Decentralised Provision of Intertemporal Public Goods with Stock Effects As Subgame Perfect Nash Equilibrium*

Anke Gerber†

Department of Economics, University of Hamburg

Philipp C. Wichardt‡

Department of Economics, University of Bonn

This Version: February 15, 2009

Abstract This paper considers the problem of decentralised provision of intertemporal public goods with stock effects and shows how the deposit based solution proposed in Gerber and Wichardt (forthcoming) applies to this case. In doing so, the present argument extends the previous results in two significant ways: First, it explicitly considers the complications that may arise in an intertemporal setting with stock effects and shows how they can be resolved. Second, the modified solution for the intertemporal case presented in this paper relies only on small ex ante deposits – large enough only to sufficiently reward full contributions in single periods; this is particularly appealing in view of applications and especially so if the respective public good exhibits stock effects.

Key words: Public Goods, Stock Effects, Climate-Change, Deposits
JEL code: H41, D02, C72

---

*Acknowledgements: We are grateful to Patrick Schmitz and Avner Shaked as well as to seminar audiences in Bielefeld, Bonn (ZEF) and Munich (LMU) for helpful comments and suggestions.

†Postal address for correspondence: Department of Economics, University of Hamburg, Von-Melle-Park 5, D-20146 Hamburg, Germany; e-mail: anke.gerber@wiso.uni-hamburg.de.

‡e-mail: philipp.wichardt@uni-bonn.de

1 Introduction

Public goods are one of the paradigm examples for the conflict between collective and individual interests. Collectively, all agents prefer having the public good to not having it despite the corresponding cost, which, as a group, they would be happy to bear. Individually, however, each single agent would prefer the others to bear of the cost and to enjoy the public good for free. Thus, it is not surprising that unregulated markets commonly fail to establish a socially optimal provision of public goods.

As a consequence of this, the optimal design of institutions and mechanisms to improve the provision of public goods have been widely discussed in the literature (e.g. Olsen, 1965; Clarke, 1971; Groves; 1973; Bagnoli and Lipman, 1989; Boadway et al., 1989; Jackson and Moulin, 1994; Varian, 1994; Kosfeld et al., forthcoming; and many others). Most of the proposed solutions, however, rely on the existence of institutions which are strong – in the sense that it can enforce transfers between the agents (e.g. taxes and/or fines) – and / or they assume that the provision of the public good can be centralised. Examples for the first category are Clarke, 1971; Groves; 1973; Boadway et al., 1989; Kosfeld et al., forthcoming; a prominent example for the latter is Bagnoli and Lipman, 1989. And although one of these assumptions is likely to be satisfied in many applications, not all relevant cases are covered. For example, present attempts to reduce global greenhouse gas emissions, which have received a lot of attention in the recent literature (e.g. Carraro, 1999; Dutta and Radner, 2004; Moslener and Requate, 2007), are obviously struggling with the decentralised provision of a (global) public good in the absence of a strong institution as there is no global government. Moreover, many local public goods which require a decentralised provision, e.g. reductions in the use of land or water resources by farmers, and which have to be provided under adverse conditions such as the lack of a reliable legal system in the background also fall into the this category.

As we have argued in a recent paper (Gerber and Wichardt, forthcoming), one possible solution for the case of decentralised provision in the absence of strong institutions is to introduce an initial deposit stage. In particular, the respective public goods game has to be modified as follows: To begin with all agents voluntarily choose whether or not to pay a deposit to some central agency, e.g. some global institution in the case of international treaties on greenhouse gas reductions or some external humanitarian organisation in the case of locally unstable conditions. – Note that both the global institution and humanitarian organisations commonly lack the power to enforce transfers between the agents. – At the end of this stage, deposits are kept if all agents have paid; otherwise they are immediately refunded. Once all deposits are paid and kept, they are only refunded to those agents who contributed their share to the public good. Accordingly, if deposits are large enough, unanimous deposit payments followed by full contributions to the public good (and eventual refunds of the deposits) constitutes a subgame perfect Nash equilibrium of the modified game.

Moreover, although somewhat more involved, a similar approach can be used for the case of intertemporal public goods which require repeated contributions over a longer time horizon. One possible solution, which we propose for a slightly simplified framework in our earlier paper, is to collect initial deposits which are large enough to reimburse the agents for their individual cost in all future periods. This seems preferable, for example, to a simple repetition of the one period game because of the transaction costs involved otherwise. A potential drawback of the "large-initial-deposits" solution, however, is that the required ex ante payments may be enormous as the deposits eventually have to cover contributions in later periods not only on the equilibrium path, i.e. if all agents continue to contribute as desired, but also off the equilibrium path. And, in particular, desired contributions off the equilibrium path may easily become exceedingly large, especially if the respective public good exhibits stock effects and the cost of its provision depends on the initial stock. Despite the theoretical validity of the "large-initial-deposits" solution, such a feature is, of course, likely to put the chance of its practical applicability beyond mitigation.

In the present paper, we therefore revisit our earlier discussion and propose a modified solution which is especially suited for the the case of intertemporal public goods with stock effects. As before, we show how outcomes that Pareto dominate the zero contribution profile can be supported as a subgame perfect Nash equilibrium of a modified game where players can conditionally commit to the public by ex ante paying some deposit to a central agency. Yet, the present discussion extends the earlier one in at least two relevant ways. First of all, the intertemporal public goods game considered is far more general; for example, it explicitly considers the possibility of stock effects. Most importantly, however, the size of the initial deposits necessary to support the implementation of the public good is substantially reduced.

According to the modified solution, deposits are simply transferred from one period to the next – in case of full compliance – and only refunded at the end of the interaction; i.e. on the equilibrium path there is no reimbursement before the final termination of the game. In case of insufficient contributions in some period, the contributors' deposits are immediately refunded, though, and new deposits are collected. Thus, players who deviate from the equilibrium path immediately forfeit their deposits. Moreover, the subsequent collection of new deposits obviates the need to cover all possible contingencies in ex ante deposits as potential changes in the targeted level of the public good can be accounted for whenever necessary. In equilibrium, initial deposits, then, only have to be marginally larger than the present value of the respective agent's net contribution cost for any (single) per period contribution on the equilibrium path in order to support full provision of the public good. The modified solution presented below, therefore, extends our earlier argument in a way that is not only theoretically valid but, hopefully, also more likely to be of practical use for intertemporal public goods problems with stock effects.

2 The Model

In this section, we present a formal discussion of our argument. To begin with, we introduce the underlying intertemporal public goods problem with stock effects. We then proceed to consider socially desired contributions and to specify a way which renders them individually rational in each single period. Finally, we show how socially desired contributions can be implemented as the outcome of a subgame perfect Nash equilibrium (SPNE) of a slightly modified game. In particular, per period interaction will be complemented by an initial deposit stage in which players can voluntarily commit to providing their share of the public good conditional on all others doing so, too.

The Underlying Public Goods Game.

