Mismatch, Rematch, and Investment

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This Version: February 28, 2009

Abstract

In markets where payoffs depend on the assignment of agents, restrictions on side payments to partners (nontransferable utility) tend to preclude efficient matchings. This provides a rationale for "associational redistribution": a rematch of agents may raise aggregate social surplus. Often individuals' productive types are determined by investments before the match. Nontransferable utility typically distorts these investments and may induce overinvestment at the top and underinvestment at the bottom; this occurs despite symmetric information about agents' characteristics. Policies mitigating the static inefficiency due to mismatch may have undesirable dynamic incentive effects. Moreover, if investment itself takes place in a matching environment (e.g. schools), the effects can be exacerbated. We study several policies of associational redistribution that have empirical counterparts, assessing the differential effects of early-stage and later-stage interventions.

Keywords: Matching, nontransferable utility, investments, policy, education.

JEL: C78, I28, H52, J78.

1 Introduction

Some of the most important economic decisions we make - where to live, which profession to enter, whom to marry - depend for their consequences

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not only on our own characteristics or "types" (wealth, skill, or temperament), but also on those of the people with whom we live or work. Often characteristics are endogenous, however, in that they are determined by individual choices and investments. By itself, the endogeneity of types is not likely to lead to market outcomes worthy of policy intervention: when there is perfect information, perfectly transferable utility, and no widespread externalities, under mild assumptions, an efficient equilibrium exists (Cole et al., 2001, Felli and Roberts, 2002).

Concerns may arise, however, when there is matching market failure, for instance due to search frictions, widespread externalities, or statistical discrimination. Such market failure may generate inefficient levels of output and investment, or undesirable degrees of inequality. The latter in particular has been cited as a justification for policy intervention that directly interferes with the sorting outcome through rematching, that is *associational redistribution* (Durlauf, 1996a). Examples include affirmative action, school integration, or certain types of labor subsidies that target the less qualified. More broadly, it has been contended in public policy debate that the market *has* failed to sort desirably: there is too much segregation (by educational attainment or racial background); certain groups appear to be "excluded" from normal participation in economic life, in turn depressing their willingness to invest in human capital. If so, possible policy remedies are affirmative action or forced integration of schools. Coate and Loury (1993) provides one formalization of this argument and also points out that equilibria where under-investment is supported by "wrong" expectations may be eliminated by affirmative action policies.

This paper emphasizes another source of inefficient stable matching: non-transferable utility within matches. In many situations circumstances place bounds on compensations to or from people we interact with, for instance through the legal framework, because of capital market imperfections and moral hazard within firms, or out of "behavioral" considerations. It is known at least since Becker (1973), that under nontransferable utility the equilibrium match need not maximize aggregate social surplus (see also Legros and Newman, 2007).

When utility is less than fully transferable, matching models display three distinct but interacting distortions that we refer to as inefficiency *of* the match, *by* the match, and *for* the match. *By-the-match* inefficiency results when the Pareto frontier for matched agents does not coincide with an iso-surplus surface; matched partners need not maximize their own joint surplus, and aggregate performance is sensitive to the distribution of surplus within matches (see e.g. Legros and Newman, 2008). *Of-the-match* inefficiency refers to the kind of mismatch pointed out by Becker: efficient matching in the market may require considerable flexibility in the distribution of surplus within a match, and nontransferable utility makes this too costly. *For-the-match* inefficiency results from the first two: since surplus shares and levels are distorted in a laissez-faire match, so are incentives to invest before it happens.

Thus nontransferable utility provides a rationale for policy intervention, at least if one accepts an "ex-ante" Pareto optimality criterion, i.e., maximizing welfare from behind a veil of ignorance, before people know their types (as in Harsanyi, 1953, Holmström and Myerson, 1983). The mismatch that associational redistribution could help to correct here does not depend on asymmetry of information about types or a concomitant role of (self-fulfilling) beliefs about the productivity of individuals with observable attributes that may be correlated with type. And quite apart from whether such policy is desirable, it is of interest to predict its likely effects; for instance, variations in aggregate surplus in the model may correspond to variations in national income across countries following different policies.

In general, the interaction of all three distortions must be taken into account for assessing policy; here, we shall shut down inefficiency by the match in order to focus on the other two distortions by assuming strictly nontransferable utility, i.e. the Pareto frontier is a single point. This also allows us to focus on policies of pure associational redistribution, since transfers such as taxes and subsidies would be hard to implement in this environment.

The setup we employ to analyze various forms of associational redistribution is as follows. Agents have a binary type reflecting whether they are privileged in terms of access to education or not. They in invest education in schools of size two, which can be integrated (heterogenous) or segregated (homogeneous). When investing agents face a fixed cost that depends on the school composition. Education investment determines the probability of a high education outcome. In the labor market agents match into firms whose output depends on members' education. The production technology is such that diversity (heterogeneity) in firms is more productive, and would be the outcome under unrestricted side payments. We model nontransferable utility in the simplest possible way: output must be shared equally within firms.

As a result the labor market segregates in educational achievement. Thus, the laissez faire equilibrium outcome is inefficient from an aggregate surplus perspective, and there is likely to be *overinvestment at the top* and *underinvestment at the bottom*: the underprivileged find investing to be too costly and the privileged receive inefficiently high rewards in the labor market.

We then evaluate several associational redistribution policies that have empirical counterparts. When segregation of the labor market in education is inefficient, an immediate remedy is an *achievement based* policy that rematches agents based on educational attainment. For instance, "Workfare" and certain European labor market policies provide wage subsidies for employing long-term unemployed or low educated youth. But in a dynamic setting with investments, a trade-off emerges. Though output increases through rematching, investment incentives are depressed: the policy raises the returns to low education outcomes and lowers the return to high outcomes. The adverse incentive effect may be partially mitigated by a rematching policy that conditions not on results of choices but on exogenous information correlated with education outcomes, such as agents' socioeconomic status. Examples of such *background based* policy are race- or gender-based affirmative action.

When agents' types enter the production function directly, returns from education investment depend positively on the quality of the match an agent obtains on the labor market. Then a background based policy may serve to encourage underprivileged agents to invest and mitigate both underinvestment at the bottom and overinvestment at the top.

Matching may be pertinent at the investment stage as well as in the labor market, since investments are often taken not in solitude but in schools or neighborhoods where peer effects matter. Thus associational redistribution both at early and late stages might be justified, and optimal timing and coordination of such interventions becomes a concern. A policy of *school integration* reduces segregation of schools with respect to background. While this policy serves to extend access to education, it does not further interfere with a laissez-faire labor market allocation. Hence, school integration is beneficial if it is cost efficient at the schooling stage, and often dominates the labor market interventions.

One can also ask how policies used in the labor market *and* the investment stage interact, and ask whether school integration is a substitute or a complement to labor market rematching policy. In addition, we can consider a policy that is dynamic in the sense that the rematch in the labor market conditions on agents choices of investment environments. Such a *club based* policy is sometimes used in regulating university access by assigning quotas to high-schools or neighborhoods (for instance the Texas 10 percent law).

The literature on school and neighborhood choice (see among others Bénabou, 1993, 1996, Epple and Romano, 1998) typically finds too much segregation in types. This may be due to market power (see e.g. Board, 2008) or widespread externalities (see also Durlauf, 1996b, Fernández and Rogerson, 2001). When attributes are fixed, aggregate surplus may be raised by an adequate policy of bribing some individuals to migrate (see also de Bartolome, 1990). Fernández and Galí (1999) compare matching market allocations of school choice with those generated by tournaments: the latter may dominate in terms of aggregate surplus when capital market imperfections lead to nontransferabilities. They do not consider investments before the match.

Peters and Siow (2002) and Booth and Coles (2009) present models where agents invest in attributes before matching on a marriage market under strictly nontransferable utility. Investments are taken in solitude, so peer effects are absent. The former finds that allocations are constrained Pareto optimal (with the production technology they study, aggregate surplus is also maximized), and does not discuss policy. The latter compares different marriage institutions in terms of their impact on matching and investments. Gall et al. (2006) analyze the impact of timing of investment on allocative efficiency. Several recent studies consider investments before matching under asymmetric information (see e.g. Bidner, 2008, Hopkins, 2005, Hoppe et al., 2008), mainly focusing on wasteful signalling, while not considering associational redistribution.

Finally, the emphasis here is on characterizing stable matches and contrasting with ones imposed by policy. Thus we shall not be concerned here with the market outcome under search frictions (Shimer and Smith, 2000, Smith, 2006), nor on mechanisms that might be employed to achieve either stable matches or ones with desirable welfare properties (e.g. Roth and Sotomayor, 1990). Stable market outcomes in this paper may, of course, be attained by way of matching mechanisms.

The paper proceeds as follows. Section 2 lays out the labor market and discusses effectiveness of policies of associational redistribution in terms of sorting, incentives, and exclusion. Section 3 presents the schooling stage and a policy of school integration. Section 4 considers effectiveness of policies at both schooling stage and labor market and introduces club based policies. Section 5 concludes, and the appendix contains the more tedious calculations.

2 A Labor Market

A labor market is populated by a continuum of agents I with unit measure. Agents are characterized by their educational attainment a which is either high h or low ℓ. Denote the measure of h agents by $q \in [0,1]$. On the labor market agents match into firms of size two and jointly produce output. Profit y in a firm depends on agents' education outcomes. Assume that profits increase in attainment,

$$y(\ell,\ell) < y(\ell,h) = y(h,\ell) < y(h,h).$$

Profits in firms have the desirability of diversity (DD) property if

$$2y(h,\ell) > y(h,h) + y(\ell,\ell).$$

The DD property holds for instance when a is real-valued and y a concave function of the sum of the types. It could also be generated by a technology that combines two different tasks, one human capital intensive and one less so, say engineering and design versus actual manufacturing, with the firm free to assign the worker to the task (Kremer and Maskin, 1996). Another possible reason for decreasing differences is diversification between two tasks, or contractual frictions, for instance if cost of capital or information rents are decreasing in the scope of the project. Denote by $w(a,a')$ the wage of an agent with educational attainment a when matching with an agent whose educational attainment is a'. Wages are positive and sum up to the firm's profit, $w(a,a') + w(a',a) = y(a,a')$. Agents derive utility from wage income.

To provide a benchmark solve now for the competitive labor market equilibrium, that is a stable match of agents into firms. With DD there exist wages $w(h,\ell) \geq 0$ and $w(\ell,h) \geq 0$ with $w(h,\ell) + w(h,\ell) = y(h,\ell)$ such that

$$w(h,\ell) \geq y(h,h)/2 \text{ and } w(\ell,h) > w(\ell,\ell)/2.$$

This implies that given wages $w(.)$ there is no distribution of profits in segregated firms such that agents in integrated firms were better off forming a segregated firm. Hence, in labor market equilibrium measure $\min\{q, 1-q\}$ of integrated firms emerge, the remainder segregates. Market wages are determined by scarcity, that is $w(h,\ell) = y(h,h)/2$ if $q > 1/2$, $w(\ell,h) = y(\ell,\ell)/2$ if $q < 1/2$, and $w(h,\ell) \in [y(h,h)/2, y(h,\ell) - y(\ell,\ell)/2]$ if $q = 1/2$.

