Increasing Block Tariffs in the Water Sector - A Semi-Welfarist Approach

by

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In this paper, we develop second-best approaches to design water tariffs. In particular, we want to test the characteristics of "increasing block tariffs" (IBTs), that are extensively applied in the water sector but clearly second-best. We compare a modfified Coase-tariff and a progressively increasing block tariff with respect to water consumption, water expenses, and utility levels. We also develop ways of including the household size into the tariff scheme. First numerical analyses show that the income groups that benefit from a progressive tariff might be smaller than expected, and that increasing block tariffs may fail to achieve "fair" cross-subsidization of low-income groups.

Keywords: water,tariffs, prices, fairness, distribution (JEL: L51, L95, H21, D40).

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1 Introduction

The pricing of water is particularly akin to political, socio-economic, and cultural influences. Empirical evidence suggests that the level and the structure of water prices rarely correspond to welfare-optimality. Instead, they are heavily influenced by country- and sector specific considerations, and redistributive aspects. This applies to industrialized countries, but even more to emerging and developing countries. Increasing block tariffs (IBTs), by which higher-income consumers cross-subsidize poorer consumers, prevail (Whittington, 2003).

In this paper, we try to explain why water pricing, as observed in reality, differs considerably from the first-best, welfarist approach of standard regulation theory. This latter literature is characterized by marginal cost pricing, such as the Coase-tariff, and based on welfare maximization (Goldman, Leland, and Sibley, 1984); it also relies on models of optimal non-linear income taxation along the lines of Mirlees (1971). However, none of the "optimal" pricing schemes is observed in practice. On the contrary, one finds a host of different applications, amongst them increasing block tariffs. We look for a rationale of these tariffs.

Our theoretical model is based on the assumption that policy makers are concerned with "fairness" and "transparency". We represent fairness by a Stone-Geary utility function that allows a subsistence amount of water to each household; in addition, we construct welfare transfers from the high-income to the lower-income households. Transparency is attained by choosing a small number of tariff groups. We also introduce asymmetric information into the model (i.e. income is not observable) and construct incentive compatible constraints. From there we calculate the "fair" and "incentive-compatible" nonlinear expenditure function for water.

The model is indeed able to explain the dominance of increasing block-tariffs, observed particularly in developing and emerging countries. Increasing block tariffs can be derived using a "fair" approach, and the construction is transparent and easy, thus in sharp contrast to the welfare approach. We also perform numerical simulation for real data from the Water sector of Bangladesh. We finish with policy conclusions; and suggest that further research should look more deeply into the "fairness" issue in order to indicate the sensitivity to the new system.

2 State of the literature: "Optimal" prices vs. empirical "second-best"

Within the broad literature on regulating and pricing in the water sector, one clearly distinguishes between "optimal" pricing schemes, and those observed in reality, which are "second-best" at best (if not third- or fourth-best). The "first-best" literature derives welfare-optimal nonuniform prices which are in general related to Ramsey-rules. A prominent example is Goldman, Leland, and Sibley (1984), who take into account income effects and the "optimal taxation" reasoning initially developed by Mirrlees (1971, 1976). Sharkey and Sibley (1993) develop optimal non-linear pricing schemes for an arbitrary number of customer types and general cost functions; the "benevolent" regulator can define welfare weights which vary over the set of customer types. Interestingly, the marginal price can be less than marginal cost if welfare weights increase with type (p. 228). Cowen and Cowen (1998) propose a radical form of price differentiation: the unregulated monopoly, that maximizes social surplus by maximizing producer rent, at the expense of consumer rent.

This "first-best" literature is challenged by what we call the "second-best" literature, both on empirical and theoretical grounds. On the empirical side, Whittington (2003) reports about the wide-spread use of increasing-block tariffs in South Asia, but which do not accomplish their main objectives, e.g. revenue sufficiency, economic signals, helping the poor. Boland and Whittington (2000) also present a critical view of IBTs. Dahan and Nisan (2007) insist on the "unintended" consequences of increasing block tariffs in urban water: since larger households, that are generally poorer, consume more water than smaller households (linear Engel curve), they are charged a higher price for water. This erodes the effectiveness of increasing block tariffs. Agthe and Billings (1987) analyze the relation between household income levels and residential water use for Tucson, Arizona. The demand models show that "under the existing increasing block rate pricing schedules, higher income households not only use more water, but have lower elasticites of demand" (p.273). [1]

[1] "Thus a uniform proportional increase will cause a larger percentage drop in water use among low income households than among high income households. Given the assumption of declining marginal utility of water use, this result leads to a policy recommendation for substantially steeper

Empirical work on the specifics of water demand is rare. Gaudin, Griffin, and Sickles (2001) analyze the "Stone-Geary" form of water demand, where a portion of water use is not responsive to price. This is certainly the most specific aspect of water that distinguishes it from all other goods. Without a minimum daily consumptin, survival is impossible. Martinez-Espineira and Nauges (2004) also apply a Stone-Geary utility function to assess if water consumption is sensitive to price control; interestingly, they find a pattern of "path-dependent" water subsistence levels. Garcia-Valinas (2005) estimates urban water demand and water costs for the Spanish muncipiality of Seville; shee finds that two-part tariffing could be a compromise betwen efficeint-but-impossible Ramsey pricing, and inefficient-but-socially-acceptabela free allocation of water to the poor.

