Bargaining Networks in International Trade

Nathalie Jorzik[1]

March 1, 2009

Abstract

This paper analyzes the impact of preferential trade agreements (PTAs) on multilateral cooperation in a "competing exporters" model with three countries that are asymmetric with respect to income. We use a network formation approach in which countries can enter bilateral and multilateral trade agreements. We investigate the structure of stable trading networks and show that global free trade is an equilibrium whenever countries' income level is relatively similar. We also model multilateral Nash bargaining under MFN and find that when all countries have the same number of PTAs, free trade is an optimal bargaining solution. Whenever the network structure is asymmetric in the way that some countries belong to very few PTAs and others are strongly networked, free trade can not be achieved. We also show that countries with few PTAs benefit more from multilateral bargaining. This result implies that the increasing number of PTAs reduces countries' incentives for multilateral cooperation.

Keywords: PTAs, most-favoured-nation (MFN) clause, multilateral stability, network formation

JEL Classification number: F13, F12, D85

[1]Alfred Weber Institut, Universität Heidelberg, Grabengasse 14, 69117 Heidelberg, Germany. E-mail : nathalie.jorzik@awi.uni-heidelberg.de.

1 Introduction

The network of international trade agreements gets more and more complex, and in addition to the efforts for multilateral cooperation within the WTO, countries are involved in regional trade agreements (RTA) that coexist with multilateralism. The number of RTAs (both custom unions and PTAs) rises steadily such that from 2002 to 2007 the number of RTAs worldwide increased from 250 to 380. Two important questions that have been analyzed in the literature are whether the process of regional trade formation will hinder or facilitate the multilateral liberalization process,[1] and will the increasing number of regional trade agreements serve as a building bloc towards global free trade or will the process of PTA formation itself converge to global free trade.

We can observe that multilateral tariff negotiations fail and multilateral tariff reduction is substituted by RTAs. Tariffs negotiated at the WTO are not welfare improving for some countries and they rather form bilateral and regional trade agreements. If, and to what extent, multilateral tariff cooperation benefits WTO member countries may depend on their regional trading network.

We consider strategic link formation of countries and investigate stable trading structures. We allow countries to form bilateral as well as multilateral links. In this aspect our paper differs from two further papers that use a network formation approach to investigate strategic stability of trading regimes, Goyal and Joshi(2006) and Furusawa and Konishi (2007). Furthermore these papers assume external tariff rates as exogenously fixed whereas in reality countries optimally adjust their tariffs when they sign a new trade agreement. In our paper we let countries endogenously adjust their optimal tariffs with respect to different network structures to investigate how these tariff adjustments affect countries' incentives to form PTAs.

Our paper also differs in the aspect that in the second part of the paper we introduce a bargaining stage after the network is in place and determine bargaining solutions under different trading structures. We use the Nash-bargaining solution to model simultaneous negotiations and consider the case when all countries have the same number of PTAs and

[1]See Panagariya (2000) and Bhagwati and Panagariya (1996) for an overview on this topic. Papers by Krugman (1992), Krishna (1998) and Freund (2000) have analyzed the impact of RTAs on the incentive for multilateral liberalization and vice versa.

the situation where countries are asymmetric with respect to linking structure. We address the question to what extent network structure and therefore the number of PTAs that a country has formed influences a country's incentives for multilateral liberalization.

In the first part of the paper we study an international trade setting as in Bagwell and Staiger (1999) with three countries and three different goods such that each country is endowed with two of the goods but has a positive demand for each of the goods such that the reason for trade is given by the positive demand function. Countries can sign bilateral and a multilateral trade agreement and determine optimal tariffs on the imports of the goods. Later we extend the framework to a multi-country setting and calculate cooperative bargaining solutions under a given network structure.
We then investigate countries' incentives to form trade agreements when they are asymmetric. We allow for asymmetry with respect to income, where we differentiate a high income country from a low income country by means of different amount of the endowed goods; a high income country is assumed to have a larger amount of endowed goods as compared to the low income country.

The main results in the paper are the following: When countries are symmetric with respect to income, global free trade, which corresponds to the network structure in which each country has a bilateral link with each country and all players have formed a multilateral link, is a stable state. This is because it can be shown that two countries always gain from signing a bilateral trade agreement. Whereas when countries are asymmetric global free trade can only be a stable state, ifr the difference in income is relatively low between all countries. Furthermore we obtain that while starting from an empty network a bilateral trade agreement is always more profitable for the lower income country. When we increase the number of countries we obtain that global free trade is still a stable state. When countries have the same number of PTAs, global free trade is an optimal bargaining result under MFN. In the asymmetric case we obtain that countries with few PTAs benefit more from multilateral tariff reduction and depend more on multilateral cooperation.

The paper proceeds as follows: First we present the basic model in section 2. In section 3 we define stability and efficiency of trading structure and calculate equilibrium tariffs for each given network structure. Then we characterize the stable and efficient networks first in the case of symmetric countries and later in the case of asymmetric countries, when countries' endowment level differs. In section 4 we introduce the bargaining stage after the network is in place and determine Nash-bargaining solutions for different network structures. In section 5 we extend the framework of section 3 and allow an arbitrary number of players.

2 The Model

2.1 Overview

In the following we will consider a three country model based on the three-good model of Bagwell and Staiger (1998) in which each country is endowed with two of the three goods. In the work of Bagwell and Staiger each country is endowed with $\frac{2}{3}$ of each good. Our model is richer in two important aspects. First we assume asymmetry with respect to endowment and hence income level. Furthermore we consider strategic link formation of countries where optimal tariffs under different trading structures are determined endogenously.