2.1 Nontransferable Utility

The example above tacitly assumed that utility was perfectly transferable on the labor market. This means agents can contract on the profit distribution within a firm without affecting productive efficiency, that is the size of the profits. For a number of plausible reasons this assumption may often be violated. Lack of access to or imperfections on the credit market, limited liability and moral hazard within the firm are one reason not to expect perfectly transferable utility. Others are incomplete contracts and renegotiation, risk aversion, legal constraints and regulation, or behavioral concerns.

To facilitate exposition we assume an extreme case of non-transferabilities, namely strictly nontransferable utility, so that only a single vector of payoffs to agents is feasible in any firm.[1] To keep notation bearable assume that profits are shared equally in firms, that is

$$y(h,h) = 2w_{hh}, y(h,\ell) = y(\ell,h) = 2w_{h\ell} = 2w_{\ell h}, y(\ell,\ell) = 2w_{\ell\ell} = 0.$$

Abbreviate $W = w_{hh}$ and $w = w_{h\ell}$ and assume that

$$W < 1.$$

Thus wages are typically bounded above by 1 which permits interpretation of investments induced by expected wages, introduced below, as probabilities.

[1]While this may be motivated for instance by Nash bargaining in renegotiations within the firm, all results in the paper are robust to allowing for some transferability by letting wages vary around equal sharing by some amount small enough.

When utility is nontransferable the equilibrium labor market allocation looks quite different. Despite $2w > W$, i.e. DD holds, integration is no longer possible in equilibrium. Suppose that a positive measure of (h, ℓ) firms form and h agents obtain wage w. Then any two h agents have a profitable deviation by starting a (h, h) firm earning W each, a contradiction to stability. Hence, under strictly nontransferable utility only homogeneous firms emerge.

As long as there are positive measures of high and low types (i.e. $0 < q < 1$), and diversity is desirable in production ($2w > W$), aggregate surplus is strictly lower when utility is nontransferable. If $q \leq 1/2$, surplus is $2qw$ if utility is transferable, while it is qW if not; if $q > 1/2$, surplus is $(1-q)2w + (2q-1)W$ if utility is transferable, and qW if not.

Nontransferability of utility may therefore distort the matching pattern and reduce aggregate surplus . Indeed this seems to provide a powerful justification for associational redistribution on the labor market. From a positive point of view, studying such interventions generates a predicted correspondence between national income and policy. Consider a policy of associational redistribution that assigns h agents to ℓ agents whenever possible. Any remaining agents match into homogeneous firms using uniform rationing. Call this an *achievement based* policy. This policy replicates the matching pattern under transferable utility and achieves an increase in aggregate surplus for any exogenously given distribution of educational attainments, as measured by q in our example. However, the non-favored group of type h is clearly worse off under the policy, which accounts for the sometimes violent opposition such policies have been met with in the past.

Active labor market policy often resembles achievement based policies, e.g. employment subsidies. By targeting the long term or young unemployed such policy effectively rematches the labor market conditional on educational achievements or rather lack thereof. In many industrialized countries some variety of wage subsidy or workfare program was used: the Targeted Jobs Tax Credit (TJTC) and later on the Work Opportunity Tax Credit (WOTC) in the US, as part of the Hartz policy reform in Germany, in the New Deal for Young People in the UK, and payroll tax subsidies for minimum wage labor contracts and wage subsidies for the unemployed young in France.

2.2 Education Investments

An often voiced critique of associational redistribution is that it arguably spoils incentives. Educational attainments, and agents' attributes on markets in general, often result from individual choices, which are affected by the anticipated rewards accruing to the various types. Therefore endogenizing types is a natural way to assess such critique. In this case q is not exogenous.

Suppose therefore that an exogenously given measure $\pi \in [0,1]$ of agents have the opportunity to invest in education. π is best understood as the fraction of the population with access to schooling. When investing, agents exert effort $e \in [0,1]$ to acquire education. They are risk neutral and effort e, which is never verifiable, comes at a utility cost $e^2/2$. Specifically, spending effort e yields a high education outcome h with probability e and a low education outcome ℓ with probability $(1-e)$. The measure of high achievers, i.e. agents with education h, q, is now given by $q = \pi e$.

The game proceeds as follows.

- Measure π of agents simultaneously choose e, the remaining agents are assigned $e = 0$.
- educational outcomes are realized and publicly observed,
- agents form a stable match (in case of laissez faire) or are matched in accordance with whichever policy is in effect.

Let $w(h)$ and $w(\ell)$ be the rationally anticipated wages of a high and a low achiever. Then, an investing agent solves

$$\max_{e} e[w(h) - w(\ell)] + w(\ell) - \frac{1}{2}e^2,$$

yielding $e = w(h) - w(\ell)$.

In the benchmark case, when utility is fully transferable, investment incentives depend on whether q is anticipated to be greater or lower than $1/2$. If $q > 1/2$, high achievers are in excess supply and have a chance of being matched with a low or a high achiever. As a pair of high achievers obtain $w(h) = W$ each, this is also what they get when matched with a low achiever,[2]

[2]Equal treatment on the labor market holds under transferability: if one high achiever gets strictly more than another one, the latter can match with the partner of the former at a wage that is slightly lower.

who gets the residual $w(\ell) = 2w - W$. Hence, investments are $e = 2(W - w)$. If $q < 1/2$, low achievers are in excess supply, obtain $w(\ell) = 0$, and high achiever match only with low achievers obtaining $w(h) = 2w$. Investments are $e = \min\{2w; 1\}$, strictly greater than in the case $q > 1/2$. In equilibrium the anticipated q coincides with its realization πe, for instance, if $q > 1/2$, $e^{TU} = 2(W - w)$ and therefore we need that $\pi 2(W - w) > 1/2$. We have the following result (all proofs missing in the text are in the appendix).

Lemma 1 *Let π be the measure of agents who are able to invest. Suppose that utility is fully transferable.*

(i) If $W - w > \frac{1}{4\pi}$, $e^{TU} = 2(W - w)$ and $q > 1/2$,

(ii) If $W - w < \frac{1}{4\pi} < \min\{w; 1/2\}$, $e^{TU} = \frac{1}{2\pi}$ and $q = 1/2$,

(iii) If $\min\{w; 1/2\} < \frac{1}{4\pi}$, $e^{TU} = \min\{2w; 1\}$ and $q < 1/2$.

When utility is strictly nontransferable, on the other hand, the labor market segregates as derived above. Therefore $w(h) = W$, $w(\ell) = 0$, and the laissez-faire investment is $e^{LF} = W$. By Lemma 1 the social return from education, and thus e^{TU}, decreases with q. Under nontransferable utility the private return from education is independent of q. Hence, given W, the difference in investment under nontransferable and transferable utility increases in q. Therefore a corollary follows from the lemma.

Corollary 1 *Comparing investment levels when utility is perfectly transferable (TU) and strictly nontransferable (LF) yields*

$$e^{LF} > e^{TU} \Leftrightarrow W > \frac{1}{2\pi}.$$

Thus, both *overinvestment* relative to the benchmark (if $\pi W > 1/2$), since $W > 2(W - w)$ and *underinvestment* (if $\pi W < 1/2$) are possible in this model. Those who have no access, of course, always underinvest relative to what they would do had they access.

2.3 Achievement Based Policy

Since mismatches due to nontransferable utility distort investment incentives, the case for associational redistribution seems even more compelling when the measure of high achievers is endogenous. This intuition is incomplete, however, because *given nontransferabilities on the labor market* enforcing the "correct" sorting may in fact worsen investment incentives.

Recall that under transferable utility the labor market wage adjusts as to provide the long market side with its autarky payoff (i.e. W for h and 0 for ℓ agents). For instance, if low achievers are in excess supply they obtain 0 and high achievers get $2w$. If utility is strictly nontransferable, however, low achievers have strictly positive payoff under an achievement based policy, since they get w with a positive probability due to uniform rationing. High achievers get w for sure. Hence, investment incentives are weaker than under laissez-faire as the return is lower in the high achievement state h and higher in the low achievement state ℓ. Indeed, it is not hard to show that in any equilibrium under forced integration low achievers must be in excess supply.

Proposition 1 *Under an achievement based policy the measure of educated agents is less than* $1/2$ *for any* $\pi \in [0,1]$. *Investment in education is*

$$e^A = \frac{1-2q}{1-q} w,$$

with $q = \frac{1}{2} - \left[\sqrt{\pi^2 w^2 + \frac{1}{4}} - \pi w\right]$. e^A *decreases and* q *increases in* π.

Clearly, $e^A < w < W = e^{LF}$ and forcing integration on the labor market worsens investment incentives. But since firms produce more output under integration than under segregation, rematching has also a positive effect on aggregate surplus. Whether an achievement based policy improves upon the laissez-faire allocation thus depends on whether the gain in output is large enough to offset investment distortions. Aggregate surplus under laissez-faire is $S^{LF} = \pi W^2/2$. An achievement based policy induces total surplus of

$$S^A = \pi e^A \left(2w - \frac{e^A}{2}\right).$$

Therefore the achievement based policy improves on laissez-faire when

$$\underbrace{e^{LF}(2w - W)}_{\text{output gain given LF effort}} > \underbrace{(e^{LF} - e^A)2w}_{\text{output loss given rematch}} \underbrace{- \frac{1}{2}((e^{LF})^2 - (e^A)^2)}_{\text{savings in costs}}. \quad (1)$$

The LHS captures the surplus added by integration under the achievement based policy keeping investment at its laissez-faire level. The RHS measures the effects on investment: a lower output given the rematch and a savings in cost due to lower incentives. Straightforward calculation shows there exists a unique value of W for which condition (1) holds with equality.

Corollary 2 *Total surplus under an achievement based policy is higher than under laissez-faire if and only if* $W \leq W_0(w, \pi)$.

The cutoff $0 \leq W_0(w, \pi) < 2w$ (as W approaches $2w$ diversity gains vanish, and the incentive losses outweigh them) increases in w and decreases in π (the larger is π, the more likely an ℓ will get w rather than 0, so investment incentives weaken from this insurance effect). Figure 1 depicts the cutoff $W_0(w, \pi)$ as a function of w that separates the areas LF and A when $\pi = 1$.

