A Golden Rule of Public Finance or a Fixed Deficit Regime?
Growth and Welfare Effects of Budget Rules

Max Groneck *‡

Center for Public Economics, University of Cologne

Revised Version: February 2009

Abstract

In this paper, we compare growth and welfare effects of various budget rules within an endogenous growth model with productive public capital, utility enhancing public consumption and public debt. We find that a fixed deficit regime does not affect the long run growth rate compared to a balanced budget while the growth rate is increased by a golden rule. Welfare effects are ambiguous. Simulations indicate that economies populated by households who have a strong tendency to smooth consumption should adhere to a balanced budget rather than a golden rule or a fixed deficit rule from a welfare point of view.

Keywords: Budget rules, golden rule of public finance, fiscal policy, endogenous growth, welfare

JEL Codes: E62, O41, H60

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* The author wishes to thank Brooke and George Lundgren as well as Robert C. Plachta for valuable comments on an earlier draft of this paper.

‡groneck@wiso.uni-koeln.de. Center for Public Economics, University of Cologne, Albertus-Magnus-Platz, 50923 Cologne, Germany. Phone: 0049-221-470 57 57 Fax: 0049-221-470 50 33.

1 Introduction

The macroeconomic effects of budget rules have been studied recently in a number of papers using growth models along the line of Futagami et al. (1993). In this framework endogenous growth stems from investment in a public capital stock, which also raises the private incentive to invest. Greiner and Semmler (1999) and (2000) incorporate the so-called golden rule of public finance into this framework. This budget rule allows the government only to run deficits if those deficits are used to finance investments in the public capital stock. Greiner and Semmler (2000) find negative long run growth effects of a strict golden rule while a more flexible regime that additionally allows interest payments to be financed by deficits yields positive growth effects. Ghosh and Mourmouras (2004a) and (2004b) study long-run welfare implications of the golden rule in the same framework. They show that the golden rule can lead to welfare improvements in contrast to the standard intertemporal budget constraint. An interesting contribution is Minea and Villieu (2005) who study a fixed deficit budget rule in the same framework and find, contrary to Greiner and Semmler (2000), that a balanced budget always leads to higher long run growth than a fixed deficit budgetary regime.[1] They solve the model numerically and study welfare effects in comparison to a balanced budget during transitional dynamics.[2]

No comparison has been made between a golden rule and a fixed deficit regime in an endogenous growth framework which is the aim of this paper. The main difference between the rules is that they differently affect the composition of government spending.[3] The golden rule stipulates that deficits can only be used to finance public investments whereas a fixed deficit rule also allows public consumption to be financed by deficits. The composition of government spending in an endogenous growth setting has been studied by Lee (1992), Devarajan et al. (1998), Turnovsky and Fisher (1995), Turnovsky (2000), Park and Philippopoulos (2004) and Park (2006). Here, the government can either invest in the public capital stock which is an input in the production function or pay for public consumption goods which increase the utility of private households. By analyzing the optimal composition between the two outlays these studies are normative. The budget is always assumed to be balanced so taxes are the only revenues of the public sector.

This paper synthesizes certain aspects of the two streams of literature but it

[1] Throughout this paper a balanced budget is considered to be a budget rule that does not allow any structural deficits.

[2] Other notable contributions in that line of research are Ghosh and Nolan (2007) and Greiner (2007).

[3] For empirical findings see Poterba (1995).

remains a positive analysis. Including public debt in the aforementioned framework, the aim is to study the impact of budget rules on the composition of government spending and through this channel the impact on growth and welfare. The paper is structured as follows: Section 2 presents a short description of the budget rules under consideration. In section 3 we lay out the basic model and derive the decentralized equilibrium. Section 4 and 5 analyze the steady state and comparative statics of the fixed deficit regime and the golden rule respective analytically. In section 6, a simulation is presented of a regime switch from a balanced budget to a fixed deficit rule and to a golden rule. The steady state effects, transitional dynamics and welfare effects of the two rules are compared. Section 7 concludes.

2 Budget Rules in Theory and Practice

For the purpose of this paper a budget rule is defined as a permanent constraint on fiscal policy, typically defined in terms of an indicator of overall fiscal performance.[4] Restrictions on fiscal policy are mainly justified in conjunction with political economic considerations.[5] Positive deficits within a range should be allowed in order to react to cyclical fluctuations and to help achieve macroeconomic stability.

Budget rules can be classified under deficit-assignment (asset-related) rules and macroeconomic rules which restrict a certain fiscal indicator such as the deficit ratio. The golden rule of public finance is a deficit-assignment rule that only allows deficits in order to finance public investments. This rule intends to foster intergenerational equity by equally dividing the burdens and benefits of public investments from one generation to the next.[6] Moreover, the golden rule sets incentives for public investments which is especially important when it comes to short-sighted politicians.[7] In practice, the golden rule has a long tradition in Germany. Great Britain implemented a golden rule in 1997.[8]

Significant examples of rules that fix certain fiscal indicators are the Maastricht Criteria of the European Union and the Stability and Growth Pact for the members of the European Monetary Union which set a deficit ceiling of three percent of GDP and require the total government debt to not exceed 60 percent of GDP.[9]

[4] See Kopits and Symansky (1998), p. 2.

[5] See, for example, Schuknecht (2004), for an overview. There are additional justifications, see e.g. Kydland and Prescott (1977).

[6] For a discussion, see Mintz and Smart (2006).

[7] See Dur et al. (1997) for a formal treatment.

[8] See Kell (2001).

[9] The deficit ceiling can be breached in exceptional cases, see Amtenbrink et al. (1997) for a

The most important rule of the Stability and Growth Pact appears to be the three percent deficit target because it is regulated on a yearly basis. The other regulations, e.g. the debt target or the requirement to achieve a budget "close to balance or in surplus", are mid-term targets. For this reason we concentrate on a fixed deficit ratio as the most well-known restricted fiscal indicator and compare this to the golden rule of public finance. The next section presents the basic model through which the rules will be analyzed.

3 The Model

Before studying the effect of budget rules it is necessary to derive the decentralised equilibrium in the economy without any budgetary regime. The model includes public consumption and public debt in the Futagami et al. (1993) framework. The representative infinitely lived agent maximizes the discounted sum of utility in the form of:[10]

$$U = \int_0^\infty u\left(c, c^s\right) e^{-\rho t} dt, \tag{1}$$

where ρ is the subjective discount rate and the instantaneous utility $u\left(c_t, c_t^s\right)$ in period t is defined as:[11]

$$u\left(c_t, c_t^s\right) = \begin{cases} \frac{\left(c^\eta \cdot (c^s)^{1-\eta}\right)^{1-\sigma} - 1}{1-\sigma}, & \text{for } \sigma \neq 1 \\ \eta \log c + (1-\eta) \log c^s, & \text{for } \sigma = 1. \end{cases} \tag{2}$$

We denote c and c^s as private and public consumption in period t respectively. The parameter σ is the inverse of the intertemporal elasticity of substitution $S = \frac{1}{\sigma}$. For $S \to 0$ households try to perfectly smooth their consumption so that consumption today and tomorrow become perfect complements whereas for $S \to \infty$ the opposite occurs. The representative households flow budget constraint is given by:

$$\dot{k} + \dot{b} = rb + (1-\tau)\, y - c - \delta^k k. \tag{3}$$

The household uses after tax income $(1-\tau)\, y$ to consume c and invest $\dot{k} + \delta^k k$, where δ^k is the rate of private capital depreciation. The household can also buy government bonds b which yield the return of rb. It is assumed that it is not allowed

detailed description. For an assesment of the rules see Buti et al. (2003). The latest reform is described by Buti (2006).

[10] All variables are functions of time. But for convenience the time index is omitted.

[11] This form of utility function is also used in Lee (1992), p. 425.

to run ponzi-games:[12]

$$\lim_{t\to\infty}\left[\exp\left[-\int_0^t r_s ds\right](k+b)\right]=0. \tag{4}$$

The production function of the representative producer exhibits constant returns to scale with deminishing returns with respect to each other:

$$y = k^{1-\alpha} g^{\alpha}. \tag{5}$$

The output y is produced with private capital k and the public capital stock g where $0 < \alpha < 1$ is the elasticity of output to public capital. Population is normalized to unity.

