R&D Tournaments with Entry Fee Auction (Work in Progress - Do not cite)[1]

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Contents

Abstract

A procurer needs an innovative good that can be provided by a number of heterogeneous sellers. Innovations are random but depend on unobservable effort and privately known types. We compare two procurement mechanisms: R&D tournaments with entry auction where an innovation is procured either employing a fixed prize or a first–price auction. We demonstrate existence of Bayesian Nash equilibria such that all players are indifferent between both mechanisms. In these equilibria players choose the same effort and the same innovations are produced.

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1. INTRODUCTION

Consider a procurement problem where a buyer needs an innovative good that can potentially be provided by many innovators. An innovation of any quality serves the procurer's needs but the procurer's profit is increasing in innovation quality.

Innovation quality is random but expected quality is increasing in the innovator's ability and R&D effort; both in the sense of first–order stochastic dominance. Ability (or type) is private information of the seller. Ability and effort are not observable.

The buyer employs a procurement mechanism that selects (a number of) innovators, provides incentives to innovate, and, finally, procures an innovation. We will consider a profit–maximizing buyer as well as a buyer who maximizes welfare.

The literature has repeatedly analyzed the profitability of two prominent R&D procurement mechanisms: fixed–prize R&D tournaments and tournaments after which an innovation is bought using a first–price (or first–score) auction.[1] These mechanisms are feasible under the usual assumption that innovation quality is not verifiable and thus not contractible. In a fixed–prize tournament, a prize is paid in return for the best innovation that is delivered at some due date. In the first–price auction, each innovator submits an innovation and a financial bid from which the procurer computes a score. Then the highest score wins and the winner receives his financial bid.

While Fullerton et al. (2002) and Che and Gale (2003) demonstrate the general superiority of the first–score auction relative to a fixed–prize tournament, Schöttner (2008) shows that a fixed–prize tournament can outperform the first–score auction. All of the above assume that entry fees are not feasible and that the sellers' types are known.

In contrast, Fullerton and McAfee (1999) consider a fixed–prize tournament problem where types are unknown. In that setting, it is vital to select the right participants for the tournament. Fullerton and McAfee (1999) propose the use of an entry auction. If carefully designed, that auction should (efficiently) identify the most able sellers. Those sellers would submit the highest bids since they expect to be the most successful in the tournament. In addition, an auction generates revenue for the procurer and restricts entry to the tournament, which is generally optimal.

[1] The first– (second–) score auction is a two–dimensional equivalent of a first– (second–) price auction. A bid has a price and a quality dimension that are combined to a score. The highest score wins the auction. See Che (1993) for an analysis of those formats in a procurement setting.

In a model that is close to (but not the same as) that of Fullerton and McAfee (1999), we compare the performance of the two mechanisms (fixed prize and first–score auction) when types are unknown and an endogenous entry fee, in the form of an entry auction, is feasible.

Our main finding is the existence of Bayesian Nash equilibria where all players are indifferent between both mechanisms. Moreover, in these equilibria the mechanisms implement the same efforts and innovations. As a corollary, revenue equivalence of the mechanisms follows.[2] Existence of these equilibria depends on appropriate choice of the model parameters.

An important issue is the optimal number of innovators (or contestants). Generally, it is optimal to restrict entry to the tournament stage. The procurer faces an tradeoff: The expected quality of the best innovation as well as total cost increase in the number of innovators engaging in R&D. It is intuitive that, potentially, any number of innovators can be optimal: if the average innovation is very profitable for the procurer he might want to let many innovators engage in R&D while if this profit is low he might prefer only one or two of them.

There is a huge literature on tournaments and on innovation (see e.g. Konrad (2007) on tournaments and Scotchmer (2004) on the economics of innovation). We briefly mention work that is closely related to the present paper. Fullerton and McAfee (1999) also analyze the use of entry auctions for selecting the most able participants for a fixed–prize tournament. Innovations are random variables that are affected by R&D effort. The heterogeneity of bidders is modeled as different marginal effort cost while in our model marginal cost is equal among innovators but sellers' types affect the distribution of their innovations. Fullerton and McAfee (1999) focus on the (in)efficiency of standard auctions but as we will see, that issue does not arise in the present model. Fullerton et al. (2002) is an experimental study that builds on the model of Taylor (1995). The tournament winner is awarded through a first–price auction. Taylor (1995) looks at a fixed–prize tournament as an optimal stopping problem, where identical innovators pay a fixed entry fee and then make a number of independent innovation draws where after each draw they decide whether to draw again. Che and Gale (2003) look at the optimal design of R&D contests assuming a deterministic innovation technology. They find that a first–score auction outperforms a fixed prize. Schöttner (2008) asks why we observe both tournaments with fixed prize and first–price auction and finds that the fixed prize can be more profitable than the auction. Che (1993) studies the use of first– and second–score auctions in procurement problems. Ding and Wolfstetter (2008)) study

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[2] By "revenue equivalence" we mean that the seller expects the same profit in both mechanisms.

the adverse selection problem that arises if the procurer cannot commit never to negotiate with inventors who circumvent the procurement mechanism. They also analyze the performance of fixed prize and first–price auction. In Tan (1992), innovators conduct R&D before the procurement stage in order to find out about production cost.

The mechanisms analyzed in the present paper are taken as given. We do not attempt to design an optimal procurement mechanism. Thus, we also do not address the question of the optimal reward structure in tournaments.[3]

Note that in our procurement setting, innovators might have an incentive to signal their types in order to influence their rivals' effort choice. In the particular equilibria that we are going to discuss, a signaling issue does not arise. But one should be careful to note that signaling is a typical issue in these procurement problems.

2. THE MODEL

There is a risk–neutral procurer (also called buyer) who needs to buy an innovative good and there is a set $\mathcal{N} := \{1, 2, \dots, N\}$ (with $N \geq 3$) of risk–neutral firms (also called sellers, or innovators). The procurer can commit to a procurement mechanism as specified below. This includes a commitment not to procure from players who circumvent the mechanism.[4]

Firm i's innovation is denoted by the random variable Y_i with realizations $y_i \in (\underline{y}, \overline{y})$, $\underline{y} \geq 0$. There, y_i is the monetary value added of the innovation for the procurer; we will also call it the quality of the innovation. This value is not verifiable but observable by the innovator and the procurer. The innovation can only be used by the procurer and is worthless for the innovator. Also, the procurer can only employ one innovation.

In particular, innovator i independently draws an innovation from the cdf $G^{a_i+e_i}$. There, G is a cdf with support $(\underline{y}, \overline{y})$ and positive continuous density, $a_i > 0$ is i's *ability* and $e_i \geq 0$ is i's research *effort*. Thus, ability and effort are perfect substitutes. For simplicity, we will often denote $k_i := a_i + e_i$. Abilities, or types, are private information of the firms. Ability models expertise or a comparative advantage to solve the problem at hand. It is denoted by random variables A_i with realizations a_i. They are independently distributed with cdf H, positive continuous density, and support $(\underline{a}, \overline{a})$. If a firm does not engage in R&D, its cost and profit are zero, while R&D activity, i.e. strictly positive effort, produces

[3] See e.g. Schweinzer and Segev (2008).

[4] See Ding and Wolfstetter (2008) where this assumptions is relaxed which gives rise to an adverse selection problem.

an innovation at cost $C(e_i) := ce_i + \gamma$ with $c, \gamma > 0$, $e_i > 0$. The fixed cost γ is the cost of employing one's ability. It can be interpreted as the cost of the minimum R&D scale necessary to solve the procurer's problem. Effort can be bought at a constant marginal cost. Thus, additional effort can be interpreted as adding office space, hiring additional personnel, or paying overtime, which might have the same cost for all firms within an industry. Firms with higher ability can generate better innovations in the sense of first–order stochastic dominance. However, if a less able firm spends more money (on effort) it can compensate for its lack of ability. All of the above is common knowledge.

Throughout the paper, random variables are denoted by upper–case letters and the corresponding realizations by the respective lower–case letters. We use the following notation for order statistics: Denote the kth highest of K independent draws from cdf H (ability) by $A_{(k:K)}$ and its cdf by $H_{(k:K)}$. The innovation of firm i is drawn from cdf $G^{a_i+e_i}$, where the kth highest of all the draws of firms $i = 1, \ldots, K$ is denoted by $Y_{(k:K)}$, and, with a slight abuse of notation (suppressing the exponents) we write $G_{(k:K)}$ for the cdf of $Y_{(k:K)}$. For example, $H_{(1:N-1)}$ is the cdf of the highest ability, $A_{(1:N-1)}$, among firm i's $N - 1$ rivals; $G_{(2:n)}$ is the cdf of the second–best innovation, $Y_{(2:n)}$, generated among n firms; and $a_{(1:N)}$ is the highest actual type.

The paper is proceeds as follows. Section 3. analyzes the fixed–prize tournament with entry auction. Section 4. looks at the scoring auction tournament with entry auction. Section 5. derives the main result employing the uniform distribution example. Section 6. provides a welfare analysis. In section 7. we discuss results and related issues. Section 8. concludes. The appendix contains some of the proofs as well as some results on order statistics that we use in the text.

3. ENTRY AUCTION AND FIXED–PRIZE TOURNAMENT

Consider the following procurement mechanism, F (for "fixed" prize). The procurer announces a uniform–price entry fee auction where the $n \geq 2$ highest–bidding participants are granted entry to an R&D tournament. They pay the $n+1$st highest bid as a non–refundable entry fee. Everybody else does not pay anything. Bids are published. The n auction winners ("contestants") compete for the fixed prize P that is awarded in return for the best innovation generated among the contestants. All other sellers are excluded from the contest (and the reward).

Recall that the n contestants have abilities a_i, simultaneously choose unobservable efforts e_i, and draw innovations from cdf $G^{a_i+e_i}$ with cost function $C(e_i) := ce_i + \gamma$ for $e_i > 0$, while cost is zero if they decide not to make a draw, $e_i = 0$.