We will denote the set of countries by N with $\mid N \mid= 3$, and the set of goods will be denoted by M with $\mid M \mid= 3$. We will assume that country $J \in N$ is endowed with zero units of good $j \in M$ and $\frac{x_J}{2}$ units of good i and k. Whereas country $I \in N$ and $K \in N$ are endowed with zero units of good i and k, respectively, and $\frac{x_I}{2}$ and $\frac{x_K}{2}$, respectively, of the other two goods. Hence we can in generally say that all countries demand each of the three goods, such that all countries $J \in N$ have to import good j from country I and K to consume it whereas J exports good i to country I and good k to country K.

Figure 1 illustrates the pattern of trade.

The utility function in country I, for all countries $I \in N$ is given by:

$$U_I = m + u(c^i) + u(c^j) + u(c^k) \tag{1}$$

where m denotes the numeraire and c^i the consumption of good i. We can derive each country I's demand for good j by:

$$D(P_I^j) = \alpha - \beta \cdot P_I^j.[2] \tag{2}$$

[2]It is well known that this demand function can be derived from a utility function U_I, where $u(c^i)$ is quadratic in c^i and additively separable in each of the three goods.

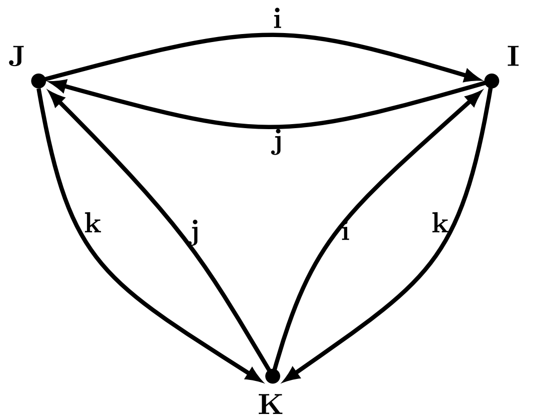

Figure 1: Pattern of trade between country I, J and K.

We have the following no-arbitrage conditions for good j that relates the price of good j in the importing country to the price in the exporting countries.

$$P_J^j = P_I^j + t_I^j = P_K^j + t_K^j, \tag{3}$$

where t_I^j and t_K^j are the tariffs that country J imposed on the imports of good j from country I and country K, respectively.
The import function of good j in country J can be calculated by :

$$IM_J^j(P_J^j) = D(P_J^j), \tag{4}$$

whereas country J's exports of good i are given by:

$$X_J^i(P_J^i) = \frac{x_J}{2} - [\alpha - \beta \cdot P_J^i]. \tag{5}$$

The market clearing condition for good j can now be written as:

$$IM_J^j(P_J^j) = X_I^j + X_K^j, \tag{6}$$

such that the total export of good j has to equal the total import of good j in market J.

2.2 Network Formation

We are interested in what networks are stable when we consider strategic link formation of the countries. For different trading structure we consider a finite number of players

$N = (1, ..., n)$ and assume $n \geq 3$. In what follows the set of players will be a set of countries.[3]
In our setting we concentrate on undirected links as in Jackson and Wolinsky (1996), which means that a link between players requires the consent of all players involved in that link.
We introduce the structure of hypergraphs to model multilateral agreements between players[4].

Definition 2.1. *Let $N = \{1, ..., n\}$ be a finite set of nodes. A pair $(N, \mathcal{L})$, where $\mathcal{L} = \{L_1, ..., L_m\}$ is a set of links, $\mathcal{L} \subseteq 2^N$ is called a hypergraph on N.*

In the following the set of nodes will represent the set of countries and the term network will be used as a synonym for the term hypergraph. A link as a subset of countries represents a trading agreement between these countries.
Since each player is linked with himself we restrict our attention to hypergraphs $(N, \mathcal{L})$ with $\mathcal{L} \subseteq \{L \in 2^N || L| \geq 2\}$. The set of all possible hypergraphs that satisfies this condition is denoted by $\mathcal{H}$.
Whenever the hypergraph contains only one group of players with $\mathcal{L} = \{N\}$, the hypergraph is called global and is denoted with $\mathcal{L}^G$.
The complete graph $\mathcal{L}^N$ is the family of subsets of N with $\mathcal{L}^N = \{L \in 2^N ||L| = 2\}$.
The *star* network, which we denote by $\mathcal{L}_I^S$, has only bilateral links from the central player $I \in N$ to each of the other players with $\mathcal{L}_I^S = \{L \in 2^N ||L| = 2 \text{ and } I \in L\}$.
Let $\mathcal{L}^e$ denote the empty network. This structure represents the situation in which countries have no trade agreements at all and therefore no trade takes places. Countries are autarkic.

The value of a hypergraph is represented by a real valued function $v : \mathcal{H} \to \mathbb{R}$, which specifies for each hypergraph $\mathcal{L} \in \mathcal{H}$ the total value $v(\mathcal{L})$ generated by $\mathcal{L}$ and will be the aggregate of individual payoffs given by a hypergraph structure.
A payoff function $Y_I : \mathcal{H} \to \mathbb{R}$ with $\sum_{I \in N} Y_I(\mathcal{L}) = v(\mathcal{L})$ assigns a payoff to each player $I \in N$ which will be the welfare a country obtains from a given trading structure. In what follows $v(\mathcal{L})$ will denote the aggregate welfare generated by network $\mathcal{L}$, and Y_I will denote

[3]Our notion allows an arbitrary number of players to form trade agreements. Whereas in what follows we will concentrate on the special case when $n = 3$.

[4]Two papers that introduce network formation and communication games on a fixed hypergraph structure are Durieu et al. (2005) and van den Nouweland et al. (1992).

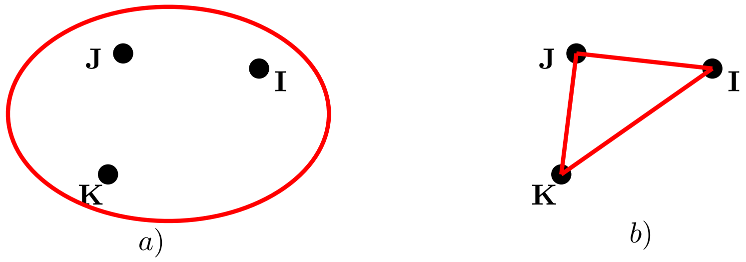

Figure 2: a) Hyperlink between I, J and K. b) Bilateral link formation.

country I's welfare.