As a starting point for our analysis, we consider the following intertemporal public goods game denoted by PG. There are n players, $i = 1, \ldots, n$. These players interact with each other for T periods; for ease of presentation, we assume $T = 2$.[1] In each period t, $t = 1, 2$, each player i receives an initial endowment $e_i^t > 0$ of the private good, which he can either consume or invest into the production of the public good. Player i's contribution to the public good in period t is denoted by c_i^t with $c_i^t \in [0, e_i^t]$ for all i; the profile of contributions made by all players in that period is denoted by $c^t = (c_1^t, \ldots, c_n^t)$. Player i's preferences over the per period consumption of the private and the public good are represented by a utility function $U_i : \mathbb{R}_+^2 \rightarrow \mathbb{R}$, where $U_i(x_i, z)$ is i's utility from consuming x_i units of the private good and z units of the public good. We assume that $U_i, i = 1, 2$ is strictly increasing and quasilinear in the private good, i.e.

$$U_i(x_i, z) = x_i + u_i(z), \ (x_i, z) \in \mathbb{R}_+^2 \tag{1}$$

for some differentiable function $u_i : \mathbb{R}_+ \rightarrow \mathbb{R}$ with $u_i' > 0$ and $u_i(0) = 0$.[2] Player i's utility from consuming (x_i^t, z^t) in periods $t = 1, 2$, then is given by the sum of discounted per period utility, i.e. by

$$U_i(x_i^1, z^1) + \delta U_i(x_i^2, z^2), \tag{2}$$

where $\delta = \frac{1}{1+r}$ is the common discount factor based on the risk-free interest rate $r \geq 0$.

As regards the public good, we assume that the stock z^t, which can be consumed in period t, depends on both the stock of the public good in period $t-1$ and the players' contributions in the respective period; i.e.

$$z^t = F(z^{t-1}, c^t), \ t = 1, 2, \tag{3}$$

[1] Assuming $T = 2$ is not crucial as the argument for the general case is analogous. However, the restriction to two periods avoids unnecessary technicalities which otherwise might obscure the main point of the argument.

[2] The simplifying assumption of a quasilinear utility function is not crucial for the argument to follow and is again made for expositional purposes only (a further comment on this assumption can be found in Section 3).

where z^0 is the initial stock of the public good and $F : \mathbb{R}_+^{n+1} \to \mathbb{R}$ with both $\frac{\partial F}{\partial z}(z, c) > 0$ and $\frac{\partial F}{\partial c_i}(z, c) > 0$ for $i = 1, \ldots, n$. Moreover, we assume that the nature of the public good z is such that z cannot be provided by a central agency but only through the players' decentralised contributions.

Regarding the players' incentives to contribute to the public good, we assume that, in both periods, positive contributions are strictly dominated by zero contributions for all players; i.e. for all $i = 1, \ldots, n$, any initial stock z^0 and all contribution profiles $c^t \in C^t := \times_{j=1}^n [0, e_j^t]$, $t = 1, 2$, it holds:

$$u_i'(z^2) \frac{\partial F}{\partial c_i^2}(z^1, c^2) < 1 \tag{4}$$

and

$$\frac{\partial F}{\partial c_i^1}(z^0, c^1) \left[u_i'(z^1) + \delta u_i'(z^2) \frac{\partial F}{\partial z}(z^1, c^2) \right] < 1 \tag{5}$$

where $z^1 = F(z^0, c^1)$ and $z^2 = F(z^1, c^2)$.

Finally, a strategy for player i in the public goods game PG is given by a tuple $\xi_i = (\xi_i^1, \xi_i^2)$, where $\xi_i^1 : \mathbb{R}_+ \to [0, e_i^1]$ and $\xi_i^2 : \mathbb{R}_+ \times C^1 \to [0, e_i^2]$. Hence, $\xi_i^1(z^0)$ is player i's contribution to the public good in period 1 given initial stock z^0, and $\xi_i^2(z^1, c^1)$ is his contribution in period 2 given initial stock z^1 and given c^1 was the contribution profile in period 1.

With these specifications, it follows immediately from a standard backwards induction argument that there is a unique SPNE of PG in which all players in both periods contribute nothing to the public good.

Proposition 1 *For any initial stock of the public good z^0, the unique subgame perfect Nash equilibrium of PG is given by zero contributions in all periods, i.e. $\xi_i^t \equiv 0$ for all i and $t = 1, 2$.*

Typically, the outcome with zero contributions by all players will be Pareto dominated by the outcome of some contribution plan $\overset{*}{\xi} = (\overset{*}{\xi}{}^1, \overset{*}{\xi}{}^2)$ with positive contributions by all players. Our concern in the following is how to implement $\overset{*}{\xi}$ while we are completely agnostic about how $\overset{*}{\xi}$ is selected.[3]

[3] $\overset{*}{\xi}$ may, for example, be the result of some unspecified bargaining process among the n players.

Socially Desired Contributions.
Regarding socially desired contributions in period t, $\overset{*}{\xi}^t$, we assume that these only depend on the current stock of the public good in period t; i.e.

$$\overset{*}{\xi}^t = \overset{*}{\xi}^t(z^{t-1}) = \left(\overset{*}{\xi}^t_1(z^{t-1}), \ldots, \overset{*}{\xi}^t_n(z^{t-1})\right), t = 1, 2. \tag{6}$$

For a given initial stock z^0, realised contributions according to $\overset{*}{\xi}$, then, are given by the tuple

$$(\overset{*}{\xi}^1(z^0), \overset{*}{\xi}^2(\overset{*}{z}^1)) , \tag{7}$$

where

$$\overset{*}{z}^1 = F(z^0, \overset{*}{\xi}^1(z^0)) \tag{8}$$

is the stock of the public good in period 1 if all players contribute the desired amount in $t = 1$; the corresponding stock of the public good at the end of period 2 according to $\overset{*}{\xi}$ is denoted by $\overset{**}{z}^2$, i.e.

$$\overset{**}{z}^2 = F(\overset{*}{z}^1, \overset{*}{\xi}^2(\overset{*}{z}^1)) . \tag{9}$$

The realised stock of the public good at the end of period 2, given some initial stock z^1 and socially desired contributions $\overset{*}{\xi}^2(z^1)$, is denoted by $\overset{*}{z}^2(z^1)$, i.e.

$$\overset{*}{z}^2(z^1) = F(z^1, \overset{*}{\xi}^2(z^1)) . \tag{10}$$

Moreover, we assume that, for any period t and any initial stock z^{t-1}, the corresponding socially desired contribution profile $\overset{*}{\xi}^t(z^{t-1})$ satisfies the following two Pareto conditions:

Pareto-1 For $t = 1, 2$, the contribution plan $\overset{*}{\xi}$ Pareto dominates zero contributions from period t onwards, i.e. for all i and all z^0, z^1, we have

$$U_i\left(e_i^2 - \overset{*}{\xi}^2_i(z^1), \overset{*}{z}^2(z^1))\right) > U_i\left(e_i^2, F(z^1, 0)\right) \tag{11}$$

and, defining $\hat{z}^1 := F(z^0, 0)$,

$$\begin{aligned} & U_i\left(e_i^1 - \overset{*}{\xi}^1(z^0), \overset{*}{z}^1\right) + \delta U_i\left(e_i^2 - \overset{*}{\xi}^2(\overset{*}{z}^1), \overset{**}{z}^2\right) \\ > \;& U_i\left(e_i^1, \hat{z}^1\right) + \delta U_i\left(e_i^2, \hat{z}^1\right) \; . \end{aligned} \tag{12}$$

Pareto-2 The contribution plan $\overset{*}{\xi}$ is time consistent in that it Pareto dominates the contribution profile which combines zero contribution in period 1 with the socially desired contributions $\overset{*}{\xi}^2(F(z^0,0))$ in period 2, i.e. for all i we have

$$\begin{aligned} & U_i\left(e_i^1 - \overset{*}{\xi}^1(z^0), \overset{*}{z}^1\right) + \delta U_i\left(e_i^2 - \overset{*}{\xi}^2(\overset{*}{z}^1), \overset{**}{z}^2\right) \\ > \ & U_i\left(e_i^1, \hat{z}^1\right) + \delta U_i\left(e_i^2 - \overset{*}{\xi}^2(\hat{z}^1), \overset{*}{z}^2(\hat{z}^1)\right) \ , \end{aligned} \tag{13}$$

where again $\hat{z}^1 = F(z^0, 0)$.