Consider stage 3, the procurement stage. All cost and effort is sunk. The best (out of n) innovations is awarded the fixed prize P. Denote $k_j = a_j + e_j$. Contestant i has produced innovation y_i and i's expected profit is (where the superscript $F3$ refers to the mechanism and the stage of the game)

$$\pi_i^{F3}(y_i) = PG^{\sum_{j=1}^{n-1} k_j}(y_i), \tag{1}$$

where $G^{\sum_{j=1}^{n-1} k_j}(y_i)$ is the probability that y_i is the best innovation.
At stage 2, contestant i chooses effort e_i. We get

$$\pi_i^{F2}(a_i, e_i) = E[\pi_i^{F3}(Y_i)] - ce_i - \gamma \tag{2}$$

$$\text{where } E[\pi_i^{F3}(Y_i)] = \int_{\underline{y}}^{\overline{y}} PG^{\sum_{j=1}^{n-1} k_j}(y_i)dG^{k_i}(y_i) = \frac{k_i}{\sum_{j=1}^{n} k_j}P \tag{3}$$

$$\Rightarrow \quad \pi_i^{F2}(a_i, e_i) = \frac{k_i}{\sum_{j=1}^{n} k_j}P - ce_i - \gamma \tag{4}$$

The profit $\pi_i^{F2}(a_i, e_i)$ is strictly concave and, if P is sufficiently large, positive for some efforts e_i. The interior solution is characterized by

$$\frac{\partial \pi_i^{F2}(a_i, e_i)}{\partial e_i} = 0 \;\Leftrightarrow\; \frac{\sum_{j \neq i} k_j}{\left(\sum_{j=1}^{n} k_j\right)^2} = \frac{c}{P} \tag{5}$$

Since the RHS of (5) is constant, we can equate the LHS of (5) for all $i = 1, \dots, n$ and the first–order condition simplifies to $k_1 = k_2 = \cdots = k_n$. Substituting back into (5), we obtain the equilibrium effort (for the unique symmetric equilibrium with positive efforts by all contestants):

$$e_i^F = \frac{(n-1)P}{n^2 c} - a_i. \tag{6}$$

By (6), $e_i^F + a_i$ is a constant for each contestant i. For later use, define

$$k^F := e_i^F + a_i = \frac{(n-1)P}{n^2 c} \tag{7}$$

Effort e_i^F is always positive if it is positive for the highest ability, $\overline{a}$. For this end, the prize must satisfy $P \geq \frac{\overline{a}cn^2}{n-1}$. Define $P_{\min}(n) := \frac{\overline{a}cn^2}{n-1}$. Insert (6) into (4), then i's expected profit at the start of the tournament is

$$\pi^{F2}(a_i) := \pi_i^{F2}(a_i, e_i^F) = \frac{P}{n^2} + ca_i - \gamma \tag{8}$$

In the following we only consider prizes $P \geq P_{\min}(n)$ because otherwise the symmetric equilibrium does not exist.[5] Also, we assume that the model parameters are such that (8) is positive. In section 5. we give an example of such parameters.

It follows that each firm i draws an innovation from the same cdf, (9), and has the same probability of winning, regardless of ability.

$$G^{a_i+e_i^F} = G^{k^F} = G^{\frac{(n-1)P}{n^2 c}}, \tag{9}$$

Now consider stage 1, where innovators bid for entry.

A participant's maximum willingness to pay for entry is equal to its equilibrium expected tournament profit conditional on entry. This profit, $\pi^{F2}(a_i)$, is strictly increasing in ability. It only depends on a firm's own ability and is thus a pure private value, while, accordingly, rivals' profits are iid random variables. Thus, we have symmetric independent private values where each firm's valuation only depends on its own type a_i. There is no signaling issue in the entry game since firms do not care about learning their rivals' abilities.

Thus, standard auction formats, like the discriminatory or the uniform–price auction, are efficient and revenue–equivalent.[6] We adopt the uniform–price format because it is the easiest to analyze, not because we recommend it. In the uniform price auction, where the n highest bidders are selected for the tournament and pay the $n+1$st highest bid, bidders have the weakly dominant strategy to bid their expected tournament profits,

$$\beta^F(a_i) = \pi^{F2}(a_i). \tag{10}$$

The usual argument applies: The proposed strategy guarantees a nonnegative expected profit since if i wins, the price is not above i's expected tournament profit. Consider bidding more than $\beta^F(a_i)$. If i was previously a winner, it is still a winner with the same profit. If it was previously a loser and is now a winner, then the previous price was at or above i's

[5] As mentioned before, the purpose of these examples is to demonstrate existence of the symmetric equilibrium. Thus, we are free to choose convenient parameters. We ignore mechanisms where P is chosen so low that some tournament entrants might decide not to innovate and instead drop out of the tournament after learning their rivals' abilities. We do not discuss optimality of that choice. It is, however, clear that a low P creates a coordination problem: who drops out and who stays in the tournament? See also Fullerton and McAfee (1999) for a similar discussion. An alternative argument is that the procurer might face a high cost of no procurement, reflected in a high value $\underline{y}$ which would make low P suboptimal (see Ding and Wolfstetter (2008) for a similar argument).

[6] These formats are standard sealed–bid multi–unit auctions with single–unit demand. See e.g. Krishna (2002, ch.13,14) for an analysis of these mechanisms in the symmetric independent private values framework.

profit and it is not lower now. Thus firm i cannot be better off now. A similar argument applies for bids below $\beta^F(a_i)$. All firms participate, and the bid $\beta^F(a_i)$ is positive and strictly increasing in ability. Thus the auction is efficient.

Firm i's equilibrium profit, if all N firms participate, is given by

$$\begin{aligned}
\pi_i^{F1}(a_i) &= \Pr\left\{a_i > A_{(n:N-1)}\right\}\left(\pi_i^{F2}(a_i) - E\left[\beta^F(A_{(n:N-1)}) \,\middle|\, a_i > A_{(n:N-1)}\right]\right) \\
&= H_{(n:N-1)}(a_i)\pi_i^{F2}(a_i) - \int_{\underline{a}}^{a_i} \pi_i^{F2}(a)dH_{(n:N-1)}(a) \\
&= \int_{\underline{a}}^{a_i} \frac{\partial \pi_i^{F2}(a)}{\partial a} H_{(n:N-1)}(a)da \\
&= c\int_{\underline{a}}^{a_i} H_{(n:N-1)}(a)da > 0, \qquad (11)
\end{aligned}$$

which confirms that all N firms participate.

Now consider stage 0, where the procurer chooses parameters P and n. The optimal choice is denoted by P^F and n^F.

By (9), the expected value of the procured innovation is

$$E[Y_{(1:n)}] = \int_{\underline{y}}^{\overline{y}} y dG(y)^{\sum_{j=1}^{n} \frac{(n-1)P}{n^2 c}} = \int_{\underline{y}}^{\overline{y}} y dG(y)^{k^F n}. \qquad (12)$$

In order to obtain that innovation, the procurer spends the prize P but also collects revenue in the entry auction, equal to n times the expected tournament profit of the firm with the $(n+1)$st highest ability. The procurer's objective is therefore:

$$\max_{n,P} \Pi^F(n,P) = E[Y_{(1:n)}] - P + nE[\beta^F(A_{(n+1:N)})] \quad \text{s.t. } P \geq \frac{\overline{a}cn^2}{n-1} \qquad (13)$$

Using (12) and (10), we write $\Pi^F(n,P)$ as

$$\Pi^F(n,P) = \int_{\underline{y}}^{\overline{y}} y dG^{\frac{(n-1)P}{nc}}(y) - P + n\left(\frac{P}{n^2} - \gamma + cE[A_{(n+1:N)}]\right) \qquad (14)$$

Remark 1 *The choice of P is irrelevant in a certain sense: By* (12)*, the mechanism implements an innovation draw from cdf $G^{\frac{(n-1)P}{nc}}$. Thus we can characterize any (expected) innovation by $K := \frac{(n-1)P}{nc}$. In order to induce "innovation K", one sets the prize $P = K\frac{nc}{n-1}$. Then the procurer's profit is a function of K and n only, i.e. the only choice variables are the number of contestants and expected innovation quality.*

$$\Pi^F(n,K) = \int_{\underline{y}}^{\overline{y}} y dG^K(y) - Kc - n\gamma + ncE[A_{(n+1:N)}] \qquad (15)$$

Going back to (14), the first term is the expected best innovation, which is increasing in n and P. The term in parentheses is the entry fee that is collected n times. The entry fee itself recollects a part of the prize P, reimburses the fixed cost, γ, and its last term depends on the $n+1$st ability which determines the entry fee. The entry fee is decreasing in n: With more contestants, it is easier to enter the tournament and it is harder to win; both reduces the willingness to pay for entry.[7] Since the entry fee is multiplied by n, it is not clear which n is optimal.

This exhibits the difficulties the procurer faces in designing the fixed–prize tournament: The optimal prize P is not easily determined, and the optimal number n may be greater than two although we have an economies–of–scale technology at the firm level.[8]

We continue the discussion in section 5. and now turn to the second mechanism.

4. ENTRY AUCTION, TOURNAMENT AND SCORING AUCTION

Here, the procurer at stage 0 announces the following mechanism, denoted by S ("scoring"): First, all interested innovators bid in a uniform–price entry auction (as in the previous section). Bids are published. The $n \geq 2$ highest bids win the auction, the winners pay the $n+1$st highest bid as a non–refundable entry fee and enter the tournament. Then those firms simultaneously choose their unobservable efforts and draw innovations. Finally, the procurer conducts a scoring auction and procures the innovation from the bidder with the highest score (details follow).

The appeal of a scoring auction as compared to a fixed prize is that it allows bidders to compete (on price) even though they do not have the best innovation and, moreover, the scoring auction provides an endogenous reward for the winner and thus the prize is not a strategic variable (and a source for errors) for the procurer. Of course, sellers anticipate that competition in a scoring auction is stronger.

We consider the first–score auction format, where bidders submit an innovation and a financial bid from which a score is computed. The highest score wins and the winner receives his financial bid (as payment for his innovation). The procurer applies the ideal scoring rule $s = y - b$, where y is the innovation and b is the seller's financial bid.[9]

[7] Technically, increasing n decreases the order of the order statistics $H_{(n+1:N)}$ and thus the expected value of ability $A_{(n+1:N)}$.