All in all, there is no consensus in neither the theoretical nor the empirical literature whether "first-best" policies are really first best. We therefore propose a structured analysis of the underlying theory in the next section.

3 The model

To construct water tariffs we assume that customers consume two goods: water and some other goods which are aggregated into a basket. Consumers differ with respect to income. We assume a continuum of income beginning with very poor households followed by an income middle class and ended by rich customers. Income is distributed according to a density function $g(y) > 0$, $\forall y \in Y = [\underline{y}, \bar{y}]$. The total number of people of income y is $Pg(y)$, where P is total number of customers. In the basic model we assume that households differ only with respect to income. Later we will include the household size into the analysis[2].

Each customer needs a subsistence level w_s, x_s to survive where w_s denotes the subsistence level of water and x_s denotes the needed level of the other good, respectively. Without loss of generality we assume that $x_s = 0$. To capture the subsistence level into the analysis we introduce the following Stone-Geary-utility function:

$$U(w, x) = (w - w_s)^{\alpha}(x - x_s)^{(1-\alpha)} \quad (1)$$

block rates to improve interpersonal equity in water pricing" (Agthe and Billings, 1987, p. 273).

[2]See section 4.3.

The water tariff system is constructed such that water expenses depend on water consumption and income. This can be denoted by a tariff plan

$$TP := \{T(y), w(y)\}, \quad \forall y \in Y = [\underline{y}, \bar{y}] \tag{2}$$

where y denotes income in the interval Y and $T(y)$ is a continuous outlay function of customers to be determined subsequently. $w(y)$ is the respective profile for water consumption. Note, that the usual tariff system $T(w)$ can be derived from (2).

Taking the TP into account, the budget constraint of households can be derived:

$$T(y) + p_x x \leq y \tag{3}$$

where p_x is the price of the other good. For simplicity we calibrate the measure of x such that $p_x = 1$. If we insert (2)and (3) into (1) we have

$$U(w(y), y - T(y)) = (w - w_s)^{\alpha}(y - T(y))^{(1-\alpha)} \tag{4}$$

The tariff system should be affordable and fair. Hence, it must depend on y. As water utilities cannot observe income (or are not allowed to ask for income details) the tarif system has to be built up in a way that customers have an incentive to tell their true income. This requires that TP has to be constructed in an incentive-compatible way. From the revelation principe we know that an incentive compatibility for the continuous case satisfies the following incentive constraint[3]:

$$y = \text{argmax}_{\tilde{y}}[U(w(\tilde{y}), y - T(\tilde{y}))] \tag{5}$$

(5) requires that $w(y)$ and $T(y)$ are chosen such that customers do report their true income to the water company. The respective properties can further be inspected if we differentiate (5) with respect to $\tilde{y}$ and, finally, set $\tilde{y} = y$.

$$U_w \dot{w}(y) - U_y \dot{T}(y) = 0, \quad \forall\ y \in Y \tag{6}$$

where dots denote the derivatives with respect to y.

Inserting the Stone-Geary-utility function yields:

$$\frac{\alpha \dot{w}(y)}{w(y) - w_s} - \frac{(1-\alpha)\dot{T}(y)}{y - T(y)} = 0, \quad \forall\ y \in Y \tag{7}$$

[3]See e. g. MasColell et al. (1995, p. 492 ff.) or Wolfstetter (1999, p. 259 ff.

(7) implicitly determines some characteristics of the admissable tariff systems. From the second order conditions[4] it also follows that

$$\dot{w}(y) > 0 \quad \text{and} \quad \dot{T}(y) > 0 \quad \forall\, y \in Y \tag{8}$$

Finally, the cost structure of the water supply has to be captured in the model. We assume the following simple cost function:

$$C(W(y)) = F + cW(y), \tag{9}$$

where F are fixed costs, c is a positive constant and

$$W(y) = P \int_{\underline{y}}^{y} w(v)g(v)dv \tag{10}$$

is the aggregated water consumption for all incomes up to y.

