Product differentiation and welfare in a mixed duopoly with regulated prices: The case of a public and a private hospital*

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February 27, 2009

Abstract

The German hospital market is characterised by price regulation and the existence of three different ownership types: public, non-profit and private (for-profit) hospitals. Using a Hotelling framework, this paper analyses the effect of different objectives of the hospitals on quality, profits and overall welfare in a price regulated duopoly with symmetric locations. In contrast to other studies on mixed oligopolies, this paper shows that in a duopoly with regulated prices privatisation of the public hospital may increase overall welfare depending on the difference of the hospitals' marginal costs and the weight of the altruistic motive.

Keywords: mixed oligopoly, price regulation, quality, hospital competition.

JEL: L13, I18, H42

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*I am grateful to Justus Haucap, Björn Kuchinke and Tobias Wenzel for comments on my research.

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1 Introduction

In many countries the hospital industry is characterised by the co-existence of different ownership types (e.g. many European countries, US). In Germany, public, non-profit and private (for-profit) hospitals compete with each other where an increasing number of public hospitals have been privatised over the last decade. Since the health care system is mainly publicly financed, regulatory authorities are interested in cost reducing and quality enhancing activities of the hospitals. To stimulate those activities, the knowledge of the hospitals' incentives is an important necessity. The literature on mixed oligopolies deals with the co-existence of two or more firms with different objectives in one market. In their seminal paper on mixed oligopolies, Merrill and Schneider (1966) assume that the public firm maximises output facing a budget constraint. Often, the public firm is assumed to follow the public owner's interest and to maximise social surplus (for example De Fraja and Delbono, 1989; Cremer, Marchand and Thisse, 1991; Nishimori and Ogawa, 2002; Matsumura and Matsushima, 2004; Willner, 2006; Ishida and Matsushima, 2009). One issue inherent to that assumption lies in the multiple principal agent problems a hospital faces.[1] As Cutler (2000) notes, key considerations in the choice of organisational form for hospitals include underlying concerns about agency problems and asymmetric information, the provision of public goods, and access to capital. At the same time, interests of major stakeholders, including administrators, staff, trustees, and community may also play a role when choosing the ownership of a hospital.

To analyse the behaviour of firms in mixed oligopolies, mostly Cournot or Bertrand models are applied assuming that goods are homogeneous and prices can be set by the firms according to their objective functions.[2] De-

[1]Non-profit firms are assumed to also follow other incentives than to purely maximise profits such as to maximise output (Gaynor and Vogt, 2003; Lakdawalla and Philipson, 1998), quality (Newhouse, 1970; Ma and Burgess, 1993; Dranove and Satterthwaite, 2000), consumers' surplus (Steinberg and Weisbrod, 2005), physicians' interests (Pauly and Redisch, 1973) or other monetary or non-monetary incentives (e.g. charity in Dranove, 1988). In his comprehensive review Sloan (2000) classifies and evaluates the theoretical and empirical literature on non-profit hospitals' behaviour until 2000.

[2]For surveys of the literature on mixed oligopolies compare De Fraja and Delbono (1990) and Nett (1993).

pending on the assumptions about the firms' differences in costs and efficiency, number of firms, and timing, it typically turns out that better allocations are achieved when public firms are present (e.g. Cremer, Marchand and Thisse, 1989) where in some cases the first-best result can be attained. With endogenous costs for investments into efficiency gains, a public monopoly would be preferred to a mixed duopoly (Nishimori and Ogawa, 2002).

In this paper, the goods (the treatments of the patients) are assumed to be differentiated. We follow one of the most important approaches to model product differentiation in markets with private firms: spatial competition à la Hotelling (Hotelling, 1929).[3] Applied to mixed oligopolies, results may differ compared to markets with homogeneous goods. Cremer et al. (1991) show that only for less than three and for more than five firms in the market, a mixed oligopoly with less than $(n + 1)/2$ public firms is preferred to a private oligopoly in a price-location game where the public firm pays higher wages and maximises social surplus under a non-negative profit constraint. However, with endogenous production costs in a mixed duopoly, privatisation of the public firm would improve welfare because it would mitigate the loss arising from excessive cost-reducing investments of the private firm (Matsumura and Matsushima, 2004). In price regulated markets such as the hospital industry, firms rather compete in quality or location than in prices (Brekke, 2004; Brekke, Nuscheler and Straume, 2006). Whilst prices and profits are easy to observe, it is difficult to measure a hospital's quality. The measurement of quality in studies of hospital competition had been in the focus of recent research (McClellan and Staiger, 1999b; Romano and Mutter, 2004; Gaynor, 2003). In Germany, quality regulation has been intensified significantly over the last ten years (introduction of minimum quantities, external quality comparisons, and internal quality management as well as the obligation to publish quality reports). However, the evaluation of these means has only started recently and has not led yet to significant results with respect to quality differences between different hospital owners (Geraedts, 2006). Empirical studies of US hospitals find weak evidence that

