

# Option pricing on target stock under multiple decision reversions

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### **Abstract**

This paper models the dynamics of the target stock price in pending merger and acquisition deals. It explicitly accounts for the possibility of multiple takeover negotiation breakdowns and resumptions. Furthermore, we develop an arbitrage-free framework for pricing European options on the target stock, and suggest ways of estimating the parameters from real data.

# 1 Introduction

Several recent spectacular takeover negotiations have drawn public attention. Some of these deals experienced multiple negotiation breakdowns and resumptions, like in the case of the attempted takeover of Yahoo by Microsoft. The price of the target stock suffered jumps of large magnitude at each of these turning points. This raises the need for an appropriate model for target stock dynamics which incorporates the possibility of multiple jumps. The main aim of such model should be pricing of options on the target stock price subject to the full reversion of the deal perspectives.

The issue of jumps was for the first time indirectly addressed in the literature by Dodd and Ruback (1977). Since then, a large number of papers has focused on the so-called abnormal returns on target stocks. From Dodd and Ruback (1977) to Schwert (1996) the empirical studies have revealed returns on the acquisition targets which exceed those predicted by a market model. The market model is a reference model, whose parameters are estimated using data from a sampling period lying considerably before the first takeover announcement (see, among others, Jensen and Ruback, 1983; Jarrell et al., 1988).

Another line of research focuses on the volatility of target stock returns around and after the bid announcement. For intraday volatility Lee et al. (1994) as well as Smith et al. (1997) documented a characteristic pattern exhibiting some rise around the announcement date and a steady decrease of volatility after a week following the dissemination of the takeover attempt. Levy and Yoder (1993) reported similar results for the implied volatility of target stock returns (derived from option prices). These papers link higher volatility around the announcement to the uncertainty of the outcome (completion or failure of the deal), but do not provide a formal model.

Later empirical studies highlight the change in the dynamics of target stock returns. Hutson and Kearney (2001, 2005) report a significant reduction of volatility and sensitivity to the market shocks of the target stock price returns after the takeover announcement. Gelman and Wilfing (2007) show that the return process switches from a high-volatility-high-beta regime to a low-volatility-low-beta regime, but also in the other direction. They establish a descriptive link from the regime to the investors' perception of the deal and suggest that regime change occurs when the market agents alter their beliefs (from "the deal is very likely to succeed" to "the deal is very likely to fail" or *vica verca*).

Theoretically the issue of jumps in the merger and acquisition context

was first addressed by Subramanian (2004). He highlights the importance of a possibility that the merger can be called off during the bid period for option pricing. However, the model of Subramanian (2004) allows only for one jump in one direction, downwards, and only in the framework of share swap deals. Upward price jumps in a takeover framework were introduced by At and Morand (2007), but also only in one direction.

We suggest a model for the target stock price with multiple jumps in both directions which can account for the abrupt change of the investors' sentiment regarding the deal. We model the price of target stock as a stochastic sequence of two jump diffusion processes, where the diffusion parameters are independent of the jump timing. In summary, target stock follows one diffusion path until it jumps to the other one and so on until the deal is completed or it ultimately failed. The time of jumps is stochastic. This framework allows us to calculate real-world and risk-neutral probability distributions of the target stock price, and hence solve for the price of a European option on the target stock. Furthermore, we suggest ways of estimating the model parameters from real data.

The rest of the paper is organized as follows: Section 2 develops the model of target stock price dynamics. Section 3 provides the solutions for option pricing. Section 4 discusses the applicability of the model. Section 5 concludes.