4 Tarif systems

In the following we turn to the issue of how to construct fair and affordable tariff systems. Here, we want to follow a semi-welfarestic approach. This approach differs from a welfarestic approach in that the optimal tarif is not the result of maximizing aggregate weighted utility[5] of all customers but, instead, introduce simple transparent rules which satisfy the notion of fairness and affordability. However, it remains welfarestic by utilizing a utility function and by securing affordability, i.e. assuring the subsistence level of water consumption.[6]

4.1 A modified Coase-Tariff

If average costs are above marginal costs a linear tarif with charges per consumption unit marginal costs is economically not viable. Either the price will be above marginal costs which leads to the Ramsey-Boiteux-pricing approach or non-linar pricing schedules are introduced. Coase (1946) was the first who dealt with the

[4]Strictly these condition must hold to guarantee a separating equilibrium, i.e. that $w(y)$ and $T(y)$ vary with respect to y. See appendix.

[5]In this respect the following approach differs from the classical literature on optimal tarif mentioned in the introduction.

[6]Note that the tariff system also guarantees the subsistence level of other goods.

latter. He proposed a simple uniform two-part tariff where the price for each unit is equal to marginal costs and an access fee is introduced such that fixed costs are covered. But this schedule can only be assured if no customers drops out of the market as a result of this two-part tariff. But it is exactly this case which is empirically relevant in developing and emerging countries. In the following we want to introduce a modified Coase tariff which takes into account that poor people cannot afford water supplied at marginal costs and,in addition, can not pay the access fee F/P.

We assume that a fraction of consumers cannot afford the subsistence level of water offered at marginal costs. Formally, there exists an interval I, such that

$$I = [\underline{y}, y_s) \quad \text{where} \quad y_s = cw_s \tag{11}$$

Since affordability must be secured water has to be provided below for the poor. This can be achieved by a non-exclusive two-part tariff (modified Coase-tariff). To begin with, for all income groups the subsistence level w_s is guaranteed. Hence, the tariff starts with

$$\{T(\underline{y}) = \underline{y}, w(\underline{y}) = w_s\} \tag{12}$$

Expenses beyond the subsistence level increase according to

$$\dot{T}(y) = (1+m)c\dot{w}(y), \quad m > 0 \tag{13}$$

where m is chosen such that the water provider breaks even. Since poor people cannot afford water provided at marginal costs and since fixed costs have to be taken into account, m will be strictly positive to induce cost coverage.

Solving the differential equation system (5) and (13) and inserting the starting condition (12) leads to the following tariff plan[7]:

$$T(y) = \underline{y} + \alpha(y - \underline{y}) \tag{14}$$

$$w(y) = w_s + \frac{\alpha}{(1+m)c}(y - \underline{y}) \tag{15}$$

Inserting (15) into (14) yields the following outlay-schedule with respect to water consumption:

$$T(w) = \underline{y} + (1+m)c(w - w_s) \tag{16}$$

[7] For details see the appendix 7.2.

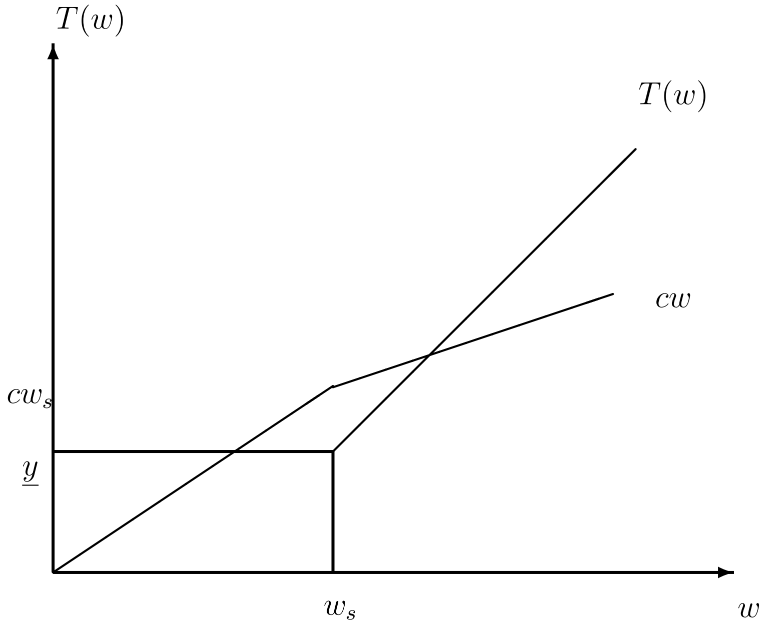

Figure 1: Modified Coase Tariff

The figure shows that the outlay schedule $T(w)$ does follow the requirement of affordability. Those with low income get their subsistence level for $\underline{y}$, i.e. those who pay less than marginal costs are quantity rationed. The provision of water to the poor leads to deficits which have to be covered by customers with high consumption. Hence, the marginal price is above marginal costs. m must be chosen such that the water provider breaks even, i.e.

$$\int_{\underline{y}}^{\bar{y}} [T(y) - cw(y)]Pg(y)dy - F = 0 \tag{17}$$

Inserting (14) and (15) yields:

$$(\underline{y} - cw_s) + \alpha \frac{m}{1+m}[E[y] - \underline{y}] = F/P \tag{18}$$

where $E[y]$ is the average income. From (18) m can be calculated. Note, that for high fixed costs F and/or a severe affordbility problem ($cw_s - \underline{y} << 0$) m might be negative. In this case the water supply is economically not viable.