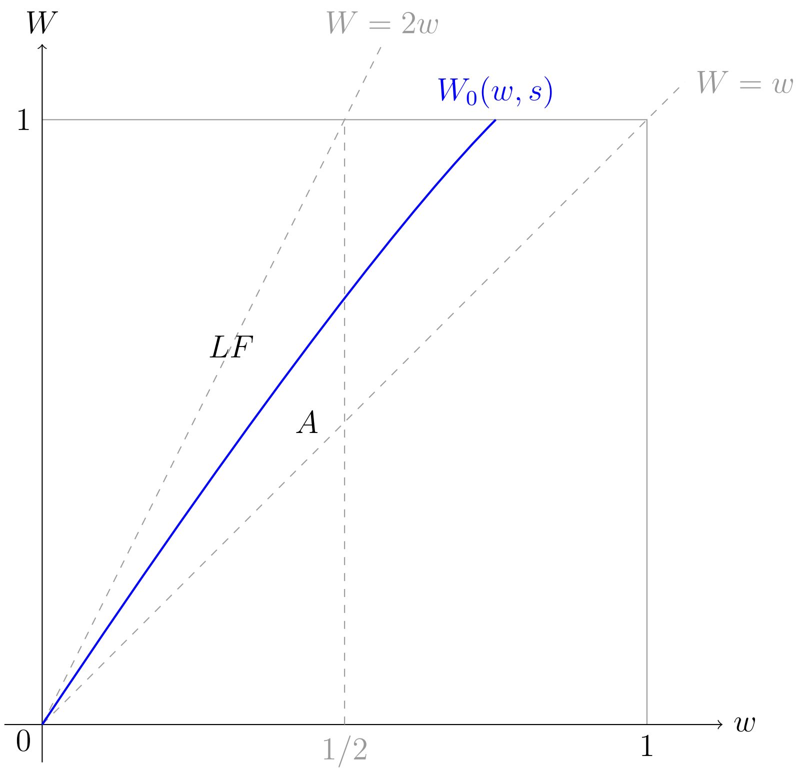


Figure 1: Laissez-faire versus Achievement Based Policies

2.4 Background Based Policy

Although an achievement based policy may increase aggregate surplus compared to laissez-faire, this is always accompanied by a downward distortion of investment incentives. Following Ramsey taxation logic a natural way to ameliorate investment distortions is therefore to condition rematching on a characteristic that is less elastically supplied than educational achievement but highly correlated with it, for instance access to education.

Suppose that whether or not an agent has access to education is observable. This may be the case when access depends on observable information such as socio-economic characteristics of agents' neighborhoods or parents, race, or gender. These characteristics can be thought of as determining whether agents are *privileged* in terms of access, or *underprivileged*. Hence, an agent i's *background*, or type, is $b_i \in \{U, P\}$. Consistent with the analysis above the measure of agents with background P is π, while the remainder $1 - \pi$ has background U. A *background based* policy integrates as many as possible U and P agents, but otherwise agents are free to choose matches.

Consider first the case $\pi \leq 1/2$. Each U agent is matched with a P agent with probability $\pi/(1-\pi)$ and with a U agent with probability $(1-2\pi)(1-\pi)$; agents of type P are matched with an agent of type U with certainty. Hence, there are measure π of (U, P) and $1/2 - \pi$ of (U, U) firms.

As under an achievement based policy a privileged agent with high achievement is matched into an integrated firm (h, ℓ) obtaining wage w for sure. A privileged low achiever, however, has now probability 0 of matching into a (h, ℓ) firm, since integration is in terms of background, and underprivileged agents become low achievers. This reduces expected payoff of a privileged low achiever compared to an achievement based policy. Hence, when $\pi < 1/2$ a background based policy induces a redistribution towards underprivileged agents (having a higher chance to obtain wage w) and stronger incentives for privileged agents. Since the measure of firms (h, ℓ) is $\pi e^B > \pi e^A$, total output is greater under a background based policy.

That is, a background based dominates an achievement based policy when $\pi \leq 1/2$. Hence, it will generate higher aggregate surplus than laissez-faire also in the neighborhood of the curve $W_0(w, \pi)$. Similar to above laissez-faire induces better incentives, but less efficient matches. One can show that there is a cutoff value $W_2(w, \pi) > W_0(w, \pi)$ such that laissez-faire yields higher total surplus than a background based policy if and only if $W > W_2(w, \pi)$. For $\pi \leq 1/2$, $W_2(w, \pi) = \sqrt{3}w$.

When $\pi > 1/2$, a privileged agent optimally invests

$$e^B = \underbrace{\frac{1-\pi}{\pi}w}_{\text{match with a } U \text{ agent}} + \underbrace{\frac{2\pi - 1}{\pi}W}_{\text{match with a high achiever } P \text{ agent}} . \tag{2}$$

As $e^A < w$ we have $e^B > e^A$, and a background based policy induces redistribution towards the underprivileged and stronger incentives for the privileged as in the case above.

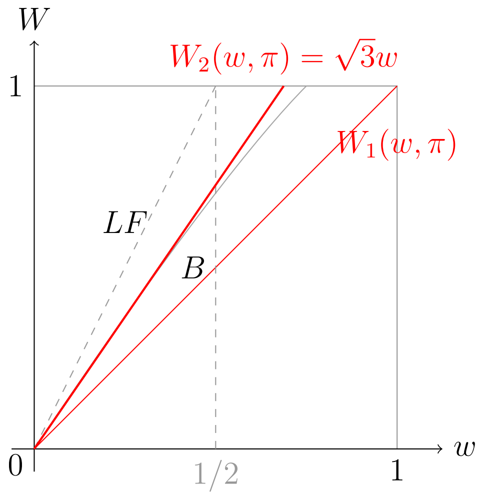
Yet now total surplus is not necessarily higher under a background based policy. Since $\pi > 1/2$ some P agents must form (P, P) matches. Since a background based policy does not condition on achievement, these agents segregate as under laissez-faire. Hence, there is a positive measure of (h, h) and (ℓ, ℓ) firms, which is inefficient from a surplus point of view. That is, for $\pi > 1/2$ a background based policy induces better incentives and a less efficient matching than an achievement based policy. An argument similar to the one in Corollary 2 yields a cutoff $W_1(w, \pi)$ such that a background based policy is preferable to one based on achievement if $W > W_1$, and the reverse is true if $W < W_1$. Since the amount of mismatch increases in π, so does $W_1(w, \pi)$, which also increases in w.

Comparing a background based policy to laissez-faire when $\pi > 1/2$ follows the same logic as in the previous section; a background based policy provides worse incentives but a more efficient matching. The cutoff $W_2(w, \pi)$ such that both policies are surplus equivalent also increases in w and in π.

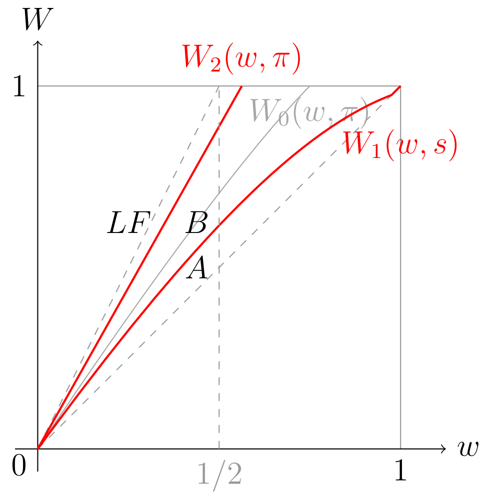
This discussion as well as some additional properties of the cutoff values are summarized in the following proposition, and illustrated in Figure 2.

Proposition 2 *There are functions $W_1(w, \pi)$, $W_2(w, \pi)$ with the properties*

- $2w \geq W_2(w, \pi) > W_0(w, \pi) > W_1(w, \pi) \geq w$,
- $W_1(w, \pi), W_2(w, \pi)$ *are increasing in w and in π,*
- $W_1(w, \pi) = w$ *for $\pi \leq 1/2$, and*
- $W_2(w, \pi) = \sqrt{3}w$ *for $\pi \leq 1/2$, $\lim_{\pi \to 1} W_2(w, \pi) = 2w$,*



(a) $\pi < 1/2$



(b) $\pi > 1/2$

Figure 2: Laissez-faire, Achievement, and Background Based Policies

such that the surplus maximizing policy is

(i) Laissez-faire when $W \geq W_2(w,\pi)$*,*

(ii) Background based when $W \in (W_2(w,\pi), W_1(w,\pi))$*, and*

(iii) Achievement based when $W < W_1(w,\pi)$*.*

2.5 Access to Education

In the preceding analysis the measure of agents who choose positive investment in education was exogenously given by π. Lack of access to education may be understood as a fixed cost that agents face when acquiring education or in production within a firm. Then individual returns to education on the labor market determine the extent of exclusion, since higher returns from investment may induce agents to invest in education. In this context downward distortion of investment incentives due to associational redistribution on the labor market may amplify exclusion from education.

2.5.1 Background in the Surplus Function

We start with the case that personal background affects agents' payoffs in the production stage. This is the case if, for instance, background measures an individual's set of useful business contacts. Suppose that now both P and U agents have access to education, but firm members' backgrounds b and b' affect production. Output in a match of agents (a, b) and (a', b') is

$$y(a, a') - 2g(b, b\prime).$$

Types in the labor market are now effectively two-dimensional. $g(.)$ denotes the cost incurred by lack of social capital, or business contacts. Assume that

$$0 = g(P, P) < g(U, P) = g(P, U) = f < g(U, U) = F.$$

Suppose that $F > w > f$, $F > 2f$, and $W - w > F - f$, i.e. the fixed cost is convex and education matters more for output than background (the case $W - w < F - f$ yields similar results).

When utility is perfectly transferable and $W - w > F - f$, all potential matches of (h, U) and (ℓ, P) agents form, and agents of the abundant type match with (h, P) agents. (ℓ, U) agents segregate. Equilibrium payoffs depend on relative scarcity.

Under laissez-faire (h, P) and (ℓ, U) agents segregate since agents' payoffs are monotone in their matches' types (a, b) on each dimension. (ℓ, P) and (h, U) agents segregate if $W-w > F-f$, which is the assumption. Education investments are therefore W for P agents and $W - F$ for U agents.

An achievement based policy exhausts all potential matches between h and ℓ agents. Given this constraint P agents segregate if possible. Similar to above P agents' invest at most w. U agents have strongest incentives when some (ℓ, P) agents remain, which yields at most investment $w - f < W - F$.

A background based policy integrates by background, leaving individuals free to segregate by attainment, using uniform rationing where applicable. Effort is $W - \min\{(1 - \pi)/\pi; 1\}f > e^P > W - F$ for P agents and $W - f > e^U > W - F$ for U agents. This is summarized in the following proposition (see the appendix for the details).

Proposition 3 *Let $F > w > f$, $F > 2f$ and $W-w > F-f$. Agents invest*

- $e^{LF} = W$ *if* $b = P$ *and* $e^{LF} = W - F$ *if* $b = U$ *under laissez-faire,*
- $e^A \leq w$ *if* $b = P$ *and* $e^A \leq w - f < W - F$ *if* $b = U$ *under an achievement based policy,*

- *$W > e^B > W - F$ if $b = P$ and $W > e^B > W - F$ if $b = U$ under a background based policy.*

This reflects the encouragement effect of affirmative action discussed by Coate and Loury (1993), inducing the underprivileged to invest more now that they expect a significant return because of the policy. Note though, that here, as elsewhere, the privileged agents' incentives are reduced. Indeed this is a general insight from our analysis: the group not favored by the policy has reduced incentives; the group favored by the policy may have improved incentives, as in this case, or not, as in the case of achievement based policy.