[3] Gabszewicz, Thisse, Fujita and Schweizer (2001) provide a comprehensive overview over location choice models.

private hospitals may provide higher quality in some local or specific markets (McClellan and Staiger, 1999a; Santerre and Vernon, 2005).[4]

This paper provides a first theoretical analysis of a price regulated mixed duopoly consisting of a public and a private (for-profit) hospital. However, due to the general setup, the results can be also applied to a duopoly of a private and a non-profit hospital or other price-regulated mixed oligopolies. As in other studies, the public hospital is assumed to maximise a linear combination of both profits and output. This assumption is considered to be more realistic and to mirror the interests of the different stakeholders better than the assumption of welfare-maximising behaviour of public hospitals.

In Section 2, preliminary assumptions will be shortly described. In Section 3, the quality choice of the two hospitals in the three scenarios (symmetric profit-maximising hospitals, symmetric altruistic hospitals and mixed duopoly) will be analysed and the comparative statics characteristics of the quality choice in equilibrium will be shown. Finally, welfare-maximising prices will be derived in Section 4. The corresponding welfare levels, consumer rent, and profits in all three scenarios will be compared with each other and with the first-best scenario in Section 5 before Section 6 concludes.

2 The model: preliminaries

The following analysis builds on the model by Brekke et al. (2006). They model competition in location and quality between two profit maximising hospitals in a price regulated market. Here, the structure of the game is as follows: In stage 0, symmetric locations are exogenously fixed before prices are set by the regulatory authority in stage 1 and hospitals compete in quality in stage 2. The game will be solved by backward induction to identify a stable Nash-equilibrium.

Assume that the two hospitals face a unit mass of consumers, distributed uniformly on the line segment $[0, 1]$. Locations $x_i, \quad i = 1, 2$ are assumed

[4] Gaynor and Vogt (2000) and Dranove and Satterthwaite (2000) review in detail the literature on antitrust and competition in mainly US health care markets, also considering differences across ownership types.

to be exogenously fixed In the hospital sector, this assumption is realistic since locations are often regulated at least in rural regions. Hospitals cannot change their location in the short or medium term because of their size and infrastructural needs and local demand. The same holds if the vertical differentiation is understood as specialisation versus diversification of the medical programs the hospitals offer. The only parameter hospitals can choose according to their respective maximisation problems is quality q_i given regulated price p. Marginal production costs c_i differ between the two hospitals and are constant with $p > c_i, i = 1, 2$. Transportation costs, which the consumers face, are quadratic in the distance between the consumer's location z and the hospital i, i.e. $t(z - x_i)^2$.

Utility function and the indifferent consumer

A consumer located at z derives the utility from getting one unit of the good from hospital i

$$U(z, x_i, q_i) = v + q_i - t(z - x_i)^2 - p \tag{1}$$

with price $p > 0$, transportation costs $t > 0$. Furthermore, the constant valuation of consuming the good v is sufficiently high such that the market is covered at any time. Due to the latter, a monopolistic hospital would always choose zero quality. A monopolist would earn non-negative profits as long as the regulated price exceeds average or marginal costs of production.[5] The indifferent consumer is located at

$$\bar{z} = \frac{1}{2}(x_1 + x_2) + \frac{q_1 - q_2}{2t(x_2 - x_1)} \tag{2}$$

For such a location to exist (i.e. to gain a bounded solution), we need to assume in the following, that the distance between the hospitals $x_2 - x_1$ does not equal zero. For reasons of simplicity, we ignore the possible existence of

[5]Brekke, Siciliani and Straume (2008) compare a monopolistic altruistic hospital with a market composed by two altruistic hospitals assuming that a fraction of consumers may not be treated due e.g. high transportation costs and capacity constraints. They find that it depends on the degree of altruism of the hospitals as to which setting would be preferred by the regulator.

mixed equilibria[6] and assume throughout that $x_2 > x_1$, namely $x_1 \in [0, \frac{1}{2} - \bar{x}]$, $x_2 \in [\frac{1}{2} + \bar{x}, 1]$, with $\bar{x} > 0$ small. The distance $\Delta = x_2 - x_1$ will be assumed to be symmetric to simplify the analysis with $x_2 = 1 - x_1$. Then, $x_1 = \frac{1}{2}(1 - \Delta)$ and $x_2 = \frac{1}{2}(1 + \Delta)$.