The modified Coase Tarif is not an increasing Block tariff. Instead, it is a simple two-part tariff and similar to what has been proposed in the literature. Boland and Whittington (2000, p. 9 sq.) and Whittington (2003, p. 70) have critizised IBTs in many respects. As an alternative, they have proposed a 'Uniform Price with Rebate' (UPR) which is rather similar to the modified Coase Tariff. In fact, the UPR is a two-part tariff where the volumetric charge is equal to marginal costs and a fixed monthly credit (fixed amount subtracted from the bill). The reason for marginal cost pricing follows from their assumption that, contrary to our model, average costs are below marginal costs, i.e. marginal costs are increasing. In fact, this is more in the spirit of Coase who thrived for marginal cost pricing.

Our tariff system can also be amended to allow for marginal cost pricing[8]. This requires to introduce an additional fixed fee, say A, if customers consume more than the subsistence level. The tariff plan TP^A has the following structure:

$$T^A(y) = \begin{cases} \underline{y} & \text{if} \quad y < \underline{y} + A \\ \underline{y} + A + \alpha(y - A - \underline{y}) & \text{if} \quad y \geq \underline{y} + A \end{cases} \tag{19}$$

The corresponding water supply function is

$$w^A(y) = \begin{cases} w_s & \text{if} \quad y < \underline{y} + A \\ w_s + A + \frac{\alpha}{c}(y - A - \underline{y}) & \text{if} \quad y \geq \underline{y} + A \end{cases} \tag{20}$$

To compare the two tariff plans the following two figures display the outlay functions and the water supply functions.

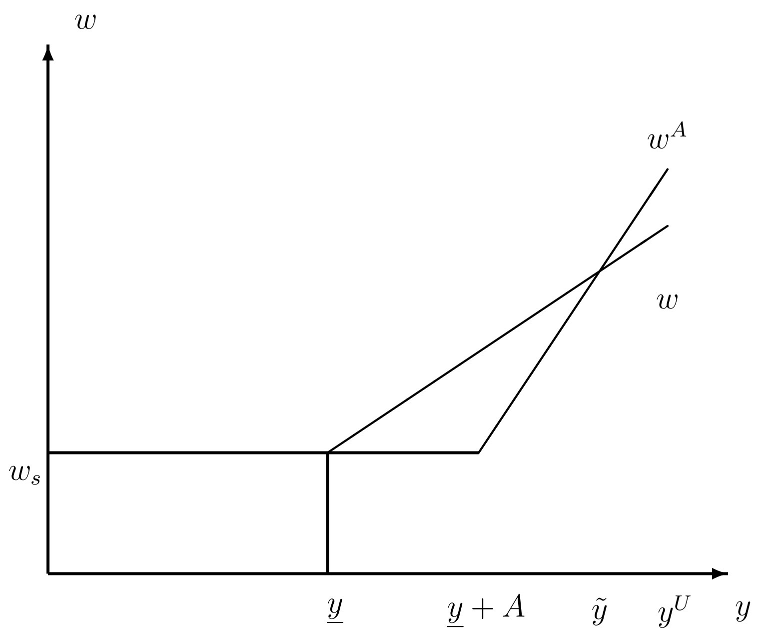

Figure 2: Comparison of water consumption

[8] We also could have considered the more general case, where a low access fee is combined with a low mark up on marginal case. Here, we confine ourselves to the two polar cases which allows to work out the distributional implications.