[8] An additional contestant incurs fixed cost γ but also contributes ability that has no marginal cost.

[9] This scoring rule is ideal in the sense that it is the most credible: it reflects the true profit of the procurer. Thus the procurer has an incentive to select the most profitable innovation, which, in equilibrium, is equal to the best innovation.

We simplify analysis be reverting to a revenue–equivalent auction format, the second–score auction.[10] We argue below why this is feasible in the present model.

In a second–score auction, the highest score wins and the winner is obliged to deliver the second highest score to the procurer. Since the winner's innovation is fixed and different from all other innovations, an amount of money is paid to or by the winner in order to adjust his score to the second highest score.

At the auction stage, all innovations are drawn and cost and effort is sunk. Thus, a bidder's decision problem amounts to choosing a bid given one's innovation and supposed distributions of his $n-1$ rivals' innovation draws. For given abilities and efforts, the distribution of innovations is completely determined which makes the standard auctions revenue–equivalent. In turn, for different standard auction formats at stage 3, the same equilibrium effort choice obtains.

Consider the bidding equilibrium. In the second–score auction, contestants have the (weakly) dominant strategy to bid $b_i = 0$, i.e. a score of $s_i = y_i$, equal to the value of their innovation. The usual argument applies: At the beginning of the auction, all cost is sunk and, by assumption, the innovation is worthless for the innovator. Given the scoring rule, a bidder wants to win the auction iff the value of his *innovation* exceeds the second–highest *score*: only then would he be paid any money in the event of winning. If his innovation were worth less than the second highest score, then in the event of winning he would have to pay money to the procurer in addition to his innovation in order to meet the second–highest score. Thus, by bidding a score equal to the value of one's own innovation, a bidder makes sure that he wins iff he wants to win.

The winner's profit is the difference between his and the second–highest score which, in the dominant–strategy equilibrium, is the difference between the value of his own and that of the best rival's innovation, $y_i - y_{1:n-1}$.

At stage 3, the auction stage, contestant i's expected profit is[11]

$$\begin{aligned}\pi_i^{S3}(y_i) &= \Pr\{y_i > Y_{(1:n-1)}\} E[y_i - Y_{(1:n-1)} | y_i > Y_{(1:n-1)}] \\ &= \int_{\underline{y}}^{y_i} (y_i - y) dG^{\sum_{j\neq i} k_j}(y) \\ &= \int_{\underline{y}}^{y_i} G^{\sum_{j\neq i} k_j}(y) dy > 0.\end{aligned} \tag{16}$$

At stage 2, the tournament stage, contestant i chooses effort e_i at cost

[10] See e.g. Che (1993) for an analysis of this auction format.

[11] Recall the notation $k_j = a_j + e_j$.

$ce_i + \gamma$ and has an expected profit of[12]

$$\pi_i^{S2}(a_i, e_i) = E[\pi_i^{S3}(Y_i)] - ce_i - \gamma \tag{17}$$

where, using (16),

$$\begin{aligned} E[\pi_i^{S3}(Y_i)] &= \int_{\underline{y}}^{\overline{y}} \pi_i^{S3}(y_i) dG^{k_i}(y_i) \\ &= \int_{\underline{y}}^{\overline{y}} \int_{\underline{y}}^{y_i} G^{\sum_{j\neq i} k_j}(y) dy dG^{k_i}(y_i) \\ &= \int_{\underline{y}}^{\overline{y}} G^{\sum_{j\neq i} k_j}(y) \int_{y}^{\overline{y}} dG^{k_i}(y_i) dy \\ &= \int_{\underline{y}}^{\overline{y}} G^{\sum_{j\neq i} k_j}(y) \left(1 - G^{k_i}(y)\right) dy \end{aligned} \tag{18}$$

Thus,

$$\pi_i^{S2}(a_i, e_i) = \int_{\underline{y}}^{\overline{y}} G^{\sum_{j\neq i} k_j}(y) \left(1 - G^{k_i}(y)\right) dy - ce_i - \gamma \tag{19}$$

Consider (19). We have

$$\frac{\partial \pi_i^{S2}(a_i, e_i)}{\partial e_i} = -\int_{\underline{y}}^{\overline{y}} G^{\sum_{j=1}^n k_j}(y) \ln(G(y)) dy - c \tag{20}$$

$$\frac{\partial^2 \pi_i^{S2}(a_i, e_i)}{(\partial e_i)^2} = -\int_{\underline{y}}^{\overline{y}} G^{\sum_{j=1}^n k_j}(y) \left(\ln(G(y))\right)^2 dy < 0 \tag{21}$$

The first–order condition for the equilibrium effort e_i is, by the above,

$$-\int_{\underline{y}}^{\overline{y}} G^{\sum_{j=1}^n k_j}(y) \ln(G(y)) dy = c \tag{22}$$

where the LHS is positive since $\ln(G(y))$ is negative. Denote $K := \sum_{j=1}^n k_j$ as the solution of (22). If the model parameters permit then K characterizes the equilibrium effort choice. In particular, if $K > \sum_{j=1}^n a_j$ then there is a positive amount of *total* effort and the allocation of that effort among firms is arbitrary. Thus, there are potentially many symmetric and asymmetric equilibria (e.g. where all firms exert the same *absolute* effort, or where effort is chosen in some relation to abilities).

Consider the symmetric equilibrium where $k_1 = k_2 = .. = k_n =: k^S$ and thus $K = nk^S$ satisfies the above equation and where firm i in equilibrium chooses $e_i^S = k^S - a_i$. Of course, this requires that k^S is sufficiently large to ensure positive efforts for all abilities, i.e. $k^S > \bar{a}$. We give feasible

[12]Profit is zero if $e_i = 0$ is chosen.

parameter values when we look at the uniform distribution example in section 5..

What makes this particular equilibrium more interesting than others? First, these effort choices are similar to the unique symmetric equilibrium with positive efforts in the fixed–prize mechanism, F, and, second, these efforts are independent of rivals' abilities and thus predicting one's rivals is not necessary. In this sense, it is the most appealing of the symmetric equilibria.[13]

Recalling (19), firm i's equilibrium profit at stage 2 is

$$\pi_i^{S2}(a_i) = \int_{\underline{y}}^{\overline{y}} G^{(n-1)k^S}(y)\left(1 - G^{k^S}(y)\right)dy - c\left(k^S - a_i\right) - \gamma. \quad (23)$$

Again, a seller's expected tournament profit is a pure private value, i.e. a function of own ability only, and, again, these symmetric effort strategies, $e_i^S = k^S - a_i$ imply that all contestants draw from the same cdf, G^{k^S} and thus have the same probability of winning, $1/n$.

Consider stage 1, the uniform–price entry auction. Similar to the result for mechanism F and by a similar argument, each bidder's willingness to pay is equal to the expected tournament profit given entry, and the dominant–strategy equilibrium bid function is

$$\beta^S(a_i) = \pi^{S2}(a_i) \quad (24)$$

At stage 1, if all N firms participate, firm i's expected profit is (the computation is similar to that of (11))

$$\begin{aligned}\pi_i^{S1}(a_i) &= \Pr\{a_i > A_{(n:N-1)}\}\left(\pi_i^{S2}(a_i) - E[\beta^S(A_{(n:N-1)})|a_i > A_{(n:N-1)}]\right) \\ &= c\int_{\underline{a}}^{a_i} H_{(n:N-1)}(a)da > 0 \quad (25)\end{aligned}$$

Thus, in our symmetric equilibrium, all N potential sellers participate.

Comparing (25) and (11) and collecting the results so far, we state

Proposition 1 *Suppose the symmetric tournament equilibria of mechanisms F and S, characterized by $k^F = e_i^F + a_i$ and $k^S = e_i^S + a_i$, respectively, for each contestant i exist.*

Then the games induced by these mechanisms have Bayesian Nash equilibria, where all N sellers participate and expect the same profits. Innovations are drawn by the n most able sellers from the same cdf, G^{k^S} and G^{k^F}, respectively, and each contestant has the same probability of winning.

[13] Of course, applying this mechanism with many equilibria does not seem to be desirable.

REMARK 2 *Proposition 1 does not depend on the (optimal) choice of the fixed prize P. Increasing the prize does not make mechanism F more desirable for the sellers. Also, Proposition 1 does not imply that the procurer's expected profit is the same in both mechanisms.*

At stage 1, the procurer's expected equilibrium profit is

$$\begin{aligned}\Pi^S(n) &= E[Y^S_{(1:n)}] - \left(E[Y^S_{(1:n)}] - E[Y^S_{(2:n)}]\right) + nE[\beta^S(A_{(n+1:N)})] \\ &= E[Y^S_{(2:n)}] + nE[\beta^S(A_{(n+1:N)})] \end{aligned} \tag{26}$$

where the first term is the (profit of the) expected innovation, the second is the amount the procurer pays to the winner of the second–score auction (the difference between the scores of the best and the second–best innovation), and the third term is the procurer's revenue from the entry auction.

5. EXAMPLE: UNIFORM DISTRIBUTION

In this section, we provide an example for model parameters such that in the above equilibria (Proposition 1), optimal choice of P is sufficient to make the procurer indifferent between both mechanisms. This result holds regardless of n, as long as the same n is used in both mechanisms. We assume uniformly distributed abilities and innovations with support $(0,1)$. Thus, $(\underline{y}, \overline{y}) = (\underline{a}, \overline{a}) = (0,1)$ and $H(x) = G(x) = x$ for $x \in (0,1)$. We make a further assumption on the cost parameters c and γ:

$$0 < \gamma < \frac{c}{N-2} \leq c < \frac{1}{(N+2)^2} < 1 \tag{27}$$

First, this assumption makes the problem economically meaningful (positive first–best effort). Second, it ensures that our symmetric equilibria exist and that the mechanisms are feasible.[14] These assumptions are sufficient for our purposes but not necessary; they are chosen for convenience.