2.3 Trading Networks

Countries only trade when there is a trade agreement between them. A network without any trade agreements represents the case of autarky. Each country consumes its own endowment and no trade takes places. This is represented by an empty network. The GATT is represented by means of a global link $L = N$ among all players as shown in Figure 2a). This represents a trading situation under MFN. In this situation each country imposes due to the non-discrimination requirement of the MFN-clause the same external tariffs on the other countries. Countries are also able to form bilateral links. In a bilateral link under MFN both trading partners reduce their tariffs to zero as per Article XXIV. Without the MFN-clause each country imposes its non-cooperative welfare maximizing Nash tariffs on its trading partner. In this case a country's optimal tariff on one trading partner might differ to the optimal tariff on another trading partner. In this situation tariff-discrimination may take place and depends crucially on the endowment and linking structure of the other countries.

In the following we analyze each country's welfare and equilibrium tariffs under different trading regimes, where the set of all possible trading regimes is represented by the set of all possible network structures for $n = 3$.

The tariff structure is determined as follows: When the set of countries has a global link that contains each of the players, each country chooses its welfare maximizing tariffs with respect to the MFN clause, such that each country levies the same tariffs on each of the countries with which they are linked multilaterally. Whereas each country imposes zero tariffs when within the multilateral link a country is bilaterally linked to another country. Without a multilateral link countries choose their non-cooperative welfare maximizing tariffs on the others where social welfare is defined as the sum of consumer surplus, producer surplus and tariff revenue over all goods and depends on the underlying linking structure. When there is no link between two countries we assume that no trade takes place. Two main questions that this paper addresses is whether the formation of PTAs increases or decreases tariffs on third countries, and whether the process of strategic link formation results in global free trade.

In what follows we will introduce a notion for stability and efficiency to characterize network structures.

3 Stable and efficient networks

Stability

In the following we will adopt a relatively weak notion of stability, to investigate which network structure can be a stable state. The notion of stability is based on the pairwise stability notion of Jackson and Wolinsky (1996) but extends the definition in a way that allows multilateral and bilateral links to form.

We introduce some notations:

- For $L \notin \mathcal{L}$, $\mathcal{L} \cup \{L\}$ is the network we obtain from $\mathcal{L}$ by adding the link L.
- For $L \in \mathcal{L}$, $\mathcal{L} \backslash \{L\}$ is the network we obtain from $\mathcal{L}$ by deleting the link L, if $L \in \mathcal{L}$

The formation of a link requires the consent of all parties involved, but severance can be done unilaterally. We introduce the following stability concept:

Definition 3.1. *A hypergraph $(N, \mathcal{L}) \in \mathcal{H}$ with $\mathcal{L} = \{L_1, ..., L_m\}$ is called multilaterally stable, if*

$$\begin{aligned} &(i)\ \ Y_I(\mathcal{L}) \geq Y_I(\mathcal{L}\backslash\{L\})\ \forall L \in \mathcal{L},\ \ \forall I \in L \textit{ and} \\ &(ii)\ Y_I(\mathcal{L} \cup \{L\}) > Y_I(\mathcal{L}) \Rightarrow \exists J \in L, \\ &\qquad \textit{such that } Y_J(\mathcal{L} \cup \{L\}) < Y_J(\mathcal{L})\ \ \forall L \notin \mathcal{L} \end{aligned}$$

The above definition describes a situation in which no country has an incentive to delete any of its existing link and no subset of countries wants to form an additional agreement.

This definition allows the formation of links with more than just two players and therefore also allows the formation of a multilateral trade agreement between all three countries.

Efficiency

In order to analyze efficient hypergraphs, we need to consider global welfare, which is given by the sum of all countries' payoffs.

Definition 3.2. *A hypergraph $(N, \mathcal{L}^*) \in \mathcal{H}$ is said to be efficient, if $v(\mathcal{L}) = \sum_{I \in N} Y_I(\mathcal{L}) \leq v(\mathcal{L}^*) = \sum_{I \in N} Y_I(\mathcal{L}^*),\ \forall (N, \mathcal{L}) \in \mathcal{H}$.*

Equilibrium Tariffs

Now we can calculate each country's equilibrium welfare level under each network structure and check which of the trading networks is stable under the multilateral stability notion. From the no arbitrage condition and market clearing condition we obtain without MFN for the prices in country I for good j in network $\mathcal{L}$:

$$P_I^j(\mathcal{L}) = \frac{\alpha}{\beta} - \frac{\sum_{K \in N_J(\mathcal{L}) \setminus \{J\}} x_K}{2\eta_J(\mathcal{L})\beta} + \frac{\sum_{K \in N_J(\mathcal{L}) \setminus \{J\} \cup \{I\}} t_K^j(\mathcal{L})}{\eta_J(\mathcal{L})} - \frac{(\eta_J(\mathcal{L}) - 1) t_I^j(\mathcal{L})}{\eta_J(\mathcal{L})} \forall J \in N_I(\mathcal{L}) \tag{7}$$

and

$$P_I^j(\mathcal{L}) = \frac{\alpha}{\beta} - \frac{x_I}{2\beta} \tag{8}$$

whenever there is no trade agreement between country I and J, where $N_I(\mathcal{L})$ denote the set of players that are linked with player I in network $\mathcal{L}$ without MFN and $\eta_I(\mathcal{L}) = |N_I(\mathcal{L})|$. First we can calculate a country 's welfare level in network $\mathcal{L}$. From consumer surplus we obtain with $I \neq J \neq K$ and $I, J, K \in N$:

$$\begin{aligned}
CS_I(\mathcal{L}) =& \sum_{j \in M} [\frac{1}{2\beta}(\alpha - \beta P_I^I)^2] \\
=& \frac{1}{2\beta}(\frac{\sum_{J \in N_I(\mathcal{L}) \setminus \{I\}} x_J}{2\eta_I(\mathcal{L})} - \frac{\beta \sum_{J \in N_I(\mathcal{L}) \setminus \{I\}} t_J^i(\mathcal{L})}{\eta_I})^2 \\
&+ \frac{1}{2\beta} \sum_{J \in N_I \setminus \{I\}} (\frac{\sum_{K \in N_J(\mathcal{L}) \setminus \{J\}} x_K}{2\eta_J(\mathcal{L})} - \frac{\beta \sum_{K \in N_J(\mathcal{L}) \setminus \{J\} \cup \{I\}} t_K^j(\mathcal{L})}{\eta_J} + \frac{\beta(\eta_J - 1) t_I^j(\mathcal{L})}{\eta_J})^2 \\
&+ \frac{1}{2\beta} \sum_{J \notin N_I(\mathcal{L})} (\frac{x_I}{2})^2.
\end{aligned}$$

Tariff revenue in country I in network $\mathcal{L}$ is given by

$$TR_I(\mathcal{L}) = \sum_{J\in N_I(\mathcal{L})\setminus\{I\}} X_J^i \cdot t_J^i$$

$$= \sum_{J\in N_I(\mathcal{L})\setminus\{I\}} t_J^i(\frac{(\eta_I-1)x_J}{2\eta_I} - \frac{\sum_{K\in N_I(\mathcal{L})\setminus\{I\}\cup\{J\}} x_K}{2\eta_I} + \frac{\beta\sum_{K\in N_I(\mathcal{L})\setminus\{I\}\cup\{J\}} t_K^i}{\eta_I} - \frac{(\eta_I-1)\beta t_J^i}{\eta_I})$$

and producer surplus:

$$\Pi_I(\mathcal{L}) = \sum_{j\in M} \Pi_I^j =$$

$$= \frac{x_I}{2}[\sum_{J\in N_I\setminus\{I\}} (\frac{\alpha}{\beta} - \frac{\sum_{k\in N_I N_J(\mathcal{L})\setminus\{j\}} x_K}{2\eta_J\beta} + \frac{\sum_{k\in N_J\setminus\{j\}\cup\{i\}} t_j^k}{\eta_J} - \frac{(\eta_J-1)t_j^i}{\eta_J}) + \sum_{j\notin N_I}(\frac{\alpha}{\beta} - \frac{x_I}{2\beta})],$$

where the last term derives from the fact that when two countries I and J have no trade agreement, then country I will consume all of its endowment of good j by itself.

In the following we calculate equilibrium tariffs for the case without MFN and we can now show that the tariff imposed on a country's trading partner J depends on the tariffs imposed on the other trading partners imports of good i and vice versa.

$$t_J^i = \frac{(4\eta_I(\mathcal{L})+2)\beta\sum_{K\in N_I(\mathcal{L})\setminus\{I\}\cup\{J\}} t_K^i + \eta_I(\eta_I-1)x_J - \eta_I\sum_{K\in N_I(\mathcal{L})\setminus\{I\}\cup\{J\}} x_K - \sum_{J\in N_I(\mathcal{L})\setminus\{I\}} x_J}{(4\eta_I^2-4\eta_I-2)\beta} \tag{9}$$

Equation (9) reflects an interesting complementary as already pointed out by Bagwell and Staiger (1998) as the tariffs imposed on country J increase with a rise in tariffs on another trading partner. It is more attractive for a country to rise tariffs on one of his trading partners when tariffs on his other trading partner are already high.[5]
Social welfare in each country in network $\mathcal{L}$ is calculated as the sum of producer surplus, consumer surplus and tariff revenue over all goods.

$$Y_I(\mathcal{L}) = CS_I(\mathcal{L}) + TR_I(\mathcal{L}) + \Pi_I(\mathcal{L}) \tag{10}$$

Under MFN we set tariffs $t_I^j = t^j$ for all countries $I \in N$ that have no additional PTA

[5]There are three different welfare effects at hand that induce a country to increase its tariffs on another country when the tariffs on third countries is high. For a detailed discussion see Bagwell and Staiger (1998) (S.7).

with country J. The prices in equilibrium are given by:

$$P_I^j(\mathcal{L}) = \frac{\alpha}{\beta} - \frac{\sum_{K\in N\setminus\{J\}} x_K}{2n\beta} + \frac{(n-\tilde{\eta}_J)t^j(\mathcal{L})}{n} - t^j(\mathcal{L}), \tag{11}$$

whenever I and J have no PTA and

$$P_I^j(\mathcal{L}) = \frac{\alpha}{\beta} - \frac{\sum_{K\in N\setminus\{J\}} x_K}{2n\beta} + \frac{(n-\tilde{\eta}_J)t^j(\mathcal{L})}{n} \tag{12}$$

else, where with $\tilde{\eta}_J(\mathcal{L})$ we denote the number of PTAs of country J with $J \in \tilde{N}_J(\mathcal{L})$.
First we can calculate a country 's welfare level in network $\mathcal{L}$. From consumer surplus we obtain with $I \neq J \neq K$ and $I, J, K \in N$:

$$\begin{aligned} CS_I(\mathcal{L}) &= \sum_{J\in N}[\frac{1}{2\beta}(\alpha - \beta P_I^j)^2] \\ &= \frac{1}{2\beta}\sum_{J\in\tilde{N}_I(\mathcal{L})}(\frac{\sum_{K\in N\setminus\{J\}} x_K}{2n} - \frac{\beta(n-\tilde{n}_J)t^j(\mathcal{L})}{n})^2 \\ &+ \frac{1}{2\beta}\sum_{J\notin\tilde{N}_I(\mathcal{L})}(\frac{\sum_{K\in N\setminus\{J\}} x_K}{2n} + \frac{\beta\tilde{n}_J t^j(\mathcal{L})}{n})^2 \end{aligned}$$

