Stochastic Revealed Preference and Rationalizability

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Abstract

This paper explorers rationalizability issues for finite sets of observations of stochastic choice in the framework introduced by Bandyopadhyay et al. (JET, 1999). Is is argued that a useful approach is to consider indirect preferences on budgets instead of direct preferences on commodity bundles. Stochastic choices are rationalizable in terms of stochastic orderings on the normalized price space if and only if there exits a solution to a linear feasibility problem. Together with the weak axiom of stochastic revealed preference the existence of a solution implies rationalizability in terms of stochastic orderings on the commodity space. Furthermore it is shown that the problem of finding sufficiency conditions for binary choice probabilities to be rationalizable bears similarities to the problem considered here.

Keywords: Stochastic choice; Rationalizability; Revealed preference; Weak axiom of stochastic revealed preference; Revealed favorability.

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1 Introduction

Bandyopadhyay, Dasgupta, and Pattanaik [1] (henceforth BDP) initiated a line of investigation in which they explord choice behavior of a consumer who chooses in a stochastic fashion from different budget sets. In BDP [2] this approach was extended by an interpretation of tuples of deterministic demand functions of different consumers as a stochastic demand function. They define a weak axiom of stochastic revealed preference which is implied by but does not imply rationalizability in terms of stochastic orderings on the commodity space.[1] In BDP [3], the authors note that

> it is not at all obvious what would be a natural stochastic translation of the familiar strong axiom of revealed preference and what would be the implications of such a 'strong axiom of stochastic revealed preference'.

It is the purpose of this paper to explore rationalizability issues, provide a necessary and sufficient condition for rationalizability in terms of stochastic orderings, and to discuss related problems.

Suppose a consumer specifies a probability for each subset of a given budget such that the probability assignments add up to unity. Suppose further that we observe these probability assignments on a finite set of budgets. Can we find conditions on the probability assignments such that, if these conditions are satisfied, we cannot reject the hypothesis that the consumer has random preference order-

[1] Formal definitions are given in Section 2.

ings and, given the budget set, optimizes on the basis of his realized preference ordering?

Alternatively, suppose we observe single choices of many anonymous consumers on a finite set of budgets, such that we observe each individual decision but do not know by which consumer the decision was made. Can we find conditions on the choices such that, if these conditions are satisfied, we cannot reject the hypothesis that the choices were made by a set of maximizing consumers?

The problem is complicated by at least two factors. Firstly, in the context of stochastic revealed preference, budget sets are infinite sets of alternatives. The stochastic choice literature is usually confined to choices from finite sets.[2] Secondly, even in the deterministic case we are not in general able to recover the entire ranking of a consumer with only a finite set of observations. This is simply because a consumer might choose a in a situation where b is not available, and chooses b in a situation where a is not available. If there are no further observations which can be used to deduce a relation between a and b via a chain of other choices, we do not know if the consumer prefers a over b. In the stochastic case we are therefore only able to deduce minimal choice probabilities; for example, we might be able to deduce that the consumer prefers a over b in at least 30% of all cases and b over a in at least 20% of all cases.

[2] Falmagne [7], who was the first to find conditions for rationalizability of stochastic choices by a probability distribution over linear preference orderings, explicitly confines himself to choices from finite sets of alternatives. Cohen [6] extends Falmagne's approach to the case of an infinite overall set of alternatives, but again, all choice sets are finite subsets of the set of alternatives.

It will be argued that a useful way to understand and analyze stochastic choices on standard budget sets is in terms of indirect preferences on the price-income space or the normalized price space. To this end Sakai's [17] conditions for indirect preferences from which a utility function can be deduced are used. That is, the problem of finding a probability measure on orderings over the available commodity bundles is transformed into the problem of finding a probability measure on orderings over the budgets on which choices are observed.