5.1 *Fixed–prize mechanism*

First, note that (8) is positive for all $P \geq P_{\min}$ by assumption $\gamma < \frac{c}{(N-2)}$ (see (27)). The first term of the procurer's profit, (14), straightforwardly becomes

$$E[Y_{(1:n)}] = \int_0^1 y \, dy^{\frac{(n-1)P}{nc}} = \frac{(n-1)P}{(n-1)P + nc}. \tag{28}$$

[14] This includes the bilateral contract, see subsection 7.7.

Also (see subsection 9.1 in the appendix),

$$E[A_{(n+1:N)}] = 1 - \frac{n+1}{N+1}. \tag{29}$$

The procurer's decision problem becomes

$$\max_{n,P} \Pi^F(n,P) \quad \text{s.t. } n \geq 2,\ P \geq P_{\min}(n) = \frac{cn^2}{n-1} \tag{30}$$

$$\text{where } \Pi^F(n,P) = \frac{(n-1)P}{cn + (n-1)P} - \frac{(n-1)P}{n} + n(c-y) - \frac{nc(n+1)}{N+1}.$$

We have that $\Pi^F(n,P)$ is strictly concave in P,

$$\frac{\partial^2 \Pi^F(n,P)}{(\partial P)^2} = -\frac{2cn(n-1)^2}{(cn+(n-1)P)^3} < 0. \tag{31}$$

Thus, for any n it is optimal to set P equal to the interior maximizer, $P_{\text{int}}(n)$, unless $P_{\text{int}}(n) < P_{\min}(n)$ in which case $P_{\min}(n)$ is optimal. By (30),

$$\frac{\partial \Pi^F(n,P)}{\partial P} = \frac{1}{n} - 1 + \frac{cn(n-1)}{(cn+(n-1)P)^2} = 0 \tag{32}$$

$$\Leftrightarrow P = \frac{n(\sqrt{c} - c)}{n-1} =: P_{\text{int}}(n) > 0 \tag{33}$$

We have $P_{\text{int}}(n) > P_{\min}(n) \Leftrightarrow c < \frac{1}{(n+1)^2}$, and this is satisfied by assumption $c < \frac{1}{(N+2)^2}$ (see (27)). Thus, the optimal prize is

$$P^F = P_{\text{int}}(n) = \frac{n(\sqrt{c} - c)}{n-1}. \tag{34}$$

Applying (34) to (6) and (28), we get the equilibrium effort and the expected best innovation at the optimal prize,

$$e_i^F = \frac{1}{n}\left(\frac{1}{\sqrt{c}} - 1\right) - a_i \tag{35}$$

$$k^F = \frac{1}{n}\left(\frac{1}{\sqrt{c}} - 1\right) \tag{36}$$

$$E[Y_{(1:n)}^F] = \int_0^1 y\, dy^{\frac{1}{\sqrt{c}}-1} = 1 - \sqrt{c}. \tag{37}$$

Note that (37) is entirely determined by the optimal choice of P and does not depend on n. The profit at the optimal prize, $\Pi^F(n, P^F)$ is

$$\Pi^F(n,P^F) = (1-\sqrt{c})^2 + n\left(c\left(1 - \frac{n+1}{N+1}\right) - y\right) \tag{38}$$

Since $\left(1-\frac{n+1}{N+1}\right) > 0$, the above is positive if $(1-\sqrt{c})^2 > n\gamma$. This is satisfied by our assumptions (see the discussion why (56) is positive).

The profit function is strictly concave in n and the interior maximizer is

$$n_{\text{int}}^F = \frac{1}{2}\left(N-(N+1)\frac{\gamma}{c}\right) \geq 2 \;\Leftrightarrow\; \frac{\gamma}{c} \leq \frac{N-4}{N+1}, \tag{39}$$

Since we have an integer constraint on n, using n_{int}^F only generates an upper bound on the maximum profit of mechanism F for $\frac{\gamma}{c} \leq \frac{N-4}{N+1}$ (as derived in (39)):

$$\Pi^F(n_{\text{int}}^F, P^F) = (1-\sqrt{c})^2 + \frac{1}{4}\left(N-(N+1)\frac{\gamma}{c}\right)\left(\frac{Nc}{N+1}-\gamma\right) \tag{40}$$

If, however, $\frac{\gamma}{c} > \frac{N-4}{N+1}$, then the corner solution $n = 2$ is optimal and profit becomes

$$\Pi^F(2, P^F) = (1-\sqrt{c})^2 - 2(\gamma - c) - \frac{6c}{N+1}. \tag{41}$$

5.2 *Scoring auction mechanism*

Applying the uniform distribution assumption, firm i's expected tournament profit, (19), becomes

$$\pi_i^{S2}(a_i, e_i) = \frac{1}{1+\sum_{j\neq i} k_j} - \frac{1}{1+\sum_{j=1}^n k_j} - ce_i - \gamma \tag{42}$$

It is strictly concave in e_i and the first–order condition is

$$\frac{\partial \pi_i^{S2}(a_i, e_i)}{\partial e_i} = 0 \quad \Longleftrightarrow \quad \sum_{j=1}^n k_j = \frac{1}{\sqrt{c}} - 1 \tag{43}$$

Thus, in any pure–strategy equilibrium, all efforts and abilities sum up to a constant. In our symmetric equilibrium, $e_i^S + a_i = k^S$, and i's equilibrium effort is

$$e_i^S = k^S - a_i = \frac{1}{n}\left(\frac{1}{\sqrt{c}} - 1\right) - a_i. \tag{44}$$

Remark 3 *Suppose the fixed prize is chosen optimally, $P = P^F$ and compare* (44) *with* (35). *The equilibrium effort is the same in both mechanisms (for the uniform distribution assumption). Thus, the expected innovations are also the same, produced at the same expected cost.*

Firm i's equilibrium tournament profit as a function of its ability, a_i, is

$$\pi^{S2}(a_i) := \pi_i^{S2}(a_i, e_i^S) = \frac{\sqrt{c}(1-\sqrt{c})^2}{n(n-1+\sqrt{c})} + ca_i - \gamma. \tag{45}$$

It is strictly increasing in a_i and strictly decreasing in n. Effort e_i^S is positive by assumption $c < \frac{1}{(N+1)^2}$ (see (27)), and (45) is positive for any ability (see the proof in the appendix).

Recall the equilibrium bid function in the entry auction, (24), and the procurer's expected profit, (26). Look at the two terms of (26) separately. The expected second-best innovation is (see subsection 9.1 in the appendix for a derivation of $G'_{(2:n)}(y)$)

$$\begin{aligned} E[Y^S_{(2:n)}] &= \int_0^1 y G'_{(2:n)}(y) dy \\ &= \int_0^1 (n-1)\left(\frac{1}{\sqrt{c}} - 1\right)\left(y^{\frac{n-1}{n}\left(\frac{1}{\sqrt{c}}-1\right)} - y^{\left(\frac{1}{\sqrt{c}}-1\right)}\right) dy \\ &= \frac{(n-1)\left(1-\sqrt{c}\right)^2}{n-1+\sqrt{c}}. \end{aligned} \tag{46}$$

The expected entry fee is

$$\begin{aligned} E[\beta^S(A_{(n+1:N)})] &= E[\pi_i^{S2}(A_{(n+1:N)})] \\ &= \frac{\sqrt{c}(1-\sqrt{c})^2}{n(n-1+\sqrt{c})} + cE[A_{(n+1:N)}] - \gamma \end{aligned} \tag{47}$$

where (see subsection 9.1 in the appendix)

$$E[A_{(n+1:N)}] = 1 - \frac{n+1}{N+1}. \tag{48}$$

Thus,

$$E[\beta^S(A_{(n+1:N)})] = \frac{\sqrt{c}(1-\sqrt{c})^2}{n(n-1+\sqrt{c})} + c\left(1 - \frac{n+1}{N+1}\right) - \gamma, \tag{49}$$

and

$$\begin{aligned} \Pi^S(n) &= \frac{(n-1)\left(1-\sqrt{c}\right)^2}{n-1+\sqrt{c}} + n\left(\frac{\sqrt{c}(1-\sqrt{c})^2}{n(n-1+\sqrt{c})} + c\left(1-\frac{n+1}{N+1}\right) - \gamma\right) \\ &= \left(1-\sqrt{c}\right)^2 + n\left(c\left(1-\frac{n+1}{N+1}\right) - \gamma\right). \end{aligned} \tag{50}$$

Since (50) is equal to (38), and recalling Remark 3, we have

Proposition 2 *Given a number $n \geq 2$ of contestants, model parameters exist such that there are Bayesian Nash equilibria of the games induced by mechanisms F and S, where all players expect the same profit, the n contestants choose the same positive efforts and the same innovations are produced in both mechanisms.*

6. WELFARE UNDER UNIFORM DISTRIBUTION

We look at the welfare properties of the two mechanisms under the uniform distribution assumption (see previous section). We derive an efficiency benchmark and analyze both mechanisms from the point of view of a welfare–maximizing buyer.

6.1 *First–best benchmark*

Recall that the procurer announces the mechanism at stage 0, simultaneously with nature's draw of abilities. In particular, the number n of "active" innovators is fixed at that stage. For a discussion of this feature refer to subsection 7.5. Taking it as given, it is appropriate to define a welfare benchmark that also fixes n before abilities are realized. We compute the benchmark as follows. First, welfare is maximized over efforts, for given abilities and n. Second, the resulting expected maximum welfare is maximized over n. Note that fixing n has two consequences: the social fixed R&D cost of ny is incurred regardless of subsequent effort choices.[15]

Consider arbitrary realizations of abilities that w.l.o.g. are ordered $a_1 > a_2 > \cdots > a_N$. For simplicity, denote $\hat{a}_n := \sum_{i=1}^n a_i$ and $\hat{e} := \sum_{i=1}^n e_i$. For given n, expected welfare, $W(n, e_1, \dots, e_n)$, is the difference between the expected best innovation and the social effort cost:[16]

$$W(n, e_1, \dots, e_n) = E[Y_{(1:n)}] - c \sum_{i=1}^{n} e_i - ny$$
$$\text{where } E[Y_{(1:n)}] = \int_{\underline{y}}^{\overline{y}} y dG(y)^{\sum_{i=1}^n a_i + e_i} \tag{51}$$

From the above, it is obvious that for given n only total effort, $\hat{e} := \sum_{i=1}^n e_i$, and total ability, $\hat{a}_n := \sum_{i=1}^n a_i$, matter, while the composition of individual efforts $e_i \geq 0$ is inconsequential.[17] Thus, we can replace the choice variables $e_1, \dots, e_n$ by $\hat{e}$ in (51).