Tariff revenue in country I in network $\mathcal{L}$ is given by

$$\begin{aligned} TR_I(\mathcal{L}) &= \sum_{J\in N\setminus\tilde{N}_I(\mathcal{L})} X_J^i \cdot t^i \\ &= \sum_{J\in N\setminus\tilde{N}_I(\mathcal{L})} t^i \cdot (\frac{(n-1)x_J}{2n} - \frac{\sum_{K\in N\setminus\{I\}\cup\{J\}} x_K}{2n} + \frac{\beta(n-\tilde{\eta}_I)t^i(\mathcal{L})}{n} - \frac{n\beta t^i}{n}) \end{aligned}$$

and producer surplus:

$$\begin{aligned} \Pi_I(\mathcal{L}) &= \sum_{j\in M}\Pi_I^j \\ &= \frac{x_I}{2}[\sum_{J\notin\tilde{N}_I}(\frac{\alpha}{\beta} - \frac{\sum_{K\in N\setminus\{J\}} x_K}{2n\beta} + \frac{(n-\tilde{\eta}_J)t^j(\mathcal{L})}{n} - t^j(\mathcal{L})) \\ &+ \sum_{J\in\tilde{N}_I\setminus\{I\}}(\frac{\alpha}{\beta} - \frac{\sum_{K\in N\setminus\{J\}} x_K}{2n\beta} + \frac{(n-\tilde{n}_J)t^j}{n})], \end{aligned}$$

where the last term derives from the fact that when two countries I and J have no trade agreement, then country I will consume all of its endowment of good J by itself.
Under MFN we can calculate country I's optimal tariff with respect to the other countries by:

$$t^i = \frac{n^2\sum_{J\in N\setminus\tilde{N}_I} x_J - (n+1)(n-\tilde{n}_I)\sum_{J\in N\setminus\{I\}} x_J}{2\beta(n-\tilde{n}_I)(2n\tilde{n}_I - n + \tilde{n}_I)} \tag{13}$$

The next example demonstrates how the equilibrium prices and tariffs are calculated under a given trading regime.

Example 3.1. *Autarky, in which no trade takes place, is represented by an empty network. All countries consume their endowments and the price of good j in market I is given by:*

$$\frac{x}{2} = \alpha - \beta \cdot P_I^j$$

Since $P_I^j = \frac{\alpha - \frac{x}{2}}{\beta}$ and tariff revenues are zero we obtain for social welfare

$$Y_I(\mathcal{L}^e) = \frac{1}{2\beta}[(\frac{x}{2})^2 + (\frac{x}{2})^2] + \frac{x}{2}(\frac{\alpha - \frac{x}{2}}{\beta}) + \frac{x}{2}(\frac{\alpha - \frac{x}{2}}{\beta}) = \frac{4\alpha x - x^2}{4\beta}$$

In the following we will consider stable networks when countries adjust their tariffs for each new link that is formed and therefore when countries choose optimal tariffs.

3.1 Symmetric Countries

In the following we will assume that countries are symmetric with respect to income such that we set $x_I = x_J = x_K$ and $n = 3$. In this section we obtain three major findings. First we show that under MFN tariffs on third countries decrease when a country increases its number of PTAs. Second we can show that global free trade can be achieved as a stable trading structure but it is not the unique stable network. Third we can show that global free trade maximizes total world welfare and is therefore efficient.
First we consider the impact of a PTA between country I and J, in which they both agree to reduce tariffs to zero, on the external tariffs on country K. We obtain from equation (13) that under MFN country I's tariffs on the other markets in network $\mathcal{L}$ is given by :

$$t^i(\mathcal{L}) = \frac{x}{2\beta(7\tilde{\eta}_I(\mathcal{L}) - 3)} \tag{14}$$

This result shows that the tariff imposed on the foreign markets decreases when country I and J increase their number of PTAs. Therefore this result is contrary to the results in Krishna (1998), who showed by means of a political economy model that PTAs decrease incentives for multilateral liberalization and tariffs on third countries increase. Furthermore we can see that a country's optimal tariff does not depend on the decision of any of the other players. Therefore country K's choice of external tariffs is not affected by a

PTA between I and J.

Without MFN, countries non-cooperatively choose welfare maximizing tariffs on the imports from countries with whom they have a bilateral trade agreement. Since countries are symmetric we can directly solve for the equilibrium tariffs in equation (9) where $t^i_J = t^i$ $\forall J \neq I$:

$$t^i(\mathcal{L}) = \frac{x}{(2 \cdot \eta_I(\mathcal{L}) + 2)\beta} \quad \forall I \in N. \tag{15}$$

where $i \neq j \neq k$.

A country imposes higher tariffs when it has fewer trade agreements and as a result of an additional bilateral trade agreement country I reduces external tariffs on third countries. This is due to the tariff complementary effect since a reduction on one of country I's trading partners increases country I's incentives to reduce tariffs on its other trading partner.

The first result on stability that we want to describe concerns the question of whether global free trade can be achieved as a stable state.