2.5.2 Solitary Investments and Fixed Cost

A second interpretation of access is in terms of fixed cost of education acquisition. Suppose, for instance, that privileged agents face no access cost to education, while underprivileged agents incur cost $F > 0$ when investing $e > 0$. In contrast to the case discussed above integrating types P and U in firms now does not affect the fixed cost incurred before the match. Let s denote the measure of agents who choose positive education investment.

Under laissez-faire individual payoff from strictly positive investment is $u^{LF} = W^2/2$ independently of s. Therefore, either $F > W^2/2$ and only the privileged invest, or $F \leq W^2/2$ and $s = 1$.

Under an achievement based policy individual payoff from a strictly positive investment depends on s through e^A and is $u^A(s) = (e^A(s))^2/2$.

Under a background based policy an agent's payoff is increasing in the investment of the agents with whom they are supposed to match. Suppose all agents invest. Supposing a symmetric equilibrium U agents solve

$$\max_e eW - \frac{e^2}{2} - F.$$

$e = W$ if $W^2/2 > F$ and 0 otherwise. P agents also optimally invest $e = W$ since they find a U agent with high achievement U with certainty if all agents invest. Hence, when $F \leq W^2/2$ there exists an equilibrium allocation under a background based policy such that $s = 1$ and $e^B(1) = W$ giving payoff $u^B(1) = (e^B(1))^2/2$. Note that there may exist other equilibria as well.[3]

[3]To see this let $s = \pi$. Denote a U agent's optimal strictly positive investment by e_u.

Using these results and the solutions for investment and market wages from above reveals that for any value of $s \geq \pi$

$$u^A(s) < u^{TU}(s) \text{ and } u^A(s) < u^{LF} \text{ and } u^B(1) = u^{LF}.$$

Since U agents invest only if $u^j(s) \geq F$, we have the following result.

Proposition 4 *Suppose that U agents face a fixed cost F when choosing $e > 0$. Then the measure of agents who choose $e > 0$ is greater under laissez-faire than under an achievement or background based policy.*

In case of transferable utility, if case (iii) of Lemma 1 applies at $s = 1$ then $u^{TU} > u^{LF}$, since $e^{LF} = W < e^{TU} = \min\{2w; 1\}$. Hence, for $F \in (W^2/2, (e^{TU})^2/2)$, access is $\pi < 1$ under laissez-faire while it is $s = 1$ under transferable utility. Suppose that we are now in case (i) of Lemma 1. Then, $u^{TU}(1) = 2w - W + 2(W - w)^2$ which is less than $W^2/2$ whenever $3W > 2w + 2$. Since $W < 1$ and $W < 2w$ this is not possible.

That is, participation in education under laissez-faire is never greater than under transferable utility. Proposition 4 states that if U agents are excluded from education under laissez-faire, associational redistribution cannot help to reduce exclusion, even when higher participation would be socially beneficial.

3 The Schooling Stage

This raises the question of whether a social planner may want to facilitate access to education by targeting the cost of access to education directly, and how such a policy interacts with labor market policies. One way of doing this may be through education expenditures and subsidies. But a potentially important alternative turns on the presence of peer effects in the investment environment: if the access cost depends on the matching of agents in school, associational redistribution at the school level may serve as a policy instrument to reduce the access cost.

Indeed most education investments are taken not in solitude but rather in a social environment where the behavior of an agents' peers influences

We show in the appendix that $e_u < W$ independently of π. Hence, for $e_u \leq F \leq W^2/2$ another equilibrium exists under a background based policy such that $s = \pi$ and $e = e^B$ if $b = P$ and $e = 0$ if $b = U$.

own behavior. This may be by way of social norms and role models, learning spill-overs in class, or pure cost externalities. For the purpose of modeling we focus on the last and assume that agents are heterogenous in cost of acquiring education, which depends on an agent's match at school. We focus on heterogeneity in cost of access to education rather than in marginal cost of acquiring education. Whereas marginal cost of education may reflect individual ability, access cost captures an agent's socioeconomic background. Let therefore $g(b, b')$ as defined above denote an agent's fixed cost for education investment. It depends on that agent's schooling environment, or *club* (b, b').

Let the labor market operate under laissez-faire in this section, implying that an agent in club (b, b') obtains payoff $\max\{W^2/2 - g(b, b'); 0\}$. This does not depend on s, the measure of agents with strictly positive investment, enabling to focus on allocation problems at the schooling level.

When utility is perfectly transferable, a U agent can compensate a P agent in a (U, P) schooling environment for the increase in access cost to f. Integrated (U, P) clubs are stable if the joint payoff exceeds the sum of segregation payoffs:

$$\max\{W^2 - 2f; 0\} \geq W^2/2.$$

That is, in the benchmark case of perfectly transferable utility there is measure $\min\{\pi; 1/2\}$ of (U, P) clubs, and $s = \min\{2\pi; 1\}$ if $4f < W^2$. If $4f > W^2$ schools segregate, i.e. the measure of (U, P) clubs is 0, and $s = \pi$.

Under nontransferable utility P agents strictly prefer a (P, P) schooling environment to a (U, P) environment as $f > 0$. Hence, an allocation with integrated (U, P) clubs cannot be stable, since any two P agents matched into (U, P) clubs have a profitable joint deviation. Thus schools segregate under laissez-faire and surplus is $S^{LF} = \pi W^2/2$ as above. That is, under nontransferable utility there is too much segregation when $4f < W^2$.

3.1 School Integration

Similar to the case in Section 2.1 the laissez-faire allocation may fail to internalize positive externalities within schooling environment when utility is (sufficiently) nontransferable. This suggests that associational redistribution at the school stage may be beneficial, in particular if it can condition on information on backgrounds. For instance, a policy of school integration that forces agents to invest in integrated (U, P) environments should raise aggregate surplus if bringing in U agents is cost efficient. A *school integration* policy matches U to P agents whenever possible, using uniform rationing to assign the remaining agents to homogeneous school environments. A prime example of this as practiced in the US is "busing." More contemporaneously, policies determining the diversity of schools in terms of pupils' backgrounds vary substantially across countries. One indicator of this is the age at which pupils are first sorted into a particular ability stratum, a policy called tracking. This age ranges from 10 in Austria and Germany to 16 and above in the US or most of Scandinavia (see Table 5.20, OECD, 2004).

Under school integration, if $\pi \leq 1/2$ the measure of (U, P) clubs is 2π, and the one of (U, U) clubs is $(1-\pi)/2$; otherwise there are measure $1-\pi$ (U, P) clubs and $\pi - 1/2$ (P, P) clubs. Supposing a strictly positive measure of agents invests,[4] laissez faire labor market wages imply investments $e^I = W$ in (P, P) clubs, $e^I = 0$ in (U, U) clubs, and $e^I = W$ if $W^2 > 2f$ and otherwise 0 in (U, P) clubs. Therefore $s^I = \min\{1; 2\pi\}$ if $W^2 > 2f$ and otherwise $s^I = \max\{0; 2\pi - 1\}$. Aggregate surplus under school integration is

$$S^I = sW^2/2 - (s - \max\{2\pi - 1; 0\})f.$$

Therefore

$$S^{LF} > S^I \Leftrightarrow W^2 < 4f.$$

This does not depend on the DD property ($W < 2w$). Hence, school integration may restore the benchmark allocation at the schooling stage under fully transferable utility when $4f < W^2$.

To give a specific example, Meghir and Palme (2005) study a schooling reform in Sweden that was implemented around 1950. The reform increased compulsory schooling by three years, abolished tracking after grade 6, and imposed a nationally unified curriculum. That is, the policy aimed at decreasing school segregation in backgrounds. The change in policy increased education acquisition (beyond the new compulsory level for highly able pupils) and labor income for individuals whose fathers had low education, while it did not significantly change education acquisition and lowered wage income for individuals whose father had high education.[5]

[4] If $\pi < 1/2$ and $w^2 < 2f$ there is another equilibrium where nobody invests. It is not robust to school integration that allows measure $\pi > \epsilon > 0$ of P agents to segregate.

[5] Segregation at school may not only apply to sorting of students. Teachers may share

4 Combining Early and Later Stage Policies

A question potentially important for policy concerns is whether effectiveness of school integration depends on the labor market policy in place. Hence, we are interested in whether associational redistribution on the labor market and at the school stage may act as complements or substitutes, that is whether they reinforce or cancel each other. Two major concerns may arise when evaluating the impact of simultaneous earlier and later stage policies. On the one hand, school integration raises the access cost of P agents, which may lead to discouragement due the investment distortion under an achievement based policy. On the other hand, integrating schools weakens the link between background and educational outcome, thus reducing the effectiveness of a background based policy.

In the following we limit our attention to cases that satisfy some parametrical assumptions on access cost.

Assumption 1 (Access Cost) *Let* $f < (W-w)^2 < W^2/2 < F < 2w^2$.

This assumption ensures that f agents find it profitable to invest when matching into integrated firms, and that high cost F agents find it optimal to invest when paid the full social benefit of turning a (ℓ, ℓ) firm into a (h, ℓ) firm.

4.1 Fully Transferable Utility Benchmark

We derive the fully transferable utility benchmark allocation for the full model. As shown above h agents get wage $w(h) \in [W, 2w]$ depending on the scarcity of h versus ℓ agents. Investment is $e^{TU} = 2(w(h) - w)$ giving payoff $2(w(h)-w)^2 + 2w - w(h) - g(b, b')$. When utility is transferable, U agents may compensate P agents for the cost externality. Integrated (U, P) clubs are stable if the joint payoff exceeds the sum of segregation payoffs:

$$\begin{aligned} (w(h)-w)^2 &> f \text{ if } F > 2(w(h)-w)^2 + 2w - w(h) \text{ and} \\ F &> 2f \text{ if } F < 2(w(h)-w)^2 + 2w - w(h). \end{aligned}$$

a preference for safe schools and motivated students, possibly to an extent that cannot be compensated by public salaries (see Hanushek et al., 2004).

Hence, Assumption 1 implies integration both on the labor market and at the schooling stage when utility is perfectly transferable. For investments two interesting cases arise as stated in the following proposition.

Proposition 5 *Suppose Assumption 1 holds. When utility is fully transferable both schools and the labor market integrate. Moreover,*

(i) if $\min\{2\pi; 1\}2(W - w) > 1/2$, $s = \min\{2\pi; 1\}$, *investments are* $e = 2(W - w)$ *and* $q > 1/2$,

(ii) otherwise $q = \min\{2w; 1/2\}$ *and*

- *if* $2w > 1/2$ *and* $\pi \geq 1/2$, $s = 1$ *and investments are* $e = 1/2$,
- *if* $2w > 1/2$ *and* $\pi < 1/2$, $s = 2\pi + \max\{0; 1/2 - 2\pi\sqrt{2F}\}$ *and investments are* $e = \min\{\frac{1}{4\pi}; \sqrt{2F}\}$.
- *if* $2w \leq 1/2$, $s = 1$ *and investments are* $e = 2w$.

Proof: In Appendix.