The government can use tax revenues τy and deficits $\dot{b}$ to finance gross public investment $\dot{g} + \delta^g g$, public consumption c^s, and to serve debt obligations rb. As in Futagami et al. (1993), the tax rate is assumed to be a flat tax on output. The government constraint is therefore:

$$\dot{b} = rb + \dot{g} + \delta^g g + c^s - \tau y. \tag{6}$$

The public capital stock evolves according to:

$$\dot{g} = i - \delta^g g, \tag{7}$$

where i is gross investment and δ^g is the rate of depreciation of the public capital stock.

In the decentralized equilibrium the household maximizes its lifetime utility (1) together with (2) subject to its budget constraint (3) and the production function (5). This leads to the equilibrium growth rate of private consumption γ^c:

$$\gamma^c = \frac{\dot{c}}{c} = \frac{1}{\sigma}\left((1-\tau)(1-\alpha)\left(\frac{g}{k}\right)^{\alpha} - \delta^k - \rho\right). \tag{8}$$

From the first order conditions one can also derive the net rate of return for private capital r:

$$r = (1-\tau)(1-\alpha)\left(\frac{g}{k}\right)^{\alpha} - \delta^k. \tag{9}$$

A necessary condition for the growth rate (8) to be positive requires the interest

[12] The no-ponzi condition corresponds to the transversality condition resulting from the decentralized optimization problem of the households.

rate to exceed the time preference rate, $r > \rho$. For utility to be bounded it is also necessary that:

$$(1-\sigma)\,\gamma^c < \rho. \tag{10}$$

The no-ponzi condition (4) leads to another restriction that the interest rate needs to be greater than the growth rate of the economy:

$$r > \gamma. \tag{11}$$

4 Fixed Deficit Regime

Fixing the deficit ratio restricts borrowing but does not earmark the deficits for a specific outlay. The rule can be represented in the following form:[13]

$$\frac{\dot{b}}{y} = m, \tag{12}$$

where m is assumed constant.

When incorporating public consumption expenditures into the model of Futagami et al. (1993) a crucial assumption is how to divide the revenues between consumption and investment. Here it is assumed, that a constant fraction θ of the tax revenues is used for net public investment, an assumption often made in the literature:[14]

$$\dot{g} = \theta\tau y. \tag{13}$$

Public consumption is therefore endogenous. Equation (6) together with (12) yields:

$$c^s = [m + (1-\theta)\,\tau]\,y - rb - \delta^g g. \tag{14}$$

In the fixed deficit regime deficits are only used for public consumption. Of course, in practice deficits might also be used to finance public investments with this rule. The extreme case is selected to outline this rule from the golden rule which allows deficits only for public investments.

[13] This rule is studied by Minea and Villieu (2005) in a model without public consumption.

[14] See Ghosh and Mourmouras (2004b), p. 628; Devereux and Love (1995), p. 237; Agénor and Yilmaz (2006), p. 13.

4.1 Dynamic System

The equilibrium growth rate of consumption is again given by the decentralized solution (8). The rates of growth for public debt $\frac{\dot{b}}{b}$ and public investment $\frac{\dot{g}}{g}$ can be derived from (12) and (13) respectively. The growth rate of private capital can be deduced from (3), (6), (13) and (14) to be:

$$\gamma^k = \frac{\dot{k}}{k} = (1-m-\tau)\left(\frac{g}{k}\right)^{\alpha} + r\frac{b}{k} - \frac{c}{k} - \delta^k. \tag{15}$$

To solve for endogenous growth solutions, the dynamic system is expressed in terms of the private capital stock. For this we define the following variables: $c_k = \frac{c}{k}$, $g_k = \frac{g}{k}$, and $b_k = \frac{b}{k}$. The dynamic system of the economy becomes:

$$\begin{aligned} \frac{\dot{c}_k}{c_k} &= \frac{\dot{c}}{c} - \frac{\dot{k}}{k} = \frac{1}{\sigma}\left((1-\tau)(1-\alpha)g_k^{\alpha} - \delta^k - \rho\right) && (16)\\ &\quad + c_k + \delta^k - (1-m-\tau)g_k^{\alpha} - rb_k \\ \frac{\dot{b}_k}{b_k} &= \frac{\dot{b}}{b} - \frac{\dot{k}}{k} = \frac{mg_k^{\alpha}}{b_k} + c_k + \delta^k - (1-m-\tau)g_k^{\alpha} - rb_k && (17)\\ \frac{\dot{g}_k}{g_k} &= \frac{\dot{g}}{g} - \frac{\dot{k}}{k} = \theta\tau g_k^{\alpha-1} + c_k + \delta^k - (1-m-\tau)g_k^{\alpha} - rb_k. && (18) \end{aligned}$$

4.2 Steady State and Comparative Statics

A stationary point of the dynamic system corresponds to a balanced growth path where all initial variables g, b, c and k grow at the same constant rate. Constant steady-state values for c_k, g_k and b_k are found by setting the dynamic system to zero. From this we obtain two conditions for the long-run growth rate depending on g_k. Combining (16), (17) and (8) leads to the growth rate of public debt

$$\gamma = \frac{mg_k^{\alpha}}{b_k}.$$

Equating (17) and (18) leads to $b_k = \frac{mg_k}{\theta\tau}$. Inserting above, we obtain the relation γ^1 for the steady-state growth rate. The second condition γ^2 comes from the Euler equation (8) of the decentralized equilibrium. The two steady state conditions are thus:

$$\begin{aligned} \gamma^1(g_k) &= \theta\tau g_k^{\alpha-1} && (19)\\ \gamma^2(g_k) &= \frac{1}{\sigma}\left((1-\tau)(1-\alpha)g_k^{\alpha} - \delta^k - \rho\right). && (20) \end{aligned}$$

The first condition (19) is the growth rate of net public investments given by (13) whereas the second condition corresponds to the growth rate of private consumption. A steady state value for g_k must satisfy these two conditions. Steady state values for c_k and b_k are found by inserting the equilibrium values for g_k and γ together with admissible parameters for $\alpha, \theta, \tau, \sigma$ and ρ into the zero set dynamic system. The resulting steady state is locally stable.[15]

We are interested in studying the impact of an increased deficit ratio on the long run equilibrium. Inspecting equations (19) and (20) it is obvious that the deficit ratio m does not appear in these conditions. It follows that increasing the deficit ratio has no effect on the long run growth rate nor on the steady state ratio of public capital.

This leads us to the following proposition:

Proposition 1 *An increase of the deficit ratio in the fixed deficit regime leaves the long run growth rate uneffected given that* $c_k^s > 0$.

What is the reason for this outcome? Deficits in the fixed deficit regime are only used for public consumption leaving the public capital stock and thus the growth rate unaffected. The no-ponzi condition (4) requires the long run interest rate to be higher than the growth rate of the economy in steady state, thus of the growth rate of debt, $r > \frac{\dot{b}}{b}$. Hence, long run interest payments on debt are higher than the deficits. The debt burden is borne by cutting public consumption expenditures instead of public investment, so that even in the long run there is no decremental growth effect present. Of course, growth neutrality of higher deficits is only possible as long as public consumption expenditures are positive.

5 Golden Rule of Public Finance

The golden rule is a deficit-assignment rule prohibiting deficits to be used for public consumption. It is thus the ultimate intention of this budget rule to influence the composition of government expenditures.[16] The rule formally states that:[17]

$$\dot{b} = \dot{g} - (1 - \varphi)\, \tau y. \tag{21}$$

[15] See appendix for a proof.

[16] See Blanchard and Giavazzi (2004).

[17] This rule is a variant of Regime (A) in Greiner and Semmler (2000), p. 368 but with a major difference which will be discussed in this section. It is also used by Ghosh and Mourmouras (2004a), p. 245, Ghosh and Mourmouras (2004b), p. 630, and Agénor and Yilmaz (2006), p. 18.

The change of debt over time $\dot{b}$ equals the change of net investment $\dot{g}$ less the fraction $(1-\varphi)$ of tax revenues τy which are used to finance public investment.[18] The fraction $0<\varphi<1$ is used for unproductive expenditures so that:

$$\varphi\tau y = c^s + \delta^g g + rb. \tag{22}$$

Unproductive expenditures consist of public consumption c^s, depreciation costs $\delta^g g$ and interest payments rb. As in Greiner and Semmler (2000), the rate of change of public investment is given by:

$$\dot{g} = \varphi_1 (1-\varphi)\,\tau y. \tag{23}$$

The parameter $\varphi_1 > 1$ determines the level of deficit financed public investment. A value of $\varphi_1 = 1.5$ means that one third of net public investment is financed by deficits. Since φ is assumed to be fixed there is a constant fraction $\varphi_1(1-\varphi)$ which is used for public investment, analogous to the fixed deficit regime.