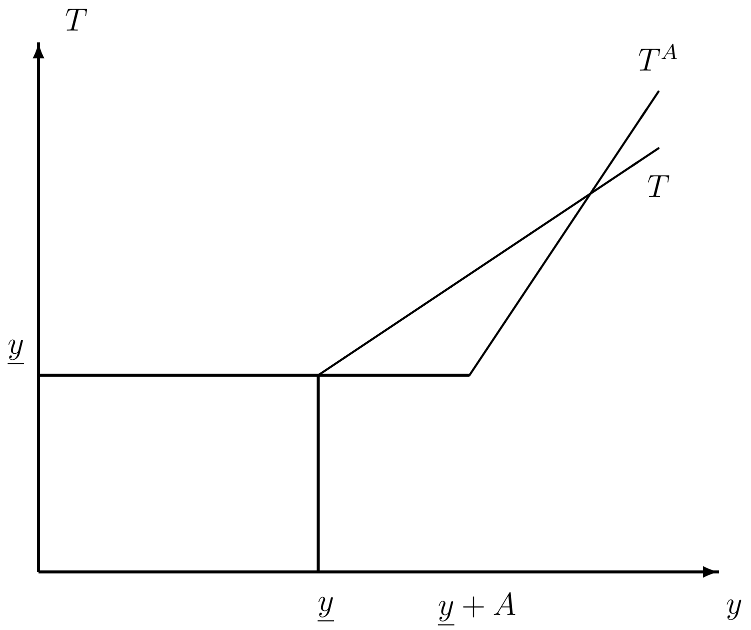

Figure 3: Comparison of water expenses

A comparison of both figures shows, that the introduction of an additional access fee A leads to lower consumption of water for a certain range of lower income. Here, customers remain on the subsistence level until income y covers both fix parts of the tariff $\underline{y} + A$. This can be referred to as a cluster effect. Income rises and water consumption remains at the subsistence level. If income is sufficiently high ($y \geq \underline{y} + A$), water consumption increases at a higher speed than under the tariff without access fee. This is due to marginal cost pricing under the A-tariff as opposed to the former tariff which covers costs by a mark up m. On the other side, expenses under the A tariff are lower for low incomes than under the tariff without additonal access fee. If we take into account both effects one can show, that for low income utility under a A-tariff is lower than under tariff with a mark up.

proposition 1 *Define $\tilde{y}$ as income where $w(\tilde{y}) = w^A(\tilde{y})$. Then there exists a $y^U > \tilde{y}$, such that $U(w^A(y^U), T^A(y^U)) = U(w(y^U), T(y^U))$. For all income $y < y^U$ utility under the A tariff is lower than under the tariff with a mark up and without an additional access fee A.*

Proof see appendix

As a result, the Coase tariff with mark up favors low income groups; the A-tariff favors the higher income groups. It is an political issue what tariff should be chosen.

This decision could also take into account the more general case, where m can be gradually reduced and at the same time A increased.

4.2 Increasing Block Tariffs

The uniform linear two-part tariff resulted from the requirement of incentive compatibility and a simple rule of proportionality in the case of $A = 0$. Everyone that consumes more than the subsistence level should contribute to the coverage of costs according to his consumption. The proportionality rule complies with the notion of fairness. The introduction of an access fee $A > 0$, however, is a per-capita approach. Each customer who consumes more than the subsistence level should contribute to the coverage of costs in a lump sum manner. The contribution does not depend on consumption and, hence, income. This tariff typ secures affordability but denies fairness aspect within the income groups that can afford more than the subsistence level.

If fairness considerations play an important part in water demand management, the introduction of cost coverage[9] might be accompanied by an explicit distributional policy, i.e. by the introduction of a progressive tariff. In the case of block tariffs this is referred to as increasing block tariffs (IBT). In the continuous case this can be achieved by introducing a mark up which depends on income. To keep calculations simple we introduce the following cost coverage mechanism:

$$\dot{T}(y) = (1+n)(y-\underline{y})^{\beta} c\dot{w}(y), \quad 0 \leq \beta < 1 \tag{21}$$

To meet the second order condition of the incentive compatibility constraint, β must be less than unity. Together with (7) this equation forms a system of non-linear differential equations which can be solved[10].

$$T^p(y) = \frac{\underline{y} + \alpha((1-\beta)y - \underline{y})}{1 - \alpha\beta} \tag{22}$$

$$w^p(y) = w_s + \frac{\alpha(y-\underline{y})^{(1-\beta)}}{(1+n)c(1-\alpha\beta)} \tag{23}$$

[9]In many developing and emerging countries the water supply is far from economically viable.

[10]The solution is shown in the appendix

where p indicates that the tarif system follows progressivity. n has to chosen such that the water company breaks even, i.e.

$$\int_{\underline{y}}^{\bar{y}} [T^p(y) - cw^p(y)]Pg(y)dy - F = 0 \tag{24}$$

Both equations guarantee affordability. If $y = \underline{y}$ is inserted into (22) and (23) it follows that $T^p(\underline{y}) = \underline{y}$ and $w^p(\underline{y}) = w_s$. If (23) is solved for y and inserted into (22) we obtain the outlay schedule:

$$T^p(w) = \underline{y} + \left(\frac{\alpha(1-\beta)}{1-\alpha\beta}\right)\left(\frac{(1+n)(1-\beta\alpha)}{\alpha}\right)^{1/(1-\beta)} (w - ws)^{1/(1-\beta)} \tag{25}$$

(25) shows the continuous case of IBTs. Since $\beta < 1$, the outlay function is convex. The philosophy of IBTs is to secure affordability of water and to implement the notion of fairness which implies the redistribution between the income groups. Those with high income should contribute to cost coverage relatively more than those with low income, thus cross-subsidizing the latter.