Profit functions

As in Brekke et al. (2006) the marginal production costs of one good and the costs of producing a certain quality can be linearly separated where quality costs are the costs of investing in higher quality not related to the marginal costs. The cost function thus only depends on quality $K(q_i)$ and is assumed to be $K(q_i) = \frac{1}{2}q_i^2$ throughout the paper to ensure the profit function to be concave and a unique maximum to exist. The profit of hospital i is defined as

$$\pi_i = (p - c_i)y_i - \frac{1}{2}q_i^2 \tag{3}$$

Furthermore, the framework is generalised by assuming that the two hospitals may differ with respect to their marginal costs (compare Cremer et al., 1989). In Germany, private hospitals do not underlie the same regulatory restrictions as public or non-profit hospitals. In contrast to public hospitals private (for-profit) hospitals are not obliged to pay wages according to collective agreements, for example. That is why, on average, private hospitals face lower personnel costs than public hospitals in Germany. Since personnel costs account for approximately 60% of total hospital costs, they are an important factor.[7] Furthermore, private hospitals face more opportunities to trade and borrow capital while non-profit and public hospitals are

[6]Bester, De Palma, Leininger and Thomas (1996) show that the Hotelling location game with quadratic transportation costs and price competition possesses an infinity of mixed strategy Nash equilibria. In these equilibria coordination failure invalidates the principle of "maximum differentiation" discovered by d'Aspremont, Gabszewicz and Thisse (1977). For a similar finding, compare Wang and Yang (2001) showing the existence of mixed equilibria in a 2 stage price-quality game.

[7]60% of the private hospitals operate less than 100 beds. In this size category, costs per full-time equivalent employee sum up to 56,000 € in public, 47,000€ in non-profit and only 43,000 € in private hospitals in 2007. However, for hospitals with more than 500 beds the costs per employee are similar across ownership types (between 52,000 and 54,000 €).

not allowed to accumulate profits because of their legal forms.[8] Let total costs $C = c_1 + c_2$ and the cost difference $D = c_1 - c_2$ where c_1 and c_2 are exogenously given. The assumption that each consumer consumes one unit of the good and the market is covered determines the market shares of the hospitals. They are defined by the location of the indifferent consumer $\bar{z}$, where $y_1 = \bar{z}$ and $y_2 = 1 - \bar{z}$ constitute the number of cases treated by the two respective hospitals.

The three scenarios

In general, a hospital's objective function is defined as $Z_i = \pi_i + \alpha_i y_i$. Here, altruistic/public hospitals are assumed to maximise their own profits to secure future existence and investments plus a fraction of their market share which depends positively on the hospital's quality. In the three possible scenarios the two hospitals behave as follows.

1. Scenario 1 (profit maximising duopoly): $\alpha_1 = \alpha_2 = 0$

   As in Brekke et al. (2006), both hospitals behave as profit-maximising private hospitals and maximise their respective objective function $Z_{pi} = \pi_i$, $i = 1, 2$.

2. Scenario 2 (altruistic duopoly): $\alpha_1 = \alpha_2 = \alpha > 0$

   Both hospitals behave symmetrically altruistic if their behaviour is in line with the objective function $Z_i = \pi_i + \alpha y_i$.

3. Scenario 3 (mixed duopoly): $\alpha_1 = \alpha > \alpha_2 = 0$

   In this scenario, the mixed duopoly is analysed. It is assumed that hospital 2 is a profit-maximising private hospital, $Z_2 = \pi_2$, while hospital 1 is a public hospital maximising the mixed objective function $Z_1 = \pi_1 + \alpha\bar{z}$.

[8] Nevertheless, cost (technical) *in*efficiency given input prices and outputs (input use) have been shown to be highest in private hospitals in the years from 2000 to 2003 (Herr, 2008).

3 Quality choice and comparative statics in the three Scenarios

3.1 Quality choice

In all three scenarios, the hospitals choose their quality levels in equilibrium such that the first order conditions

$$\frac{dZ_i}{dq_i} = \frac{p - c_i + \alpha_i}{2t\Delta} - q_i = 0$$

are fulfilled. Thus, the hospital's quality level in the Nash-equilibrium can be derived to be

$$q_i = \frac{p - c_i + \alpha_i}{2t\Delta}, \tag{4}$$

which is uniquely defined since $\frac{d^2 Z_i}{dq_i^2} < 0$ and $t > 0$, $\Delta > 0$. The quality level in equilibrium is independent of the other hospital's quality and only depends on the price mark-up, the consumers' transportation costs and distance and the degree of altruism α_i. The first hospital provides higher quality ($q_1 > q_2$) if $c_1 - c_2 < \alpha_1 - \alpha_2$, i.e. if the cost difference is smaller than the difference in the degrees of altruism.

If $\alpha_i = 0$, $i = 1, 2$, the equilibrium collapses to a private profit-maximising duopoly (*Scenario 1*) in which the first hospital sets higher quality as long as $c_1 < c_2$ and vice versa given symmetric locations. In *Scenario 2* both hospitals will produce higher quality than in Scenario 1, since they value market shares and thus consumer's utility more than purely profit maximising hospitals.