Under the uniform distribution example, (51) simplifies to

$$W(n, \hat{e}) = \frac{\hat{a}_n + \hat{e}}{1 + \hat{a}_n + \hat{e}} - c\hat{e} - ny \tag{52}$$

[15] Observe that it does not make sense to "fix" some n and later, after abilities become known, decide to make use of a lower number $n' < n$ of innovators in order to save the fixed cost y if that is more profitable. Then we could as well say that we "fix" $n = N$ (all innovators) and later decide how many to employ. But then "fixing n" is meaningless.

[16] Note that by the assumed ordering of abilities, $a_1 > a_2 > \cdots > a_N$, welfare $W(n, e_1, \dots, e_n)$ only makes use of the n most able innovators, which is efficient.

[17] This is due to the constant and equal marginal cost of effort across firms.

Since $W(n, \hat{e})$ is strictly concave in $\hat{e}$, the unique maximizer is

$$\hat{e}^* = \frac{1}{\sqrt{c}} - 1 - \hat{a}_n \tag{53}$$

It is also positive: By $a_i < 1 \Rightarrow \hat{a}_n < n$,

$$\hat{e}^* = \frac{1}{\sqrt{c}} - 1 - \hat{a}_n > \frac{1}{\sqrt{c}} - 1 - n > 0 \iff c < \frac{1}{(n+1)^2} \tag{54}$$

The latter is satisfied by assumption $c < \frac{1}{(N+2)^2}$, see (27). The fact that (54) is positive implies that it is optimal to spend additional effort regardless of the realized abilities $a_1, \ldots, a_n$.
By (53), the expected value of the innovation is completely determined, i.e. the innovation is drawn from $G^{\hat{e}^*+\hat{a}_n} = G^{\frac{1}{\sqrt{c}}-1}$, regardless of n:

$$E[Y_{(1:n)}] = \int_{\underline{y}}^{\overline{y}} y dG^{\frac{1}{\sqrt{c}}-1}(y) = 1 - \sqrt{c} \tag{55}$$

Inserting (53) into (52), we get

$$W(n, \hat{e}^*) = (1 - \sqrt{c})^2 + c\hat{a}_n - ny \tag{56}$$

By assumption (27), (56) is positive: Ignore the positive term $c\hat{a}_n$. By (27), $c < \frac{1}{(N+2)^2}$. This implies $\frac{c}{N-2} < \frac{(1-\sqrt{c})^2}{N}$. This, together with $y < \frac{c}{N-2}$ (by (27)) implies $y < \frac{(1-\sqrt{c})^2}{n}$ which means $W(n, \hat{e}^*) > 0$.
It remains to determine the integer n, i.e. the maximization problem is

$$\max_{n \in [1,N]} E[W(n, \hat{e}^*)] \tag{57}$$

where $W(n, \hat{e}^*)$ is given by (56). We have

$$\begin{aligned} E[W(n, \hat{e}^*)] &= E\left[(1 - \sqrt{c})^2 + c\sum_{i=1}^{n} A_{(i:N)} - ny\right] \\ &= (1 - \sqrt{c})^2 - ny + cE\left[\sum_{i=1}^{n} A_{(i:N)}\right]. \end{aligned} \tag{58}$$

Since $E[A_{(i:N)}] = \frac{N+1-i}{N+1}$ (see subsection 9.1 in the Appendix),

$$\begin{aligned} E\left[\sum_{i=1}^{n} A_{(i:N)}\right] &= \sum_{i=1}^{n} E\left[A_{(i:N)}\right] \\ &= \frac{N}{N+1} + \frac{N-1}{N+1} + \cdots + \frac{N-(n-1)}{N+1} \\ &= \frac{1}{N+1}\left(N + (N-1) + \cdots + (N-(n-1))\right) \\ &= \frac{n(2N-n+1)}{2(N+1)}. \end{aligned} \tag{59}$$

Therefore,

$$E[W(n,\hat{e}^*)] = (1-\sqrt{c})^2 + c\frac{n(2N-n+1)}{2(N+1)} - n\gamma \tag{60}$$

which is quadratic and strictly concave in n. The global maximizer is

$$n^*_{\text{int}} = \frac{1}{2} + N - (N+1)\frac{\gamma}{c}. \tag{61}$$

Inserting n^*_{int} into (60) gives an upper bound on maximum welfare:

$$W(n^*_{\text{int}},\hat{e}^*) = (1-\sqrt{c})^2 + \frac{(c(2N+1)-2\gamma(N+1))^2}{8c(N+1)} \tag{62}$$

Since (60) is quadratic in n, the welfare maximum is found by rounding n^*_{int} to the nearest integer (of the range $1,\dots,N$) and inserting into (60).
In the following subsections, we consider a welfare-maximizing buyer who employs mechanisms F and S under the same informational and incentive constraints as the profit-maximizing buyer in sections 3. to 5..

6.2 *Fixed-prize mechanism*

Consider a welfare-maximizing buyer who employs mechanism F. We have shown that the equilibrium effort is given by (6) as long as the prize satisfies $P \geq \frac{cn^2}{n-1}$ (uniform distribution).
The expected welfare of mechanism F, as a function of n and P, is the expected difference of the best innovation, $Y_{(1:n)}$, and the corresponding social R&D cost given that in equilibrium the n most able sellers provide effort as in (6). Recall (28) for the computation of $E[Y_{(1:n)}]$.

$$\begin{aligned} W^F(n,P) &= E\left[Y_{(1:n)} - \sum_{i=1}^{n}\left(c\left(\frac{(n-1)P}{n^2c} - A_{(i:N)}\right) + \gamma\right)\right] \\ &= \frac{(n-1)P}{(n-1)P+nc} - \frac{(n-1)P}{n} + cE\left[\sum_{i=1}^{n} A_{(i:N)}\right] - n\gamma \end{aligned} \tag{63}$$

where, again, $E\left[\sum_{i=1}^{n} A_{(i:N)}\right] = \frac{n(2N-n+1)}{2(N+1)}$ (see (59)). Comparing (60) and (63) as functions of P we see that both differ only in a constant and thus have the same optimal P which is given by (34). Inserting (34) into (63), we get welfare as a function of n:

$$W^F(n) = (1-\sqrt{c})^2 + \frac{cn(n+1)}{2(N+1)} + n(c-\gamma) \tag{64}$$

It is easily verified that this is equal to (60). Thus, the welfare-maximizing buyer actually achieves the welfare benchmark as long as the optimal n

(obtained by rounding (61) to the nearest integer) is in the range $2, \dots, N-1$. Note that the benchmark permits $n = 1$ and $n = N$ which is not feasible in mechanism F. Thus, mechanism F can almost always implement first best and is then optimal if the buyer is a welfare–maximizer.

Comparing (39) and (61) (taking into account assumption (27)), we note that, in comparison with the profit–maximizing buyer, the welfare–maximizing buyer chooses a larger n.

6.3 *Scoring auction mechanism*

Recall two results of the previous subsection: If the buyer is a welfare maximizer and employs mechanism F, then equilibrium efforts and the optimal prize are the same as with a profit–maximizing buyer. This implies that, for both types of buyer and for the same n, inventors would submit the same bids in the auction. From this we can conclude that the two mechanisms, again, induce the same expected procurer's profit. It is easy to show that this is indeed the case:

Consider a welfare–maximizing buyer as in subsection 6.2. Recall that equilibrium effort for mechanism S is given by (44). The expected welfare $W^S(n)$ of mechanism S is the expected difference of the best innovation, $Y_{(1:n)}$, and the corresponding social R&D cost given that in equilibrium the n most able sellers provide effort as in (44). Recall that $E[Y^S_{(1:n)}] = 1 - \sqrt{c}$ is independent of n.

$$W^S(n) = E\left[Y^S_{(1:n)} - \sum_{i=1}^{n}\left(c\left(\frac{1}{n}\left(\frac{1}{\sqrt{c}} - 1\right) - A_{(i:N)}\right) + \gamma\right)\right] \tag{65}$$

$$= (1 - \sqrt{c})^2 + cE\left[\sum_{i=1}^{n} A_{(i:N)}\right] - n\gamma \tag{66}$$

where, again, $E\left[\sum_{i=1}^{n} A_{(i:N)}\right] = \frac{n(2N-n+1)}{2(N+1)}$ (see (59)). Thus, $W^S(n)$ is equal to (60) and we conclude that mechanism S implements the welfare benchmark as long as the welfare–maximizing n (obtained by rounding (61) to the nearest integer) is in the range $2, \dots, N-1$. Note again that benchmark 2 permits $n = 1$ and $n = N$ which is not feasible in S.

7. DISCUSSION

7.1 *Comparison of stage 3*

By Proposition 2, given the optimal prize P^F and the same number of contestants, innovators draw their innovations from the same distribution in both mechanisms. This implies that all expected innovations are the same. Is the procurer's profit at stage 3 also the same, i.e. does she procure the innovations for the same price?

Consider the (implicit) prize that is determined by the scoring auction in mechanism S. Obviously, using the same prize, say $\hat{P}$, in mechanism F will result in the same expected procurer's profit at the procurement stage if the expected best innovation is same in both cases. However, the equilibrium price of the scoring auction mechanism is not equal to the optimal fixed prize P^F. It can be verified that in equilibrium, the buyer's expected profit at the third stage of mechanism S exceeds that of mechanism F, $\Pi^{S3}(n) > \Pi^{F3}(n, P^F)$ (for any $n \geq 2$), where

$$\begin{aligned}\Pi^{F3}(n, P^F) &= E[Y^F_{(1:n)}] - P^F \\ &= 1 - \sqrt{c} - \frac{n(\sqrt{c} - c)}{n - 1} \qquad (67)\\ \Pi^{S3}(n) &= E[Y^S_{(1:n)}] - \left(E[Y^S_{(1:n)}] - E[Y^S_{(2:n)}]\right) \\ &= E[Y^S_{(2:n)}] \\ &= \frac{(n-1)(1-\sqrt{c})^2}{n - 1 + \sqrt{c}} \qquad (68)\end{aligned}$$

Thus, sellers expect a larger reward in mechanism F. But that is anticipated and results in larger bids in the entry auction (as can be easily checked). Put differently, the scoring auction is more competitive since sellers compete in price and quality, while in the fixed–prize mechanism always the best innovator wins. But, as a result, employing the scoring mechanism makes sellers less aggressive at the entry stage since there is less to gain. This endogenous adjustment of entry fees makes the mechanisms equivalent.