In case (i) social returns to education are high enough for $q > 1/2$ when all agents in (U, P) and (P, P) clubs, while (U, U) agents do not. In case (ii) social returns are high enough to induce all agents, even those in (U, U) clubs, to invest when $q < 1/2$, but not when $q > 1/2$. Hence, $q = 2w$ if all agents invest but $w < 1/4$. Otherwise $q = 1/2$ and the market price adjusts to make (U, U) agents indifferent between investing or not.

That is, if the number of privileged agents and the value added in (h, h) firms, $W - w$, are sufficiently great, the underprivileged underinvest and the privileged overinvest in education under laissez-faire compared to the benchmark allocation. Otherwise all agents underinvest. This means the effects of nontransferable utility vary with the characteristics of the economy. Abundance of low access cost agents and a technology that values skilled labor input best describes an industrialized country, whereas the reverse seems true in developing economies. Maintaining this interpretation, our results indicate that non-transferable utility exacerbates inequality of opportunity in industrialized countries by discouraging underprivileged agents, while the discouragement effect is universal for developing economies.

4.2 Achievement Based Policy and School Integration

Suppose now that an achievement based policy is used on the labor market in conjunction with integration at school. As above an agent optimally chooses

$$e^{AI} = \frac{1-2q}{1-q}w. \tag{3}$$

if $q \leq 1/2$, where q is the measure of h agents. e^{AI} depends on s^{AI} via q, and indeed $e^{AI}(s) = e^{A}(s)$, but $s^{A} = \pi$ whereas s^{AI} is endogenous. By Proposition 1 $q < 1/2$. Aggregate surplus under a combination of achievement based and school integration policies is

$$S^{AI} = s^{AI} e^{A}(s^{AI}) \left(2w - \frac{e^{A}(s^{AI})}{2}\right).$$

An agent in a (U, P) schooling environment invests if

$$\left(\frac{1-2q}{1-q}\right)^2 \frac{w^2}{2} > f.$$

Investments of agents in (U, P) clubs depend on q and determine s^{AI}, which in turn affects q. If f is small enough, agents in (U, P) clubs invest, otherwise they do not, with adverse consequences for total surplus.

Proposition 6 *Under school integration and an achievement based policy*

(i) in case $\sqrt{2f} < e^{A}(\min\{2\pi; 0\})$*:* $e^{AI} < e^{A}$, $q^{AI} > q^{A}$, $s^{AI} = \min\{2\pi; 1\} > s^{A}$, *and* $S^{AI} > S^{A}$,

(ii) in case $\sqrt{2f} > e^{A}(\max\{2\pi - 1; 0\})$*:* $e^{AI} > e^{A}$, $q^{AI} < q^{A}$, $s^{AI} = \max\{2\pi - 1; 0\} < s^{A}$, *and* $S^{AI} < S^{A}$.

Proof: In Appendix.

There may arise the case, e.g. when $\pi \leq 1/2$ and $w^2/2 < f < (W - w)^2$, that school integration induces zero investments, given an achievement based policy on the labor market, since incentives to invest are depressed.

4.3 Background Based Policy and School Integration

Turn now to a combination of a background based policy on the labor market and integration at school. Then measure $2\min\{\pi; 1-\pi\}$ of agents are matched into (U,P) clubs with access cost f. The labor market policy exhausts all possible matches between U and P agents, using uniform rationing when necessary, but given this constraint agents segregate. Supposing a symmetric equilibrium (see appendix for the full argument) the measure of h agents with background U equals the measure of h agents with background P required to match with U agents. Therefore U agents in in (U,P) schooling environments and all P agents solve

$$\max_e eW - \frac{e^2}{2} - g(b,b').$$

That is, $e^{BI} = W$ and the labor market payoff is $W^2/2 - f > 0$. This generates the symmetric outcome assumed above, and the following proposition.

Proposition 7 *Let a school integration policy be in place. Then, in a symmetric equilibrium, a background based policy yields the laissez faire allocation on the labor market,*

$$e^{BI} = W = e^{LF} = e^I, s^{BI} = \min\{2\pi; 1\} = s^I, S^{BI} = S^I.$$

Proof: In Appendix.

Hence, a background based labor market policy induces the same allocation as a laissez-faire labor market when schools integrate. This is since background carries less information about achievement as schools become more integrated. Some empirical evidence links the degree of tracking to dependence of students' educational attainments on parental background.[6]

4.4 Club Based Policies

A particular intriguing policy of associational redistribution on the labor market that conditions on the school environment or club (b,b'), which we call a *club based* policy. For instance, this could be the socio-economic characteristics of neighborhoods individuals live in, or the performance rank of the school attended. This may be of particular relevance when it is infeasible to learn agents' cost types, for instance due to legal or informational

[6]See e.g. Schütz et al. (2008), Brunello and Checchi (2007) or Ammermüller (2005) who find that dependence of students' outcomes on their socioeconomic backgrounds depends positively on earlier start of tracking, and number of tracks or private schools.

constraints. A notable example is the Texas Top 10 Percent law used to admit students into the University of Texas. Others include measures of placing students from disadvantaged neighborhoods or schools in firms, e.g. school-to-work-policies like the School-to-work Opportunities Act in the US.

A club based policy in our model could work as follows:

- Agents are free to choose schools,
- alumni of (P, P) schools must match with alumni of (U, U) schools whenever possible, using uniform rationing when necessary,
- alumni of (U, P) schools are unrestricted by the planner in choosing a partner on the labor market.

Denote the measure of (U, U) schools by s_u, and the one of (P, P) schools by s_p. Then the measure of agents in (U, P) clubs is $1 - s_u - s_p$. Suppose first that all environments are segregated, $s_u = 1 - \pi$ and $s_p = \pi$. Then a P agent matches to a U agent with probability $(1 - \pi)/\pi$ if $\pi > 1/2$ and with certainty if $\pi \leq 1/2$. Since $F > W^2/2$ agents in (U, U) environments do not invest and P agents solve

$$\max_e ew - \frac{e^2}{2} \text{ if } \pi \leq 1/2 \text{ and}$$
$$\max_e e\left(\frac{s_u}{s_p}w + \left(1 - \frac{s_u}{s_p}\right)W\right) - \frac{e^2}{2} \text{ if } \pi > 1/2.$$

Interior solutions satisfy $e^C = W - (s_u/s_p)(W - w)$ if $\pi > 1/2$, and $e^C = w$ otherwise. Hence, $e^C = e^B$ if $s_p = \pi$ and $s_u = 1 - \pi$, and a club based policy coincides with a background based policy when schools segregate.

Let us focus on the case $\pi > 1/2$.[7] To verify whether school segregation is an equilibrium, suppose a pair of agents match into a (U, P) school. On the labor market these agents may segregate in education outcome (the measure of h agents is positive as $\pi > 1/2$). Hence, their investments solve

$$\max_e eW - \frac{W^2}{2} - f.$$

[7]Otherwise some integration remains an equilibrium outcome, although multiplicity of equilibria becomes an issue, see the appendix.

$e = W$ generates payoff $W^2/2 - f > 0$, so (U,P) agents invest. Agents segregate into schools if payoffs are higher in segregated than in integrated schools for P or U agents, that is if

$$(e^{CB})^2 > W^2 - 2f \text{ or } 2e^{CB}w > W^2 - 2f,$$

since (U,U) have expected payoff $e^C w$ due to uniform rationing. The first condition implies the second as $2w > e^C$. Therefore schools segregate and background and club based policies coincide if and only if

$$2f > \frac{1-\pi}{\pi} 2w(W-w) - W(2w-W). \tag{4}$$

If this is not the case, then agents in (P,P) clubs obtain higher payoff in (U,P) clubs when $s_p = \pi/2$. Therefore $s_u + s_p < 1$ if (4) does not hold. Let $s_u = 0$ and suppose $s_p \geq 1/2$ for the moment, then a (U,P) agent is matched to a (P,P) agent with certainty, and, assuming (P,P) agents invest, solves

$$\max_e eW - c(e,U,P).$$

As $W^2 > 2f$, (U,P) and (P,P) agents invest. When deviating to segregated schools U agents obtain ew, P agents obtain $W^2/2$. Since $W^2/2 > W^2/2 - f$ all P agents in (U,P) clubs can profitably deviate to a (P,P) school. An analogous argument applies to the case $s_p < 1/2$. Hence, $s_u > 0$ and $s_p > 0$.

(U,P) agents segregate on the labor market and invest $e = W$. Incentive compatibility for school sorting binds for P agents, so that s_p and investment of (P,P) agents makes them indifferent between (U,P) and (P,P) schools, see Appendix for details. This implies the following proposition.

Proposition 8 *If $2f < \frac{1-\pi}{\pi} 2w(W-w) - W(2w-W)$, a club based policy*
(i) induces integration at school, $s_p + s_u < 1$ and $s_u > 0$, and on the labor market, as the measures of (ℓ,ℓ), (ℓ,h), and (h,h) firms are all positive,
(ii) generates investments $e = 0$ in (U,U), $e = W$ in (U,P) and $e = (W^2 - 2f)/(2w) > e^B$ in (P,P) clubs.
Otherwise club based and background based policies coincide.

The next proposition evaluates welfare under a club based policy.

Proposition 9 *Let $W^2 > 6f$. There exists $\pi^* > 1/2$ such that for all $1/2 \leq \pi < \pi^*$ a club based policy dominates all other labor market policies (achievement based, background based, and laissez faire) if and only if*

$$W^2 - 2f > 2w \left(W - \frac{1-\pi}{\pi}(W - w) \right).$$

Proof: In Appendix.

That is, if access cost in integrated schools is small enough and the measure of privileged close to 1/2, a club based policy dominates other labor market policies whenever it induces desegregation at school.[8] It successfully trades off investment incentive and output effects from rematching. Negative incentive effects of integration on the labor market are curbed by conditioning on club membership rather than on achievement. Negative effects of school integration due to reducing quality of screening are counteracted by incentive compatibility of the sorting equilibrium, which requires positive measures of (U, U), (U, P), and (P, P) clubs. Finally, a club based policy reaps at least part of the benefits from rematching both at school and on the labor market, since the all firm and club constellations have positive measure.

Proposition 9 also applies when firm profits do not satisfy DD, i.e. $2w < W$. Then segregation on the labor market maximizes output all else equal.[9] Even in this case, when fixed costs in integrated schools are low enough to admit school integration under a club based policy, this policy dominates the laissez-faire allocation.

An illuminating example of club based policies is admission of high school graduates to public universities in Texas. In late 1996, Texas state universities abolished affirmative action based on race in response to the Fifth Circuit Court decision in *Hopwood vs. Texas.* In 1997 the Texas Top 10 Percent law was instituted with the stated aim to preserve minority attendance rates. This scheme guarantees automatic admission to Texas state universities for students who graduate among the best ten percent of their class. Since Texan high schools were highly segregated this was expected to counteract any adverse effect of abolishing affirmative action to campus

[8] Indeed it dominates a background based policy unconditionally.