## 2 The Model

We start from the well-known Samuelson-Rosenthal (1986) formula for the target stock price, which is simply the weighted average of the discounted expected bid- and fallback-prices:

$$P(t) = \alpha_t \cdot E[V_T | \varphi(t)] e^{-r_{RA1}(T-t)} + (1 - \alpha_t) \cdot E[P^*(T) | \varphi(t)] e^{-r_{RA2}(T-t)}, \quad (1)$$

where  $0 \leq \alpha_t \leq 1$  denotes the subjective takeover probability that the deal is concluded at time  $T$ ,  $V(T)$  stays for the bid price to be paid at date  $T$ ,  $P^*(t)$  represents the *fallback* or fundamental price of the target stock, and  $\varphi(t)$  denotes the information set at time  $t$ .  $r_{RA1}$  and  $r_{RA2}$  denote the corresponding risk-adjusted discount rates.

For the dynamics of  $P^*(t)$  we assume a geometric Brownian motion:

$$dP^*(t) = \mu^* \cdot P^*(t) \cdot dt + \sigma^* \cdot dW_1(t), \quad (2)$$

where  $\mu^* \in \mathbb{R}$ ,  $\sigma^* > 0$  and  $dW_1(t)$  denotes the increment of the standard Wiener process.

The bid price depends on the bid condition. It is the sum of the market value of the number of shares (denoted by  $\gamma$ ) of the bidding company to be swapped for one share of the target at the time of transaction closure  $T$  plus the cash amount, offered to the target stock holders  $\delta$ :

$$V_T = \gamma \cdot P^B(T) + \delta, \quad (3)$$

where  $\gamma, \delta \geq 0$  and  $P^B(T)$  denotes the price of one share of bidder's stock. This general setting allows to model the dynamics under all usual bid constellations: if  $\gamma, \delta > 0$  mixed bid results,  $\gamma > 0, \delta = 0$  yields a pure share swap, and a constellation  $\gamma = 0, \delta > 0$  is addressed as a cash bid in the literature.

For the dynamics of the bidder stock price we also assume a geometric Brownian motion:

$$dP^B(t) = \mu^B \cdot P^B(t) \cdot dt + \sigma^B \cdot dW_2(t), \quad (4)$$

where  $\mu^B \in \mathbb{R}$ ,  $\sigma^B > 0$  and  $dW_2(t)$  denotes the increment of another standard Wiener process.

Inserting eq. (3) into the eq. (1) and solving for the expectations yields:

$$\begin{aligned} P(T) &= \alpha_t \cdot \left[ \gamma P^B(t) + \delta e^{-r(T-t)} \right] + (1 - \alpha_t) \cdot P^*(t), \\ &= G(P^B(t), P^*(t), \alpha_t, t) \end{aligned}$$

where  $r$  denotes the risk-free rate.

To compute the stochastic differential of  $P(t)$  as a function of two stochastic variables and time we apply the multivariate Ito-lemma and get the instantaneous changes of the target stock  $dP(t)$ :

$$\begin{aligned} dP(t) &= \left[ \delta \cdot e^{-r \cdot (T-t)} + \gamma \cdot P^B(t) - P^*(t) \right] \cdot d\alpha_t + \alpha_t \cdot \gamma \cdot dP^B(t) \\ &\quad + \alpha_t \cdot r \cdot e^{-r \cdot (T-t)} \cdot \delta \cdot dt + (1 - \alpha_t) \cdot dP^*(t). \end{aligned} \quad (5)$$

Equation (5) shows that the most crucial change is induced by a one unit change in  $\alpha_t$ . As the information on the possible deal outcome arrives irregularly and with a rather low frequency, it is intuitive to model the subjective probability as a jump process. For the tractability of the formal derivations we assume that the subjective probability can take only two

states: certainty of deal completion ( $\alpha_t = 1$ ) and the overall belief that the deal is condemned to fail ( $\alpha_t = 0$ ). The intensity of the full reversal of the investors' sentiment is  $\lambda$ . In this context we assume that the probability of changing their minds from disbelief to belief ( $\Delta\alpha_t = 1$ ) is equal to the probability of changing their minds in the opposite direction ( $\Delta\alpha_t = -1$ ) during time periods of the same length. Consider some time  $t = 0$  at which the investors do not believe in merger  $\alpha_{0=0}$ , however there are some rumours about the potential takeover (which justify  $\lambda > 0$ ). Then the current price is simply the fundamental price of the target stock:

$$P(0) = P^*(0).$$

The instantaneous change is given by the drift and the stochastic part of the fundamental process, augmented by the jump process accounting for the eventual sentiment switch:

$$\begin{aligned} dP(0) &= \mu^* P(0-) dt + \sigma^* dW_1(t) + (\gamma P^B(0-) + \delta e^{-rT} - P^*(0-)) d\mathfrak{k} \\ &= \mu^* P(0-) dt + \sigma^* dW_1(t) + (J_1(0-) - 1) P(0-) d\alpha_t, \end{aligned}$$

where

$$J_1(t-) = \frac{\gamma P^B(t-) + \delta e^{-r(T-t)}}{P^*(t-)}$$

denotes the size of the gross relative upward jump. Thus, in the infinitely small time period the price of the target stock can either stay on its fundamental path (with probability  $1 - \lambda dt$ ) or jump upwards to the bid price path (with probability  $\lambda dt$ ).

In this framework the price of the target stock has to end up on one of the two paths also in any later time point  $T_0$ , which is however prior to the deal consummation (or ultimate failure) date  $0 < T_0 \leq T$ . As the recent history of M&As shows negotiations may experience multiple decision reversions, we allow an infinite number of jumps (i.e. investors can reverse their assesment of deal success an infinite number of times) in our framework. Then in the case when in  $P(0)$  the current market's conviction is "no deal", the target stock price is equal to its fundamental value in  $T_0$  if the investors will not change their mind or change it an even number of times. As the probability of no jump in the considered period is  $\exp[-\lambda T_0]$  and the jump probability

is  $1 - \exp[-\lambda T_0]$ , the probability of such a set of events is given by:

$$\begin{aligned} \Pr(P(T_0) = P^*(T_0) | \alpha_0 = 0) &= \exp[-\lambda T_0] \sum_{i=0}^{\infty} (1 - \exp[-\lambda T_0])^{2i} \quad (7) \\ &= \frac{1}{2 - \exp[-\lambda T_0]}. \end{aligned}$$

Correspondingly, the probability of ending up on the bid price path in  $T_0$  corresponds to the probability of the opinion reversal an uneven number of times:

$$\begin{aligned} \Pr(P(T_0) = \tilde{V}_{T_0} | \alpha_0 = 0) &= \exp[-\lambda T_0] (1 - \exp[-\lambda T_0]) \times \quad (8) \\ \sum_{i=0}^{\infty} (1 - \exp[-\lambda T_0])^{2i} &= \frac{1 - \exp[-\lambda T_0]}{2 - \exp[-\lambda T_0]}, \end{aligned}$$

where  $\tilde{V}_{T_0} = \delta \cdot e^{-r \cdot (T - T_0)} + \gamma \cdot P^B(T_0)$ .

Note that landing on the same path (Eq. (7)) is more probable than changing the path (Eq. (8)). These results stay in line with intuition: The probabilities diverge, if the jump intensity is low or the forecast horizon is short. In the extreme case, if the jump intensity is zero, the target stock price will stay on the fundamental path with unit probability. In contrast, if jumps are highly expected and the time horizon is long, both outcomes are equally probable.

Thus, the subjective distribution of the target stock price at time  $T_0$  based on the investors' knowledge as of  $t = 0$  is a mixture of the bid price and fundamental value distributions with the weights specified by the jump intensity:

$$\begin{aligned} F(P(T_0) = x | \varphi(t)) &= \frac{1}{2 - \exp[-\lambda T_0]} \cdot \int_{-\infty}^x f(P^*(T_0)) dP^*(T_0) \quad (9) \\ &+ \frac{1 - \exp[-\lambda T_0]}{2 - \exp[-\lambda T_0]} \cdot \int_{-\infty}^x f(\tilde{V}_{T_0}) d\tilde{V}_{T_0}. \end{aligned}$$

If we change the starting state from no confidence in deal completion to full confidence in deal completion we will only need to switch the weights in Eq. (9) in front of the integrals to obtain the new solution for the distribution function.