Proposition 3.1. *When countries are symmetric, global free trade (the complete network with a global link) is a stable network.*

Proof. We have to show multilateral stability for the free trade network. The first condition for multilateral stability is satisfied since no additional link can be formed. We have to show that no player has an incentive to delete any of his trade agreements. First it can be shown that no player wants to delete the global link since this will result in the complete network and the loss in welfare gains will be $\triangle Y_I = Y_I(\mathcal{L}^G \cup \mathcal{L}^N) - Y_I(\mathcal{L}^N) = \frac{505x^2}{8712\beta} > 0$ for each country $i \in N$. We can additionally observe that no player will delete any of his bilateral links since this will again result in a lower welfare as $Y_I(\mathcal{L}^G \cup \mathcal{L}^N) > Y_I(\mathcal{L}^G \cup \mathcal{L}^S_J)$ for all $J \neq I$. □

To obtain an intuition why free trade is a stable state consider the effect of a trade agreement between player I and J on country I's welfare when we consider equilibrium tariffs in the network that contains the trade agreement and the welfare level under equilibrium tariffs in the network that does not contain the trade agreement between I and J.

Proposition 3.2. *Global free trade and the global link with a PTA between one pair of players are the unique stable states.*

For an intuition of this result we first show that without MFN each pair of countries always gains from an additional trade agreement and therefore wants to form as many links as possible:

$$
\begin{aligned}
& Y_I(\mathcal{L}) - Y_I(\mathcal{L}\backslash\{I,J\}) \\
=& \frac{1}{2\beta}(\frac{(\eta_I-1)x}{2\eta_I} - \frac{(\eta_I-1)x}{\eta_I(2\eta_I+2)})^2 - \frac{1}{2\beta}(\frac{(\eta_I-2)x}{2(\eta_I-1)} - \frac{(\eta_I-2)x}{(\eta_I-1)2\eta_I})^2 \\
+& \frac{x}{2}(\frac{\alpha}{\beta} - \frac{(\eta_J-1)x}{2\eta_J\beta} + \frac{(\eta_J-2)x}{\eta_J\beta(2\eta_J+2)} - \frac{(\eta_J-1)x}{\eta_J\beta(2\eta_J\beta(2\eta_J+2)}) - \frac{x}{2}(\frac{\alpha}{\beta} - \frac{x}{2\beta}) \\
+& \frac{(\eta_I-1)x}{\beta(2\eta_I+2)}(\frac{x}{2\eta_I} - \frac{x}{\eta_I(2\eta_I+2)}) - \frac{(\eta_I-2)x}{2\beta\eta_I}(\frac{x}{2(\eta_I-1)} - \frac{x}{(\eta_I-1)2\eta_I}) \\
+& \frac{1}{2\beta}(\frac{(\eta_J-1)x}{2\eta_J} - \frac{(\eta_J-2)x}{\eta_J(2\eta_J+2)} + \frac{\eta_J-1)x}{(2\eta_J+2)\eta_J})^2 - \frac{1}{2\beta}(\frac{x}{2})^2 \\
=& \frac{x^2}{4\beta(\eta_I+1)^2\eta_I^2}(\eta_I+\eta_I^2) + \frac{x^2}{8\beta(\eta_J+1)^2} > 0
\end{aligned}
$$

Therefore two players always want to sign a trade agreement. Since in the proof of Proposition 3.1 we showed that the complete network cannot be stable, we can conclude that without MFN there exists no stable network.

Furthermore we can observe that global free trade is not the unique stable network. Another stable trading structure consists of a PTA between one pair of countries under MFN, e.g. country I and J. To see why this is a stable structure consider that starting from the global link each pair of players will benefit from a trade agreement. But now country K prefers not to form a bilateral link. This is due to the fact that country K's welfare will decrease when the other countries are already linked bilaterally. To see this consider that country K's gain in consumer surplus in good k with $\frac{1}{2\beta}(\frac{x}{3})^2 - \frac{1}{2\beta}(\frac{x}{4})^2 > 0$ and its gain in profit with $\frac{8x^2}{44\beta} - \frac{x^2}{6\beta} > 0$ is smaller than its losses in tariff revenue from the additional PTA and in consumer surplus in the market of the exported good. The additional gain in consumer surplus and profit can not compensate the loss in consumer surplus and tariff revenue that would result from an additional PTA with either country I or J. Furthermore the deletion of the global link is not profitable either since tariff revenue decreases. The existing PTA between I and J cause that country K's welfare losses are higher than its welfare gains that country K obtains from the opening of its market with either I or J.

Efficiency

Next we investigate the nature of the efficient networks.

Proposition 3.3. *Global free trade is the unique efficient network.*

Proof. Under global free trade total world welfare is given by:

$$\sum_{I\in N} Y_I(\mathcal{L}^{G\cup N}) = \sum_{I\in N}[\frac{1}{2\beta}((\frac{x}{3})^2+(\frac{x}{3})^2+(\frac{x}{3})^2)] + \sum_{I\in N}\frac{x}{2}(\frac{2\alpha}{\beta}-\frac{2x}{3\beta}) = \frac{3\alpha x}{\beta}-\frac{x^2}{2\beta}$$

In comparison under MFN it can be verified that world welfare in network $\mathcal{L}$ can be calculated from equation (14):

$$\begin{aligned}
&\sum_{I\in N} Y_I(\mathcal{L})\\
=&\sum_{I\in N}\frac{1}{2\beta}[\tilde{\eta_I}\cdot(\frac{x}{3}-\frac{(3-\tilde{\eta_I})x}{2(7\tilde{\eta_I}-3)3})^2+(3-\tilde{\eta_I})(\frac{x}{3}+\frac{x\sum_{i\in N}}{2(7\tilde{\eta_I}-3)\cdot 3})^2]\\
+&\sum_{I\in N}(3-\tilde{\eta_I})(\frac{x}{2\beta(7\tilde{\eta_I}-3)})(\frac{x}{6}+\frac{x(3-\tilde{\eta_I})}{3\cdot 2(7\tilde{\eta_I}-3)}-\frac{x}{2(7\tilde{\eta_I}-3)})\\
+&\sum_{I\in N}[(3-\tilde{\eta_I})(\frac{x}{2}(\frac{\alpha}{\beta}-\frac{x}{3\beta}+\frac{(3-\tilde{\eta_I})x}{3\cdot 2(7\tilde{\eta_I}-3)}-\frac{x}{2(7\tilde{\eta_I}-3)})\\
+&(\tilde{\eta_I}-1)(\frac{x}{2}(\frac{\alpha}{\beta}-\frac{x}{3\beta}+\frac{(3-\tilde{\eta_I})x}{3\cdot 2\beta(7\eta_J\tilde{eta}_I-3)})]\\
=&\frac{3\alpha x}{\beta}-\frac{x^2}{8}\sum_{I\in N}(\frac{65\tilde{\eta_I}^2-55\tilde{\eta_I}+12}{\beta(7\tilde{\eta_I}-3)^2})
\end{aligned}$$