It is also shown that the rationalizability problem bears similarities to the problem of finding necessary and sufficient conditions for rationalizability of binary choice probabilities; this is specifically true for stochastic revealed preference conditions based on partial relations between alternatives. That is, a set of conditions sufficient for rationalizability is likely to also be applicable to the strand of literature concerned with binary choice. No finite sets of necessary and sufficient conditions for each number of alternatives is known, and Fishburn [8] showed that the set of conditions on the choice probabilities that are sufficient for rationalizability regardless of the number of alternatives must be infinite. This poses some problems for the framework considered here.

The remainder of the paper is organized as follows. Section 2 introduces the notation, and recalls the relevant work by BDP and Sakai. Section 3 introduced a linear feasibility problem which is solvable if and only if the choices are rationalizable in terms of stochastic orderings on the normalized price space. Combined with the weak axiom of stochastic revealed preference it implies the existence of probability distribution of orderings on the commodity space. Problems, in particular with connection to binary choices, are discussed. Section 4 concludes.

2 Preparations

2.1 Notation and Basic Concepts

Let ℓ be the number of commodities, and let $X = \mathbb{R}^{\ell}_{+}$ be the *commodity space*.[3] The *normalized price space* P is defined by

$$P = \Big\{p : p = (p_1, p_2, \ldots, p_\ell) \text{ and } p_i = \rho_i / w \quad (i = 1, 2, \ldots, \ell) \\ \text{for some } (\rho_1, \rho_2, \ldots, \rho_\ell, w) \in \mathbb{R}^{\ell}_{++} \times \mathbb{R}_{++}\Big\},$$

where ρ_i denotes the price of commodity i and w denotes the consumer's income; for most of the paper we shall assume that we observe consumption decisions on a finite set of n budgets. A *budget set* can then be defined by $\{x \in X : px \leq 1\}$. We will denote the budget sets as $B^i = B(p^i)$ and the upper bound of budget sets as $\bar{B}^i = \{x \in X : p^i x^i = 1\}$, where superscripts index the observation. Furthermore $\mathcal{B} \subseteq 2^X$ denotes the family of all budget sets, i.e. $\mathcal{B} = \bigcup \{B(p) : p \in P\}$.

Let h be a nonempty *demand correspondence (function)* on $\mathcal{B}$ which assigns to each B a nonempty subset $h(B)$. For most of the paper, we shall assume that h is a

[3]Notation: $\mathbb{R}^{\ell}_{+} = \{x \in \mathbb{R}^{\ell} : x \geqq 0\}$, $\mathbb{R}^{\ell}_{++} = \{x \in \mathbb{R}^{\ell} : x > 0\}$, where " $x \geqq y$ " means " $x_i \geq y_i$ for all i ", and $x \neq y$ ", and " $x > y$ " means " $x_i > y_i$ for all i ". Note the convention to use subscripts to denote scalars or vector components and superscripts to index bundles.

singleton, and denote $x^i = (x^i_1, x^i_2, \ldots, x^i_\ell) = h(B^i)$. Furthermore we shall assume that the entire income is spent, such that $h(B^i) = h(\bar{B}^i)$.

Let $R \subseteq X \times X$ be a binary relation on X. If $p^i x^i \geq p^i x$ then $\{x^i, x\} \in R$ and we say that the observation x^i is *directly revealed preferred* to x. For brevity, we write $x^i R x$. The observation x^i is *revealed preferred* to x, written $x^i R^* x$, if either $x^i R x$ or for some sequence of bundles $(x^j, x^k, \ldots, x^m)$ it is the case that $x^i R x^j$, $x^j R x^k$, …, $x^m R x$. In this case R^* is the *transitive closure* of the relation R, i.e. $R^* = \bigcup_i R^i$. Let $\mathcal{R}$ be the set of all orderings over X.[4]

The *weak axiom of revealed preference* (WARP) asserts that R is asymmetric: For all $x, x' \in X$, $x \neq x'$, xRx' implies $\neg(x'Rx)$, where $\neg$ means "not true". The *strong axiom of revealed preference* (SARP) asserts that the transitive closure of R, R^*, is asymmetric: xR^*x' implies $\neg(x'R^*x)$.