7.2 *Social choice functions*

We briefly discuss our results in the spirit of *social choice* functions.[18] [19] In our setting, a social choice consists of efforts for all sellers, and transfers for all players, including the procurer. By Propositions 1 and 2, both mechanisms can implement the same allocation of efforts, the same innovations and the same *expected* transfers (as functions of one's own type) for all N sellers and the same expected transfer for the procurer. However, the particular transfers are generally different between both mechanisms.

[18] See, e.g., Mas-Colell et al. (1995, ch. 23).

[19] We cannot state a social choice *function* that is implemented by our mechanisms since there is no mapping from types into transfers: a part of the transfers, the reward for the contest winner, is a function of innovations that are drawn randomly. Thus, our mechanisms map types into efforts and there is another mapping from types and innovations into transfers.

These results (for the uniform distribution example) go beyond ”revenue–equivalence“: The players do not only expect the same profits, but those profits are also produced with the same efforts by the same sellers and result in the same innovations.

7.3 *A direct procurement mechanism*

Our equivalence result (Propositions 1 and 2) can be interpreted as follows: Both mechanisms implement the outcome of a particular equilibrium of a direct incentive–compatible procurement mechanism. We briefly sketch what that direct mechanism looks like.

In the present problem, incentive compatibility has two dimensions: truthful reporting of types and choice of the prescribed effort. In order to induce effort, payments must depend on innovations quality.

Consider a direct mechanism that maps types into efforts, collects a type–dependent payment from each seller and, in addition, pays a reward for the best innovation. The mechanism induces a sequential game. At stage 0, nature draws abilities and the mechanism is announced, including the number n of contestants. At stage 1, sellers report their types, $\hat{a}_i$. Reports are published and become common knowledge.[20] The procurer prescribes positive efforts for the contestants and zero effort for the other sellers (they also get zero payments). At stage 2, contestants choose effort and draw innovations. At stage 3, they present their innovations, payments are made and the game ends. (Then the procurer employs the best innovation.) Payments are composed of a type–dependent payment (entry fee) and an innovation–dependent payment: a reward $p(y)$ for the best innovation.

Consider stage 3. The reward is a positive function of (the quality of) all innovations, $p(y) := p(y_1, y_2, \ldots, y_n)$. It may be constant in one, several or all arguments. Thus, a fixed prize, $p(y) := P$, is included. Contestant i's expected profit is (denote the vector of rivals' innovations by y_{-i} and denote $k_j := a_j + e_j$)[21]

$$\begin{aligned} \pi_i^3(y_i) &= \Pr\{y_i > Y_{(1:n-1)}\} E[p(Y) | y_i > Y_{(1:n-1)}] \\ &= \int_{\underline{y}}^{y_i} \cdots \int_{\underline{y}}^{y_i} p(y_i, y_{-i}) dG^{k_1}(y_1) \cdots dG^{k_{n-1}}(y_{n-1}) \end{aligned} \tag{69}$$

[20] Recall that in our symmetric equilibrium, it does not matter whether type reports are published or not since effort strategies are independent of information about rivals' types.

[21] It can be straightforwardly verified that (1) and (16) can be computed from (69). Also note that the subscripts of $k_1, \ldots, k_{n-1}$ in (69) refer to i's rivals and not to the $n-1$ highest abilities.

At stage 2, contestant i's expected profit is

$$\begin{aligned}\pi_i^2(a_i, e_i) &= E[\pi_i^3(Y_i)] - ce_i - \gamma \\ &= \int_{\underline{y}}^{\overline{y}} \pi_i^3(y_i) dG^{k_i}(y_i) - ce_i - \gamma\end{aligned} \tag{70}$$

Consider (70). The expected reward, $E[\pi^3(Y_i)]$, is entirely determined by k_1, ..., k_n. Since $\frac{dk_i}{de_i} = 1$, the first derivative of $E[\pi^3(Y_i)]$ w.r.t. e_i is again entirely a function of k_1, ..., k_n. The derivative of the remaining terms of (70) w.r.t. e_i is equal to c, a constant. Thus, if there is a pure–strategy equilibrium that satisfies each player's first–order condition (derived from (70)), then i's optimal choice is some k_i as a function of the rivals' k_js only.

Again, we restrict attention to the symmetric equilibrium candidate where $k_i = k_j =: k^*$ for all contestants i, j.

Contestant i's probability of winning the prize resembles the Tullock contest success function.[22]:

$$\begin{aligned}\Pr\{Y_i > Y_{(1:n-1)}\} &:= \int_{\underline{y}}^{\overline{y}} G^{\left(\sum_{j\neq i} k_j\right)}(y) dG^{k_i}(y) = \frac{k_i}{\sum_{j=1}^{n} k_j} \in (0,1) \\ k_i = k_j = k^* &\Rightarrow \Pr\{Y_i > Y_{(1:n-1)}\} = \frac{1}{n}\end{aligned} \tag{71}$$

Thus, in equilibrium, each contestant is equally likely to win. Again, (70) is a pure private value, characterized by the constant k^* and one's own ability a_i. Thus, the expected tournament profit function is the same for all contestants, denoted by $\pi^2(a_i)$.

In order to induce truthful reporting of types, the procurer sets an appropriate payment for each of the n contestants that depends on the ability of the (reportedly) most able player who is *not* selected for the R&D stage.[23] Thus, a contestant pays the expected profit of the player that is crowded out by him.

Suppose the procurer selects the n (reportedly) most able players as contestants for the prize $p(y)$ (with random tie breaking) and each of them has to pay a fee of $f(n) := \pi^2\left(a_{(n+1:N)}\right)$, while the other $N-n$ players do not pay anything. Then truthful reporting of types is a weakly dominant strategy for all sellers: At stage one, when players are asked for their types (abilities), they face a simple decision problem (similar to that in a Vickrey auction for a pure private value). Truthful reporting of types ensures a nonnegative profit since in case of being selected as one of the

[22] Note that we neither assumed this function nor does it require a specific distribution assumption. See Jia (2008) on the stochastic foundations of the Tullock contest success function.

[23] In auction settings, that player is sometimes called the marginal bidder.

n contestants, the fee one has to pay cannot exceed one's true expected continuation profit. If truthful reporting makes player i a winner, then reporting a higher ability does not change anything. If a lower report changes anything, then player i becomes a loser but prefers being a winner. If truthful reporting makes i a loser, then lowering i's report does not change anything while raising it might also not change anything, but if it does, then i becomes a winner but the expected continuation profit does not cover the fee.

Thus, the game induced by the direct mechanism has a Bayesian Nash equilibrium where all N sellers participate, report their types truthfully, the n most able sellers are selected for the R&D stage and exert the requested effort.

The equilibrium expected profit of a player with ability a_i is

$$\begin{aligned}\pi^1(a_i) &= \Pr\{a_i > A_{(n:N-1)}\}\left(\pi^2(a_i) - E\left[f(n) \mid a_i > A_{(n:N-1)}\right]\right)\\ &= H_{(n:N-1)}(a_i)\pi^2(a_i) - \int_{\underline{a}}^{a_i} \pi^2(a)dH_{(n:N-1)}(a)\\ &= \int_{\underline{a}}^{a_i} \frac{\partial \pi^2(a)}{\partial a} H_{(n:N-1)}(a)da\\ &= c\int_{\underline{a}}^{a_i} H_{(n:N-1)}(a)da > 0,\end{aligned} \tag{72}$$

equal to (11) and (25). In order to derive the last step, note that in equilibrium, $\pi^2(a_i)$ (see (70)) contains the argument a_i only in the middle term, ce_i, where $e_i = k^* - a_i$. We know that a_i is eliminated from the first term (see the discussion above), $E[\pi^3(Y_i)]$, since a_i enters only as part of the sum $a_i + e_i$ which in equilibrium is replaced by the constant k^*. Since (72) is positive, it is confirmed that everybody participates.

The above analysis shows that the exact form of the reward, $p(y)$, is inconsequential. The fees, $f(n)$, are functions of contestants' expected continuation profit. The reward, $p(y)$, is part of that expected profit. A more generous reward implies higher entry fees and vice versa. What is important, however, is the fact that the best innovator gets the reward. We conclude that the two mechanisms can implement the above equilibrium (outcome) of the direct incentive–compatible procurement mechanism.

7.4 *Signaling*

Our procurement problem, in principle, exhibits a signaling issue. Players may have an incentive to signal their ability at the entry stage in order to influence their potential tournament rivals' effort choices.

In the literature, it is sometimes assumed that the private information becomes common knowledge before the tournament stage (e.g Fullerton and McAfee (1999)). This assumption might be justified e.g. in industries where players know each other such that they are sufficiently well informed as soon as the identity of their rivals is revealed.

In the present paper, we focussed attention on a symmetric tournament equilibrium that was unique in the sense that bidders play strategies that do not require predicting their rivals' types.

Although signaling is clearly an important issue, the relevance of an equilibrium without signaling can be justified: In complex decision problems, players may have to revert to simple heuristic strategies instead of trying to infer all required information. This may be due to time, cognitive or cost constraints etc. In this sense, if making use of signaled information is not feasible, then these signaling equilibria are not relevant and, instead, simple strategies based on one's own information might form the appropriate solution.

The analysis in section 4. has shown that the scoring auction mechanism has other, but, as we argued, less plausible, equilibria, where signaling is indeed an issue. We also mentioned that that mechanism, having many equilibria, cannot be recommended.