It is interesting to observe that the progressive tariff system of (22) and (23) may fail to achieve this goals. Similar to the Coase-Tariff, the scheme secures affordability; but the redistributive effects can be almost nil. In the following, we compare the progressive tariff with the Coase tariff with respect to fairness aspects. Specifically, we ask which income groups fare better under the two regimes.

To begin with, we compare (14) with (22). It is easy to show, that $T(y) > T^p(y), \forall y \in (\underline{y}, \bar{y}]$. The comparison of water consumption is somewhat more complicated, since water consumption in both regimes depend on the cost adders m and n respectively. Setting $w(y)$ (15) equal to $w^p(y)$ (23) and solving for y leads to the upper bound of y

$$y_{cs} = \underline{y} + \left(\frac{1+m}{(1-\alpha\beta)(1+n)}\right)^{1/\beta} \tag{26}$$

(The subscript stands for the cross subsidy case). For all incomes that are lower or equal to this boundary, we know that the utility as measured by the Stone-Geary-function is higher under the IBT than under Coase tariff. This follows from $T(y) > T^p(y), \forall y \in (\underline{y}, \bar{y}]$ and $w(y) \leq w^p(y), \forall y \in (\underline{y}, y_{cs}]$. But even for income higher than y_{cs} we may have an increase in utility for the low income group.

proposition 2 *There exists a non-empty interval* $(\underline{y}, y_{Ucs})$, $\underline{y} < y_{cs} < y_{Ucs}$, *for which* $U(w(y), y - T(y)) < U(w^p(y), y - T^P(y))$, *i.e. the utility under a progressive tariff is higher than under the Coase tariff. All other income groups will loose utility under the progressive tariff.*

To identify the income range for which the utility under the progressive tariff is higher than under Coase tariff we insert the tariff schedules (14), (15) and (22), (23) into the Stone-Geary function and set $U(w(y), y - T(y)) = U(w^p(y), y - T^P(y))$. Solving for y yields:

$$y_{Ucs} = \underline{y} + (\frac{1}{(1-\alpha\beta})^{1/(\alpha\beta)} \left(\frac{1+m}{1+n}\right)^{1/\beta} \tag{27}$$

The proposition shows that for $y \in (\underline{y}, y_{Ucs})$, utility of customers under a progressive tariff will be higher than under the Coase-tariff. Although there always will exists such an interval, the income group that benefits from a progressive tariff might be very small and should be compared to those income groups that will loose under a progressive tariff. The following figure shows for a numerical example the graphs for both utility functions with respect to y. Income ranges between $\underline{y} = 15$ and $\bar{y} = 10000$. Marginal costs are assumed $c = 2$, the subsistence level of water is $w_s = 15$, fixed costs are $F = 800000$ and total population is $P = 1$ million. The income distribution is modelled by a truncated Pareto distribution.[11].

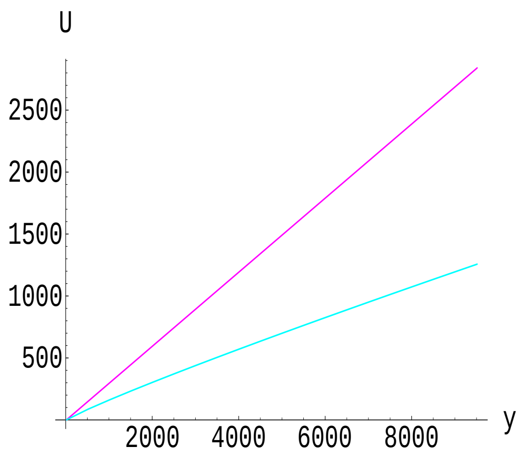

Figure 4: Comparing utility levels

[11] The next version of the paper will contain a real-world simulation for the water system in Bangladesh

In this figure the income range for which $U(w(y), y - T(y)) < U(w^p(y), y - T^P(y))$ is not visible.[12]. The range is $(15, 16.1)$ which represents 7.2 % of the customer population. For all other customers (i.e. 92.8 %) the progressive tariff will lead to less welfare. While this is intended for the upper income range, it certainly should not be the case for the lower income range. In the numerical example average income is 87.5. Almost all customers with an income lower than 87.5 would prefer the Coase tariff.

Why not increase progression, i.e. lifting the parameter β? The numerical example shows that an increase of β may narrow the income interval of those who benefit from the progressive tariff and, at the same time, lead to an increase in utility losses for all other incomes. Hence, the management of a water company needs a lot of experience to gauge the right parameter value.