2.2 Indirect Revealed Preference and Revealed Favorability

There is a notion of *indirect revealed preference* due to Sakai [17], Little [10], and Richter [15].[5] We will rely on Sakai's definitions and use the concept of *revealed favorability* in the following sense: Let $\mathbf{F} \subseteq \mathcal{B} \times \mathcal{B}$ be a binary relation on $\mathcal{B}$. If $x^j \in B^i$ then there has to be an element $x \in B^i$ which is at least as good as x^j,

[4] We use the term "ordering" in the same sense as BDP. An ordering over $\mathbb{R}^\ell_+$ is binary relation R over $\mathbb{R}^\ell_+$ satisfying: (i) reflexivity: for all $x \in \mathbb{R}^\ell_+$, xRx; (ii) connectedness: for all distinct $x, y \in \mathbb{R}^\ell_+$, xRy or yRx; and (iii) transitivity: for all $x, y, z \in \mathbb{R}^\ell_+$, $[xRy$ and $yRz]$ implies xRz.

[5] Sakai [17] calls the relations on the price-income space *revealed favorability relations* and defines weak and strong axioms of revealed favorability by analogy with WARP and SARP. Little [10] calls his relations *indirect preference relations* and employs the Congruence Axiom due to Richter [14]. See also Varian [18], who explores the possibilities of ordinal comparisons between budgets in empirical analysis.

and we say that budget B^i is *revealed more favorable* than budget B^j. Given a set of observations on a consumer, we define the relation $\mathbf{F}^1$ as $B^i\,\mathbf{F}^1\,B^j$ if $x^j \in B^i$ and $B^i \neq B^j$. Let $\mathbf{F}$ be the *transitive closure* of the relation $\mathbf{F}^1$. Let $\mathcal{F}$ be the set of all orderings on $\mathcal{B}$.

The *weak axiom of revealed favorability* (WARF) asserts that $\mathbf{F}^1$ is asymmetric: For all $B, B' \in \mathcal{B}$, $B\,\mathbf{F}^1\,B'$ implies $\neg(B'\,\mathbf{F}^1\,B)$. The *strong axiom of revealed favorability* (SARF) asserts that the transitive closure of $\mathbf{F}^1$, $\mathbf{F}$, is asymmetric: $B\,\mathbf{F}\,B'$ implies $\neg(B'\,\mathbf{F}\,B)$.

2.3 Stochastic Revealed Preference and its Weak Axiom

Next we recall the relevant part of the concepts used by BDP [1, 3].

A *stochastic demand function* (SDF) is a rule g, which, for every normalized price vector $p \in P$ specifies exactly one probability measure q over the class of all subsets of the budget set $B = B(p)$. Let $q = g(p)$, where g is an SDF, and let A be a subset of a budget set $B(p)$. Then $q(A)$ is the probability that the bundle chosen by the consumer from the budget set $B(p)$ will be in the set A.

A stochastic demand function g is *degenerate* if and only if, for every normalized price vector $p \in P$, there exists $x \in B(p)$ such that, for every subset A of $B(p)$, $x \in A$ implies $q(A) = 1$ and $x \notin A$ implies $q(A) = 0$, where $q = g(p)$.

A stochastic demand function g satisfies the *weak axiom of stochastic revealed preference* (WASRP) if and only if, for all pairs of normalized price vectors p and p', and for all $A \subseteq B \cap B'$

$$q\left(B \backslash B'\right) \geq q'(A) - q(A), \tag{1}$$

where $q = g(p)$, $q' = g(p')$, $B = B(p)$ and $B' = B(p')$.

A stochastic demand function which satisfies $q(\bar{B}) = 1$ is called *tight* [3]. The analysis here is confined to tight demand.

A stochastic demand function g satisfies *rationalizability in terms of stochastic orderings* (RSO) if and only if there exists a probability measure r defined on $\mathcal{R}$ such that, for every normalized price vector p and every subset A of $B = B(p)$

$$\begin{aligned} q(A) = r[\{R \in \mathcal{R} : &\text{there is a unique } R-\text{greatest element in } B \\ &\text{and that element is in } A\}] \end{aligned} \tag{2}$$

where $q = g(p)$. BDP [1] show that RSO implies but is not implied by WASRP.