Put differently, while condition Pareto-1 ensures that the socially desired contribution profile leads to a Pareto improvement over zero contributions, Pareto-2 implies that players do not benefit from procrastinating the implementation of $\overset{*}{\xi}$.[4]

Having specified socially desired contributions, we proceed to ask how per period incentives would have to be modified if the socially desired was to made individually rational.

Providing Appropriate Per Period Incentives.
Obviously, one way to induce players to fully comply with the socially desired contribution is to sufficiently reward them for doing so or to sufficiently punish them for not doing so. The purpose of the subsequent discussion, therefore, is to determine what is "sufficient" in single periods. In the next step, we then show how the game can be modified in a way such that, for all players, the ex ante implementation of their own "sufficient" rewarding/punishment scheme is part of a subgame perfect Nash equilibrium.

One way to induce player i to contribute $\overset{*}{\xi}_i(z^{t-1})$ in period t given initial stock z^{t-1} is to offer him a sufficiently high reward $d_i^t(z^{t-1})$ for doing so. In particular, let $d_i^t(z^{t-1}), t = 1, 2,$ be such that for each player i and any period t contribution

[4] Note that Pareto-2 is plausible in view of applications such as climate change (or the conservation of other resources) where it is often argued that the level of the public good decreases exponentially over time if nothing is done.

profile $\xi^t_{-i}(z^{t-1})$ of the other players we have:

$$\begin{aligned} d_i^2(z^1) \;>\; & U_i\Big(e_i^2, F\left(z^1,(0,\xi^2_{-i}(z^1,c^1))\right)\Big) \\ & -U_i\left(e_i^2-\overset{*}{\xi}{}_i^2(z^1), F\left(z^1,(\overset{*}{\xi}{}_i^2(z^1),\xi^2_{-i}(z^1,c^1))\right)\right) \end{aligned} \tag{14}$$

for all period-1 contribution profiles c^1, and

$$\begin{aligned} d_i^1(z^0) \;>\; & U_i(e_i^1,\tilde z^1)-U_i(e_i^1-\overset{*}{\xi}{}_i^1(z^0),\hat z^1) \\ & +\delta\Big[U_i\left(e_i^2-\overset{*}{\xi}{}_i^2(\tilde z^1),\overset{*}{z}{}^2(\tilde z^1)\right) \\ & \qquad -U_i\left(e_i^2-\overset{*}{\xi}{}_i^2(\hat z^1),\overset{*}{z}{}^2(\hat z^1)\right)\Big] \end{aligned} \tag{15}$$

where $\tilde z^1 = F(z^0,(0,\xi^1_{-i}(z^0)))$ and $\hat z^1 = F(z^0,(\overset{*}{\xi}{}_i^1(z^0),\xi^1_{-i}(z^0)))$. Then, by definition, for any initial stock z^1 in period 2, $d_i^2(z^1)$ suffices to render $\overset{*}{\xi}{}_i^2(z^1)$ strictly dominant for player i in period 2 irrespective of the other players' contributions in that period. Similarly, by definition, $d_i^1(z^0)$ suffices to render $\overset{*}{\xi}{}_i^2(z^0)$ strictly dominant for player i in period 1 irrespective of the other players' contributions in period 1, given that play continues with the socially desired contributions in period 2. Moreover, the above condition remains unchanged if $\overset{*}{\xi}{}_i^t(z^{t-1})$ is to be implemented by fining any deviation of player i with $d_i^t(z^{t-1})$ because utility is quasilinear in the private good. Accordingly, we obtain:

Lemma 1 *For period 2 and any corresponding initial stock z^{t-1}, it is strictly dominant for all players to contribute $\overset{*}{\xi}{}_i^2(z^1)$ in period 2 if either doing so is rewarded with $d_i^2(z^1)$ or not doing so is fined with $d_i^2(z^1)$. Moreover, for period 1 and any corresponding initial stock z^0, it is strictly dominant for all players to contribute $\overset{*}{\xi}{}_i^1(z^0)$ in period 1 if either doing so is rewarded with $d_i^1(z^0)$ or not doing so is fined with $d_i^1(z^0)$, provided that period 2 contributions are given by $\overset{*}{\xi}{}^2$.*

Implementing Full Contributions with Small Ex Ante Deposits.

In the sequel, we finally show how, for any period t and any initial stock z^{t-1}, socially optimal contributions can be implemented as part of a SPNE of a modified game. In particular, let PG^* be the following modification of PG. As before, there are n players who interact with each other for 2 periods. In each period the players have to decide on their contributions to some public good in the way specified above. However, different from PG, per period interaction now is complemented by a conditional deposit stage which, if present, precedes the contribution stage of the respective period; in the deposit stage, players can voluntarily commit to the public good.

Proceedings in the deposit stage are as follows. If there is a deposit stage in period t, which by definition is always the case in period 1, players simultaneously decide whether or not to pay a certain deposit D_i^t to some central agency; we assume that the respective amounts can be borrowed at the per period riskfree interest rate r if necessary. The exact value of D_i^t depends on the initial stock z^{t-1} in period t; in particular, for any initial stock z^0

$$D_i^1(z^0) := \max\{d_i^1(z^0), \delta d_i^2(\overset{*}{z}{}^1)\} \tag{16}$$

and for all z^1

$$D_i^2(z^1) := d_i^2(z^1) \; ; \tag{17}$$

i.e. $D_i^t(z^{t-1})$ corresponds to the maximum present value of all rewards $d_i^\tau(z^{\tau-1})$ necessary to induce full contributions in period τ, $t \leq \tau \leq 2$. If all players decide to pay $D_i^t(z^{t-1})$, deposits are kept by the central agency and the deposit stage is said to be *completed successfully*. If some player does not pay the required deposit, then all deposits paid are immediately refunded and the deposit stage is not completed successfully. After the deposit stage, the game moves on to the contribution stage of period t.