5.1 Dynamic System

The rates of growth of the variables form the dynamic system with a golden rule. Here the equations will immediately be expressed in relation to the private capital stock. The growth rate of public capital from (23) is given by:

$$\gamma^g = \frac{\dot{g}}{g} = \varphi_1 (1-\varphi)\,\tau g_k^{\alpha-1}. \tag{24}$$

Combining the budget constraint of the private households (3) with the golden rule restrictions (21), (22) and (23) yields the growth rate of private capital:

$$\gamma^k = \frac{\dot{k}}{k} = (1-(\varphi+\varphi_1(1-\varphi))\,\tau)\,g_k^{\alpha} + rb_k - c_k - \delta^k. \tag{25}$$

The growth rate of debt is obtained from (21) and (23):

$$\gamma^b = \frac{\dot{b}}{b} = \frac{(\varphi_1-1)(1-\varphi)\,\tau g_k^{\alpha}}{b_k}. \tag{26}$$

[18] The golden rule is modelled as a net investment rule unlike in Germany where gross investments are allowed to be financed by deficits.

The dynamic system with a golden rule thus becomes:

$$\begin{aligned}
\frac{\dot{c}_k}{c_k} &= \frac{\dot{c}}{c} - \frac{\dot{k}}{k} = \frac{1}{\sigma}\big((1-\tau)(1-\alpha) g_k^{\alpha} - \delta^k - \rho\big) \qquad (27)\\
&\quad + c_k + \delta^k - r b_k - (1 - (\varphi + \varphi_1 (1-\varphi))\tau) g_k^{\alpha} \\
\frac{\dot{b}_k}{b_k} &= \frac{\dot{b}}{b} - \frac{\dot{k}}{k} = \frac{(\varphi_1 - 1)(1-\varphi)\tau g_k^{\alpha}}{b_k} \qquad (28)\\
&\quad + c_k + \delta^k - r b_k - (1 - (\varphi + \varphi_1 (1-\varphi))\tau) g_k^{\alpha} \\
\frac{\dot{g}_k}{g_k} &= \frac{\dot{g}}{g} - \frac{\dot{k}}{k} = \varphi_1 (1-\varphi)\tau g_k^{\alpha-1} \qquad (29)\\
&\quad + c_k + \delta^k - r b_k - (1 - (\varphi + \varphi_1 (1-\varphi))\tau) g_k^{\alpha}.
\end{aligned}$$

5.2 Steady State and Comparative Statics

As in the fixed deficit regime, steady state values are obtained by setting the dynamic system to zero. The Euler equation (8) together with a combination of the zero set dynamic system lead to two conditions of the long run growth rate γ only depending on the variable g_k :

$$\gamma^1 (g_k) = \varphi_1 (1-\varphi)\tau g_k^{\alpha-1} \qquad (30)$$

$$\gamma^2 (g_k) = \frac{1}{\sigma}\big((1-\tau)(1-\alpha) g_k^{\alpha} - \delta^k - \rho\big). \qquad (31)$$

The first condition γ^1 again corresponds to the growth rate of public capital in (24) The steady state is locally stable.[19]

We now study the impact of higher deficits with the golden rule of public finance. This is represented by an increase of the policy parameter φ_1 which results in higher deficit financed public investments. The main difference to the fixed deficit regime is easily seen from equation (30): While condition (31) is unaffected by a higher φ_1 the impact on (30) - given the value for g_k - is positive:[20]

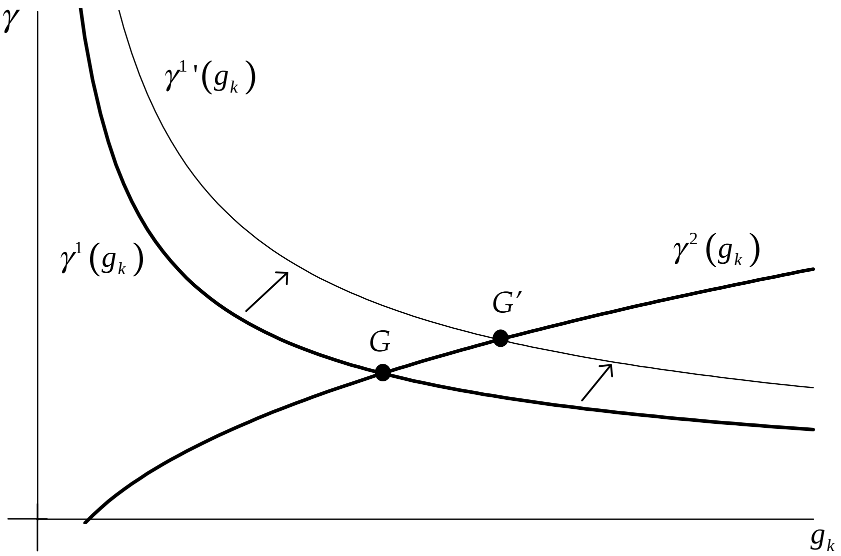
$$\left.\frac{d\gamma^1}{d\varphi_1}\right|_{g_k \ given} = (1-\varphi)\tau g_k^{\alpha-1} > 0$$

The comparative static effects are shown in figure 1, where equations (30) and (31) are represented grafically. An increase of public investments financed by deficits shifts the $\gamma^1(g_k)$-curve outwards and leaves the $\gamma^2(g_k)$-curve unchanged. The equilibrium moves from G to G', with a higher growth rate and a higher public capital

[19] See appendix for a proof.

[20] A detailed proof of this result is given in the appendix.

Figure 1: Steady state and comparative statics with the golden rule



ratio.

This leads us to:

Proposition 2 *An increase of the deficit ratio in the golden rule of public finance increases the growth rate given that* $c_k^s > 0$*.*

This proposition contrasts previous findings of the literature. Minea and Villieu (2005)[21] find that higher deficits for public investment can never increase the long run growth rate. The growth effect in Greiner and Semmler (2000)[22] is not clear cut analytically, however they show a negative impact in numerical simulations. The reason for the opposite outcome compared to Greiner and Semmler (2000) is the parameter φ which is endogenous in their model[23] whereas in our model it is constant. Endogenizing the fraction of tax revenues which are used for non-productive public expenditures has the following effect: Rising interests because of higher debt accumulation also raises the fraction φ of tax revenues devoted to unproductive expenditures. Consequently public investments fall and the higher debt burden in this case has to be borne by public investments. In our model φ is constant, so for equation (22) still to be met with higher interests on public debt, public consumption expenditures need to adjust. Here, as in the fixed deficit regime,

[21] see proposition 1, p. 11.
[22] see proposition 1, p. 372.
[23] Greiner and Semmler (2000) use the notation φ_0 for φ, see equation (20) on p. 370.