2.4 Indirect Preferences and Stochastic Choice

To extend WASRP to a stronger condition analogous to SARP it seems necessary to be able to utilize transitive closures of preference relations. But when we observe probability measures over all subsets of given budgets it is difficult to interpret these measures in terms of preference relations between elements of X. It is more obvious how to interpret the observations in terms of indirect preference relations or revealed favorability relations between elements of $\mathcal{B}$: We can interpret $q^j(B^j \cap B^i)$

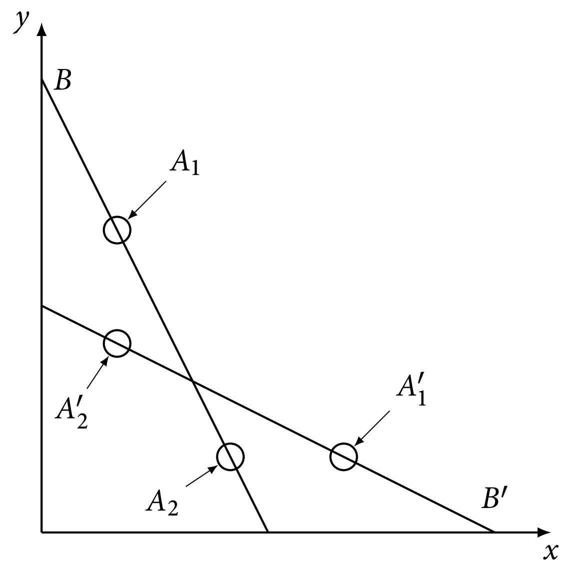

Figure 1: An example

as the minimal share of the consumer's indirect preference relations which rank budget B^i over budget B^j.

Consider Figure 1. Suppose on budget B the consumer assigns the probabilities $q(A_1) = 6/8$ and $q(A_2) = 2/8$ to the sets A_1 and A_2 respectively. On budget B' he assigns the probabilities $q'(A_1') = 4/8$ and $q'(A_2') = 4/8$. Clearly he reveals that at least 2/8 of his preference orderings rank B' over B, and at least 4/8 of his preference orderings rank B oder B'.

Now suppose the indicated subsets of the budgets are singletons. The observed probability assignments are consistent with a consumer who has three different preferences R_a, R_b, R_c, such that A_1 is the R_a-greatest element of B and A_2' is the R_a-greatest element of B' and the preference R_a is realized with probability 4/8; A_1 is the R_b-greatest element of B and A_1' is the R_b-greatest element of B' and the preference R_b is realized with probability 2/8; A_2 is the R_c-greatest element of B and A_1' is the R_c-greatest element of B' and the preference R_c is realized with probability 2/8.

When considering indirect preferences, the only conditions imposed by the observed probability assignments are that the consumer has an indirect preference which ranks budget B over B' and is realized with a probability of at least 4/8, and an indirect preference which ranks budget B' over B and is realized with a probability of at least 2/8. For example the consumer could have two different indirect preferences $\mathbf{F}_a$ and $\mathbf{F}_b$, such that B is the $\mathbf{F}_a$-greatest element of $\{B, B'\}$ and the preference $\mathbf{F}_a$ is realized with probability 5/8; B' is the $\mathbf{F}_b$-greatest element of $\{B, B'\}$ and the preference $\mathbf{F}_b$ is realized with probability 3/8.

3 Rationalizability

3.1 Rationalizability in Terms of Stochastic Orderings on the Normalized Price Space

We say that a stochastic demand function g satisfies *rationalizability in terms of stochastic orderings on the normalized price space* (RSOP) if and only if there exists a probability measure f defined on $\mathcal{F}$ such that this assignment over preferences could have generated the observed stochastic choices. That is, we can use f to generate the observed stochastic demand: For budgets $B^1, B^2, \ldots, B^m$

$$f\left[\left\{\mathbf{F} \in \mathcal{F} : \{B^i\}_{i=1}^{m-1} \mathbf{F} B^m\right\}\right] \geq q^m \left(B^m \bigcap_{i=1}^{m-1} B^i\right),$$

