longer be independent from tax evasion if the possibility to form a cartel is taken into account. Changes in the critical discount factor imply that for some discount factors, collusion may occur under one of the two settings but not in the other. Firm and industry activity depend strongly on whether collusion occurs. Consequently, we establish a link between tax evasion and firm activity levels and thereby contribute to the literature which casts doubt on the separability of activity levels and tax evasion.

Finally, we highlight that isolated enforcement may have counter-intentional effects. Whereas stricter cartel law enforcement helps to tackle tax evasion, an increase in the audit probability may make it easier to stabilize collusion and, in the end, even produce more tax evasion if firms evade more in the collusive state. As a consequence, we point to possible detrimental effects of decentralized decision-making in the enforcement realm.
References