7.5 *Why not choose n after the entry fee auction?*

We took the two mechanisms, fixed prize and scoring auction, as given, and added an entry auction as in Fullerton and McAfee (1999). The procurer chooses the number of contestants, n, before the entry auction, i.e. before being able to infer the seller's abilities. This is also reflected in our welfare benchmark. It is a typical feature of multi–unit auctions that the number of objects to be sold is announced before the auction.[24]

However, one might ask if this is an optimal design since the procurer could use the information collected in the entry auction to optimally adjust the number of (costly) contestants. We give two reasons to justify the design we analyzed.

The first one is the observation that e.g. government procurers are often restricted to predefined rules of procurement, like specifying the (minimum) number of offers to elicit. This is done in order to increase transparency and prevent corruption of the government agents to whom the task of procurement is delegated.

The second reason is that choosing the number of contestants ex ante makes the game considerably simpler to play for the sellers:

[24]In our case, an object means entry to the tournament.

Suppose mechanisms F and S are modified sucht that the procurer announces n after the entry auction (and, in mechanism F, the prize P^F; alternatively, the buyer might announce a prize function $P^F(n)$ at stage 0; then the uncertainty is only about n). The apparent weakness of this design is that sellers are not allowed to express their willingness to pay for different n. Just consider that potentially any number between $n = 2$ and $n = N - 1$ might be chosen by the seller, depending on the realizations of abilities. If sellers can make just one entry bid, then they have to bid very cautiously in order to avoid overbidding for higher n, while at the same time they are underbidding if a lower n is selected. This might reduce the buyer's entry fee income although there might still be an equilibrium that identifies the most able sellers.

Now consider the more appealing modification of F and S where sellers are invited to submit bids contingent on the subsequent choice of n (and in mechanism F, the buyer announces a prize function $P^F(n)$ at stage zero), i.e. seller i submits a bid vector $\beta_i = (\beta_i(2), \beta_i(3), \ldots, \beta_i(N - 1))$. Then the buyer, after observing the bids, selects the most profitable n and collects the entry fees of a uniform–price auction with the bids $\beta_1(n), \beta_2(n), \ldots, \beta_N(n)$. An equivalent way of describing this game is to say that the sellers take part in $N - 2$ different entry fee auctions and then the buyer selects one of them (and the corresponding n and $P^F(n)$) to be payoff–relevant.

A complication of that design is that the sellers need beliefs about how the buyer chooses n if he receives contradictory or inconsistent signals about abilities: just suppose the observed bid functions cross, e.g. bidder i submits the highest bid for $n = 2$ but the second–highest bid for $n = 3$.

Although truthful bidding (for each n) seems to be a straightforward equilibrium candidate for the entry auction(s), we now have a potential incentive to deviate: Suppose everybody bids the true expected tournament profit (as in our basic game) for each n. Suppose the buyer then chooses some $n = \tilde{n}$. Then, say, seller j's bid, $\beta_j(\tilde{n})$ is the $\tilde{n}+1$st–highest bid for $\tilde{n}$ which implies that j sets the entry fee while not being selected as a contestant. Now if seller j deviates by reducing his bid $\beta_j(\tilde{n})$ then the entry fee for $n = \tilde{n}$ decreases and $\tilde{n}$ becomes less attractive for the buyer.[25] If this induces the buyer to choose a larger n then the deviation is profitable for j who then enters the tournament with a positive expected continuation profit. However, since there is uncertainty about which n will be selected, it is not easy to pin down a profitable deviation. Reducing one's bid for the n that is going to be selected is dominated, as in the basic game.

[25]The feasible range for that "deviation" depends on the next–lowest bid. If j undercuts the next–lowest rival then that rival sets the entry fee and j's bid becomes irrelevant.

7.6 *Entry fees*

Following Fullerton and McAfee (1999), we adopted an entry auction as a means of selecting the most able participants, collect revenue and restrict entry to the tournament. The alternative, a fixed entry fee, would be an additional strategic variable and thus a source for errors. The auction provides an endogenous entry fee.

Recall that we analyzed the uniform–price auction format only because of its simplicity. In the equilibria we discussed, a discriminatory auction, where bidders pay their bids, is revenue–equivalent.

Consider that format. In some procurement settings, bid preparation is very costly and sellers are only willing to participate if there is some kind of reimbursement also for unsuccessful bidders.[26] Then the bids in a discriminatory auction might be requests for reimbursement of preparation cost where the winners are those who require the least amounts. An alternative interpretation is that bids are not taken literally but are (sunk) cost of bid preparation or development of a prototype. Then one might expect that more able applicants, who believe to have a larger probability of winning, would invest more at this stage and can thus be identified. The fact that in our model the losers do not pay anything might be interpreted as a reimbursement of the losers' cost by the buyer. This would encourage participation and prevent mixed strategies. However, with that interpretation the procurer would not be able to collect money at the entry stage, as in our model.

7.7 *Bilateral contracts*

The following analysis applies to a context where, contrary to our assumption, the innovation is verifiable and, thus, innovation quality is contractible. We briefly discuss the use of bilateral contracts (B) for our uniform distribution assumption in order to compare the performance of a bilateral contract with that of our two tournament mechanisms.

As in mechanism S, consider the scoring rule $s = y - b$. We also use the same entry auction, which, since $n = 1$, collapses to a standard Vickrey auction. The auction winner, say, i, draws an innovation y_i and, in return, receives a prize $p(y_i)$.

Consider the prize function $p(y) = y$ where the procurer hands over the entire profit of the innovation to the innovator. Thus, the innovator has the monopoly rights on the innovation profit (residual claimant); in contrast to the tournament mechanisms, the auction winner gets a prize for sure. These are the optimal effort incentives since the innovator has

[26] Fan and Wolfstetter (2008) analyze a procurement problem where bid preparation is costly and the procurement game induces mixed participation strategies.

all relevant information (about ability), bears all cost and receives the entire profit of the innovation.[27] The procurer's profit is equal to the single entry fee that is paid by the auction winner.

Since the analysis is similar to that of the other mechanisms, we abbreviate the presentation.

At stage 3, the innovator receives profit $\pi^{B3}(y_i) = y_i$. At stage 2, i's expected profit is

$$\pi^{B2}(a_i, e_i) = E[Y] - ce_i - \gamma = \frac{k_i}{1+k_i} - ce_i - \gamma \tag{73}$$

It is strictly concave and the interior maximizer is $e_i^B = \frac{1}{\sqrt{c}} - 1 - a_i$. Again, $e_i^B > 0$ by assumption $c < \frac{1}{(N+2)^2}$ (see (27)) and the resulting expected innovation is the same as in mechanisms F and S. Reinserting into (73) we get the equilibrium profit

$$\pi^{B2}(a_i) := \pi^{B2}(a_i, e_i^B) = (1 - \sqrt{c})^2 + ca_i - \gamma \tag{74}$$

Again, the equilibrium bid is $\beta^B(a_i) = \pi^{B2}(a_i)$. At stage 1, a bidder with ability a_i expects profit

$$\begin{aligned}\pi^{B1}(a_i) &= H_{(1:N-1)}(a_i)\pi^{B2}(a_i) - \int_{\underline{a}}^{a_i} \pi^{B2}(a) dH_{(1:N-1)}(a) \\ &= c\int_{\underline{a}}^{a_i} H_{(1:N-1)}(a) da \end{aligned} \tag{75}$$

Comparing (75) with (11) and (25) these expressions are equivalent, but $n = 1$ in (75). Thus, (75) is smaller, i.e. a seller's expected profit is lower in the bilateral contract mechanism, B.

The procurer collects a single entry fee (equal to the equilibrium bid of the firm with second–highest ability), gets the innovation y_i, and pays out the prize $p(y_i) = y_i$ to the innovator. Expected profit is

$$\Pi^B = E[Y] - E[Y] + E[\beta(A_{(2:N)})] = E[\beta(A_{(2:N)})] \tag{76}$$

where

$$E[\beta(A_{(2:N)})] = E[\pi^{B2}(A_{(2:N)})] = (1 - \sqrt{c})^2 + cE[A_{(2:N)}] - \gamma \tag{77}$$

and (see subsection 9.1 in the Appendix), $E[A_{(2:N)}] = 1 - \frac{2}{N+1}$. Therefore,

$$\Pi^B = (1 - \sqrt{c})^2 + c\left(1 - \frac{2}{N+1}\right) - \gamma \tag{78}$$

Again, this is positive: recall the argument why (56) is positive and note that $\left(1 - \frac{2}{N+1}\right) > 0$.

[27] Alternatively, one can show that $p(y) = y$ is optimal within a class of increasing prize functions $p(y)$ that contain it as an element, e.g. it is easily shown that for $p(y) = \frac{1}{s}y^s$ with $s > 0$ it is optimal to set $s = 1$.