i.e. the sum over all indirect preferences which rank all budgets in $\{B^i\}_{i=1}^{m-1}$ higher than B^m is greater than or equal to the choice probability assigned to the part of B^m that intersects with all $\{B^i\}_{i=1}^{m-1}$. Furthermore,

$$f\left[\left\{\mathbf{F} \in \mathcal{F} : B^m \mathbf{F} \{B^i\}_{i=1}^{m-1}\right\}\right] \leq q^m \left(B^m \setminus \bigcup_{i=1}^{m-1} B^i\right),$$

i.e. the sum over all indirect preferences which rank all budgets in $\{B^i\}_{i=1}^{m-1}$ lower than B^m is less than or equal to the choice probability assigned to the part of B^m that does not intersects with any B^i in $\{B^i\}_{i=1}^{m-1}$.

Because the number of different indirect preferences is finite if the number of observations is finite, it is straightforward to test, at least in principle, for RSOP. Let $N = \{1, 2, \ldots, n\}$ be the set of indices of the observed budgets. Let $S(N)$ be the set of all ordered n-tuples of indices in N, i.e. the set of the $\eta = n!$ permutations of N. The elements of $S(N)$ will be indicated by σ, and more explicitly as $\sigma_i = \langle a, b, \ldots, e\rangle$ and $\sigma_i(1) = a$, $\sigma_i(2) = b$, etc. Let $\pi_i = \pi(\sigma_i)$ be the probability assigned to the ordering σ_i.

We now define the following linear feasibility problem:

$$\text{find} \qquad \Pi = (\pi_1, \pi_2, \ldots, \pi_\eta) \tag{FP.1}$$

$$\text{satisfying} \qquad \pi_i \geq 0 \;\text{ for all } i = 1, 2, \ldots, \eta \tag{FP.2}$$

$$\sum_{i=1}^{\eta} \pi_i = 1 \tag{FP.3}$$

$$\sum_{\{i:\sigma_i(j)<\sigma_i(k)\forall j\in M\}} \pi_i \geq q^k\left(B^k \bigcap_{j\in M} B^j\right) \tag{FP.4}$$

$$\sum_{\{i:\sigma_i(j)>\sigma_i(k)\forall j\in M\}} \pi_i \leq q^k\left(B^k \setminus \bigcup_{j\in M} B^j\right) \tag{FP.5}$$

for all $1 \leq i \leq \eta$ and all nonempty $M \subset N$ and all $k \in N, k \notin M$

Note that $\sum_{\{i:\sigma_i(j)<\sigma_i(k)\forall j\in M\}} \pi_i$ denotes the sum over all probability assignments over preferences which rank *all* $j \in M$ higher than k, excluding preferences which rank one or more $j \in M$ lower than k.

Theorem 3.1 The following conditions are equivalent:

(1) there exists a probability measure f over the set of all orderings on $\mathcal{B}$ that rationalizes the stochastic choices $\{q(B^i)\}_{i=1}^n$;

(2) the linear feasibility problem (FP) has a solution.

Proof. Follows immediately from the definition of RSOP. Note that the theorem bears similarities to Block and Marschak [4, Theorem 3.1]. □

3.2 Rationalizability in Terms of Stochastic Orderings on the Commodity Space

Sakai [17, Theorem 6] shows that if the entire income is spent, the (deterministic) demand at every normalized price vector is a singleton, and the demand function satisfies SARP, then a (direct) utility function can be deduced from the favorability relation.Because WARP implies single valued demand, SARF and WARP together imply the existence of a utility function that rationalizes the demand. So it is not surprising that analogously the existence of a solution to (FP) and satisfaction of WASRP imply RSO:

Theorem 3.2 The stochastic demand function g satisfies rationalizability in terms of stochastic orderings (on the commodity space) if the linear feasibility problem (FP) has a solution and the weak axiom of stochastic revealed preference is satisfied.