However, the exact proceedings in any period-t contribution stage of PG^* now depend on the players' earlier deposit decisions. In particular, if the deposit stage was not completed successfully in any previous period $\tau \leq t$, then the contribution stage in period t is exactly as specified in PG and afterwards the game moves on to period $t+1$, if $t < 2$, or terminates if $t = 2$. If the deposit stage is completed successfully in some period t, though, things change. First of all, deposits are deducted from the players' period t endowment of the private good. Moreover, the players' period t contributions to the public good now have more complex consequences. In particular, if $c_i^t \neq \overset{*}{\xi}{}_i^t(z^{t-1})$ for some player i, this player forfeits his deposit, while the deposit of any player i with $c_i^t = \overset{*}{\xi}{}_i^t(z^{t-1})$

is refunded; for the sake of argument, we assume that forfeited deposits are "burned" – despite the negative effect on greenhouse gas emissions.[5] Moreover, if $t = 1$ (and $c_i^t \neq \overset{*t}{\xi}_i(z^{t-1})$ for some player i), period 2 again starts with a new deposit stage; if $t = 2$ the game simply terminates. By contrast, if $c^t = \overset{*t}{\xi}(z^{t-1})$, i.e. if all players contribute the socially desired amount to the public good in period t, then

a) if $t = 2$, the central agency refunds *all* deposits and the game terminates;

b) if $t = 1$,

- the central agency keeps a share $\delta d_i^2(\overset{*1}{z})$ of each player's deposit ($\overset{*1}{z} = F(z^0, \overset{*1}{\xi}(z^0))$), refunds $\Delta_i^1 = D_i^1(z^0) - \delta d_i^2(\overset{*1}{z}) \geq 0$ (this is feasible by definition of D_i^1) and then the game moves on to period 2;
- remaining deposits are transferred from period 1 to period 2, thereby earning interest according to the common market rate r so that at the start of period 2 the central agency is endowed with deposits $D_i^2(\overset{*1}{z})$ for each player i;
- period 2 immediately starts with the contribution stage the proceedings of which are as described above for the case with full deposit payments in some period t.

Summing up, PG^* differs from PG in that per period interaction in PG^* is complemented by an conditional deposit stage which, if present, precedes the contribution stage of the respective period and in which players can voluntarily commit to the public good. The commitment is binding only if made by all players; otherwise all deposits are refunded and the next period starts with a new deposit stage. Once deposits are paid by all players, individual deviations from the socially desired level are costly in that defectors loose their deposit. Moreover, in case of deviations from the socially desired level, all contributors' deposits are refunded so that the previous general commitment is lost and the next period starts with a new deposit stage. If all deposits are paid in some period and full contributions are made from then on, no further deposit stages

[5] For a discussion of alternative, more efficient ways to use forfeited deposits, such as refunds to contributors, see Gerber and Wichardt (forthcoming).

are conducted. Thus, the deposit stage is present with certainty only in period 1 and takes place in period 2 if and only if

- either the deposit stage was not completed successfully in period 1,
- or the deposit stage was completed successfully in period 1, but $c^1 \neq \overset{*}{\xi}{}^1(z^0)$; i.e. at least some player did not make the desired contribution in period 1, so that deposits were refunded and no deposits are kept by the central agency at the start of period 2.

Technically, the presence of the deposit stage in period t is determined by an index variable ψ^{t-1} which can take values either 1 or 0. By definition $\psi^0 := 0$, $\psi^1 := 1$ if and only if all deposits were paid and $c^1 = \overset{*}{\xi}{}^1(z^0)$ in period 1, and $\psi^1 := 0$ otherwise. Thus, ψ^{t-1} indicates whether the central agency at the start of period t already possesses the necessary deposits ($\psi^{t-1} := 1$) or not ($\psi^{t-1} := 0$).

Finally, regarding the players' strategies in PG^*, these now have to specify two things:

1. a deposit decision $\theta_i^t \in \{0, D_i^t(z^{t-1})\}$ for any period t with $\psi^{t-1} = 0$ and any z^{t-1};
2. a contribution decision for each period t which is now conditional on both
   a) the initial stock z^{t-1}
   b) the history of the game, i.e. in period 2 the players' period 1 contributions c^1, and
   c) whether or not the central agency is in the possession of adequate deposits for all players (which is the case if either $\psi^{t-1} = 1$ or $\psi^{t-1} = 0$ and $\theta_i^t = D_i^t(z^{t-1})$ for all i).

Figure 1 provides a graphical summary of the overall proceedings in PG^*.

Different from PG, then, full contributions to the public good from period 1 onwards are part of a SPNE of PG^* in which all players immediately pay the required deposits and make the full socially desired contributions to the public good in subsequent each period.