[9] Also under fully transferable utility the labor market segregates in education, and wages coincide with those under strictly nontransferable utility. Hence, this result requires (sufficiently) nontransferable utility at the schooling stage, but not on the labor market.

diversity, tacitly assuming that composition was not affected by the policy change. Kain et al. (2005) report that

> Hopwood had a devastating effect on minority enrollment in Texas selective public universities, reducing the African-American and Hispanic share of entering classes by 37 percent and 21 percent between 1996 and 1998.

That is, after about two decades of affirmative action in Texas its removal triggered a sudden reversal to segregation. This may indicate that affirmative action policies were ineffective in changing beliefs, or that segregation in higher education was not entirely belief-based. Kain et al. (2005) further conclude that the Texas Top 10 percent law was not effective in preserving campus diversity since the top slots were disproportionally taken by non-minority students. Long (2004) confirms both observations in a broader study covering US-wide substitution of affirmative action by high school quotas.

Parents appear to have reacted to incentives, as Cullen et al. (2006) report some evidence of strategic sorting by good students into worse peer-groups in Texas. This appears to be consistent with our model where a club based policy may induce a rematching of schools. If diversity at school is desirable from a social planner's point of view, the Texas Top 10 Percent Law seems a fine example of unintended, yet beneficial consequences.

5 Conclusion

We presented a framework to analyze policies of associational redistribution on the labor market and at school. The framework imposes strictly nontransferable utility serving to focus on the interaction of matching patterns and investment incentives. It remains silent, however, about another source of inefficiencies when utility is transferable, but not perfectly so. Then competition may require inefficient sharing of surplus (see e.g. Legros and Newman, 2008) which in turn affects investment incentives. Pursuing this topic appears to be an important task for future research.

In the present approach policies aim at replicating the fully transferable utility matching outcome, that is integration, as a benchmark. In a more complex derivation of nontransferable utility, nontransferabilities may affect the optimal matching, however. See Gall et al. (2008) for an example when information rents decrease in the scope of the project, so that the optimal matching involves integration when there is asymmetric information generating nontransferabilities, but segregation under perfect information.

Labor market policies need to trade off output efficient sorting and provision of adequate incentives for pre-match investments. Conditioning labor market re-matching on observable information not subject to individual choice, such as background, appears beneficial when it is linked to education outcome. Early stage intervention, i.e. at school, does not distort incentives and provides benefits when integrating schools is cost efficient. In that sense early stage policies are more effective than later stage policies.

Earlier and later stage policy are interdependent, however. School integration may limit the informational content provided by individual background and reduce the effectiveness of screening, rendering background based policies obsolete. When an achievement based policy is used on the labor market school integration may discourage investment due to low returns to education. Moreover, optimal policies may depend on characteristics of the economy. For instance, if privileged agents are scarce, a background based policy dominates an achievement based policy. This does not hold for economies where the privileged abound, suggesting that the use of achievement based policies should be restricted to developed economies.

Finally, we identify a labor market policy that looks promising in terms of trading off incentive provision and efficient sorting: a club based policy rematches the labor market conditioning on individual school choices. It yields some integration both on the labor market and at school while inducing higher investments than other policies. This result is particular encouraging since there is no reason to expect this policy to be optimal. The nature of optimal mechanisms of associational redistribution in sequential assignment markets when utility is nontransferable remains an open question.

A Mathematical Appendix

A.1 Proof of Lemma 1

When maximizing expected utility $u = ew(h) + (1-e)w(\ell) - \frac{e^2}{2}$, a necessary condition for optimal investment is $e^{TU} = 2(w(h) - w)$.

We have established in the text that if $q > 1/2$, agents with education h are abundant and obtain wage $w(h) = W$, agents with ℓ obtain $w(\ell) = 2w - W$. Hence, $e^{TU} = 2(W - w)$ and the realized $q = \pi 2(W - w)$, which is greater than $1/2$ only if $W - w > 1/(4\pi)$.

If $q < 1/2$, h agents are scarce, so that $w(h) = 2w$ and $w(\ell) = 0$. As $e^{TU} = \min\{2w; 1\}$, $q = \pi e^{TU} < 1/2$ only if $\min\{2w; 1\} < 1/(2\pi)$.

Finally, if $q = 1/2$ a continuum of wages is consistent with a stable allocation: $w(h) \in [W, 2w]$ and $w(\ell) = 2w - w(h)$. Agents choose $e^{TU} = 2(w(h) - w)$. $q = \pi 2(w(h) - w)$ is equal to $1/2$ only if $w(h) = w + 1((4\pi)$. Therefore $e^{TU} = 1/(2\pi)$ in this case.

Suppose first $q = \pi 2(w(h) - w) > 1/2$. This is only consistent with $\pi 2(W - w) > 1/2$. $q < 1/2$ is only consistent with $\pi \min\{2w; 1\} < 1/2$. For intermediate cases, that is $\pi(W - w) < 1/4 < \pi \min\{w; 1/2\}$, $q = 1/2$. Therefore $e = 2w(h) - 2w = 1/(2\pi)$, that is $w(h) = w + \pi/4$.

A.2 Proof of Corollary 1

When $W - w > 1/(4\pi)$, the property DD ($2w > W$) implies that $W > 1/(2\pi)$. In this case, $e^{LF} - e^{TU} = 2w - W > 0$.

When $\min\{w; 1/1\} < 1/(4\pi)$, $W < 1/(2\pi)$ since $2w > W$ and $W < 1$. In this case, $e^{LF} - e^{TU} = W - 2w < 0$.

In the intermediate case $W - w < 1/(4\pi) < \min\{w; 1/2\}$, $e^{TU} = 1/(2\pi)$ and therefore $e^{LF} - e^{TU}$ is positive only if $W > 1/(2\pi)$.

A.3 Proof of Proposition 1

Given an achievement based policy, which assigns h agents to ℓ agents whenever possible, an agent chooses effort e to solve

$$\begin{aligned} &\max_e \quad e\left(\frac{1-q}{q}w + \frac{2q-1}{q}W\right) + (1-e)w - \frac{e^2}{2} \text{ if } q > 1/2, \\ &\max_e \quad ew + (1-e)\frac{q}{1-q}w - \frac{e^2}{2} \text{ if } q \leq 1/2. \end{aligned} \tag{5}$$

Supposing $q > 1/2$, a necessary condition for investment is

$$e = \frac{2q-1}{q}(W-w).$$

In equilibrium $\pi e = q$ must hold. Since e above increases in q and πe increases in π, it is sufficient to verify that $q > 1/2$ can occur when $\pi = 1$. $e = q$ implies

$$q^2 - 2(W-w)q + (W-w) = 0$$

but the discriminant is $(W-w)^2 - (W-w) = (W-w)(W-w-1) < 0$ since $W < 1$. Therefore in any equilibrium $q \leq 1/2$ and

$$e^A = \frac{1-2q}{1-q}w < w. \tag{6}$$

Replacing e^A by q/π and solving for q yields the expression in the proposition (the other solution is greater than 1). Clearly, the solution is less than 1/2. Since $e^A < w$ both $e^A < e^{LF}$ and $e^A < e^{TU}$. Under an achievement based policy investments satisfy (6). With $q = \pi e^A$

$$e^A = w + \frac{1}{2\pi} - \sqrt{w^2 + \frac{1}{4\pi^2}}.$$

Simple calculations show that the derivative of e^A with respect to π is negative, and the derivative of $q = \pi e^A$ with respect to π is positive.

A.4 Proof of Corollary 2

The condition $S^A > S^{LF}$ holds, if e^A solves the quadratic equation ${e^A}^2 - 4we^A + W^2 < 0$. Solving yields

$$e^A > 2w - \sqrt{4w^2 - W^2}. \tag{7}$$

Since $e^A < w$, $W^2 \geq 3w^2$ implies $S^A < S^{LF}$, that is $W_0 < \sqrt{3}w$. Finally, using (1), $W_0(w,\pi)$ solves

$$W(2w - W) = (W - e^A)2w - \frac{1}{2}(W^2 - {e^A}^2) \tag{8}$$

By Proposition 1, differentiating q with respect to w, and using $q = \pi e^A$, e^A is an increasing function of w. Therefore the RHS of (8) decreases in w, and increases in W since $W < 1$. The LHS increases in w and decreases in W since $w < W$. Hence as w increases, W must increase to restore equality. Hence, $W_0(w,\pi)$ increases in w. It decreases in π, since e^A decreases in π, hence the RHS of (8) increases in π, and W must decrease to restore equality.

A.5 Proof of Proposition 2

As a background based policy does not constrain matching in educational achievements, the labor market segregates in achievements *given* the policy constraints on backgrounds. Hence, P agents solve

$$\begin{cases} \max_e ew - \frac{e^2}{2} & \text{if } \pi \leq 1/2 \\ \max_e e\left(\frac{1-\pi}{\pi}w + \frac{2\pi-1}{\pi}W\right) - \frac{e^2}{2} & \text{if } \pi > 1/2. \end{cases}$$

Interior solutions satisfy $e^B = w$ if $\pi \leq 1/2$, and $e^B = W - (1-\pi)(W-w)/\pi$ if $\pi > 1/2$. That is, $e^{LF} > e^B \geq w > e^A$ for $\pi \in (0,1)$.

A.5.1 Derivation of the Cutoff $W_1(w,\pi)$

For $\pi < 1/2$ a background based dominates an achievement based policy. While both policies induce exactly the same matching – each P is matched with a U and there is the same measure of (h,ℓ) firms for a given e – since $e^B > e^A$, there are more integrated firms and, as $2w > W$, surplus is higher. Therefore $W_1(w,\pi) = w$ as claimed.

If $\pi > 1/2$ screening by background loses its effectiveness as a measure $2\pi - 1$ of P agents segregate in education outcome, unlike under an achievement based policy. Total surplus under a background based policy is

$$S^B = e^B\left((2\pi - 1)W + (1-\pi)2w - \pi\frac{e^B}{2}\right). \tag{9}$$

$S^B > S^A$ if and only if

$$\pi(e^B - e^A)\left(2w - \frac{1}{2}(e^B + e^A)\right) > (2\pi - 1)e^B(2w - W). \tag{10}$$

The LHS captures the gain through better incentive provision by a background based policy, while the RHS gives the benefit from rematching by an achievement based policy. Since (10) strictly relaxes as W increases,

$$S^B > S^A \Leftrightarrow W > W_1(w, \pi).$$

Straightforward calculation shows that $W_1(w, \pi)$ increases in π for $\pi \in [1/2, 1]$. $W_1(w, 1) = W_0(w, 1)$, as for $\pi = 1$ a background based policy implies the laissez-faire outcome. Therefore $W_0(w, \pi) > W_1(w, \pi)$ for $\pi < 1$ and the difference decreases in π.