4.3 Including the household size

A major shortcoming of IBTs in practice is that they do not take into account the size of households. Some tariff system construct the first block taking into account a best guess of household size of the poor. In the following we want to explicitly introduce the number of household members into the tariff. Two methods are conceivable:

- The tariff is based on reported income and the reported number of household members. This requires a tariff schedule of the form $T(y, n), W(y, n)$, where y is total household income and n the houshold size. T and W are defined for households (not for the single member). This scheme is very difficult to design if there is no additional information available. It requires to solve partial differential equations. The resulting tariff schemes are very sensitive with respect to the relevant parameters. Of course, if reliable information on income and household size is available, first best tariffs can be implemented.[13]

[12] The steeper the graph indicates the utility function under a Coase tariff, the lower graph the utility function under a progressive tariff.

[13] In Chile, for example, some districts have implemented a so called means tested approach where households have to verify their income and their size. If they fall short of certain social standards they receive water for a highly subsidized price, see Gmez-Lobo and Contreras (2003)

- The tariff is based solely on reported income. This approach does not require households to report their size. The number of household members is estimated utilizing econometric methods. The resulting size function depends on income and is included in the tariff scheme.

In the following we take the second approach and include the household size function into the tariff system. The size function is assumed as follows[14]:

$$n(y) = \underline{n} + (\bar{n} - \underline{n}) \left(\frac{y - \underline{y}}{\bar{y} - \underline{y}} \right)^{\gamma}, \quad \gamma < 1 \tag{28}$$

where $\underline{n}$ ($\bar{n}$ is the householde size of the lowest (highest) income group and $\underline{n} > \bar{n}$. The incentive compatibility constraint can be derived by applying (5) to the whole household, i.e.

$$y = \text{argmax}_{\tilde{y}}[n(y)U(W(\tilde{y}/n(y) - w_s, (y - T(\tilde{y}))/n(y))] \tag{29}$$

Households try to maximize aggregate utility by chosing the optimal message $\tilde{y}$. If the tariff system $T(y), W(y)$ is incentive compatible $\tilde{y} = y$. The first order condition requires:

$$U_w \dot{W}(y) - U_y \dot{T}(y) = 0, \quad \forall\, y \in Y \tag{30}$$

The second order condition requires $\dot{W}(y) > 0$ and $\dot{T}(y) > 0$. Utilizing the Stone-Geary-utility function (30) can expressed as:

$$\frac{\alpha \dot{W}(y) n(y)}{W(y) - n(y) w_s} - \frac{(1 - \alpha)\dot{T}(y)}{y - T(y)} = 0, \quad \forall\, y \in Y \tag{31}$$

To derive the tariff schedule we have to add a mechanism that prescribes how much households have to contribute to the coverage of costs. We confine our analysis to the Coase tariff[15]. Hence, we assume

$$\dot{T}(y) = (1 + k)c\dot{W}(y) \tag{32}$$

[14]In the following we disregard economies of scale of water consumption with respect to household size.

[15]In a follow-up paper we will include progressive tariffs.

(28), (31) and (32) form a system of non-linear differential equations which can analytically be solved.

$$T(y) = \alpha(y - \underline{y}) + (1-\alpha)(1+k)c(\underline{n} - \bar{n})w_s \left(\frac{y - \underline{y}}{\bar{y} - \underline{y}}\right)^{\gamma} + \underline{y} \quad (33)$$

$$W(y) = \underline{n}w_s + \frac{\alpha}{(1+k)c}(y - \underline{y}) + (1-\alpha)(\underline{n} - \bar{n})w_s \left(\frac{y - \underline{y}}{\bar{y} - \underline{y}}\right)^{\gamma} \quad (34)$$

Notice that (33) and (33) satisfy the second order conditions. Contrary to the single member household case[16] the tariff plan is degressive if the schedule controls for the household size.

proposition 3 *The tariff functions $T(y)$ and $W(y)$ increase on a diminishing rate with respect to income.*

Proof: It is straightforward to differentiate (33) and (34) twice and to assertain that both second degree derivatives are negative.

Including the household size into the Coase tariff leads to a declining increase in both, outlay and consumption, of households. This is due to the assumed decrease in household size with respect to income. Under the Coase tariff for single member households higher income increases water consumption proportionally; if the tariffs allows for the household size, the lower household size increases consumption per capita leading to the diminishing increase of water consumption (and outlays).
Note however, that the outlay schedule $T(W)$ remains linear.

proposition 4 *The outlay schedule $T(W)$ is linear. From (33) and (34) it follows that $T'(W) = (1+k)c = constant$.*

Proof:

From (30) and (32) it follows:

$$\dot{T}/\dot{W} = \frac{dT}{dW} = U_w/U_y = (1+k)c \quad (35)$$

5 Conclusions

In this paper we have compared different tariff systems that deviate from welfare-optimal first-best approaches. Our analysis is motivated by the fact that welfare-

[16] Confer (14) and (15).

optimal pricing of water does not exist anywhere around the world, and that understanding the trade-offs between second-best approaches is also a useful exercise. Affordability of water consumption can be obtained via (modified) Coase tariffs, and increasing block tariffs (IBTs). Peculiarly, progressive tariffs fulfill the "efficient" cross-subsidization of subsistance levels only partially. It is not certain that the increasing block tariff yields a higher utility under a progressive tariff than the Coase tariff. Also, numerical simulations show that the results depend strongly upon the gauged parameters. We have also shown that the household size can be included in the analysis, either through direct reporting, or through an econometric estimation of a relation between income and household size.
Further research should address the sensitivity of the welfare of different income groups to the selected parameters, and confirm (or reject) our proposals on the role of household size. We shall also pursue the numerical work by real-world estimations of the model using data from Bangladesh. Last but not lest, more empirical work on Stone-Geary demand for water is necessary to underpin the theoretical findings, both for water-poor areas in developed countries, and in emerging and developing countries.