Proof. Identify a deterministic demand function h with a degenerate stochastic demand function. Under WARP and SARF, xRx' if and only if $B\,\mathbf{F}\,B'$, where $x = h(B)$ and $x' = h(B')$. By virtue of Sakai's [17] Theorem 5, there exists a function $v : \mathcal{B} \to \mathbb{R}$ such that $B\,\mathbf{F}\,B'$ implies $v(B) > v(B')$. When h is a one-to-one correspondence, we can define a function $\lambda : X \to \mathcal{B}$ such that $\lambda(x) = h^{-1}(x)$. Then there exists a function $u : X \to \mathbb{R}$ such that $u(x) = v(\lambda[x]) > v(\lambda[x']) = u(x')$, so u rationalizes R (Sakai [17, Theorem 6]).

In the stochastic case RSOP implies that there exist functions $v_{\mathbf{F}} : \mathcal{B} \to \mathbb{R}$ such that $B\,\mathbf{F}\,B'$ implies $v_{\mathbf{F}}(B) > v_{\mathbf{F}}(B')$, for all $\mathbf{F}$ with $f(\mathbf{F}) > 0$. Define a set of functions $g_R : \mathcal{B} \to X$ for $R \in \mathcal{R}$ such that $g_R(B) = \{x \in X : x \text{ is the } R-\text{greatest element in } B\}$. Note that WASRP implies that each $g_R(\cdot)$ is uniquely invertible. This is because for two budgets $B = B'$, WASRP excludes $q(A) \neq q'(A)$ for all $A \subseteq B, B'$.

Then under WASRP and RSOP, for every indirect preference preference $\mathbf{F}$ there are direct preferences R such that $x\,R\,x'$ if and only if $B\,\mathbf{F}\,B'$, where $x = g_R(B)$ and $x' = g_R(B')$. Define a set of functions $\mu_R : X \to \mathcal{B}$ such that $\mu_R(x) = g_R^{-1}(x)$. Then in analogy to Sakai's Theorem there exist functions $u_R : X \to \mathbb{R}$ such that $u_R\left(g_R[B]\right) = v_{\mathbf{F}}\left(\mu_R[g_R(B)]\right) > v_{\mathbf{F}}\left(\mu_R[g_R'(B')]\right) = u_R\left(g_R'[B']\right)$. □

3.3 Problems and Open Questions

Consider the following construction: A budget B^i is *revealed more favorable by degree* $\varphi(i,j)$ than B^j if

$$\varphi(i,j) = \max\Bigg\{ q^j\left(B^j \cap B^i\right),\ q^j\left(B^j \cap B^{M(1)}\right) + \sum_{k=1}^{m-1} q^{M(k)}\left(B^{M(k)} \cap B^{M(k+1)}\right) + q^{M(m)}\left(B^{M(m)} \cap B^i\right) - m \Bigg\}, \tag{3}$$

where the maximum is over all sets of indices $M \subseteq N \backslash \{i,j\}$. Then obviously

$$\varphi(i,j) + \varphi(j,i) \leq 1 \tag{4}$$

is a necessary condition for RSOP. It may seem to be a reasonable conjecture that the condition is also sufficient, but unfortunately it is not, as will be shown below. But first note the following:

Claim 3.1 Identify a deterministic demand function with a degenerate stochastic demand function. For that demand function, condition (4) is equivalent to the strong axiom of revealed favorability.

Proof. In the deterministic case, $B^i \,\mathbf{F}\, B^j$ is equivalent to $\varphi(i,j) = 1$. To see this, note that (i) $\varphi(i,j) \in \{0,1\}$, (ii) $B^i \,\mathbf{F}^1\, B^j$ is equivalent to $q^j\left(B^j \cap B^i\right) = 1$, and (iii) $B^i \,\mathbf{F}\, B^j$ is equivalent to $q^j\left(B^j \cap B^{M(1)}\right) = 1$, $q^{M(1)}\left(B^{M(1)} \cap B^{M(2)}\right) = 1$, ..., $q^{M(m)}\left(B^{M(m)} \cap B^{M(i)}\right) = 1$ for some $M \subset N$. So condition (4) is equivalent to asymmetry of **F**. □