### 3 Option pricing

In order to derive the option pricing formulas a risk-neutral distribution is needed. For this purpose we first provide martingale equivalents of the price process in Eq. (6). Applying the change of measure and compensating for jumps one obtains:

$$dP(t)' I_{\alpha_{t-}=0} = (r - d - \lambda(J_1(t-) - 1)) P(t-) dt + \sigma^* P(t-) dW_1(t) + (J_1(t-) - 1) P(t-) d\alpha_t,$$

where  $d$  denotes the instantaneous dividend yield. Switching the current state condition (to a situation when the market participants are absolutely certain of deal consummation) provides for a similar process:

$$dP(t)' I_{\alpha_{t-}=1} = (r - d - \lambda(J_2(t-) - 1)) P(t-) dt + \sigma^B \gamma P^B(t-) dW_2(t) + (1 - J_2(t-)) P(t-) d\alpha_t,$$

where

$$J_2(t-) = \frac{1}{J_1(t-)} = \frac{P^*(t-)}{\gamma P^B(t-) + \delta e^{-r(T-t)}}.$$

The price of the call expiring in  $T_0$  on the target stock at  $t=0$  (if the deal is considered improbable) is thus:

$$C(0, T_0, K) \cdot I_{\alpha_0=0} = \frac{1}{2 - e^{-\lambda T_0}} \cdot \int_K^\infty f_1(P(T_0) | 0) (P(T_0) - K) dP(T_0) + \frac{1 - e^{-\lambda T_0}}{2 - e^{-\lambda T_0}} \cdot \int_K^\infty f_2(P(T_0) | 0) (P(T_0) - K) dP(T_0)$$

where  $f_1$  is the risk-neutral density of the price of the target stock at  $T_0$  if the takeover is perceived as failed:

$$f_1(P(T_0) | 0) = \frac{1}{\sigma^* P^*(T_0) \sqrt{2\pi T_0}} \exp \left\{ -\frac{\left( \ln \left[ \frac{P(T_0)}{P(0)} \right] - \left( r - \frac{\sigma^{*2}}{2} - d \right) T_0 \right)^2}{2\sigma^{*2} T_0} \right\}$$



and  $f_2$  is the density of the price if the deal is on as of  $T_0$ :

$$f_2(P(T_0)|0) = \frac{1}{\sigma^B \gamma P^B(T_0) \sqrt{2\pi T_0}} \times \exp \left\{ -\frac{\left( \ln \left[ \frac{P(T_0) - \delta e^{-r(T-T_0)}}{\gamma P^B(0)} \right] - \left( r - \frac{\sigma^{B2}}{2} - d \right) T_0 \right)^2}{2\sigma^{B2} T_0} \right\}$$

Solving the integrals in Eq. (10), if the density  $f_2$  exists (i.e.  $\gamma > 0$ ) yields:

$$C(0, T_0, K) I_{\alpha_0=0} = \frac{1}{2 - e^{-\lambda T_0}} \cdot \left\{ P(0) e^{-dT_0} \Phi(x_1) - K e^{-rT_0} \Phi(x_1 - \sigma^* \sqrt{T_0}) \right\} \quad (11)$$

$$+ \frac{1 - e^{-\lambda T_0}}{2 - e^{-\lambda T_0}} \cdot \left\{ (\gamma P^B(0) + \delta e^{-rT}) e^{-d} \Phi(x_2) - K e^{-rT_0} \Phi(x_2 - \sigma^B \sqrt{T_0}) \right\},$$

where  $\Phi(\cdot)$  denotes the distribution function of the standard normal distribution at point  $\cdot$  and

$$x_1 = \frac{\ln \left( \frac{P(0)}{K} + \left( r - d + \frac{1}{2} \sigma^{*2} \right) T_0 \right)}{\sigma^* \sqrt{T_0}}$$

$$x_2 = \frac{\ln \left( \frac{\gamma P^B(0)}{K - \delta e^{-rT}} \right) + \left( r - d + \frac{1}{2} \sigma^{B2} \right) T_0}{\sigma^B \sqrt{T_0}}.$$