LEMMA 1 *Assume uniform distribution of types and innovations (see* (27)*) and suppose innovations are verifiable. If the R&D cost exhibits a sufficiently large "economies of scale" effect, then the bilateral contract (B) is superior to the tournament mechanisms (F and S); otherwise the tournaments are more profitable. In particular,*

$$B \succ \{F, S\} \iff \frac{\gamma}{c} > \frac{N-3}{N+1}. \tag{79}$$

PROOF Recall the equivalence results for F and S and denote both by T for "tournament". Consider n^T, the optimal number of tournament contestants. For $n^T_{\text{int}} < 2$ (which is equivalent to $\frac{\gamma}{c} > \frac{N-4}{N+1}$, we get the corner solution $n^T = 2$. Then the tournament profit is (see (41))

$$\Pi^T(2) = (1-\sqrt{c})^2 - 2(\gamma - c) - \frac{6c}{N+1}. \tag{80}$$

Then,

$$\Pi^B > \Pi^T(2) \iff \frac{\gamma}{c} > \frac{N-3}{N+1}. \tag{81}$$

Thus, we established that the tournament is superior in the range $\frac{N-4}{N+1} < \frac{\gamma}{c} < \frac{N-3}{N+1}$ while the bilateral contract is superior for $\frac{\gamma}{c} > \frac{N-3}{N+1}$.
Now consider $n^T_{\text{int}} > 2$ (which is equivalent to $\frac{\gamma}{c} < \frac{N-4}{N+1}$), taking into account that n is an integer. We show that $\Pi^T(n^T_{\text{int}} + 1) > \Pi^B$. Together with the facts that 1) the tournament profit $\Pi^T(n)$ is strictly concave in n, and 2) $n^T_{\text{int}} < N - 1$, this implies that there always exists an integer number of contestants in the range $[n^T_{\text{int}}, n^T_{\text{int}} + 1]$ that does not exceed $N-1$, where the tournament profit is above that of the bilateral contract.

$$\Pi^T\left(n^T_{\text{int}} + 1\right) = (1-\sqrt{c})^2 + \frac{1}{4}\left(N - (N+1)\frac{\gamma}{c} + 2\right)\left(c\frac{N-2}{N+1} - \gamma\right) \tag{82}$$

$$\Pi^T\left(n^T_{\text{int}} + 1\right) - \Pi^B = \frac{((N-4)c - (N+1)\gamma)\,(Nc - (N+1)\gamma)}{4c(N+1)} \tag{83}$$

Consider the RHS of (83). The denominator is a product where both factors are positive by assumption $n^T_{\text{int}} > 2$. □

One might think that whenever the bilateral contract is superior to the tournament, then this is due to the special prize function $p(y) = y$. However, it can be shown that the bilateral contract ($n = 1$) strictly profit dominates a tournament ($n \geq 2$) if the same prize function $p(y) = y$ is used in both.[28]

[28] A proof is not provided here since the computation is somewhat tedious and does not provide much insight. A corresponding Mathematica file can be provided by the author.

Now consider the welfare–maximizing buyer. It can be straightforwardly verified that mechanism B implements the welfare benchmark if the welfare–maximizer is $n^* = 1$, i.e. if $n^*_{\text{int}} \le \frac{3}{2}$ (see (60) and (61)). We have $E[Y] = 1 - \sqrt{c}$.

$$W^B = E\left[Y - c\left(\frac{1}{\sqrt{c}} - 1 - A_{(1:N)}\right) - \gamma\right] \tag{84}$$

$$= (1 - \sqrt{c})^2 - c\frac{N}{N+1} - \gamma \tag{85}$$

Thus, whenever $n^* = 1$ is the welfare–maximizer (in the sense of our benchmark), mechanism B is optimal.

Note that since the interior optimal n, n^*_{int}, is rounded to the nearest integer, we have that mechanism B is optimal if $n^*_{\text{int}} \le \frac{3}{2}$ (since then $n^* = 1$) while the two tournament mechanisms, F and S, are optimal if $\frac{3}{2} \le n^*_{\text{int}} \le N - \frac{1}{2}$ (since then $2 \le n^* \le N - 1$).

Comparing the above results with those of section 6., we note that the welfare–maximizing buyer finds an optimal mechanism among mechanisms F, S, and B, unless it is optimal (first–best) to let all existing inventors engage in R&D ($n^* = N$). In particular, mechanisms F and S are revenue–equivalent and optimal if $n^* \in [2, N-1]$ while mechanism B is optimal if $n^* = 1$. The restriction of optimality to $n^* < N$ is simply due to the fact that the bidding equilibrium in the entry auction breaks down if all N inventors are admitted to the tournament.

8. CONCLUSION

We considered procurement of an innovation when innovations are random and depend on heterogeneous and unobservable innovators' effort and ability. We also assumed that the innovation is not verifiable.

We looked at two procurement mechanisms that combine well–known elements: entry auctions, R&D tournaments, fixed prizes and scoring auctions. An entry fee auction was adopted in order to select the most able innovators, restrict entry and collect entry fees. The procurer's only strategic variable is the number of winners. After that auction a tournament takes place and the best resulting innovation is procured using either a fixed prize or a scoring auction. In the former, the procurer has to set an appropriate prize while in the latter, the best innovator's reward is determined endogenously.

We demonstrated existence of particular Bayesian Nash equilibria where the allocation of efforts, the innovations, and all players' expected profits are the same under both mechanisms. These equilibria do not involve a signaling issue since equilibrium efforts only depend on the respective player's own type. At the tournament stage, that symmetric equilibrium is the unique equilibrium with positive efforts of all contestants in the fixed–prize mechanism while the scoring–auction mechanism had multiple symmetric equilibria. Thus, one would favor the fixed–prize design.

The results imply that as long as the best innovation receives a reward, the particular form of the reward does not matter. In contrast to "pure" fixed–prize or scoring–auction tournaments, that result relies on the presence of the entry auction that allows bidders to adjust their bids to the subsequent procurement method. We also saw that there was a degree of freedom in choosing an auction format for the entry auction.

As an interpretation of our equivalence result, we argued that both mechanisms can be seen as implementations of the same equilibrium (outcome) of a larger class of (direct incentive–compatible) mechanisms. This, again, implies that particular features of the mechanisms are arbitrary.

The (expected) equivalence result was only shown under the uniform distribution assumption and convenient model parameters. However, it seems intuitive that a more competitive procurement stage, i.e. a scoring auction instead of a fixed prize, reduces the willingness to pay for participation. Thus, having a stage where bidders pay for entry makes the choice of the procurement mechanism more arbitrary.

We also showed that sellers might expect the same profit in both mechanisms, and that result did not depend on the uniform distribution assumption and it also did not depend on the optimal choice of the fixed prize. The latter might be somewhat surprising. The entry auction stage allows sellers to adjust bidding to the expected profits that arise from the subsequent game. The bidding equilibrium is a result of the sellers' competition with each other. A more generous reward leads to more aggressive bidding. This might explain why sellers' expected profits do not depend on the choice of the fixed prize: different profit opportunities are "competed away" between the sellers.

Under the uniform distribution assumption, our example mechanisms were constrained optimal for a welfare–maximizing procurer. In our symmetric equilibrium, effort and the expected innovation were the same for welfare– and profit–maximizing procurers. This is another reason that makes that particular equilibrium appealing.

9. APPENDIX

9.1 *Order statistics*

1. Demonstrate $E[A_{(i:N)}] = \frac{N+1-i}{N+1}$ for the uniform distribution example. We have

$$E[A_{(i:N)}] = \int_{\underline{y}}^{\overline{y}} a dH_{(i:N)}(a) = \underline{y} - \int_{\underline{y}}^{\overline{y}} H_{(i:N)}(a)da \\ = 1 - \int_0^1 H_{(i:N)}(a)da. \tag{86}$$

By a standard result on order statistics,

$$H_{(i:N)}(a) = \sum_{j=0}^{i-1} \binom{N}{j} a^{N-j}(1-a)^j \tag{87}$$

By (87), we have for any $k \in \{1, \ldots, N-1\}$),

$$H_{(k+1:N)}(a) - H_{(k:N)}(a) = \binom{N}{k} a^{N-k}(1-a)^k. \tag{88}$$

Repeated integration by parts of the RHS of (88) yields

$$\int_0^1 \binom{N}{k} a^{N-k}(1-a)^k da = \frac{1}{N+1}. \tag{89}$$

Combining (88) and (89) yields

$$\int_0^1 H_{(i:N)}(a)da = \frac{1}{N+1} + \int_0^1 H_{(i-1:N)}(a)da = \ldots \\ = \frac{i-1}{N+1} + \int_0^1 H_{(1:N)}(a)da = \frac{i}{N+1}. \tag{90}$$

Thus,

$$E[A_{(i:N)}] = 1 - \frac{i}{N+1} = \frac{N+1-i}{N+1}. \tag{91}$$

2. Derive $G'_{(2:n)}(y)$ for the uniform distribution on $(0,1)$. For mechanism S, recall that in equilibrium $k^S = k = a_i + e_i^S = \frac{1}{n}\left(\frac{1}{\sqrt{c}} - 1\right)$ is a constant for the $i = 1, \ldots, n$ tournament participants (see (44)).

$$G_{(2:n)}(y) = \Pr\{Y_{(2:n)} < y\} \\ = G^{nk}(y) + nG^{(n-1)k}(y)(1 - G^k(y)) \\ = nG^{(n-1)k}(y) - (n-1)G^{nk}(y) \tag{92}$$

$$G'_{(2:n)}(y) = nk(n-1)G'(y)\left(G^{(n-1)k-1}(y) - G^{nk-1}(y)\right) \tag{93}$$

If G is the uniform distribution on $(0,1)$ we have

$$\begin{aligned} G'_{(2:n)}(y) &= nk(n-1)\left(y^{(n-1)k-1} - y^{nk-1}\right) \\ &= (n-1)\left(\frac{1}{\sqrt{c}} - 1\right)\left(y^{\frac{n-1}{n}\left(\frac{1}{\sqrt{c}}-1\right)-1} - y^{\left(\frac{1}{\sqrt{c}}-2\right)}\right) \end{aligned} \tag{94}$$

9.2 *Proof that* (45) *is positive*

PROOF It is sufficient to show that $y < \frac{\sqrt{c}(1-\sqrt{c})^2}{n(n-1+\sqrt{c})}$. Since $y < \frac{c}{N-2}$ by assumption (27), we only need to show that

$$\frac{c}{N-2} < \frac{\sqrt{c}(1-\sqrt{c})^2}{n(n-1+\sqrt{c})}. \tag{95}$$

Recall that $n \leq N-1$. We replace n by $N-1$ in (95). We will demonstrate that

$$\frac{c}{N-2} < \frac{\sqrt{c}(1-\sqrt{c})^2}{(N-1)(N-2+\sqrt{c})}. \tag{96}$$

which implies (95). In order to do this, we first write (96) as:

$$\frac{\sqrt{c}(N-1)(N-2+\sqrt{c})}{N-2} < (1-\sqrt{c})^2. \tag{97}$$

Next, we use assumption $c < \frac{1}{(N+2)^2}$ (see (27)): We replace $\sqrt{c}$ by $\frac{1}{N+2}$ on both sides. Note that this makes the LHS greater and the RHS smaller. We get an inequality in N that is satisfied for $N \geq 3$ (as we assume). □

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