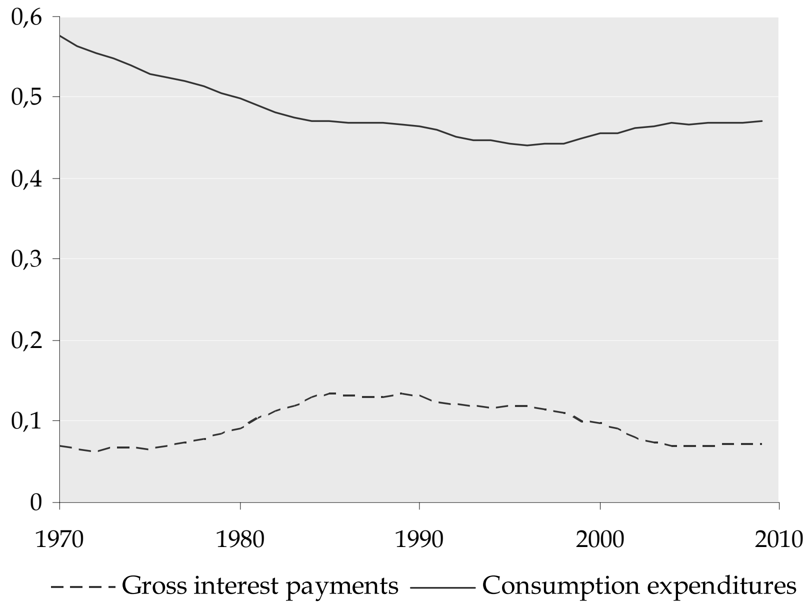
public consumption is an endogenous variable with:

$$c_k^s = \varphi \tau g_k^\alpha - \delta^g g_k - r b_k. \tag{32}$$

Higher interests on debt resulting from an increase in φ_1 lead to lower public consumption expenditures whereby public investments are increased. Consequently, the golden rule substitutes public consumption by public investments in this model. Even though this formulation of the golden rule is used in a variety of papers, the crucial difference of the parameter φ being exogenous or endogenous has not been made clear in the literature so far.[24]

The central assumption for the results above is that higher debt burdens are served with a cut in public consumption expenditures. Generally, public investments tend to be adjusted due to revenue losses resulting, for example, from an economic downturn. Public investment outlays are disposable in the short run so they can be reduced more easily than public consumption expenditures. However, this is an argument for the short term. Higher interests on debt evolve gradually, reducing the budget in the mid and long run. Assuming consumption expenditures as the adjustment variable in the longer run seems realistic. Figure 2 displays con-

Figure 2: Public consumption and interest payments relative to total public expenditures in OECD countries



[24]In contrast, Ghosh and Mourmouras (2004b), p. 630, claim that their golden rule in equation (14) matches regime (A) in Greiner and Semmler (2000), p. 368, which is apparently not the case as was shown above.

sumption expenditures relative to total government outlays compared to the interest payments as a ratio of total expenditure for the OECD countries from 1970-2009.[25] The figure indicates that increased interest payments on debt as a percentage of total expenditures were accompanied by a lower consumption expenditure ratio and vice versa. There is no doubt that raising debt obligations also leads to a fall in public investments.[26] However, this paper concentrates on the other extreme of how a government can react to higher interest payments. Additionally, our formulation for the golden rule has been used in a series of papers. Interestingly enough, substituting utility increasing public consumption with productive expenditures has complex welfare effects. These impacts on welfare together with transitional dynamics resulting from positive deficits is what we turn to in the next section.

6 Numerical Simulations

To study the macroeconomic effects of the budget rules more clearly we refer to numerical simulations. The base case is choosen to be the balanced budget with a zero deficit. We analyze steady state effects and transitional dynamics of a transition to a fixed deficit regime and a golden rule respectively.

6.1 Base Case and Calibration

As a reference point we choose the balanced budget which allows no debt or deficits so that

$$\dot{b} = b = 0.$$

The budget rules in the model above both have the balanced budget as a special case. The fixed deficit rule becomes a balanced budget by setting the fixed deficit ratio to zero ($m = 0$) while for the golden rule the parameter φ_1 needs to be one ($\varphi_1 = 1$) which means that public investments are solely financed by tax revenues. To compare the fixed deficit with the golden rule it is necessary to assume the fraction of tax revenues devoted to public investments (see equations (13) and (21)) to be the same in both rules:

$$\theta = 1 - \varphi.$$

Table 1 presents the chosen parameters for the calibrated model. The values are stylized and mainly chosen so that a large range of parameters of the elasticity of

[25] Source: Organization for Economic Co-operation and Development (2008).

[26] Another possible reaction to raising interest payments is to increase the taxrate, of course.

Table 1: Calibrated parameter values

$S = 0.4$	Intertemporal elasticity of substitution
$\rho = 0.1$	Subjective discount rate
$\eta = 0.2$	Utility weight factor for private consumption
$\alpha = 0.4$	Share of public capital
$\tau = 0.4$	Income tax rate
$\delta^k = \delta^g = 0.05$	Depreciation rates
$k_0 = 1$	Initial private capital
$\theta = 0.1$	Fraction of taxes for public investments (fixed deficit)
$m = 0.01$	Deficit ratio (fixed deficit)
$\varphi = 0.9$	Fraction of taxes for non-productive expenditures (golden rule)
$\varphi_1 = 1.25$	Parameter for deficit financed investments (golden rule)

substitution are consistent with the solvability condition (11). The value for the intertemporal elasticity of substitution S is also used by Turnovsky (2004) and is in the middle of the range of empirical findings.[27] The weight factor for public consumption in the utility function $(1-\eta)$ is in the middle of what is chosen in the literature.[28] The output elasticity with respect to public capital α is in line with the findings of Aschauer (1989).[29] The rates of depreciation δ^g and δ^k and the investment tax revenue ratio θ (and $1-\varphi$) are broadly consistent with German data. The fixed deficit rule is studied with a deficit ratio m of one percent. Correspondingly, the policy parameter φ_1 for the golden rule is chosen to lead to a one percent deficit. From the condition $\frac{\dot{b}}{y} = (\varphi_1 - 1)(1-\varphi)\tau$ in (21) and (23) this is found by setting $\varphi_1 = 1.25$. The private capital in time zero k_0 is normalized to unity.

6.2 Steady State Effects

Table 2 lists the steady state values for the base case consisting of a balanced budget, the fixed deficit and the golden rule. The presented variables are the growth rate γ, public capital g_k, public debt b_k and private and public consumption c_k and c_k^s all in relation to the private capital stock.

The balanced budget leads to a growth rate of $\gamma = 0.0556$ and a corresponding public capital ratio of $g_k = 0.5775$. The values are found by solving the system (19)

[27] See Biederman and Goenner (2008) for a literature review.

[28] The weight factor ranges from 0.075 to 0.3. See Baier and Glomm (2001), Footnote 21, p. 2032, Park and Philippopoulos (2004), p. 657 and Turnovsky (2000), p. 202.

[29] The study of Aschauer (1989) has been criticized for methological shortcomings. Subsequent studies found elasticities which are much lower though still positive. See Romp and De Haan (2007) for a review.

Table 2: Steady state values

	γ	g_k	b_k	c_k	c_k^s
Balanced Budget	0.0556	0.5775	0.0	0.3761	0.2601
Fixed Deficit	0.0556	0.5775	0.1444	0.4026	0.2337
Golden Rule	0.0632	0.6769	0.1354	0.4265	0.2392

and (20) with $m = 0$.[30] The private consumption ratio in steady state results from relation (15) or (25) with the equilibrium growth rate γ:

$$c_k = (1 - \tau)\, g_k^{\alpha} - \gamma - \delta^k.$$

The public consumption ratio comes from relation (14) or (32) with the chosen parameters.

Introducing a fixed deficit regime leaves the long run growth rate and the public-private capital ratio uneffected, as was expected. The deficit is only used for public consumption so that there is no long run growth effect. A positive deficit ratio leads to a positive debt ratio. Private consumption in relation to the private capital stock also increases whereas the public consumption ratio falls in the long run. The latter is due to the fact that debt obligations are served by cutting public consumption.

The golden rule however, exhibits positive long run growth effects; the growth rate increases to $\gamma = 0.0632$ and the public capital ratio g_k is also higher. The public debt ratio is now positive and private consumption also increases as a measure of private capital, whereas the public consumption ratio falls. The rise in the ratio of private consumption is more profound than with a fixed deficit while the fall in the public consumption ratio is less strong. The positive growth effect comes from the fact that with the golden rule the deficits are used for public investments which boost private investments and the rate of growth of the economy. This effect is even present in the long run because the interests on debt only lead to a fall in public consumption. The increase of private consumption because of higher output and higher returns from government bonds outweighs the increase of the private capital stock, so that the ratio c_k also increases.

6.3 Transitional Dynamics

The transitional dynamics of a regime change from a balanced budget to a fixed deficit and to a golden rule respectively is interesting as there are different short run

[30] Solving the system (30) and (31) for $\varphi_1 = 1$ yields the same result.

effects of the variables to be expected than in the long run. Moreover, the effects are very different between the two budget rules under consideration. The economy is assumed to be in a steady state equilibrium with a balanced budget initially and an unanticipated policy change to a new budget rule occurs at time $t = 0$.