If, however, we consider a pure cash bid ( $\gamma = 0$ ), there is only one possible price realization if markets are sure at time  $T_0$  about deal conclusion at  $T$ , namely  $\delta e^{-r(T-T_0)}$ . In this case we get a simpler alternative to Eq. (11):

$$C(0, T_0, K) I_{\alpha_0=0} = \frac{1}{2 - e^{-\lambda T_0}} \cdot \left\{ P(0) e^{-dT_0} \Phi(x_1) - K e^{-rT_0} \Phi(x_1 - \sigma^* \sqrt{T_0}) \right\}$$

$$+ \frac{1 - e^{-\lambda T_0}}{2 - e^{-\lambda T_0}} \cdot \left\{ \delta e^{-rT} - K e^{-rT_0} \right\}.$$

Eq. (12) is intuitively tractable: the call enables us to gaining the payoff in  $T_0$  which is simply the difference between the bid price of target stock and the strike price, if the markets are convinced of deal success. Similar to the distribution result (Eq. (9)) in order to adapt the call option price to initial confidence in the deal conclusion (i.e. to  $\alpha_0 = 1$ ) only switching of the probability weightings in Eq. (11) or (12) is required.

## 4 Discussion

In order to be able to apply this model for the estimation of option prices the knowledge of  $r, d, \sigma^*, \sigma^B$ ; of the deal terms  $\gamma$  and  $\delta$  as well as of the jump intensity  $\lambda$  is required. Furthermore, it is necessary to know, whether the markets perceive the outstanding deal as certain ( $\alpha_0 = 1$ ) or completely improbable ( $\alpha_0 = 0$ ).

The first six parameters can be directly or indirectly observed in most cases: The risk-free rate  $r$  is usually approximated by the US-Treasury bill rate. Also, the dividend yield of the target stock  $d$  can be calculated from the financial reports of the target company. The respective volatilities of the fundamental process  $\sigma^*$  and the bidder stock price  $\sigma^B$  can be calculated from historical returns or as implied volatilities of corresponding at-the-money options for the period prior to the first deal announcement (as suggested in Subramanian 2004). The knowledge of the deal terms restricts the applicability of our model to those targets, for which the bid terms have become public: either through official announcements or rumours in the financial press.

The last two parameters would require some pre-calculations. As for the current state of investors' beliefs, there are numerous studies, starting from Samuelson and Rosenthal (1986) and Brown and Raymond (1986), which provide tools to assess the markets' current sentiment regarding the success of the deal. One precise tool is suggested by Gelman and Wilfing (2007), who analyze the target stock price dynamics with Markov-Switching, assuming two different regimes with respect to market beta and idiosyncratic variance. They show that before the announcement date a regime with high market beta and high idiosyncratic volatility of target stock returns prevails, whereas after the announcement, especially after further deal supportive information arrives, target stock returns switch to a regime with a low beta and low volatility. If it is possible to perform Markov-Switching estimation of the target stock returns, then using the terminology of Gelman and Wilfing (2007), one could assume that  $\alpha_t = 0$  if the probability of the high-beta-high-volatility regime is close to 1 and inversely,  $\alpha_t = 1$  if the probability of this regime is close to zero.