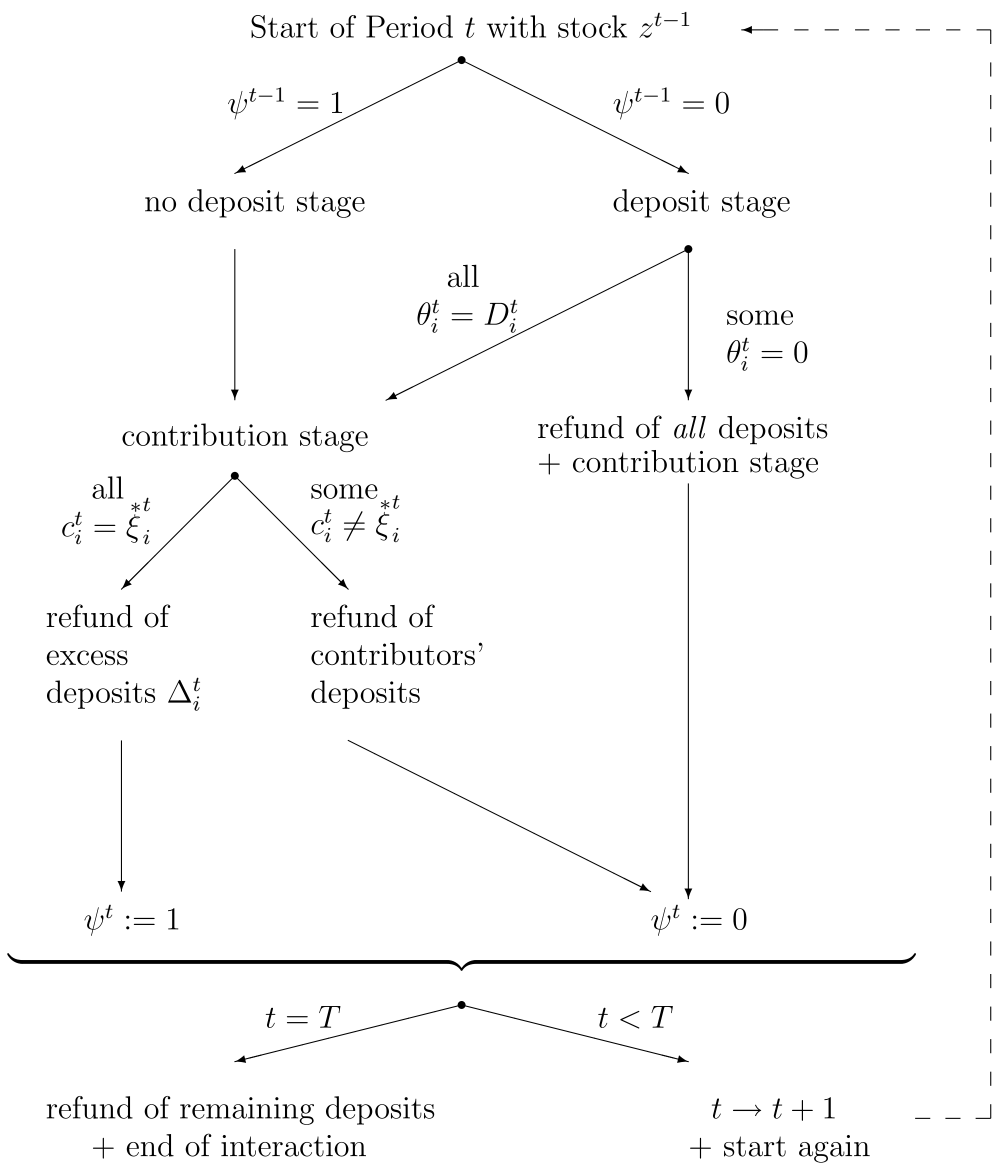

Figure 1: Graphical illustration of per period proceedings in PG^* ($T = 2$). The initial deposit stage is conditional on $\psi^{t-1} = 0$; $\psi^0 = 0$ by definition. If $\psi^{t-1} = 0$ and $\theta_i^t = 0$ for some i, then all paid deposits are refunded, contributions are made, and $\psi^t := 0$. If either $\psi^{t-1} = 0$ and $\theta_i^t = D_i^t$ for all i or $\psi^{t-1} = 1$, then: (a) if $c^t = \overset{*}{\xi}{}^t(z^{t-1})$, only excess deposits ($\Delta_i^t$) are refunded and $\psi^t := 1$; (b) if $c^t \neq \overset{*}{\xi}{}^t(z^{t-1})$, all contributors' deposits are refunded and $\psi^t := 0$. Finally, if $t < T$, interest is paid on any remaining deposits and period $t+1$ is entered; if $t = T$ all remaining deposits are refunded (without further interest) and PG^* terminates.

Proposition 2 *Let PG^* be as specified above. Moreover, let $\overset{*}{\xi}$ satisfy conditions Pareto-1 and Pareto-2 specified in Equations (11), (12) and (13), and let $d_i^t(z^{t-1})$ satisfy Equation (14), resp. (15). Then the following strategy profile constitutes a subgame perfect Nash equilibrium of PG^*:*

For each player i and every period t with initial stock z^{t-1}

- *if $\psi^{t-1} = 0$, i.e. if there is a deposit stage in period t, then*

$$\theta_i^t = D_i^t(z^{t-1})$$

- *period t contributions are given by:*

$$c_i^t = \begin{cases} \overset{*t}{\xi}_i(z^{t-1}), & \text{if } \psi^{t-1} = 1 \\ & \text{or } \psi^{t-1} = 0 \text{ and } \theta_j^t(z^{t-1}) = D_j^t(z^{t-1}),\ j = 1, \ldots, n \\ 0, & \text{else} \end{cases}$$

Moreover, on the equilibrium path there only is a deposit stage in period 1, which is completed successfully, and realised contributions are given by $(\overset{*1}{\xi}(z^0), \overset{*2}{\xi}(\overset{*1}{z}))$, i.e. in all periods all players make the socially desired contributions to the public good.

Lemma 2 *The equilibrium strategies specified in Proposition 2 are stage-wise dominant in the following sense: The equilibrium contributions to the public good are strictly dominant in each period t, given that play continues with the equilibrium strategies in all future periods $\tau > t$ (if any), and the deposit decision is weakly dominant in each period t, given that players' period t contributions to the public good are given by their equilibrium strategies and given that play continues with the equilibrium strategies in all future periods $\tau > t$ (if any).*

In order to see why the statements of Proposition 2 and Lemma 2 are correct, the main point to note is the following: Once all players have paid the required deposit in some period t, failing to contribute the socially desired amount in period t costs the respective player more (his deposit) than it earns (the cost of the contribution) provided that play continues in period $t+1$ as specified in the proposition; this simply follows from the choice of the deposits (cf. Lemma 1). Accordingly, full contributions in period t are the best each player can do as long as he (rationally) expects play in any future period $\tau > t$ to follow the equilibrium strategies. Moreover, as deposits are transferred to the next period after full contributions to the public good, the incentive structure naturally repeats itself. Thus, all players paying the required deposits in some period induces full contributions to the public good from then on. Furthermore, by definition, the socially desired contribution profile Pareto dominates the zero contribution profile in any period and for any initial stock in that period (cf. Pareto-1 and Pareto-2). Consequently, everyone paying his deposit whenever there is a deposit stage and full socially desired contributions to the public good thereafter is a SPNE of PG^* as it also supports the players' assumed expectations about play in future periods. The following proof formalises this intuitive argument.

Proof. In order to formally prove the statement of Proposition 2, we give a backward argument starting in period $T = 2$. The proof also covers the additional claim in Lemma 2.

Period $t = 2$: There are two cases we have to distinguish for period 2: (a) $\psi^1 = 0$ (there is a deposit stage in period 2) and (b) $\psi^1 = 1$ (there is no deposit stage in period 2). Consider case (a) first; this case effectively corresponds to the 1-period public goods game considered in Gerber and Wichardt (forthcoming). By construction, for any initial z^1, if all players i have paid the deposit $D_i^2(z^1)$, contributing $c_i^2 = \overset{*}{\xi}{}_i^2(z^1)$ in the contribution stage of period 2 is strictly dominant for each player i; this is an immediate consequence of Lemma 1 and the fact that any player i choosing $c_i^2 \neq \overset{*}{\xi}{}_i^2(z^1)$ forfeits his deposit. Moreover, if there exists at least one player i who has not paid the deposit $D_i^2(z^1)$ all deposits are refunded immediately and playing $c_i^2 = 0$ in the contribution stage of period 2 is strictly dominant for each player i. Then, given z^{t-1}, $\theta_i^2 = D_i^2(z^1)$ is weakly dominant for each player i in the deposit stage of period 2 as by assumption $U_i(e_i^2, F(z^1, 0)) < U_i(e_i^2 - \overset{*}{\xi}{}_i^2(z^1), \overset{*}{z}{}^2(z^1))$ for all i; $\theta_i^2 = D_i^2(z^1)$ is only weakly dominant because the own deposit payment has no effect as long as at least some other player does not pay his deposit. Finally, in case (b), i.e. if $\psi^1 = 1$, the incentive structure in the contribution stage of period 2 is identical to the situation with a successfully completed deposit stage discussed above so that again $c_i^2 = \overset{*}{\xi}{}_i^2(z^1)$ is strictly dominant for all i. Hence, the actions specified in

Proposition 2 for period 2 constitute a Nash equilibrium for period 2 irrespective of the initial stock z^1 of the public good in that period.