Numerical simulations of endogenous growth models are usually performed by linearizing the dynamic system around the steady state to study the dynamics of the linear system.[31] The method over-predicts short run responses of the variables and can lead to huge deviations of the true system. Hence, policy changes might be judged incorrectly.[32] For this reason we use the relaxation-algorithm proposed by Trimborn et al. (2008) which is well suited for endogenous growth models with more than one state variable.[33]

6.3.1 Dynamics of the Growth Rates

Figure 3 compares the dynamics of the growth rate of public capital g, private consumption c, private capital k and public debt b for the fixed deficit regime and the golden rule. All growth rates start with the value $\gamma = 0.0556$ and exhibit an immediate jump after the policy change except for the growth rate of consumption which moves gradually after the shock. With the fixed deficit rule the rates eventually evolve to their initial value while the growth rates with the golden rule all meet at a higher value of $\gamma^{GR} = 0.0632$.

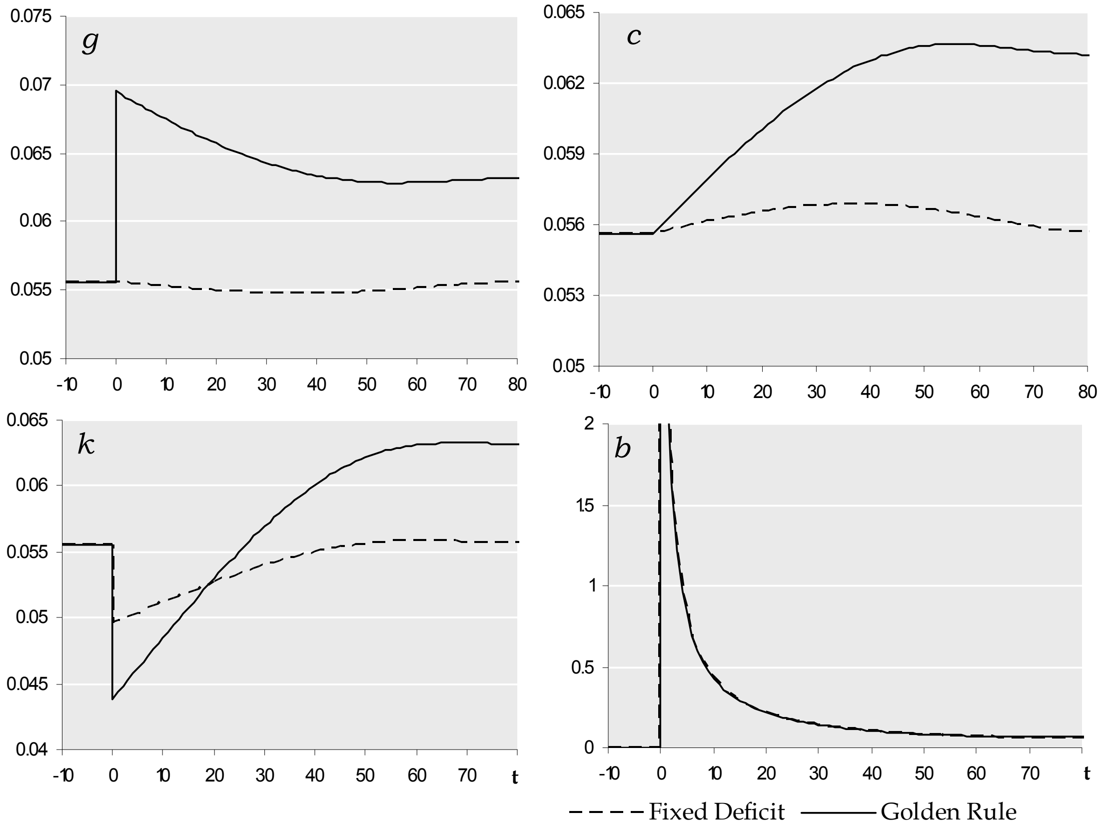
The crucial difference between the two rules is the development of the growth rate of public investment. The golden rule leads to an immediate jump of the rate of growth of public capital and also a higher growth rate in the long run. In the fixed deficit rule this growth rate slightly falls in the medium run while maintaining the same value in the long run. The movement of the growth rate of private capital is more profound with the golden rule. An increase of the deficit ratio immediately shifts ressources away from the private sector, causing an initial decline in the rate of private investment. This can be seen from equation (15) for the fixed deficit rule and (25) for the golden rule where the relevant parameters m and φ_1 both have a negative impact on γ^k. Over time the growth rate increases again. With fixed deficits the rising rate of private capital comes from increased income of the households because of higher returns on bonds. These two effects compensate for each other in the long run. The golden rule however, also leads

[31] This is done e.g. by Minea and Villieu (2005) and Greiner (2007).

[32] For a comparison of the linearization method and a more complex iteration see Atolia et al. (2008).

[33] All simulations are performed with Matlab 7.0.

Figure 3: Dynamics of the growth rates with the golden rule vs. the fixed deficit regime

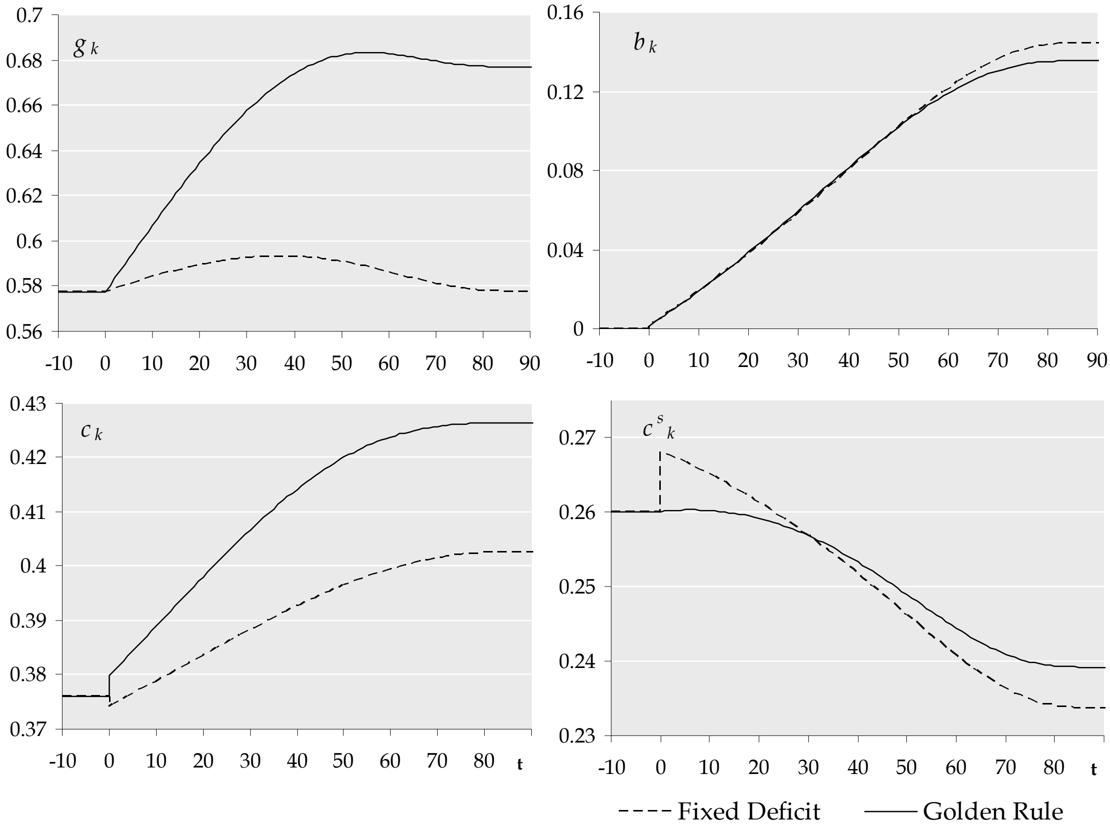


to the accumulation of public capital and thus higher output, so private capital growth eventually exceeds the initial value. Additional public capital formation also increases the interest rate (see (9)) leading to more consumption and less savings. Thus, the consumption growth rate (8) increases. In the fixed deficit rule the lower rate of private investment leads to a lower output. This explains the temporary decline of the growth rate of public capital because private capital eventually grows faster again. Because of the temporarily higher marginal product of private capital the growth rate of private consumption in the fixed deficit regime is higher in the medium term until it falls back to its initial value. With the golden rule however, the rate of private consumption gradually increases along with the increased marginal products to a higher value. The growth rate of public debt initially jumps to very high values but diminishes fast eventually being higher with the golden rule than with the fixed deficit rule.