A "system of binary probabilities" $\left[\alpha_{ij} : i,j \in \{1,2,\ldots,n\}, i \neq j, \alpha_{ij} + \alpha_{ji} = 1\right]$ is said to be "induced by rankings" (rationalizable) if there is a probability distribution on the set of $n!$ orderings of $\{1,2,\ldots,n\}$ such that, for all distinct i and j, α_{ij} is the sum of all probabilities attached to orderings which rank i over j (cf. Fishburn [8]). The so-called *triangular condition*

$$\alpha_{ij} + \alpha_{jk} + \alpha_{ki} \leq 2 \tag{5}$$

and its generalization

$$\alpha_{M(1)\,M(2)} + \alpha_{M(2)\,M(3)} + \ldots + \alpha_{M(m)\,M(1)} \leq m - 1 \tag{6}$$

for all sets of indices $M \subseteq N$ of length m is a necessary condition for rationalizability.[6] It was also conjectured to be a sufficient condition for rationalizability by Marschak [11]. In an unpublished paper, McFadden and Richter [12] provided a counterexample for $n = 6$.[7] Later on, Fishburn [8] observed that the set of conditions on the choice probabilities that are sufficient for rationalizability regardless of n must be infinite.

This poses some problems for the framework considered here. Consider the counterexample of McFadden and Richter [12] applied to the framework of stochastic revealed preference: For $n = 6$, let

$$\begin{aligned} \alpha_{12} = \alpha_{14} = \alpha_{34} = \alpha_{36} = \alpha_{56} = \alpha_{52} = 1 \\ \alpha_{21} = \alpha_{41} = \alpha_{43} = \alpha_{63} = \alpha_{65} = \alpha_{25} = 0 \\ \alpha_{ij} = 1/2 \text{ for all other } i, j \end{aligned}$$

where $q^j\left(B^j \cap B^i\right) = \alpha_{ij}$. Then the triangular condition and its generalization are satisfied, and so is condition (4); but (FP) has no solution. Indeed, with $q^j\left(B^j \cap B^i\right) = \alpha_{ij}$, conditions (6) implies (4) because

$$\begin{aligned} \varphi(i,j) + \varphi(j,i) = {} & \alpha_{j\,M^i(1)} + \alpha_{M^i(1)\,M^i(2)} + \ldots + \alpha_{M^i(m)\,i} \\ & + \alpha_{i\,M^j(1)} + \alpha_{M^j(1)\,M^j(2)} + \ldots + \alpha_{M^j(m)\,j} \\ & - m^i - m^j, \end{aligned} \tag{7}$$

[6] For the generalized form, see for example Cohen and Falmagne [5]. In the case of binary probabilities, the generalized form can be deduced from the triangular condition.

[7] A revision of the paper was later published as McFadden [13].

where M^i and M^j, $|M^i| = m^i$ and $|M^j| = m^j$, are the sets of indices used to construct $\varphi(i,j)$ and $\varphi(j,i)$, and with (7) and condition (6) we obtain

$$\begin{aligned} \varphi(i,j) + \varphi(j,i) + m^i + m^j &\leq (m^i + 1) + (m^j + 1) - 1 \\ \Leftrightarrow \varphi(i,j) + \varphi(j,i) &\leq 1. \end{aligned}$$

While it might also be possible that exploitation of the particularities of the framework of BDP, e.g. linearities of budgets, helps to find finite sets of necessary and sufficient conditions for stochastic revealed preference without applicability to the binary probability problem[8], the results of this section suggest that conditions for RSOP based on definitions for a partial revealed favorability relation between budgets suffer from similar problems as the conditions for rationalizability of binary probabilities. Therefore a "strong axiom of stochastic revealed favorability" could possibly also solve the problem of finding a finite set of necessary and sufficient conditions for systems of binary probabilities for each particular n.