The additional asymmetry of *Scenario 3* comes from the assumption that $\alpha_1 = \alpha > 0$ for the first hospital and $\alpha_2 = 0$ for the second (pure profit maximiser). Then,

$$\begin{aligned} q_1^a &= \frac{p - c_1 + \alpha}{2t\Delta} \\ q_2^a &= q_2^p = \frac{p - c_2}{2t\Delta} \end{aligned}$$

and $q_1^a > q_2^a$ if $\alpha > c_1 - c_2 = D$. In other words, due to the underlying cost structure it is possible that the private hospital produces at a higher quality level than the public hospital.

3.2 Comparative statics with respect to transportation costs, price, and distance

The comparative statics results hold similarly for all three scenarios which only differ in their magnitudes of α_i and the levels of qualities and price.

Change in location

The higher the distance, the lower the two quality levels in equilibrium. This result complies with basic competition theory.

$$\frac{dq_1^s}{d\Delta} = \frac{dq_2^s}{\Delta} = -\frac{(p - c_i + \alpha_i)}{2t\Delta^2} < 0 \tag{5}$$

For the two symmetric scenarios the only difference between the two hospitals' reactions is determined by the cost difference.

Change in consumers' transportation costs

If the marginal transportation costs increase given the distance remains the same, switching to the other hospital will become more expensive. This decrease in competition leads to a decrease in quality enhancing investments.

$$\frac{dq_i^s}{dt} = -\frac{p - c_i + \alpha_i}{2t^2\Delta} < 0 \tag{6}$$

Change in regulated price

Regarding the price effect on the hospital's quality the comparative static result can easily be derived from the first order condition:

$$\frac{dq_1^s}{dp} = \frac{dq_2^s}{dp} = \frac{1}{2t\Delta} > 0 \tag{7}$$

As expected, a price increase will lead to higher quality levels for both hospitals to regain the second best utility level for the consumers suffering the price increase. This result holds independent of α_i. Note that the regulated prices and thus the magnitudes of change differ across the scenarios, though.

Change in the degree of altruism

An increase in the weight of the altruistic motive α leads to an increase in quality provided.

$$\frac{dq_i^s}{d\alpha_i} = \frac{dq_2^s}{d\alpha_i} = \frac{1}{2t\Delta} \tag{8}$$

In the mixed duopoly, the public hospital will increase its quality as shown in Equation (8) while the private hospital's quality does not depend on the public hospital's valuation of the market share. It will thus not change.

Empirical studies analysing the effect of competition on quality of hospitals have shown mixed results. Kessler and McClellan (2000) analyse the impact of competition on both costs and patient health outcomes in the US. They find that whilst the welfare effects of competition in the 1980s were ambiguous, post-1990 competition was welfare improving. Looking at mergers, Hamilton and Ho (1998) do not find any effect of mergers on mortality of either heart attack or stroke. Propper, Burgess and Green (2004) show that higher competition between hospitals is associated with higher death rates in the English NHS.

4 Setting welfare-maximising prices

In the first step of the game the regulatory authority sets welfare-maximising prices in each of the three Scenarios. The corresponding second-best results are compared to the first-best that will be derived first.

4.1 First-best solution

The welfare function

If symmetric locations are assumed, the welfare function simplifies to

$$\begin{aligned} W &= v - \frac{1}{12}t + \frac{1}{4}t\Delta(1-\Delta) - \frac{1}{2}C + \frac{1}{2}(q_1 + q_2) - \frac{1}{2}\left(q_1^2 + q_2^2\right) \\ &\quad + \frac{1}{4\Delta t}\left((q_1 - q_2)^2 - 2\,(q_1 - q_2)\,D\right), \end{aligned} \tag{9}$$

Note that the overall welfare does not depend on the price chosen by the regulatory authority. Only the distribution between consumer rent and producer rent differs with the price.

4.1.1 Choice of welfare-maximising quality

To find the welfare-maximising first-best quality level of the two hospitals, the first derivatives of the welfare function with respect to q_1 and q_2 are set to zero. After rearranging, the welfare maximising quality levels are,

$$q_1^w = \frac{t\Delta - 1 - D}{2\,(t\Delta - 1)} \tag{10}$$

$$q_2^w = \frac{t\Delta - 1 + D}{2\,(t\Delta - 1)}, \tag{11}$$

where $D = c_1 - c_2$, $t\Delta > \frac{1}{2}$ for a local maximum to exist and $t\Delta > 1 + |D|$ or $t\Delta < 1 - |D|$ for both quality levels to be non-negative. The latter two restrictions ensure that a finite quality level exists ($t\Delta \neq 1$). The difference between the quality levels is $q_1^w - q_2^w = -\frac{D}{t\Delta - 1}$, which only depends on the hospitals' locations and their marginal costs. In the optimum, the public hospital's quality is higher than the private hospital's if $c_1 < c_2$ and $t\Delta > 1$ or vice versa. If marginal costs are equal, the welfare maximising quality levels are $q_1 = q_2 = \frac{1}{2}$ for both hospitals. Market shares are non-negative if $t\Delta > 1 + |D|$ (non-negative quality) and $t\Delta(t\Delta - 1) > D > t\Delta(1 - t\Delta)$. Total welfare is independent of the price level. However, hospitals should at least be able to produce at a non-negative profit level. In general,