6.3.2 Dynamics of the Ratios

Figure 4 depicts the variables in percentage of the private capital stock of the two budget rules and compares their dynamics. The public capital ratio g_k increases

Figure 4: Dynamics of the ratios with the golden rule vs. the fixed deficit regime



gradually on a higher level with the golden rule, because deficits were used to finance public investments which are also not cut in the long run to meet the debt obligations. In the fixed deficit regime the public capital ratio raises in the medium term and then falls back to its initial value. This temporary movement is due to the stronger fall of private relative to public capital (see figure 3). Both budget rules yield positive debt ratios compared to the balanced budget, eventually being a little higher with a fixed deficit. The trend of the private and public consumption ratios c_k and c_k^s move in equal directions in both rules in the long run. The main difference can be found in the immediate response. The golden rule leads to an upward jump of the private consumption ratio c_k while the ratio falls in the fixed deficit regime. The effect on the public consumption ratio c_k^s is reversed. Here, the ratio jumps up in the fixed deficit regime and gradually declines with the golden rule. This can be explained with the assumption that deficits are used for public consumption only in the fixed deficit regime. With rising debt the public consumption ratio falls to a lower level, because debt obligations are met by lowering public consumption expenditure. Since the debt ratio is higher in the long run with fixed deficits, the public consumption ratio is eventually lower.

The different impact of the budget rules on private consumption is more complex. Positive deficits raise the consumption ratio with both budget rules in the long run but the immediate jump after the policy change in time $t = 0$ is downward. The golden rule initially leads to an upward jump of the ratio while with the fixed deficit rule the consumption ratio initially falls (see figure 4). The reason for this different outcome is as follows: deficits have two effects on the household's response of present consumption: an intertemporal substitution effect and an income effect. The income effect due to returns from government funds is positive for both budget rules. A golden rule additionally increases output, thus, the income effect is higher than with the fixed deficit rule. The intertemporal substitution effect works in the opposite direction. With increasing interest rates the willingness to save is higher so the substitution effect leads to less consumption today in favor of future consumption. In the short run this effect outweighs the (lower) positive income in the fixed deficit rule while in the golden rule the income effect dominates. The choice of the intertemporal elasticity of substitution S crucially affects the consumption behavior of households. As mentioned above, the higher the value for S the more the households are willing to substitute consumption tomorrow for present consumption, thus the substitution effect becomes stronger.

6.4 Welfare Effects

Welfare effects are calculated by comparing the utility paths from a balanced budget to a new budget rule being either a fixed deficit or a golden rule, both leading to a deficit ratio of one percent. We compare the steady state instantaneous utility maintaining a balanced budget $u_t\left(c_t^{zero}, (c_t^s)^{zero}\right)$ with the utility path resulting from a transition to a steady state with a new budget rule $u_t\left(c_t^{pos}, (c_t^s)^{pos}\right)$. This method is not just a comparison of steady state welfare but takes the whole transition path into account.[34] To judge the welfare effect we use the present value of instantaneous net welfare defined as:

$$\Delta u_t = \left[u_t\left(c_t^{pos}, (c_t^s)^{pos}\right) - u_t\left(c_t^{zero}, (c_t^s)^{zero}\right)\right] \cdot \exp\left(-\rho \cdot t\right). \tag{33}$$

[34] A similar procedure for calculating welfare effects is performed by Minea and Villieu (2005), pp. 14 and Greiner (2007), pp. 471.

The time path of Δu_t gives the welfare effects of a regime switch over time. The total net welfare ΔU is calculated by taking the integral:

$$\Delta U = \int_0^{\infty} \Delta u_t dt. \tag{34}$$

The total net welfare cannot be compared for different elasticities of substitution because the utility level greatly varies. For this reason the total net welfare effect will be presented in a ratio of initial total welfare:

$$\Delta U \text{ (in \%)} = \frac{\Delta U}{U^{zero}}. \tag{35}$$

The sign of ΔU provides a criterion to jugde the welfare impact of deficits: if the sign is positive the welfare of the economy is increased by the new rule.

To evaluate utility of the private households we need to find the paths of private and public consumption. We first turn to the calculation of time paths within the steady state with a balanced budget. Private consumption c_t^{zero} is calculated as:

$$c_t^{zero} = k_0 \cdot c_k^{zero} \cdot \exp\left(\gamma^{zero} \cdot t\right),$$

where c_k^{zero} and γ^{zero} are the constant steady state consumption ratio and the growth rate respectively and k_0 is the initial value of the private capital stock. The public consumption path with a balanced budget $(c_t^s)^{zero}$ is found by first calculating the time path of private capital with:

$$k_t^{zero} = k_0 \cdot \exp\left(\gamma^{zero} \cdot t\right).$$

Multiplying this with the known steady state public capital ratio $(c_k^s)^{zero}$ yields the time path of public consumption:

$$(c_t^s)^{zero} = (c_k^s)_t^{zero} \cdot k_t^{zero}. \tag{36}$$

Inserting these time paths into the utility function (2) we get the utility path if the economy stays in a steady state with a balanced budget regime $u_t\left(c_t^{zero}, (c_t^s)^{zero}\right)$. Lifetime utility is obtained from the integral according to (1) and will be labeled U^{zero}.

Evaluating private and public consumption after the introduction of a new budget rule is more complex. The adoption of a new rule leads to transitional dynamics so the growth rates and the ratios are not constant anymore. The time path of private consumption after a regime switch with positive deficits c_t^{pos} can be calculated as follows:

$$c_t^{pos} = k_0 \cdot c_{k0}^{pos} \cdot \exp\left(\int_0^t \gamma_s^{pos} ds\right),$$

where the instantaneous growth rate γ_s^{pos} is calculated with the time path of g_k from the simulation inserted in (8). The initial jump of consumption is the first value of the capital ratio from the simulation which will be labeled as c_{k0}^{pos}.

Again the time path for public consumption is given by (36). Now the time path of the ratio for both rules is calculated with (14) and (32) respectively using the values for g_k and b_k from the simulation. Private capital k_t evolves with:

$$k_t^{pos} = k_0 \cdot \exp\left(\int_0^t \gamma_s^k ds\right),$$

where the of γ_s^k is calculated with (15) and (25) respectively, additionally using the simulated time path for c_k. Instantaneous utility is again defined as in (2) and is labeled $u_t\left(c_t^{pos}, (c_t^s)^{pos}\right)$ whereas the lifetime utility (1) is labeled U^{pos}. The welfare effects of a reform of the budget rules from a balanced budget with zero deficits to a fixed deficit rule with $m = 0.01$ and to a golden rule with $\varphi_1 = 1.25$ are presented in table 3.[35] The net welfare effects of the transition to a fixed deficit