4 Conclusion

The weak axiom of stochastic revealed preference, as introduced by Bandyopadhyay et al. [1], is a necessary but not sufficient condition for stochastic demand behavior to be rationalizable in terms of stochastic orderings on the commodity

[8] Suppose the commodity space is restricted to the positive orthant of the two-dimensional Euclidean space. Then, in analogy to deterministic revealed preference (see Rose [16], Heufer [9]), WASRP might imply RSO.

space. It was the purpose of this paper to explore rationalizability issues and to show how one can, in principle, test whether or not a finite set of observations of stochastic choice is rationalizable by stochastic orderings.

To this end the problem of finding a probability measure over orderings on the commodity space was transformed into a problem of finding a probability measure over orderings on the normalized price space. The advantage of this indirect approach is that it avoids the problems resulting from the infinity of the set of alternatives a consumer chooses from when facing a budget set defined in the usual way. Furthermore, it is interesting to note that rationalizability in terms of stochastic orderings on the normalized price space and the weak axiom of stochastic revealed preference together imply rationalizability in terms of stochastic orderings on the commodity space.

In Section 3.3 similarities with binary probability systems were pointed out. In particular it was shown that conditions based on partial revealed favorability relations are likely to suffer from similar problems as the conditions for rationalizability of binary probabilities.

References

[1] Bandyopadhyay, Taradas, Indraneel Dasgupta, and Prasanta K. Pattanaik, "Stochastic Revealed Preference and the Theory of Demand," *Journal of Economic Theory* 84 (1999), 95–110

[2] ———, "Demand Aggregation and the Weak Axiom of Stochastic Revealed Preference," *Journal of Economic Theory* 107 (2002), 483–489

[3] ———, "A General Revealed Preference Theorem for Stochastic Demand Behavior," *Economic Theory* 23 (2004), 589–599

[4] Block, H. D. and Jacob Marschak, "Random Orderings and Stochastic Theories of Responses," in I. Olkin, S. G. Ghurye, W. Hoeffding, W. G. Madow, and H. B. Mann (editors), "Contributions to Probability and Statistics," Stanford: Stanford University Press (1960), 97–132

[5] Cohen, Michael and Jean-Claude Falmagne, "Random Utility Representation of Binary Choice Probabilities: A New Class of Necessary Conditions," *Journal of Mathematical Psychology* 34 (1990), 88–94

[6] Cohen, Michael A., "Random Utility Systems – The Infinite Case," *Journal of Mathematical Psychology* 22 (1980), 1–23

[7] Falmagne, Jean-Claude, "A Representation Theorem for Finite Random Scale Systems," *Journal of Mathematical Psychology* 18 (1978), 52–72

[8] Fishburn, Peter C., "Binary Probabilities Induced by Rankings," *SIAM Journal on Discrete Mathematics* 3 (1990), 478–488

[9] Heufer, Jan, "Revealed Preference and the Number of Commodities," *Ruhr Economic Papers* #36, University of Dortmund, Discussion Paper (2007)

[10] Little, James T., "Indirect Preferences," *Journal of Economic Theory* 20 (1979), 182–193

[11] Marschak, Jacob, "Binary-Choice Constraints and Random Utility Indicators," in "Mathematical Methods in the Social Sciences," Stanford: Stanford University Press (1959), 312–329

[12] McFadden, D. and M. K. Richter, "Revealed Stochastic Preference," (1970), Unpublished manuscript, University of California, Department of Economics

[13] McFadden, Daniel L., "Revealed Stochastic Preference: A Synthesis," *Economic Theory* 26 (2005), 245–264

[14] Richter, Marcel K., "Revealed Preference Theory," *Econometrica* 34 (1966), 635–645

[15] ———, "Duality and Rationality," *Journal of Economic Theory* 20 (1979), 131–181

[16] Rose, Hugh, "Consistency of Preference: The Two-Commodity Case," *Review of Economic Studies* 25 (1958), 124–125

[17] Sakai, Yasuhiro, “Revealed Favorability, Indirect Utility, and Direct Utility,” *Journal of Economic Theory* 14 (1977), 113–129

[18] Varian, Hal R., “The Nonparametric Approach to Demand Analysis,” *Econometrica* 50 (1982), 945–972