$$\pi_1^w = (p - c_1)\frac{-D - t\Delta + t^2\Delta^2}{2t\Delta(t\Delta - 1)} - \frac{(t\Delta - D - 1)^2}{8(t\Delta - 1)^2} \text{ and} \tag{12}$$

$$\pi_2^w = (p - c_2)\frac{D - t\Delta + t^2\Delta^2}{2t\Delta(t\Delta - 1)} - \frac{(t\Delta + D - 1)^2}{8(t\Delta - 1)^2}. \tag{13}$$

Both profits are positive if the price mark-up is sufficiently high. As will be shown below, this is fulfilled even with a low price ($p^w = p^s$) if the non-zero-profit conditions of the other three scenarios derived in Subsection 4.2 hold. The optimal welfare level will be

$$W^w = v - \frac{1}{12}t + \frac{1}{4} - \frac{1}{2}C + \frac{1}{4}t\Delta(1 - \Delta) + \frac{D^2}{4(t\Delta - 1)t\Delta} \tag{14}$$

independent of the regulated price. It can be easily shown that the overall welfare in the first-best setting increases, the lower transportation costs, distance and marginal costs and the higher the cost-difference (if $t\Delta > 1$). The latter results from different influences. If the cost difference increases, the quality of the disadvantaged hospital will decrease and it will thus attract less patients than the hospital with the cost advantage. Furthermore, the assumption of symmetric locations may also play a role preventing the hospitals to move in different directions. Overall, the effect is positive.

4.2 Price regulation when location is fixed

In a second-best setting, hospitals behave according to their objective functions and choose the quality levels derived in Section 3 where the outcomes only differ by the degree of altruism α_i.

For bounded solutions to exist in all four scenarios, we need to assure concavity of the objective functions by assuming that distance and transportation costs are sufficiently high. In all scenarios as well as in the first-best case, a stable equilibrium exists with given welfare-maximising prices if the resulting profits, market shares and quality levels are non-negative. An example for a sufficient condition ensuring a simultaneous equilibrium in all settings is given by $t\Delta > t\widetilde{\Delta} = 1 + (c_1 + c_2) + \alpha$.[9] In the following, all equilibria are analysed under this assumption. If the hospitals are close to each other or transportation costs are low, competition will be fierce leading the hospitals to overbid each other until one or both of the hospitals exit the market. In that case, no bounded solution exists to the maximisation problem and we cannot identify a unique equilibrium.

4.2.1 Prices, quality, profits, and welfare in Scenarios 1 and 2

The welfare function will be maximised by the price setting authority with respect to the quality choice of the hospitals of the second stage. Inserting the quality levels of the two symmetric Scenarios 1 and 2, the second-best prices are defined by

$$\text{Scenario 1: } p^p = t\Delta + \frac{1}{2}C \tag{15}$$

$$\text{Scenario 2: } p^s = t\Delta + \frac{1}{2}C - \alpha \tag{16}$$

where in Scenario 2 both hospitals value their own output equally much ($\alpha_1 = \alpha_2 = \alpha$). In the private profit-maximising duopoly the price is higher ($p^p > p^s$)to induce the hospitals to produce at a higher quality level. The second derivative of the welfare function with respect to p is negative in both scenarios letting us conclude that the prices are in the respective local maxima of the welfare functions. The resulting quality levels correspond with each other in the two scenarios with $q_1^p = q_1^s = \frac{1}{2} - \frac{1}{4t\Delta}D$ and $q_2^p = q_2^s = \frac{1}{2} + \frac{1}{4t\Delta}D$. Thus, the higher price induces both profit-maximising hospitals to produce at the same quality level as if they were also considering output in their objective function. The first hospital's quality is lower than the quality of the second hospital if $c_1 > c_2$. Inserting the corresponding quality and price levels into the profit functions, we get for the profit maximising

[9] Scenario 1 requests the least restrictive constraint with $t\Delta > \frac{1}{2}|D| + \frac{1}{4}$.

duopoly (Scenario 1)

$$\begin{aligned}\pi_1^p &= \pi_1^s + \frac{1}{2}\alpha(1 - \frac{1}{2(t\Delta)^2}D) \text{ and} &(17)\\ \pi_2^p &= \pi_2^s + \frac{1}{2}\alpha(1 + \frac{1}{2(t\Delta)^2}D) &(18)\end{aligned}$$

and for the altruistic duopoly (Scenario 2)

$$\begin{aligned}\pi_1^s &= \frac{1}{32(t\Delta)^2}D\,(8\alpha - 4t + 3D) - \frac{1}{8}\,(4\alpha - 4t\Delta + 2D + 1) \text{ and} &(19)\\ \pi_2^s &= -\frac{1}{32(t\Delta)^2}D\,(8\alpha - 4t\Delta - 3D) - \frac{1}{8}\,(4\alpha - 4t\Delta - 2D + 1) &(20)\end{aligned}$$

The equilibrium is stable if simultaneously $\pi_i \geq 0$ and $y_i \geq 0$.