This is maximal, whenever $\tilde{\eta_I}=3$ for all $I\in N$.
Without MFN global welfare can by calculated as:

$$\begin{aligned}
&\sum_{I\in N} Y_I(\mathcal{L})\\
=&\sum_{I\in N}\frac{1}{2\beta}[(\frac{(\eta_I-1)x}{2\eta_I}-\frac{(\eta_I-1)x}{(2\eta_I+2)\eta_I})^2+(\eta_I-1)(\frac{(\eta_I-1)x}{2\eta_I}-\frac{x}{(2\eta_I+2)\eta_I}+\frac{(\eta_I-1)x}{(2\eta_I+2)\eta_I})^2]\\
+&\sum_{I\in N}\frac{x}{2}[(3-\eta_I)(\frac{\alpha}{\beta}-\frac{x}{2\beta})+(\eta_I-1)(\frac{\alpha}{\beta}-\frac{(\eta_I-1)x}{2\beta\eta_I}+\frac{(\eta_I-2)x}{\beta\eta_I(2\eta_I+2)}-\frac{(\eta_I-1)x}{\beta(2\eta_I+2)\eta_I})]\\
+&\frac{x(\eta_I-1)}{(2\eta_I+2)\beta}(\frac{x}{2\eta_I}+\frac{x}{(2\eta_I+2)\eta_I}-\frac{(\eta_I-1)x}{(2\eta_I+2)\eta_I})+(\frac{x}{2})^2(\frac{(3-\eta_I)}{2\beta})\\
=&\frac{3\alpha x}{\beta}-\frac{x^2}{8}\sum_{I\in N}(\frac{\eta_I^4-12\eta_I^3-3\eta_I^2+9\eta_I-9}{\beta\eta_I^2(\eta_I+1)^2}).
\end{aligned}$$

It can easily be verified that $\sum_{I\in N} Y_I(\mathcal{L})$ is maximal whenever $\eta_I=3\ \forall I$ such that the complete network generates the highest welfare with $v(\mathcal{L})=\frac{3\alpha x}{\beta}-\frac{33x^2}{64\beta}<\sum_{I\in N} Y_I(\mathcal{L}^{G\cup N})$. This completes the proof. □

3.2 Asymmetric Countries

Since P^j is the free trade price without any tariffs in each country for good j we can calculate with $x_I < x_J < x_K$ each country's income as $E_I = \frac{x_I}{2}P^j + \frac{x_I}{2}P^k < \frac{x_J}{2}P^i + \frac{x_J}{2}P^k < \frac{x_K}{2}P^j + \frac{x_K}{2}P^i = E_K$ such that we can interpret country I as the low income country and country K as the high income country.

From equation (9) we can calculate country I's tariff without MFN on imports from country J whenever I and J have a trade agreement:

$$t^i_J(\mathcal{L}) = \frac{x_J}{6\beta}, \tag{16}$$

whenever country $K \notin N_I(\mathcal{L})$, and

$$t^i_J(\mathcal{L}) = \frac{3x_J - x_K}{16\beta} \tag{17}$$

else, such that the tariffs depend on both countries' income level but not on the other countries' tariffs. We can observe that country I sets a higher tariff on the imports of good i when the importing country J has a higher income level relatively to country K.

When we start with an empty network we investigate whether two countries I and J want to form a trade agreement. Here we can make the first observation:

Lemma 3.1. *Starting from an empty network a trade agreement between two countries is always more profitable for the lower income country.*

Proof. Let's consider a trade agreement between I and J. Country I's welfare increase will be $\triangle Y_I = \frac{3x_J^2}{72\beta} + \frac{x_I^2}{72\beta} > 0$ whereas for country J we obtain $\triangle Y_J = \frac{3x_I^2}{72\beta} + \frac{x_J^2}{72\beta}$ which is smaller than $\triangle Y_I$ whenever $x_I < x_J$. Therefore the lower income country benefits more from a trade agreement. □

Now we investigate the incentives for two countries I and J to sign a PTA under MFN tariffs. We consider the MFN tariffs in a trading structure where country I and J have a trade agreement versus the MFN tariffs for country I and J under the network without a PTA. This analysis is similar to the one in section 3.1. From equation (13) tariffs are now given by:

$$t^i(\mathcal{L}) = \frac{9\sum_{J\in N\setminus\tilde{N}_I(\mathcal{L})} x_J - 4(3-\tilde{n}_I(\mathcal{L}))\sum_{J\in N\setminus\{I\}} x_J}{2\beta(3-\tilde{n}_I(\mathcal{L}))(6\tilde{n}_I(\mathcal{L}) - 3 + \tilde{n}_I(\mathcal{L}))}, \tag{18}$$

such that each country levies the same tariffs on each of its trading partners.

First we investigate the effect of the opening of the markets on the welfare components.