6 Appendix

6.1 Second-order conditions to the IC-constraint

Define

$$G_{\tilde{y}}(\tilde{y}, y) = U_w(w(\tilde{y}), y - T(\tilde{y}))\dot{w}(\tilde{y}) - U_y(w(\tilde{y}), y - T(\tilde{y}))\dot{T}(\tilde{y}) = 0, \tag{36}$$

From (36) the optimal message $\tilde{y}$can be derived. A comparative static analysis yields:

$$G_{\tilde{y}\tilde{y}}(\tilde{y}, y)\frac{d\tilde{y}}{dy} + G_{\tilde{y}y}(\tilde{y}, y) = 0 \tag{37}$$

where $G_{\tilde{y}\tilde{y}} < 0$ to secure sufficiency of the first order conditions and $\frac{d\tilde{y}}{dy} = 1$ by construction of the incentive compatible functions $w(y), T(y)$. Hence, $G_{\tilde{y}y}(\tilde{y}, y) = -U_{yy}\dot{T} > 0$. Since $U_{yy} < 0$ it follows that $\dot{T} > 0$ and hence, by (36)$\dot{w} > 0$.

6.2 Coase-Tariff

The differential equation system (7) and (13) can be solved by inserting the latter equation into the former. This yields

$$\alpha(y - T(y)) = (1-\alpha)(w(y) - w_s)(1+m)c \tag{38}$$

Solving (38) and (13) yields

$$\begin{aligned} T(y) &= (1-\alpha)c(1+m)w_s + \alpha(y - M) + M & (39)\\ w(y) &= (1-\alpha)w_s + \frac{\alpha(y-M)}{c(1+m)} & (40) \end{aligned}$$

where M is an integration constant to be determined. Inserting the initial conditions (12) yields $M = \underline{y} - c(1+m)w_s$. Re-inserting into (39) and (40) yields the solution (14) and (15).

6.3 Proof of proposition 1

y^U is determined by the equation $U(w^A(y^U), T^A(y^U)) = U(w(y^U), T(y^U))$. Inserting the Stone-Geary-utility function yields

$$\left(\frac{\alpha}{(1+m)c}\right)^{\alpha}(1-\alpha)^{1-\alpha}(y - \underline{y}) = \left(\frac{\alpha}{c}\right)^{\alpha}(1-\alpha)^{1-\alpha}(y - \underline{y} - A) \tag{41}$$

and after some rearrangements

$$y^U - \underline{y} = \frac{-A}{1-(1+m)^{\alpha}} \tag{42}$$

From $w(y) = w^A(y)$ it follows

$$\tilde{y} - \underline{y} = \frac{1+m}{m}A \tag{43}$$

Comparing (42) and (43) and recalling $\alpha < 1$ shows that $y^U > \tilde{y} > \underline{y}$.

6.4 Deriving the continous IBT

The differential equation system (7) and (21) can be solved by inserting the latter equation into the former. This yields

$$\alpha(y - T(y)) = (1-\alpha)(w(y) - w_s)(1+n)c(y - \underline{y})^{\beta} \tag{44}$$

Differentiating (44) with respect to y yields

$$\alpha(1-\dot{T}(y)) = (1-\alpha)\dot{w}(y)(1+n)c(y-\underline{y})^{\beta} + (1-\alpha)\beta(w(y)-w_s)(1+n)cy^{\beta-1} \quad (45)$$

Solving (45) and (21) yields

$$T(y) = \frac{(1-\alpha)\underline{y}+\alpha(1-\beta)y}{1-\alpha\beta} - \frac{(1-\alpha)c(1+n)(y-\underline{y})^{\alpha\beta}M1}{\alpha} + M2 \quad (46)$$

$$w(y) = w_s + \frac{\alpha(y-\underline{y})^{1-\beta}}{c(1+m)(1-\alpha\beta)} + (y-\underline{y})^{\beta(\alpha-1)}M1 \quad (47)$$

where $M1$ and $M2$ are integration constants. Recalling the initial conditions (12) and inserting into (46) and (47) allows to determine both constants. Re-inserting into the latter two equations yields (22) and (23).

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