Proposition 1 *Let $p^s = t\Delta + \frac{1}{2}C - \alpha$ and $t\Delta > \alpha + \frac{1}{2}C + \frac{1}{2}$ with two altruistic hospitals. Then, the Nash equilibrium is stable with non-negative profits, quality levels and market shares of both hospitals. In the private duopoly with $p^p = t\Delta + \frac{1}{2}C$, it suffices that $t\Delta > \frac{1}{2}|D| + \frac{1}{4}$ for a stable and unique Nash-equilibrium in pure strategies to exist.*

Since the second best quality levels are equal across scenarios, welfare is independent of the price, and distance is exogenously fixed, welfare is equally high in both symmetric settings

$$\begin{aligned}W^p &= W^s &(21)\\ &= v - \frac{1}{12}t - \frac{1}{2}C + \frac{1}{4} + \frac{1}{4}t\Delta(1-\Delta) + \frac{1}{16(t\Delta)^3}D^2\,(3t\Delta + 1)\,.\end{aligned}$$

The distribution of consumer rent and profits differs, though, since prices and profits are lower and the consumer rent is higher if both hospitals behave altruistically.

4.2.2 Prices, quality, profits, and welfare in the mixed duopoly (Scenario 3)

In the mixed duopoly, quality levels differ between the two hospitals. The welfare maximising price is

$$p^a = t\Delta + \frac{1}{2}C - \frac{1}{2}\alpha \tag{22}$$

with $p^s < p^a < p^p$. The price will always be higher in the mixed duopoly than in the symmetric altruistic duopoly to induce the private hospital to produce at a higher quality level. For positive market shares of both hospitals, $2(t\Delta)^2 > D - \alpha$ and $2(t\Delta)^2 > -(D - \alpha)$ need to be assured which is given if $p > c_i \Leftrightarrow 2t\Delta > \alpha + |D|$ and $t\Delta > 1$. The corresponding quality levels $q_1^a = \frac{1}{2} - \frac{D-a}{4t\Delta}$ and $q_2^a = \frac{1}{2} + \frac{D-a}{4t\Delta}$ are higher and lower, respectively, than the levels in the two symmetric scenarios. Inserting the quality and price levels into the profit functions, we get

$$\pi_1^a = \frac{1}{32(t\Delta)^2}\left(4t\Delta - 5\alpha - 3D\right)(\alpha - D) + \frac{1}{4}\left(2t\Delta - \alpha - D - \frac{1}{2}\right) \tag{23}$$

and

$$\pi_2^a = \frac{1}{32(t\Delta)^2}\left(4t\Delta - 3\alpha + 3D\right)(-\alpha + D) + \frac{1}{4}\left(2t\Delta - \alpha + D - \frac{1}{2}\right) \tag{24}$$

Both hospitals stay in the market if profits $\pi_i > 0$ and market shares $y_i \geq 0$.

Proposition 2 *Let $p^a = t\Delta + \frac{1}{2}C - \frac{1}{2}\alpha$ and $t\Delta > \frac{1}{2}\alpha + \frac{1}{2}C + 1$. Then, the Nash equilibrium is stable with non-negative profits, quality levels and market shares of both hospitals. The public hospital's quality is higher than the private hospital's if $D < \alpha$, i.e. if the difference in marginal costs is lower than the degree of altruism. The private hospital earns higher profits than the public hospital if $D < \alpha\frac{\alpha - t\Delta}{\alpha - t\Delta - 2t^2\Delta^2}$ which is possible even if $D > \alpha$.*

The welfare level in the mixed duopoly is given by.

$$W^a = v - \frac{1}{12}t - \frac{1}{2}C + \frac{1}{4} + \frac{1}{16(t\Delta)^3}\left((\alpha - D)^2 - t\Delta\,(\alpha - D)\,(\alpha + 3D)\right) \quad (25)$$

5 Welfare analysis

Assume in the following that $t\Delta > \widetilde{t\Delta} = 1 + \frac{1}{2}C + \alpha$ to enable comparisons across all four scenarios (including the first-best scenario). Furthermore, let $c_i^p = c_i^s = c_i^a = c_i$, $i = 1, 2$.

5.1 Compare resulting welfare levels

Given second best prices in the two symmetric scenarios, quality and welfare levels are of the same magnitude, no matter whether hospitals take into account market shares or only maximise profits. Furthermore, comparing (21) and (25) it can be shown that

$$W^p = W^s > W^a \Leftrightarrow -\alpha\frac{t\Delta - 1}{2(1 + t\Delta)} < D$$

Let $c_1 > c_2$. Then, the welfare level in the mixed duopoly is below the level in the two symmetric scenarios. In this case, a private duopoly would provide higher welfare than a mixed market due to its symmetric structure.