Finally, we show that $W \geq \sqrt{3}w$ implies $S^B > S^A$. Since e^B increases and e^A decreases in π, both output and incentive effect in condition (10) move in the same direction. Since W_1 increases in π, $W_1(w, \pi)$ may not be monotone in w. Solving the quadratic expression $S^B > S^A$ for e^A yields

$$e^A < 2w - \sqrt{4w^2 - \frac{2}{\pi}S^B}. \tag{11}$$

$2S^B > 3\pi w^2$ gives a sufficient condition:

$$\begin{aligned} & 2((2\pi - 1)W + (1 - \pi)w)((2\pi - 1)W + (1 - \pi)3w)/2 > 3\pi^2 w^2 \\ \Leftrightarrow \quad & (2\pi - 1)W^2 + 4(1 - \pi)Ww > 3w^2. \end{aligned}$$

Solving this quadratic expression in W yields the condition

$$W > \left(\frac{\sqrt{3(2\pi - 1) + 4(1 - \pi)^2}}{2\pi - 1} - 2\frac{1 - \pi}{2\pi - 1}\right) w.$$

Since $(\sqrt{3(2\pi - 1) + 4(1 - \pi)^2} - 2(1 - \pi))/(2\pi - 1) < \sqrt{3}$ a sufficient condition for $S^B > S^A$ is $W \geq \sqrt{3}w$, that is $W_1(w, \pi) < \sqrt{3}w$.

A.5.2 Derivation of the Cutoff $W_2(w, \pi)$

We now compare the background based policy to laissez-faire when $\pi \leq 1/2$. Under laissez-faire there are $\pi e^{LF}/2$ firms of type (h, h) contributing to total output and the surplus is $S^{LF} = \pi W^2/2$. With a background based policy there are measure πe^B of (h, ℓ) firms and total surplus is $S^B = \pi 3w^2/2$. Therefore, when $\pi \leq 1/2$, $S^{LF} > S^B$ if and only if $W > \sqrt{3}w$; hence $W_2(w, \pi) = \sqrt{3}w$ as claimed.

Consider now the case $\pi > 1/2$. In this case, $S^{LF} > S^{BB}$ if and only if

$$(e^{LF} - e^B)\left[(1-\pi)2w + (2\pi-1)W - \frac{\pi}{2}\left(e^{LF} + e^B\right)\right] > (1-\pi)e^{LF}(2w - W). \tag{12}$$

Manipulating condition (12) and solving for W yields

$$W > \frac{4\pi - 2 + \sqrt{1 - 4\pi + 7\pi^2}}{3\pi - 1} w := W_2(w, \pi).$$

Clearly, $W_2(w, \pi)$ increases in w and simple calculation reveals that W_2 increases also in π. Note that $W_2(w, \pi) \to 2w$ as $\pi \to 1$. Bounds on W_2 for $\pi \in [1/2, 1]$ are given by

$$\sqrt{3}w = W_2(w, 1/2) \leq W_2(w, \pi) \leq W_2(w, 1) = 2w.$$

$W_2(w, \pi) \geq \sqrt{3}w$ implies in particular that $W_2(w, \pi) > W_0(w, \pi)$ (see the proof of Corollary 2).

A.6 Proof of Proposition 3

The case of laissez-faire has been established in the text.

Under an achievement based policy agents segregate in background when possible. By Proposition 1 less than half of the P agents become educated. The remaining (ℓ, P) agents match with (h, U) agents if possible, since $w - f > 0$. Therefore $e < e^A(\pi) \leq w$ for P agents, since (ℓ, P) agents' expected wages exceed $w\pi e^A/(1 - \pi e^A)$. $e \leq w - f$ for U agents, which holds with equality if (ℓ, P) outnumber (h, U) agents.

Turn now to a background based policy. Denote effort investments by e_P for P and by e_U for U agents. Start with the case $\pi > 1/2$. That is, a P has a chance of $(2\pi - 1)/\pi$ of being able to match with a P agents. Otherwise he matches with a P agent, and, if own achievement is h, obtains a h match with probability $\min\{e_U/e_P; 1\}$, and with probability $\max\{(e_U - e_P)/(1 - e_P); 0\}$

if own achievement is ℓ. Hence, a P agent's effort choice solves

$$\max_e e\left(\frac{2\pi-1}{\pi}W+\frac{1-\pi}{\pi}\left(\min\left\{\frac{e_U}{e_P};1\right\}(W-w)+w-f\right)\right)$$
$$+(1-e)\frac{1-\pi}{\pi}\max\left\{\frac{e_U-e_P}{1-e_P};0\right\}(w-f)-\frac{e^2}{2}.$$

A U agent solves

$$\max_e e\left(\min\left\{\frac{e_P}{e_U};1\right\}(W-w)+w-f\right)+(1-e)\max\left\{\frac{e_P-e_U}{1-e_U};0\right\}(w-f)-\frac{e^2}{2}.$$

Note that investment incentives of P and U agents can only be aligned if $\pi=1/2$. Therefore $e_P\neq e_U$ for $\pi\neq 1/2$. Suppose $e_P>e_U$. Then

$$\begin{aligned} e_P &= W-\frac{1-\pi}{\pi}\left(\frac{e_P-e_U}{e_P}(W-w)+f\right)>w-f, \\ e_U &= W-w+\frac{1-e_P}{1-e_U}(w-f)>W-F. \end{aligned}$$

Suppose $e_P<e_U$. Then

$$\begin{aligned} e_P &= W-\frac{1-\pi}{\pi}\left(f+\frac{e_U-e_P}{1-e_P}(w-f)\right)>W-F, \\ e_U &= w-f+\frac{e_P}{e_U}(W-w)>w-f. \end{aligned}$$

Under the assumption DD ($2w>W$) education inputs are substitutes, so that both a regime where P agents invest a lot and U agents hardly invest, and the reverse may emerge in equilibrium. Indeed both regimes are possible for π sufficiently close to 1/2.

Let now $\pi<1/2$. That is, a P agent matches with a U agent with certainty, and, if own achievement is h, obtains a h match with probability $\min\{e_U/e_P;1\}$, and with probability $\max\{(e_U-e_P)/(1-e_P);0\}$ if own achievement is ℓ. Hence, a P agent's effort choice solves

$$\max_e e\left(\min\left\{\frac{e_U}{e_P};1\right\}(W-w)+w-f\right)+(1-e)\max\left\{\frac{e_U-e_P}{1-e_P};0\right\}(w-f)-\frac{e^2}{2}.$$

A U agent is assigned to a P agent with probability $\pi/(1-\pi)$ and otherwise matches with a U agent. Therefore a U agent solves

$$\max_e e\left(\frac{1-2\pi}{1-\pi}(W-F)+\frac{\pi}{1-\pi}\left(\min\left\{\frac{e_P}{e_U};1\right\}(W-w)+w-f\right)\right)$$
$$+(1-e)\frac{\pi}{1-\pi}\max\left\{\frac{e_P-e_U}{1-e_U};0\right\}(w-f)-\frac{e^2}{2}.$$

Again first order conditions imply that $e_P \neq e_U$ for $\pi \neq 1/2$. Suppose $e_P > e_U$. Then

$$\begin{aligned} e_P &= w - f + \frac{e_U}{e_P}(W - w) > w - f, \\ e_U &= W - \frac{1-2\pi}{1-\pi}F - \frac{\pi}{1-\pi}\left(w - \frac{1-e_P}{1-e_U}(w-f)\right) > W - F. \end{aligned}$$

Otherwise, if $e_P < e_U$

$$\begin{aligned} e_P &= W - w + \frac{1-e_U}{1-e_P}(w-f) > W - F, \\ e_U &= \frac{1-2\pi}{1-\pi}(W - F) + \frac{\pi}{1-\pi}\left(\frac{e_P}{e_U}(W-w) + w - f\right) > W - F. \end{aligned}$$

Again both regimes are possible for π sufficiently close to 1/2.

A.7 Proof of Proposition 5

As established in the text integration is a stable labor market outcome. When investing an agent solves

$$\max_e ew(h) + (1-e)w(\ell) - \frac{e^2}{2} - g(b, b'),$$

where market wages are $w(\ell) = 2w - w(h)$ and (i) $w(h) = W$ if $q > 1/2$, (ii) $w(h) \in [W, 2w]$ if $q = 1/2$, and (iii) $w(h) = 2w$ if $q < 1/2$. Corresponding optimal interior investments are

$$e = \begin{cases} e_0 = 2(W - w) & \text{if } q > 1/2 \\ [2(W - w), \min\{2w; 1\}] & \text{if } q = 1/2 \\ e_1 = \min\{2w; 1\} & \text{if } q < 1/2 \end{cases}$$

Strictly positive investment is profitable if $e^2/2 > g(b, b')$.

Denote by ρ the (endogenous) measure of agents in (U, U) schools. $q > 1/2$ implies $e = 2(W - w)$, so that (U, U) agents do not invest as $F > W^2/2$. This is only consistent if $(1-\rho)e > 1/2$, that is $(1-\rho)2(W - w) > 1/2$.

$q \leq 1/2$ implies $e = \min\{2w; 1\}$, so that (U, U) agents invest as $F < 2w^2$. This is only consistent if $\min\{2w; 1\} \leq 1/2$, that is $w \leq 1/4$.

If $1/4 \geq w \geq W - 1/(4(1-\rho))$, $q = 1/2$ must hold. To have $q = 1/2$ either $(1-\rho)2(w(h) - w) = 1/2$ if $1/(4(1-\rho)) \in [W - w, \sqrt{F}]$, or $2(w(h) - w)^2 = F$

and measure $1/2-(1-\rho)\sqrt{F}$ of (U,U) agents invest $e=\sqrt{F}$, leaving them indifferent between $e>0$ and $e=0$.

The measure ρ is determined at the school stage. If $q>1/2$ payoffs at the school stage are given by

$$\begin{aligned}&2w-W \text{ if } g(b,b')=F,\\ &2w-W+2(W-w)^2-g(b,b') \text{ if } g(b,b')<F,\end{aligned}$$

Hence, a U agent values a P agent at $2(W-w)^2-f$, and a P agent values a U agent at $-f$. Hence, schools integrate, that is $\rho=\max\{1-2\pi;0\}$, if

$$(W-w)^2>f,$$

which is implied by Assumption 1.

Using this and $(1-\rho)2(W-w)\geq 1/2$ gives condition (i) in the statement.

If $q<1/2$ $e=2w$ for all agents as above and schools integrate as $2f<F$.

If $q=1/2$, payoffs at school are $2w-w(h)+2(w(h)-w)^2-g(b,b')$ if $g(b,b')<F$ and otherwise $2w-w(h)$, or $F-g(b,b')$ for all agents. Since $2f<2(W-w)^2\leq 2(w(h)-w)^2\leq F$ under Assumption 1, schools integrate and $\rho=\max\{1-2\pi;0\}$. Therefore, if $\pi\geq 1/2$ all agents invest $e=2(w(h)-w)=1/2$. Let $\pi<1/2$. If $1/(4\pi)\leq\sqrt{2F}$ only agents with $g(.)<F$ invest $e=1/(4\pi)$, otherwise $e=\sqrt{2F}$ and also measure $1/2-2\pi\sqrt{2F}$ of (U,U) agents invest. This completes the argument for part (ii).