Table 3: Welfare effects from a balanced budget to a new budget rule

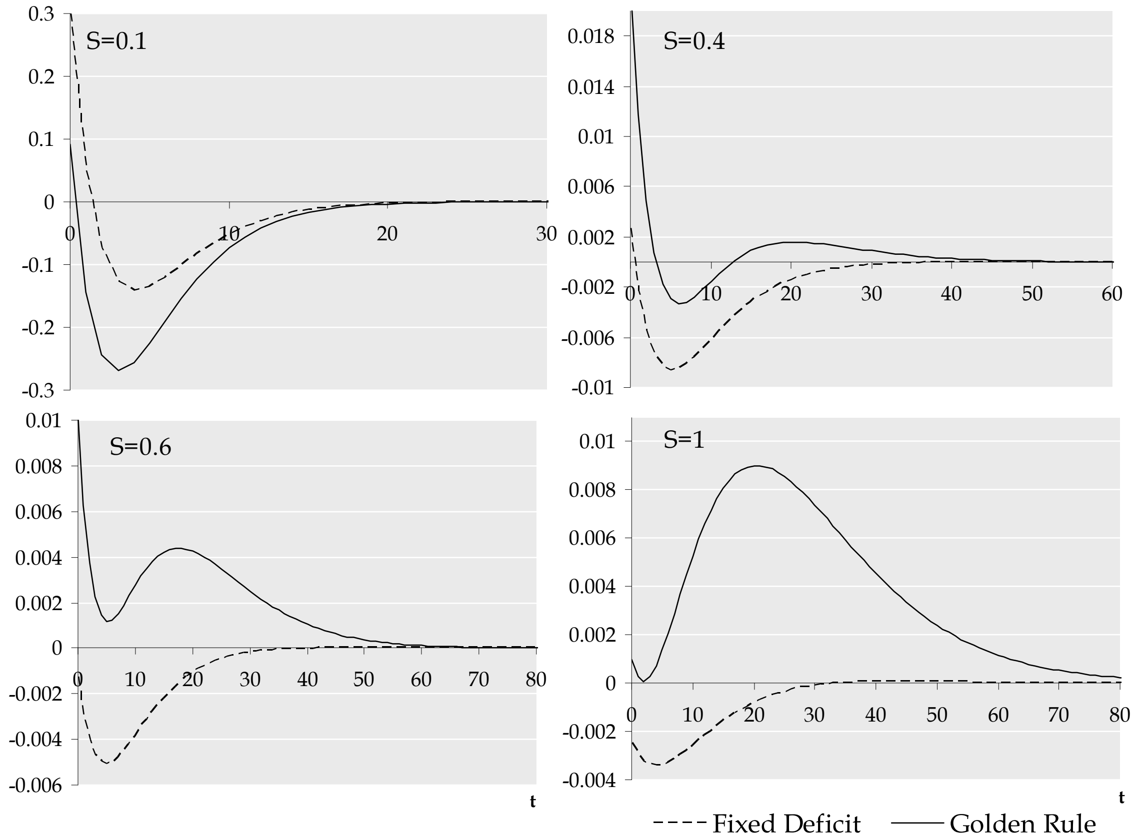
S	U^{zero} Balanced Budget	U^{pos} Fixed Deficit Rule	Golden Rule	ΔU Fixed Deficit Rule	Golden Rule
0.1	−31.2337	−31.9414	−33.0677	−2.27%	−5.87%
0.2	−17.2327	−17.4940	−17.5105	−1.52%	−1.61%
0.4	−9.6990	−9.8045	−9.6502	−1.09%	0.5%
0.6	−7.4670	−7.5389	−7.3327	−0.97%	1.8%
1	−5.5294	−5.5807	−5.2236	−0.93%	5.53%

rule are all negative. The lower the intertemporal elasticity of substitution S the higher the net welfare loss due to the fixed deficit regime being up to −2.27 percent with an elasticity of $S = 0.1$. Welfare effects from a transition to a golden rule are ambiguous. A golden rule leads to welfare losses of −5.87 percent for low elasticities, however this result turns over for higher elasticities. The net welfare gain from a golden rule amounts to 5.53 percent (with $S = 1$). The turning point for the golden rule is an elasticity of substitution of $S > 0.331$ for the given parameters. Low

[35] The time horizon is chosen to be $T = 100$ periods.

elasticity values imply that households treat consumption today and tomorrow as complements. Thus the results indicate that if households have a strong tendency to smooth consumption, the introduction of a golden rule leads to welfare losses in the economy. Only if households are willing to substitute consumption today and tomorrow, introducing a golden rule would be beneficial. Figure 5 shows the

Figure 5: Dynamics of net welfare with the golden rule vs. the fixed deficit regime



dynamics of the net welfare effect resulting from a transition to the fixed deficit rule and to the golden rule respectively (see equation (33)). The short run effects are positive for low values of S in both cases but for different reasons. The positive effect with the golden rule is due to the upward jump of private consumption while with the fixed deficit rule it is the upward jump of public consumption which outweighs the negative impact from the downward jump of private consumption that is less profound with low S-values. With higher elasticities the short run effect becomes negative for the fixed deficit rule and remains positive for the golden rule. The main effect on welfare with higher elasticities is due to a much longer adjustment process of the economy. This means that the positive income effect on consumption with the golden rule is much more distinct.

7 Conclusion

This paper shows growth and welfare effects of budget rules within an endogenous growth model with productive public capital, welfare improving public consumption expenditures and public debt. We compare a fixed deficit regime and a golden rule of public finance. For the former it is assumed that deficits are only used for public consumption while the latter allows public deficits only for public investments. This comparison points to an important aspect of budget rules: namely their impact on the composition of government expenditures. The transition from a base case with a balanced budget to both of the two rules is simulated. We show that in the long run positive deficits in the fixed deficit regime do not affect the growth rate while the growth rate is increased with a golden rule. These findings are in contrast to previous studies due to the assumption that rising debt obligations are borne by cutting public consumption instead of investments. This leads to complex welfare effects since public consumption expenditures deliver utility to the private households. Welfare effects from a transition to a fixed deficit regime are negative. Introducing a golden rule has ambiguous welfare effects being only positive with high elasticities of intertemporal substitution. The results indicate that economies populated by households who have a strong tendency to smooth consumption should maintain a balanced budget from a welfare point of view.

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Appendix

Local stability with a fixed deficit regime

To confirm local stability of the steady state with a fixed deficit rule we calculate the Jacobian matrix. The dynamic system with a fixed deficit is given by (16), (17) and (18) in the main text. The Jacobian is defined as:

$$J \equiv \begin{bmatrix} \frac{d\dot{c}_k}{dc_k} & \frac{d\dot{c}_k}{db_k} & \frac{d\dot{c}_k}{dg_k} \\ \frac{d\dot{b}_k}{dc_k} & \frac{d\dot{b}_k}{db_k} & \frac{d\dot{b}_k}{dg_k} \\ \frac{d\dot{g}_k}{dc_k} & \frac{d\dot{g}_k}{db_k} & \frac{d\dot{g}_k}{dg_k} \end{bmatrix}$$

The matrix is calculated by differenciating the differential equations for the variables c_k, b_k and g_k. This yields:

$$J = \begin{bmatrix} c_k & -rc_k & c_k \alpha g_k^{\alpha-1}\left((1-\tau)(1-\alpha)\left(\frac{1}{\sigma}-b_k\right)-(1-\tau-m)\right) \\ b_k & -\frac{mg_k^{\alpha}}{b_k}-rb_k & \alpha g_k^{\alpha-1}\left[m(1-b_k)+b_k(1-\tau)(1-(1-\alpha)b_k)\right] \\ g_k^i & -rg_k & \left(-\alpha(1-m-\tau)-(1-\alpha)\left(\alpha(1-\tau)b_k+\frac{\theta\tau}{g_k}\right)\right)g_k^{\alpha} \end{bmatrix}$$

with the interest rate $r = (1-\alpha)(1-\tau)\left(g_k^i\right)^{\alpha} - \delta^k$. The steady state is locally stable if the eigenvektor of the system has one positive and two negative eigenvalues λ_i. This is equivalent to a positive determinant of the Jacobian matrix $|J|$. With the parameters of the text ($\alpha = \tau = 0.4$; $\delta^k = \delta^g = 0.05$; $\rho = 0.1$; $\theta = 0.1$; $m = 0.01$) we first calculate the steady state values for different values of the intertemporal elasticity of substitution S:

Table 4: Steady state values, fixed deficit rule

	g_k	c_k	b_k
$S = 0.2$	0.9629	0.5635	0.2407
$S = 0.4$	0.5775	0.4026	0.1444
$S = 0.6$	0.4397	0.3322	0.1099
$S = 1$	0.3224	0.2607	0.0806
$S = 1.5$	0.2596	0.2145	0.0649

The resulting eigenvalues and the determinant are given by:

Table 5: Eigenvalues and determinant of the Jacobian, fixed deficit rule

	λ_1	λ_2	λ_3	$\lvert J\rvert$
$S=0.2$	0.2058	-0.0238	-0.0238	0.0012
$S=0.4$	0.2246	-0.0758	-0.0758	0.0018
$S=0.6$	0.2166	-0.0967	-0.0967	0.0022
$S=1$	0.2038	-0.0975	-0.1437	0.0029
$S=1.5$	0.1950	-0.1775	-0.1001	0.0035

The results show two negative and one positive eigenvalue corresponding with a positive determinante of the Jakobian for various elasticities. Hence, the steady-state is saddle-path stable.