Naturally, the first-best setting gives the highest welfare level since with Equations (21), (14) and (25) and $t\Delta > \widetilde{t\Delta} > 1$ the comparison shows

$$\begin{aligned} W^s - W^w &= W^p - W^w = -\frac{D^2\,(t\Delta + 1)^2}{16(t\Delta)^3(t\Delta - 1)} \\ W^a - W^w &= -\frac{(\alpha(1 - t\Delta) - D\,(t\Delta + 1))^2}{16(t\Delta)^3\,(t\Delta - 1)} \end{aligned}$$

The first-best result can be reached in the symmetric Scenarios 1 and 2 if $D = 0$ that means if marginal costs are equal across hospitals. Comparing the two symmetric settings, it is rather a political decision whether the public authority prefers to support producers by privatising both hospitals or to enlarge consumer rent. In the mixed duopoly, the first-best can only be reached if $t\Delta = \frac{\alpha - D}{\alpha + D} > \widetilde{t\Delta}$, thus if $c_1 \ll c_2$. In the case that the public hospital has a big cost advantage, a mixed setting would increase welfare compared to the symmetric settings. This may be due to the assumption that hospitals are nevertheless located symmetrically between 0 and 1 which decreases transportation costs. However, if the private hospital had the cost advantage, quality levels in the mixed duopoly would low such that the first-best outcome cannot be reached even with regulated prices and symmetric locations.

5.2 Compare resulting consumer rents

In a duopoly with symmetric locations, the consumer rent is defined in general as

$$\begin{aligned} K &= \int_0^{\bar{z}} (v + q_1 - t\,(x - x_1)^2 - p)dx + \int_{\bar{z}}^1 (v + q_2 - t\,(x - x_2)^2 - p)dx \\ &= v - \frac{1}{12}t - p + \frac{1}{2}(q_1 + q_2) + \frac{1}{4}t\Delta(1 - \Delta) + \frac{1}{4\Delta t}\,(q_2 - q_1)^2 \end{aligned} \qquad (26)$$

Since $\frac{\partial K}{\partial q_i} = \frac{q_i - q_j}{2t\Delta} + \frac{1}{2} > 0$ if $q_i - q_j > -t\Delta$, for at least one hospital $i \neq j$ the consumer rent would be maximal if quality increased to infinity or distance is close to zero (leading to infinitely high quality via high competition between the hospitals).[10] However, given the quality choice by the hospitals and inserting second best prices consumer rent in the case of two profit-maximising hospitals ($\alpha_i = 0$) is defined by

$$K^p = v - \frac{1}{12}t - \frac{1}{4}t\Delta(3 + \Delta) + \frac{1}{2} - \frac{1}{2}C + \frac{1}{16t^3\Delta^3}D^2 \qquad (27)$$

In Scenario 2 ($\alpha > 0$ for both hospitals) the consumer rent is higher, namely

$$K^s = K^p + \alpha \qquad (28)$$

[10]The consumer rents and profits of the three Scenarios are not compared with the first-best setting since in the latter any arbitrary price would lead to maximal welfare.

The consumer rent in an altruistic duopoly is higher due to higher quality and lower regulated prices. In the mixed duopoly it holds that

$$K^a = K^p + \frac{1}{2}\alpha + \frac{1}{16(t\Delta)^3}\left(\alpha - 2D\right)\alpha \tag{29}$$

Proposition 3 *Assume that a sub-game perfect Nash equilibrium exists where both hospitals are active in the market in all three scenarios, i.e. transportation costs and distance are sufficiently high with* $t\Delta > \frac{1}{2}C + \alpha + 1$. *Then,* $K^s > K^a > K^p$.

For an analysis of consumer rents with lower transportation costs, compare Appendix 6.1.

5.3 Compare resulting profits

The profits of the first two Scenarios are easy to compare with each other. Since welfare levels coincide but prices are higher in the duopoly with two profit-maximising hospitals, profits will also be higher than in the symmetric altruistic case if the cost difference between the hospitals is sufficiently low. From (17), (18), (19) and (20) it can be shown that $\pi_i^p > \pi_i^s$ if $|D| < 2(t\Delta)^2$. Compared to the mixed duopoly the following Proposition can be derived.

Proposition 4 *Assume that a sub-game perfect Nash equilibrium exists where both hospitals are active in the market, i.e. transportation costs and distance are sufficiently high with* $t\Delta > \frac{1}{2}C + \frac{1}{2} + \alpha$. *Then,* $\pi_i^p > \pi_i^a > \pi_i^s$ *for* $i = 1, 2$.

See Appendix 6.2 for a comparison of the respective profit functions .