Period $t = 1$: Now consider period $t = 1$ and assume that play after period 1 continues as described in Proposition 2; from the above analysis for $t = 2$ we already know that this constitutes a Nash equilibrium in any subgame of PG^* starting in period 2.[6] Since $\psi^0 = 0$ (by definition) there is a deposit stage in period 1. Moreover, if all players pay the deposit $D_i^1(z^0)$, contributing $c_i^1 = \overset{*}{\xi}{}_i^1(z^0)$ in period 1 is strictly dominant for all i provided that play continues in period 2 as specified in the proposition. The dominance immediately follows from Lemma 1 and the choice of $D_i^1(z^0)$ (Equation 16), because deviating players lose $D_i^1(z^0) \geq d_i^1(z^0)$ in period 1 while contributors lose $D_i^1(z^0)$ in period 1 but then either receive $D_i^1(z^0)$ at the end of period 1 (if some other player does not contribute) or they receive Δ_i^1 at the end of period 1 and $d_i^2(\overset{*}{z}{}^1)$ at the end of period 2 (if all players fully contribute in period 1 and play continues as specified in Proposition 2); note that both refund schemes are equivalent as utility is quasilinear in the private good and $D_i^1(z^0) = \Delta_i^1 + \delta d_i^2(\overset{*}{z}{}^1)$ by definition. Also, by the same token as before, $c_i^1 = 0$ is strictly dominant for all i if some player i has not paid the deposit $D_i^1(z^0)$. Consequently, paying $D_i^1(z^0)$ again is weakly dominant for each player i in the deposit stage of period 1, provided that play continues in period 2 as specified in the proposition, because from Pareto-2 it follows that implementing $\overset{*}{\xi}{}^1(z^0)$ in period 1 is strictly better than $c_i^1 = 0$ for all i and implementing $\overset{*}{\xi}{}^2(z^1)$ in period 2 (which would happen otherwise according to the strategies specified in the proposition).

The remaining statement about the action profile on the equilibrium path is immediate. ■

3 Discussion

This section is devoted to a discussion of the solution proposed above for intertemporal public goods games with stock effects. First, we reconsider the assumption of quasilinear utility fuctions. In a second step, we discuss a modification in the requirement on per period deposits which yields a strengthening of the dominance

[6] For a general proof (i.e. $T > 2$) we would have give a complete backwards induction argument; i.e. we would consider a general period t here, for some $t < T$, and assume that play in all later periods was as specified in the proposition.

argument about individual contributions albeit at the expense of the deposits' size. Third, we argue how the new solution that only relies on small ex ante deposits compares to the "large-initial-deposits" solution proposed in Gerber and Wichardt (forthcoming) for a simplified intertemporal public goods game. Fourth, we consider in how far a possible additional participation decision for the whole mechanism would affect our argument. Finally, the section concludes with a brief comment on issues regarding renegotiation proofness.

The Effects of Quasilinear Preferences.
For the purpose of the preceding discussion we have assumed preferences to be quasilinear in the private good; cf. Equation (1). The assumption simplified the argument in so far as it allowed to freely transfer amounts of the private good between periods without affecting the player's utility, given that the interest is added or deducted appropriately. However, the assumption of quasilinear utility is not crucial for the argument at large. In fact, Proposition 2 holds for general utility functions if deposits are chosen appropriately. The conditions expressed in (14) and (15) would look different in this case but the later argument remains essentially the same.

Dominance of Contributions vs. Deposit Size.
In Equations (14) and (15) deposits were chosen such that in each period it is a strictly dominant strategy for all players to contribute the desired amount to the public good *if future play follows the equilibrium strategies* (cf. Proposition 2). Although this requirement is sufficient to prove Proposition 2, it may nonetheless be desirable to strengthen the dominance requirement for equilibrium contributions in certain settings thereby making per period desired contribution a strictly dominant strategy for any player *independent of the actions chosen in future periods*. This stronger requirement can be met if, instead of (14) and (15), we impose the following conditions on deposits: For any contribution profile c^1 in period 1 and any strategy profile ξ^2 in period 2,

$$\begin{aligned} d_i^2(z^1) \quad > \quad & U_i\Bigg(e_i^2 - \xi_i^2(z^1, c^1), F\left(z^1, \xi^2(z^1, c^1)\right)\Bigg) \\ & -U_i\left(e_i^2 - \overset{*}{\xi}{}_i^2(z^1), F\left(z^1, (\overset{*}{\xi}{}_i^2(z^1), \xi_{-i}^2(z^1, c^1))\right)\right) \qquad (18) \end{aligned}$$

for all z^1, and

$$\begin{aligned} d_i^1(z^0) \quad > \quad & U_i\left(e_i^1 - c_i^1, \tilde{z}^1\right) - U_i\left(e_i^1 - \overset{*}{\xi}{}_i^1(z^0), \hat{z}^1\right) \\ & + \delta\left[U_i\left(e_i^2 - \xi_i^2(\tilde{z}^1, c^1), F\left(\tilde{z}^1, \xi^2(\tilde{z}^1, c^1)\right)\right)\right. \\ & \qquad \left. - U_i\left(e_i^2 - \xi_i^2(\hat{z}^1), F\left(\hat{z}^1, \xi^2(\hat{z}^1, c^1)\right)\right)\right] \end{aligned} \tag{19}$$

where $\tilde{z}^1 = F(z^0, c^1)$ and $\hat{z}^1 = F(z^0, (\overset{*}{\xi}{}_i^1(z^0), \xi_{-i}^1(z^0)))$. Then, for any initial stock z^1 in period 2, player i is reimbursed by $d_i^2(z^1)$ for contributing $\overset{*}{\xi}{}_i^2(z^1)$ in period 2 irrespective of the other players' contributions in that period. Similarly, player i is reimbursed by $d_i^1(z^0)$ for contributing $\overset{*}{\xi}{}_i^1(z^0)$ in period 1 irrespective of the other players' contributions in period 1 and irrespective of how play continues in period 2. Lemma 1, then, can be reformulated as follows:

Lemma 3 *For any period t, $t = 1, 2$, and any corresponding initial stock z^{t-1}, it is strictly dominant for all players to contribute $\overset{*}{\xi}{}_i^t(z^{t-1})$ if either doing so is rewarded with $d_i^t(z^{t-1})$ or not doing so is fined with $d_i^t(z^{t-1})$, where $d_i^t(z^{t-1})$ satisfies Equation (18), resp. (19).*

Moreover, using Lemma 3 the proof of the following proposition is analogue to the proof of Proposition 2:

Proposition 3 *Let PG^* be as specified in Section 2 with $\overset{*}{\xi}$ satisfying conditions Pareto-1 and Pareto-2. Moreover, let $d_i^t(z^{t-1})$ satisfy Equation (18), resp. (19). Then, the following strategy profile constitutes a subgame perfect Nash equilibrium of PG^*:*

For each player i and every period t with initial stock z^{t-1}:

- *if $\psi^{t-1} = 0$, i.e. if there is a deposit stage in period t, then*

$$\theta_i^t = D_i^t(z^{t-1})$$

- *period t contributions are given by:*

$$c_i^t = \begin{cases} \overset{*}{\xi}{}_i^t(z^{t-1}), & \textit{if } \psi^{t-1} = 1 \\ & \textit{or } \psi^{t-1} = 0 \textit{ and } \theta_j^t(z^{t-1}) = D_j^t(z^{t-1}),\ j = 1, \ldots, n \\ 0, & \textit{else} \end{cases}$$

Moreover, if the deposit stage was successfully completed in period t, then for any player i it is a strictly dominant strategy to contribute $\overset{*}{\xi}{}_i^t$ in period t and $\overset{*}{\xi}{}_i^\tau$ in all periods $\tau > t$, if there is no deposit stage in period τ.[7]

The advantage of the statement of Proposition 3 compared to that of Proposition 2 lies in the fact that the additional dominance of the desired contributions to the public good renders it an "almost" dominant strategy for all players to pay the required deposit: As long as a player believes that his opponents will never play a strictly dominated strategy — which is a minimal requirement on the opponents' rationality — paying the required deposit is a weakly dominant strategy.[8] The reason is that once everyone has paid the required deposit, a player can be sure that everyone will contribute the desired amount to the public good and by Pareto-1 and Pareto-2 the resulting outcome is strictly better than the outcome that results if the player does not pay the deposit.