The Markov-Switching approach provides also transition probabilities from one regime to another. If one can establish direct linkages between the estimated regimes and the market sentiment  $\alpha_t$  (as above), one can interpret the off-diagonal transition probabilities as the first approximation of the jump intensity parameter  $\lambda$ . E. g., the average estimate of the transition probabilities in both directions reported in Gelman and Wilfing (2007) is

1.17%. Plugging it as  $\lambda$  into Eq. (7) yields the probability of landing on the same path as plotted in Fig. (1).

$$\Pr(\alpha_{T_0} = i | \alpha_0 = i) = \frac{1}{2 - e^{-0.0117 \times T_0}}, \quad i = 0, 1.$$

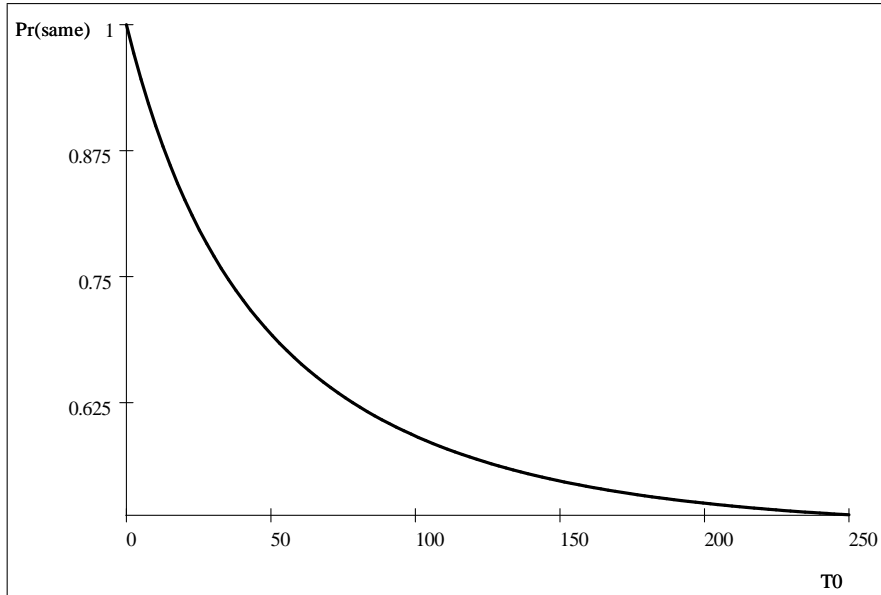


Figure 1. The probability of landing on the same path in  $T_0$

Thus, if markets are fully certain about deal completion at one point in time, its about 2:1 that they will be still sure in 60 days. However there is about 49%-probability that they will be of the opposite opinion in a year.

Instead of the pre-estimation approach one can follow Subramanian (2004) and calibrate  $\lambda$  on the set of all current option prices on the target stock with the same expiration date. Furthermore, if it turns out that the calibrated  $\lambda$  stays in a stable relation to the Markov-Switching transition probability, one could exploit it and solely rely on the analysis of the underlying dynamics after some learning period (needed to estimate the relation). It gives an opportunity, as opposed to the approach of Subramanian (2004), to price the options without extracting information from the concurrent option prices

## 5 Conclusions

In this paper we present a model for the dynamics of the target stock price in the case of mixed bids. Also, we develop an arbitrage-free framework to

price options on such stock explicitly accounting for the possibility that the deal may be called off and resumed an infinite number of times. Therefore, we model the target stock price process as a sequence of two jump diffusions: with each jump the process switches to the other stochastic diffusion path. As both paths are generated by processes with known parameters, we analytically derive the risk-neutral distribution of the future target stock price. The price of an European call option on target stock follows from this distribution. The resulting model is simpler and intuitively more tractable than the approach of Subramanian (2004). Furthermore, it is applicable for a larger scope of deals, including cash and mixed bids. Moreover, it allows not only to explain the option pricing ex-post, but also to price options given the information on past option prices and the current price of the underlying.

Empirical testing of the quality of the suggested model could be an interesting line of research. Furthermore, theoretical generalizations, such as eliminating the restriction that the bid stock volatility is unaffected by the deal, can improve its applicability.

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