Local stability with a golden rule

The dynamic system with the golden rule is given by equations (27), (28) and (29).of the main text. The Jacobian matrix with a golden rule is given by:

$$J^G = \begin{bmatrix} c_k & -rc_k & a_{13} \\ b_k & -\frac{(\varphi_1-1)(1-\varphi)\tau g_k^\alpha}{b_k} - rb_k & a_{23} \\ g_k & -rg_k & a_{33} \end{bmatrix} \tag{37}$$

with:

$$\begin{aligned} a_{13} &= c_k\alpha g_k^{\alpha-1}\left((1-\tau)(1-\alpha)\left(\frac{1}{\sigma}-b_k\right)-(1-(\varphi+\varphi_1(1-\varphi))\tau)\right) \\ a_{23} &= \alpha g_k^{\alpha-1}\left(\tau(1-\varphi)\left[\varphi_1(1-b_k)-1\right]+b_k\left((1-\tau\varphi)-(1-\tau)(1-\alpha)b_k\right)\right) \\ a_{33} &= \left((\alpha-1)\varphi_1(1-\varphi)\frac{\tau}{g_k}-\alpha\left((1-(\varphi+\varphi_1(1-\varphi))\tau)-(1-\tau)(1-\alpha)b_k\right)\right)g_k^\alpha \end{aligned}$$

and the interest rate. With the parameters of the text ($\alpha=\tau=0.4$; $\delta^k=\delta^g=0.05$; $\rho=0.1; \varphi=0.9$; $\varphi_1=1.25$) the resulting steady state values are for different values of the intertemporal elasticity of substitution S given by:

Table 6: Steady state values, golden rule

	g_k	c_k	b_k
$S = 0.2$	1.1468	0.6029	0.2294
$S = 0.4$	0.6769	0.4265	0.1354
$S = 0.6$	0.5095	0.3485	0.1019
$S = 1$	0.3674	0.2681	0.0735
$S = 1.5$	0.2914	0.2154	0.0583

From this we can calculate the eigenvalues and the determinant which also confirm saddle-path stability:

Table 7: Eigenvalues and determinant of the Jacobian, golden rule

	λ_1	λ_2	λ_3	$\|J\|$
$S = 0.2$	0.2306	-0.0307	-0.0307	0.0016
$S = 0.4$	0.2423	-0.0852	-0.0852	0.0023
$S = 0.6$	0.2301	-0.1079	-0.1079	0.0029
$S = 1$	0.2113	-0.1116	-0.1567	0.0037
$S = 1.5$	0.1970	-0.1922	-0.1161	0.0044

Comparative Statics with a golden rule

Higher deficits represented by a rise of the parameter φ_1 influence the growth rate as well as the public to private capital ratio g_k. Differentiating the growth rate (31) leads to:

$$\frac{\partial \gamma}{\partial \varphi_1} = \frac{1}{\sigma}\left(1-\tau\right)\left(1-\alpha\right)\alpha g_k^{\alpha-1} \cdot \frac{\partial g_k}{\partial \varphi_1}. \tag{38}$$

Equation (38) shows, that the impact on the growth rate depends on the influence on the capital ratio $\frac{\partial g_k}{\partial \varphi_1}$. To calculate this we need to implicitly differentiate the zero set dynamic system (27), (28) and (29). First we first convert the system into two implicit functions $F(\cdot)$ and $F_1(\cdot)$. From $\frac{\dot{c}_k}{c_k} = 0$ in (27) it follows that:

$$c_k = rb_k + \left(1 - \left(\varphi + \varphi_1\left(1-\varphi\right)\right)\tau\right) g_k^{\alpha} - \frac{1}{\sigma}\Big(\left(1-\tau\right)\left(1-\alpha\right) g_k^{\alpha} - \delta^k - \rho\Big) - \delta^k.$$

Inserting into $\frac{\dot{b}_k}{b_k} = 0$ and $\frac{\dot{g}_k}{g_k} = 0$ in (28) and (29) leads to:

$$\begin{aligned} F(\cdot) &= 0 = \frac{\left(\varphi_1 - 1\right)\left(1-\varphi\right)\tau g_k^{\alpha}}{b_k} - \frac{1}{\sigma}\Big(\left(1-\tau\right)\left(1-\alpha\right) g_k^{\alpha} - \delta^k - \rho\Big) \\ F_1(\cdot) &= 0 = \varphi_1\left(1-\varphi\right)\tau g_k^{\alpha-1} - \frac{1}{\sigma}\Big(\left(1-\tau\right)\left(1-\alpha\right) g_k^{\alpha} - \delta^k - \rho\Big). \end{aligned}$$

Totally differentiating gives us:

$$\begin{bmatrix} \frac{\partial F}{\partial b_k} & \frac{\partial F}{\partial g_k} \\ \frac{\partial F_1}{\partial b_k} & \frac{\partial F_1}{\partial g_k} \end{bmatrix} \cdot \begin{bmatrix} \frac{\partial b_k}{\partial \varphi_1} \\ \frac{\partial g_k}{\partial \varphi_1} \end{bmatrix} = \begin{bmatrix} -\frac{\partial F}{\partial \varphi_1} \\ -\frac{\partial F_1}{\partial \varphi_1} \end{bmatrix}, \tag{39}$$

where the coefficient matrix corresponds to the Jacobian J^G from (37). Applying cramers rule leads to:

$$\frac{\partial g_k}{\partial \varphi_1} = \frac{|J_{g_k}|}{|J^G|}.$$

The matrix J_{g_k} above is calculated by exchanging the second column of the Jacobian with the right hand side of equation (39):

$$\frac{\partial g_k}{\partial \varphi_1} = \frac{1}{|J^G|} \cdot \left(-\frac{\partial F_1}{\partial \varphi_1}\right)\frac{\partial F}{\partial b_k} + \frac{\partial F_1}{\partial b_k}\frac{\partial F}{\partial \varphi_1}.$$

The determinant from J^G is defined as:

$$\begin{aligned}
\left|J^G\right| &= \left|\begin{array}{cc} \frac{\partial F}{\partial b_k} & \frac{\partial F}{\partial g_k} \\ \frac{\partial F_1}{\partial b_k} & \frac{\partial F_1}{\partial g_k} \end{array}\right| \\
&= \frac{\partial F}{\partial b_k}\frac{\partial F_1}{\partial g_k} - \frac{\partial F_1}{\partial b_k}\frac{\partial F}{\partial g_k}.
\end{aligned}$$

Differentiating the matrix gives us a positive determinant:

$$\begin{aligned}
\left|J^G\right| &= \left(-\frac{(\varphi_1-1)(1-\varphi)\tau g_k^{\alpha}}{b_k^2}\right)\left((\alpha-1)\varphi_1(1-\varphi)\tau g_k^{\alpha-2} - \frac{1}{\sigma}\alpha(1-\tau)(1-\alpha)g_k^{\alpha-1}\right) \\
&= \frac{(\varphi_1-1)(1-\varphi)\tau g_k^{\alpha}}{b_k^2}(1-\alpha)\varphi_1(1-\varphi)\tau g_k^{\alpha-2} \\
&\quad + \frac{(\varphi_1-1)(1-\varphi)\tau g_k^{\alpha}}{b_k^2}\frac{1}{\sigma}\alpha(1-\tau)(1-\alpha)g_k^{\alpha-1} \\
\left|J^G\right| &> 0.
\end{aligned}$$

The sign of $|J_{g_k}|$ is also positive:

$$\begin{aligned}
|J_{g_k}| &= \left(-\frac{\partial F_1}{\partial \varphi_1}\right)\frac{\partial F}{\partial b_k} + \frac{\partial F_1}{\partial b_k}\frac{\partial F}{\partial \varphi_1} \\
&= -\left(-\frac{(\varphi_1-1)(1-\varphi)\tau g_k^{\alpha}}{b_k^2}\right)\cdot(1-\varphi)\tau g_k^{\alpha-1} > 0.
\end{aligned}$$

This leads to an overall positive relation between φ_1 and g_k :

$$\begin{aligned}
\frac{\partial g_k}{\partial \varphi_1} &= \underbrace{\frac{1}{\det J^G}}_{>0}\cdot\underbrace{\left[\left(-\frac{\partial F_1}{\partial \varphi_1}\right)\frac{\partial F}{\partial b_k} + \frac{\partial F_1}{\partial b_k}\frac{\partial F}{\partial \varphi_1}\right]}_{>0} \\
\frac{\partial g_k}{\partial \varphi_1} &> 0
\end{aligned}$$

Together with equation (38) it is shown that the growth effect of higher deficits with a golden rule, represented by a higher φ_1, is positive.