However, introducing the additional dominance requirement regarding desired contributions is not for free. Instead, it immediately increases the size of the required deposit as more future contingencies have to be covered by the "threat" to lose ones deposit. To see this consider the special case of a linear public goods game, where

$$U_i(x_i, z) = x_i + az, i = 1, \ldots, n, \tag{20}$$

for some a with $\frac{1}{n} < a < 1$. Assume that all players i have identical endowments

[7]Observe that once the deposit stage is successfully completed in period t, there will be no deposit stage in any period $\tau > t$ if all players play their strictly dominant strategy, i.e. contribute the desired amount to the public good in each period $\tau \geq t$.

[8]If a player expected the opponents to always pay the required deposits but to nonetheless never contribute to the public good (which is strictly dominated), not paying the deposit would of course still be the best response.

$e_i^t = e > 0$ in all periods $t = 1, \ldots, T$, and assume that

$$F(z,c) = \sum_{i=1}^{n} c_i \tag{21}$$

for all z and all contribution profiles c, i.e. the public good is given by the sum of the players' contribution and there are no stock effects. Then, it is immediate to see that $\overset{*}{\xi} = (\overset{*1}{\xi}, \ldots, \overset{*T}{\xi})$ with $\overset{*t}{\xi}_i(z^{t-1}) = e$ for all z^{t-1}, all t and all i satisfies the corresponding versions of Pareto-1 and Pareto-2 for arbitrary $T \geq 2$. In order to implement $\overset{*}{\xi}$ using the T-period version of the mechanism defined in Section 2, $d_i^t = d_i^t(z^{t-1})$ has to satisfy

$$d_i^t > (1-a)e \tag{22}$$

(cf. the corresponding versions of Equations (14) and (15) for arbitrary $T \geq 2$). Hence, the period t deposit, D_i^t, that is necessary in order to implement full contributions of the public good *in subgame perfect Nash equilibrium* (cf. Proposition 2) has to satisfy

$$D_i^t = d_i^t > (1-a)e \tag{23}$$

for all $t = 1, \ldots, T$. In particular, the initial deposit in period 1 for any player i has to satisfy

$$D_i^1(z^0) > (1-a)e. \tag{24}$$

By contrast, the corresponding versions of Equations (18) and (19) for arbitrary $T \geq 2$ require d_i^t to satisfy

$$\begin{aligned} d_i^t &> e(1-a)\sum_{\tau=0}^{T-t}\delta^\tau + ea(n-1)\sum_{\tau=1}^{T-t}\delta^\tau \\ &= e(1-a)\frac{1-\delta^{T-t+1}}{1-\delta} + ea(n-1)\frac{\delta-\delta^{T-t+1}}{1-\delta}. \end{aligned} \tag{25}$$

[9]

To understand (25) observe that the reward d_i^t has to be large enough to cover the maximum possible utility loss player i can suffer when contributing e to the public good in period t compared to the case where he does not contribute e in t. Now, the worst thing that can happen to player i after contributing in period t is that in all future periods $\tau > t$ he contributes e to the public good, while everyone

[9] By definition $\sum_{\tau=1}^{T-t}\delta^\tau = 0$ if $t = T$.

else contributes zero. The best that can happen to player i after not contributing e in period t is that in all future periods $\tau > t$ everyone else contributes e, while he himself contributes zero. Hence, the maximum aggregate future loss in utility from contributing e in period t is $(e + a(n-1)e)\sum_{\tau=1}^{T-t}\delta^{\tau} - ae\sum_{\tau=1}^{T-t}\delta^{\tau}$. Adding this to the maximum loss in period t itself (which is given by $e(1-a)$) yields Equation (25).

Hence, the period t deposit, D_i^t, that is necessary in order to implement full contributions of the public good *in dominant strategies* (cf. Proposition 3) has to satisfy

$$D_i^t = d_i^t > e(1-a)\frac{1-\delta^{T-t+1}}{1-\delta} + ea(n-1)\frac{\delta-\delta^{T-t+1}}{1-\delta}. \tag{26}$$

Accordingly, the initial deposit in period 1 has to satisfy

$$D_i^1 > e(1-a)\frac{1-\delta^{T}}{1-\delta} + ea(n-1)\frac{\delta-\delta^{T}}{1-\delta}. \tag{27}$$

Since

$$e(1-a)\frac{1-\delta^{T}}{1-\delta} > e(1-a) \tag{28}$$

it is obvious that the implementation of full contributions in dominant strategies requires higher deposits than the implementation in subgame perfect equilibrium only. For example, if $\delta = 0.95$ and T is large, then the required deposit in case of implementation in dominant strategies is more than 20 times as large as the required deposit in case of implementation in subgame perfect equilibrium. Hence, there is a trade-off between the size of the deposit and the robustness of the implementation result. But in any case the required deposit is smaller than the one in the "Large-Initial-Deposit" solution proposed in Gerber and Wichardt (forthcoming) as we will argue in the following.

Comparison with the "Large-Initial-Deposits" Solution

The solution proposed in Gerber and Wichardt (forthcoming) for a linear intertemporal public goods problem as analysed above requires that the deposits collected in any deposit stage suffice to reward players *in each single subsequent period* in which they make the full socially desired contribution. Hence, the required initial deposit for player i is

$$\widehat{D}_i^1 = \sum_{t=1}^{T} \delta^{t-1} d_i^t, \tag{29}$$

where d_i^t either satisfies (22) or (25) depending on whether the implementation is in subgame perfect equilibrium or in dominant strategies. From Equations (23) and (26) it is immediate that $\widehat{D}_i^1 > D_i^1$, the latter being the required initial deposit in the mechanism proposed in this paper.

The advantage of the "large initial deposits mechanism" as proposed in Gerber and Wichardt (forthcoming) is that, once the deposit stage has been completed successfully, there never will be another deposit stage – even if some player fails to make the desired contribution in some period. Defecting players will simply forfeit their refund for the respective period but no collection of new deposits is necessary as this contingency was already accounted for in the large initial deposits; i.e. the remaining deposits still suffice to provide the required rewards from that period onwards given the resulting stock of the public good. The disadvantage of the large initial deposits mechanism, however, is that in early periods large amounts of money are bound to the central agency. And, once the common frictions on financial markets are acknowledged, this of course will be costly for the players. Accordingly, requiring huge initial deposits, even if repaid with interest later, is likely to have a tangible deterrent effect on the players' willingness to commit to the public good in particular in early periods.