6 Conclusion

In industries which are characterised by asymmetric information about for example quality, a plurality of different ownership types can be observed. The empirical literature on differences in outcomes such as efficiency, quality and costs of different ownership types is vast. One essential issue underlying those studies is the need to assume a hospital's behaviour. This paper analyses in a simple theoretical framework, whether and in which respect different objectives lead to different quality outcomes. Furthermore, it shows which market structure would be preferred from a welfare perspective assuming that the public hospital maximises a linear combination of its profits and the number of patients treated.

The analysis shows that the regulatory authority is able to set prices in the private as well as in the public duopoly such that quality levels, market shares and welfare coincide between the two different scenarios independent of the degree of altruism. First-best can be reached if the hospitals face equal marginal costs. These scenarios only differ in the distribution of the rents between hospitals and patients. Privatisation or nationalisation of both hospitals is thus solely a political decision in these symmetric scenarios.

Compared to the mixed duopoly, a private duopoly will be preferred whenever the public hospital faces higher marginal costs than the private competitor. This result derived in a price regulated setting conflicts with the result by Cremer et al. (1991) who state that a mixed duopoly would be superior to a private duopoly in a price-location game although the public firm faces higher wages and thus higher marginal costs. Here, in the mixed duopoly, first-best can only be reached if the public hospital has a big cost advantage compared to the private (for-profit) hospital. Although the model is very simple and based on a duopolistic setup, this result may have an implication for the mixed hospital industry. Since public hospitals face higher costs in Germany as well as in other countries, first-best may not be achievable with regulated prices if hospitals follow different objectives.

Further possible generalisations of this model include the introduction of endogenous costs, location choice, choice of slack, and the extension to more than two competitors. To derive policy implications, it is essential to identify the objectives of different ownership types empirically. Additionally, empirical studies of competition and quality should be conducted for German hospitals where it is important to not only account for prices, and costs, but also for quality simultaneously.

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Appendix

6.1 Comparison of consumer rents with high and low transportation costs

Comparing the consumer rents without obeying the necessary constraint on transportation costs and distance, we can identify three different orders of magnitude shown in the table below. In the case of high transportation costs (1 and 2), the order is clear, the symmetric altruistic scenario is preferred by the consumers with $K^s > K^a > K^p$. For low transportation costs, the asymmetric setting can lead to lowest (3) and highest (4) consumer rent depending on the relative marginal costs of the two hospitals.

	$D > \frac{1}{2}\alpha$	$D < \frac{1}{2}\alpha$
$t\Delta > \frac{1}{2}\sqrt[3]{(-2D+\alpha)}$ if $D < \frac{1}{2}\alpha$	1	2
$t\Delta > \frac{1}{2}\sqrt[3]{(2D-\alpha)}$ if $D > \frac{1}{2}\alpha$	$K^s > K^a > K^p$	$\mathbf{K}_s > \mathbf{K}_a > \mathbf{K}_p$
$t\Delta < \frac{1}{2}\sqrt[3]{(-2D+\alpha)}$ if $D < \frac{1}{2}\alpha$	3	4
$t\Delta < \frac{1}{2}\sqrt[3]{(2D-\alpha)}$ if $D > \frac{1}{2}\alpha$	$K^s > K^p > K^a$	$K^a > K^s > K^p$

As expected, two profit maximising hospitals set quality levels such that the consumer rent is always lowest across scenarios. Since it is assumed that $t\Delta > t\tilde{\Delta}$, only cases 1 and 2 will be observed in equilibrium.

6.2 Comparison of profits

The hospital's profits of the asymmetric setting (23) and (24) are lower than the profits of the profit maximising hospitals in the symmetric setting (17) and (18) if

$$\pi_1^p - \pi_1^a > 0 \Leftrightarrow 8t^2\Delta^2 - 4t\Delta > -5\alpha + 2D$$

The second hospital maximises profits in both scenarios but faces different competitors.

$$\pi_2^p - \pi_2^a > 0 \Leftrightarrow 8t^2\Delta^2 + 4t\Delta > 3\left(\alpha - 2D\right)$$

The profits of the first of the two symmetric altruistic hospitals (19) are lower than the public hospital's profits of the asymmetric market (23) if

$$\pi_1^s - \pi_1^a < 0 \Leftrightarrow 8t^2\Delta^2 + 4t\Delta > 5\alpha + 6D$$

The profits of the second symmetric hospital (20) are lower than the private hospital's profits of the asymmetric equilibrium (24) if

$$\pi_2^s - \pi_2^a < 0 \Leftrightarrow 8t^2\Delta^2 - 4t\Delta > -3\alpha - 2D$$

In the stable Nash equilibrium it is assumed that transportation costs and distance are sufficiently high with $t\Delta > \frac{1}{2}C + \frac{1}{2} + \alpha$. Thus, the above inequalities are fulfilled in equilibrium and $\pi_i^p > \pi_i^a > \pi_i^s$.