A.8 Proof of Proposition 6

By (3) given s an agent with fixed cost f invests if

$$e^{AI}>\sqrt{2f}\Leftrightarrow e^{A}(s)>\sqrt{2f}.$$

Since $e^A(s)$ strictly decreases in s, $e^A(\max\{2\pi-1;0\})<\sqrt{2f}$ implies that (U,P) agents do not invest if $s=\max\{2\pi;0\}$ which is consistent therewith. If $e^A(\min\{2\pi;1\})>\sqrt{2f}$ (U,P) agents invest at $s=\min\{2\pi;1\}$. For intermediate f, s is defined by $e^A(s)=\sqrt{2f}$ making (U,P) agents indifferent between investing or not, which is consistent with $\max\{2\pi;0\}<s<\min\{2\pi;1\}$. Aggregate surplus under AI is

$$\begin{aligned}&\max\{2\pi-1;0\}e^{AI}\left(2w-\frac{e^{AI}}{2}\right) \text{ if } e^A(\max\{2\pi-1;0\})<\sqrt{2f},\\ &\min\{2\pi;1\}e^{AI}\left(2w-\frac{e^{AI}}{2}\right)-2\min\{\pi;1-\pi\}f \text{ if } e^A(\min\{2\pi;1\})>\sqrt{2f}.\end{aligned}$$

$S^A > S^{AI}$ if and only if $q^A(2w - e^A(s^A)/2) > q^{AI}(2w - e^A(s^{AI})/2)$. For $e^A(\max\{2\pi - 1; 0\}) < \sqrt{2f}$, $q^A = \pi e^A(\pi) > q^{AI}$ and $e^A(\pi) < e^A(s^{AI})$, therefore $S^A > S^{AI}$. Let now $e^A(\min\{2\pi; 1\}) > \sqrt{2f}$ and suppose $\pi \leq 1/2$ first. Then a sufficient condition for $S^{AI} > S^A$ is

$$\begin{aligned} & 2w\left(2e^A(2\pi) - e^A(\pi)\right) > 2(e^A(2\pi))^2 - \frac{(e^A(\pi))^2}{2} \\ \Leftrightarrow \quad & 4\pi w(q^{AI} - q^A) > (q^{AI})^2 - (q^A)^2, \end{aligned}$$

which must be true since $e^A(2\pi) < e^A(\pi) < w$. In case $\pi > 1/2$ a sufficient condition for $S^{AI} > S^A$ is

$$2w\left(q^{AI} - q^A\right) > \left(\frac{3}{2} - \pi - \frac{1}{2\pi}\right)(q^A(1))^2 + \frac{(q^{AI})^2 - (q^A)^2}{2\pi}.$$

This is implied by

$$w\left(1 - \frac{q^A}{q^{AI}}\right) > \left(3 - 2\pi - \frac{1}{\pi}\right) q^{AI}.$$

Since $w \geq e^{AI} = q^{AI}$ and for $1/2 < \pi \leq 1$

$$2\pi + \frac{1}{\pi} - 2 > 1 > \frac{q^A}{q^{AI}},$$

$S^{AI} > S^A$ follows.

Proof of Proposition 7

Suppose both U and P agents in (U, P) environments invest. The measure of h agents with a U background is $\min\{\pi; 1-\pi\}e$, since agents in (U, U) environments with access cost F do not invest. The measure of h agents with background P is πe. The measure of P agents required to match with U agents is $\min\{\pi; 1-\pi\}$. Hence, if all agents invest, both U and P agents with education h encounter an agent with h and the required background for sure. Therefore U and P agents solve

$$\max_e eW - \frac{e^2}{2} - g(b, b'). \tag{13}$$

That is, $e^{BI} = W$ and the labor market payoff is $W^2/2 - f > 0$. Hence, agents facing access cost f or smaller find it indeed profitable to invest whereas

(U, U) do not, which is consistent with our assumption. Hence, $e^{BI} = e^{LF} = e^I$, $s^{BI} = s^I$, and $S^{BI} = S^I$ where the last statement is implied by the preceding two.

Analogously to the case of background in the surplus function existence of asymmetric equilibria, where either U or P agents invest more, cannot be precluded. These equilibria may generate higher output as when investments differ in background a positive measure of integrated firms emerges on the labor market.

A.9 Omitted Details for Proposition 8

In the text it has been shown that $s_u > 0$ and $s_p + s_u < 1$. If $\pi > 1/2$ then $s_p = s_u + 2\pi - 1$ and a (U, U) agent is matched to a (P, P) agent for sure, does not invest, and obtains $e^P w$. For a (U, P) agent, who may match to a (P, P) or a (U, P) agent, positive investment solves

$$\max_e eW - \frac{e^2}{2},$$

supposing that at least (P, P) agents invest. Since $f < W^2/2$ all (U, P) agents invest $e = W$. In this case a (P, P) agent solves

$$\max_e e\left(\frac{s_p - s_u}{s_p}W + \frac{s_u}{s_p}w\right) - \frac{e^2}{2},$$

and therefore

$$e_p = \frac{s_p - s_u}{s_p}(W - w) + w.$$

That is, (U, P) invest more than (P, P) agents. No agent has an incentive to change schools if

$$W^2 - 2f \geq e_p^2 \text{ and } W^2 - 2f \geq 2e_p w,$$

with at most one strict inequality. Since $e_p < 2w$ the second condition must bind, that is $W^2 - 2f = 2e_p w > e_p^2$. This determines measures s_u and s_p since $s_u = s_p + 1 - 2\pi$ by feasibility, so that

$$s_p = (2\pi - 1)2w\frac{W - w}{W^2 - 2f - 2w^2}.$$

Note that $W^2 - 2f > 2w\left[W - \frac{1-\pi}{\pi}(W-w)\right]$ implies $2\pi - 1 < s_p < \pi$ and thus $0 < s_u < 1-\pi$. Hence, measure $s_u e_p > 0$ of (h,ℓ), measure $(1-s_u-s_p)W + (s_p - s_u)e_p > 0$ of (h,h), and measure $(s_u+s_p)W - s_p e_p > 0$ of (ℓ,ℓ) firms form.

Briefly consider the case $\pi \leq 1/2$. Suppose that $s_p = \pi$ and $s_u = 1-\pi$ implying payoffs $w^2/2$ for (P,P) and $w^2\pi/(1-\pi)$ for (U,U) agents. Let a pair of agents matches into a (U,P) school. Since (P,P) agents are scarce these agents also match on the labor market implying optimal investments e_u, e_p satisfy $e_u = w - e_p(2w-W)$ and vice versa. That is, $e = w/(1+2w-W) < w$. Hence, segregation can be supported as an equilibrium outcome if

$$w^2\left(\frac{\frac{3}{2}+2w-W}{(1+2w-W)^2} - \max\left\{\frac{1}{2};\frac{\pi}{1-\pi}\right\}\right) > f.$$

Let now $s_u < 1-\pi$ and $s_p < \pi$. Then (U,P) agents segregate and invest W, (P,P) invest w, and (U,U) agents invest 0 and obtain payoff $w^2 s_p/s_u$, where $s_p/s_u = 1-(1-2\pi)/s_u$. Hence, $s_u < 1-\pi$ can hold in equilibrium if

$$W^2 - w^2 \geq 2f \text{ and } W^2 - 2w^2\left(1 - \frac{1-2\pi}{s_u}\right) \geq 2f.$$

Hence, whenever $W^2 - 2f \geq w^2$ there exist $s_u < 1-\pi$ such that $W^2 - 2w^2\left(1-\frac{1-2\pi}{s_u}\right) \geq 2f$, in particular $s_u = 1-2\pi$, that is full school integration, can be supported as an equilibrium outcome.

A.10 Proof of Proposition 9

From above we know $e_p = (W^2-2f)/2w$. Assume that

$$W^2 - 2f > 2w\left[W - \frac{1-\pi}{\pi}(W-w)\right]. \tag{14}$$

Then aggregate surplus under a club based policy can be written as

$$\begin{aligned} S^C &= 2s_u e_p w + (s_p - s_u)e_p W + (1-s_p-s_u)\left(\frac{W^2}{2} - f\right) - s_p\frac{e_p^2}{2} \\ &= s_p\frac{e_p^2}{2} + s_u e_p w + 2(\pi - s_p)\left(\frac{W^2}{2} - f\right) = s_p\frac{e_p^2}{2} + (1-s_p)\left(\frac{W^2}{2} - f\right). \end{aligned}$$

Note that $S^C = s_p e_p^2/2 + (1-s_p)we_p$. We continue to show that for each labor market policy there exists some $\pi^* > 1/2$ with the property stated in the proposition.

Comparing to laissez faire, $S^C > S^{LF}$ if

$$\left(1 - s_p\left(1 - \frac{e_p}{2w}\right)\right)\left(\frac{W^2 - 2f}{2}\right) > \pi\frac{W^2}{2}.$$

That is,

$$1 - \pi\frac{W^2}{W^2 - 2f} > s_p\left(1 - \frac{e_p}{2w}\right), \tag{15}$$

Since $s_p \leq \pi$ (with equality if (14)) holds with equality), this is implied by

$$\frac{1}{\pi} > 1 + \frac{W^2}{W^2 - 2f} - \frac{W^2 - 2f}{4w^2}.$$

The RHS is less than 2 if $W^2 < 6f$, so that there is $\pi^* > 1/2$ such that (14)) holds with equality and $S^C > S^{LF}$. Hence, for any $1/2 \leq \pi < \pi^*$ (14)) continues to hold and $s_p < \pi$, which implies $S^C > S^{LF}$ at π.

Concerning an achievement based policy, $S^C > S^A$ if and only if

$$\left(1 - s_p\left(1 - \frac{e_p}{2w}\right)\right) we_p > \pi e_A(\pi)\left(2w - \frac{e^A(\pi)}{2}\right).$$

Using that $e_p \geq w$ a sufficient condition is

$$\left(1 - \frac{s_p}{2}\right) w^2 > \pi e_A(\pi)\left(2w - \frac{e^A(\pi)}{2}\right).$$

If (14) holds $s_p \geq \pi$, so that $S^C > S^A$ if

$$\frac{1}{\pi} > \frac{1}{2} + \frac{e_A(\pi)}{w}\left(2 - \frac{e^A(\pi)}{2w}\right).$$

As $e_A < w$ there is $\pi^* > 1/2$ such that this condition holds with equality for π^* and with strict inequality for $1/2 \leq \pi < \pi^*$. Since S^C decreases in s_p and increases in e_p, the above condition holds for all $\pi \geq 1/2$ so that (14) holds.

Comparing to a background based policy, $S^C > S^B$ if and only if

$$s_p\frac{e_p^2}{2} + (1 - s_p)we_p > \pi\frac{(e^B)^2}{2} + (1 - \pi)we^B. \tag{16}$$

If (14) holds with equality, $s_p = \pi$ and $e_p = e_B$, and the above inequality holds with equality. This implies, as the LHS of (16) increases in e_p and $e_p = (W^2 - 2f)/2w$ that if (14) holds with strict inequality sign the same is true for the above condition. Hence, $S^C > S^B$ whenever (14) holds with strict inequality, and $\pi^* = 1$.

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