Moreover, the deterrent effect of the cost of large initial deposits is particularly problematic in cases where socially desired contributions satisfy *increasing urgency* in the sense that the required contributions are weakly decreasing in the current stock of the public good but weakly increasing in the time parameter t; i.e. if, for all t, for all z and all i, we have:

$$\frac{\partial \overset{*}{\xi}{}_i^t(z)}{\partial z} < 0 \tag{30}$$

and

$$\overset{*}{\xi}{}_i^t(z) < \overset{*}{\xi}{}_i^{t+1}(z)\ . \tag{31}$$

In effect, increasing urgency essentially implies that the failure to comply in some period does not lessen one's burden in future periods. And, indeed, from an applied point of view this is rather plausible to assume (consider e.g. reductions in green house gas emissions). Yet, if the public good in question satisfies increasing urgency, the individual cost of the socially desired contributions in later periods may become large as the possibility that there were no sufficient contributions in earlier periods has to be accounted for. Accordingly, the required rewards $d_i^t(z^{t-1})$ may easily become large off the equilibrium path. As a consequence, the initial deposits $\widehat{D}_i^1$, which are the sum of maximum of the future rewards $\max_{z^{t-1}} d_i^t(z^{t-1})$ for all periods t, may become enormous.

The present solution, by contrast, remedies this deficiency. In particular, it only necessitates the initial payment of deposits $D_i^t(z^{t-1})$ which correspond to the present value of the largest reward necessary in any future period (cf. Equations 16 and 17). Thus, only the largest present value of the deposits required for later periods is taken instead of adding up all present values. Moreover, future periods are considered based on the most positive assumption regarding the development of the public good, namely that all players always contribute the full desired amount. Potentially cost-raising effects of inferior contributions in some period are only accounted for once they have been observed. Admittedly, accounting for such deviations, then, requires a new deposit stage in the following period which may be costly itself. However, as insufficient contributions and the resulting additional deposit stage are an off-equilibrium event, this nevertheless seems preferable in view of applications – in particular for intertemporal public goods that cover a long horizon (T large) and for which socially desired contributions exhibit increasing urgency.

Voluntary Participation.

For our discussion in Section 2 we have taken as given the set of players in PG. Based on this, we then have argued how a socially desired level of the public good in question can be implemented if all these players participate in a modified game PG^* where they first have to decide whether or not they commit to the later provision of the public good. Moreover, we have argued that in this case initial commitment by all players followed by full contributions to the public good is a SPNE of the modified game.

However, a valid question to ask is what happens if participation in the whole game is voluntary. For example, it might be that the participation of a subset of the set of all players already suffices in order to implement a level of the public good that Pareto dominates the zero contribution profile. If players then could opt out before the game starts, the remaining players may still have an incentive to implement the public good and the players who do not participate free-ride on the contribution of others.

Note however, that voluntary participation does not prevent the provision of the public good using the mechanism we have proposed in this paper. If we add a stage 0 to our game, where players simultaneously decide whether to participate in the mechanism, conditional on the set of players who participate, then it is straightforward to show that there exists a SPNE, where some group of players participates and contributes to the public good.[10]

Renegotiation-Proofness
A potential problem of many SPNE of repeated games is that they are supported by off equilibrium path threats that are costly for all and not only the deviating player. In such cases, it seems plausible to assume that players rather renegotiate and follow some new equilibrium path which Pareto dominates the execution of the costly of path threats instead of sticking to their earlier strategies. If such renegotiations were possible, however, this would immediately undermine the ex ante credibility of the SPNE.

Addressing the issue of renegotiations, Farrell and Maskin (1989) introduce the notion of renegotiation-proofness for SPNE of repeated games. While, strictly speaking, we cannot apply this concept here since both PG and PG^* are not repeated games (due to the stock effects), we can nevertheless observe that renegotiations are not an issue in our case because any punishment that is entailed in PG^* is not costly at the moment of its execution. In fact, players voluntarily pay their initial deposits and, once some player deviates from the socially desired contribution, he simply forfeits his claims to get his deposit back. No potentially costly ex post collection of fines is necessary. Thus, renegotiations in the sense considered by Farrell and Maskin are not an issue here.

[10]Cf. Kosfeld et al., forthcoming, who study such a participation stage in the context of the provision of public goods if sanctioning of free-riders is possible.

4 Conclusion

In the present paper, we have addressed the problem of a decentralised provision of an intertemporal public good with stock effects. Extending our earlier discussion (Gerber and Wichardt, forthcoming), we have argued how the provision such a public good can be implemented as a subgame perfect Nash equilibrium of a modified game in which players pay an ex ante fee. This fee serves as a credible commitment to the public good although it essentially only covers the respective player's contribution cost in a single period.

Apart from considering a more general framework, the main contribution of the present paper compared to its predecessor lies in the reduction of the size of the ex ante deposits which are necessary to implement the desired contributions to the public good. As we have argued in Section 3, the effect is particularly visible in cases where the public good in question is one with stock effects and where socially desired contributions exhibit increasing urgency, i.e. where failure to implement the public good in early periods reduces the players' welfare by increasing the cost of later contributions. As both stock effects of the public good and increasing urgency of socially desired contributions are present in many applications, in particular when it comes to the provision or preservation of common resources (e.g. a stable climate), we hope that the present discussion provides some helpful insights for the practical solution of such problems.

References

Bagnoli, M., Lipman, B.L., 1989. "Provision of Public Goods: Fully Implementing the Core Through Private Contributions," The Review of Economic Studies 56, pp. 583-601.

Boadway, R., Pestieau, P., Wildasin, D., 1989. "Tax-Transfer Policies and the Voluntary Provision of Public Goods," Journal of Public Economics 39, pp. 157-176.

Carraro, C., 1999. *International Environmental Agreements on Climate Change*, Kluver Academic Publishers, Dordrecht, The Netherlands.

Clarke, E.H., 1971. "Multipart Pricing of Public Goods," Public Choice 11, pp. 17-33.

Dutta, P., Radner, R., 2004. "Self-enforcing climate change treaties," Proceedings of the National Academy of Sciences 101, pp. 5174-5179.

Farrell, J., Maskin, E., 1989. "Renegotiation in Repeated Games," Games and Economic Behaviour 1, pp. 327-360.

Gerber, A., Wichardt, P., forthcoming. "Providing Public Goods in the Absence of Strong Institutions," Journal of Public Economics, article in press.

Groves, T., 1973. "Incentives in Teams," Econometrica 41, pp. 617-631.

Jackson, M., Moulin, H., 1992. "Implementating a Public Project and Distributing its Cost," Journal of Economic Theory 57, pp. 125-140.

Kosfeld, M., Okada, A., Riedel, A., forthcoming, "Institution Formation in Public Goods Games," American Economic Review, article in press.

Moslener, U., Requate, T., 2007. "Optimal Abatement in Dynamic Multi-Pollutant Problems When Pollutants Can Be Complements or Substitutes," Journal of Economic Dynamics and Control 31, pp. 293-2316.

Olsen, M., 1965. *The Logic of Collective Action*, Harvard University Press, Cambridge, MA.

Varian, H.R., 1994. "Sequential Contributions to Public Goods," Journal of Public Economics 53, pp. 165-186.