The change in tariff revenue for country I is given by:

$$\triangle TR_I = TR_I(\mathcal{L}^G \cup \{\{I,J\}\}) - TR_I(\mathcal{L}^G) = (\frac{5x_K - 4x_J}{22 \cdot \beta})(\frac{4x_K - x_J}{22}) - (\frac{(x_J + x_K)^2}{16 \cdot 8\beta}) \quad (19)$$

For country J this is given by:

$$\triangle TR_J = TR_J(\mathcal{L}^G \cup \{\{I,J\}\}) - TR_J(\mathcal{L}^G) = (\frac{5x_K - 4x_I}{22 \cdot \beta})(\frac{4x_K - x_I}{22}) - (\frac{(x_I + x_K)^2}{16 \cdot 8\beta}) \quad (20)$$

A comparison of equation (19) and (20) shows that the change in tariff revenue from the opening of the markets is lower for the lower income market.
For the change in profit of country I we obtain:

$$\triangle \Pi_I = \Pi_I(\mathcal{L}^G \cup \{\{I,J\}\}) - \Pi_I(\mathcal{L}^G) = \frac{x_I}{6}(\frac{5x_K - 4x_I}{22\beta} + \frac{x_I + x_K}{16\beta}) \quad (21)$$

The effect of a PTA on producer surplus is positive, whenever $x_I < x_K$ and negative else and thus depends on the relation of the income level between market I itself and the income level of the market that does not belong to any PTA.
Country I's change in consumer surplus is given by:

$$\triangle CS_I = \frac{1}{2\beta}[(\frac{5x_J + 2x_K}{22})^2 - (\frac{(x_J + x_K)}{8})^2 + (\frac{5x_I + 2x_K}{22})^2 - (\frac{9(x_I + x_K)}{48})^2] \quad (22)$$

From equation (19), (21) and (22) we can now calculate the change in welfare level of country I from a PTA with country J.

$$\triangle Y_I = -\frac{1}{61\,952}\frac{-46x_I x_K + 721x_I^2 - 632x_J^2 + 1296x_K x_J - 1015x_K^2 - 512x_I x_J}{\beta}$$

The above expression shows that whether the effect is positive or negative depends on the relation betwen all three countries' income level. For the case of symmetric countries we could show in section 3.1 that this effect is always positive. Whenever the income level between all three countries differs a lot, this expression might become negative for one of the countries and a PTA under MFN is not formed.

Lemma 3.2. *Under MFN starting from a global link two countries always benefit from the formation of a PTA, whenever their income level is relatively similar.*

To see whether the lower or the higher income country has a higher gain in an additional PTA we have to compare the change in country J's welfare level with the change in country I's welfare level and obtain that the higher income country benefits more from a PTA.

$$\triangle Y_I - \triangle Y_J = -\frac{1}{5632}\frac{-122x_I x_K + 123x_I^2 - 123x_J^2 + 122x_K x_J}{\beta} > 0 \quad (23)$$

Therefore we can conclude that country I benefits more from a PTA than country J and derives a larger benefit from the opening of the foreign market, whenever $x_I > \frac{122}{123}x_K - x_J$.

Proposition 3.4. *Global free trade is a stable state whenever the income level of all three countries are not too different.*

The first condition is obviously fulfilled since no additional link can be added.
The second condition for stability can be derived, when we calculate a country's welfare change when the global link is deleted and when a PTA is deleted. We obtain that global free trade is stable whenever $\frac{1}{4608}\frac{350x_I^2-202x_Ix_K-97x_K^2-97x_J^2+272x_Jx_K-202x_Ix_J}{\beta} > 0$ and $-\frac{1}{8712}\frac{-340x_I^2+412x_Ix_K-440x_Jx_K+275x_J^2+64x_K^2}{\beta} > 0$. The complete proof is shown in the appendix.

Remark 3.1. *When countries' income level differs a lot it is not quite clear which structure is stable. This tremendously depends on the relation between the income levels. For example a bilateral link between I and J can be stable, whenever country I (country J) does not want to add an additional bilateral link with player K. In the symmetric case we observed that this will never be the case since countries form more and more links, whereas in the asymmetric case it has to be fulfilled that* $Y_I(\{I,J\}) - Y_I(\{I,J\},\{I,K\}) = -\frac{1}{576}\frac{8x_I^2+3x_J^2+27x_K^2-18x_Jx_K}{\beta}$ *which can be positive, whenever* x_K *is large enough.*

4 Bargaining under MFN

Countries are involved in a bargaining process within the WTO to agree on a commonly reduced multilateral tariff $t^*(\mathcal{L})$. We investigate a bargaining stage after the network is in place and investigate whether under the Nash bargaining solution all countries improve or whether the solution is below the threat point. We consider Nash-bargaining to get rid of the problem of which country proposes first in a multilateral trading round. In the following we will assume that the threat point of country I under network $\mathcal{L}$ is given by its non-cooperative welfare level as calculated in section 3. We will assume that negotiations fail when the threatpoint for any of the players levies a higher payoff than the Nash-bargaining solution. We want to investigate the effect of the network structure on the bargaining solution and whether countries that are asymmetrically linked benefit differently from the bargaining solution.
We start with the case in which two symmetric countries share a bilateral link with non-cooperative tariffs and investigate the cooperative Nash bargaining solution. We have to

maximize the Nash bargaining product:

$$\max_{t^*} V(\mathcal{L}) = [(Y_I(t^*(\mathcal{L})) - Y_I(\mathcal{L}))(Y_J(t^*(\mathcal{L}) - Y_J(\mathcal{L}))] = \frac{1}{20\,736}\frac{\left(36t^2\beta^2 - x^2\right)^2}{\beta^2} \tag{24}$$

such that $Y(t^*)$ lies within the bargaining set and $Y_I(t^*) > Y_I \; \forall I \in N$, such that $Y_I(\mathcal{L})$ denotes the threat point for country I in network $\mathcal{L}$ that corresponds to the non-cooperative welfare level under MFN in case negotiations fail and $Y_I(t^*(\mathcal{L}))$ denotes the welfare level under Nash-bargaining tariffs. In this formula it is already assumed, that countries have equal bargaining power. Maximizing equation (24) with respect to t^* yields the following first order condition:

$$\frac{1}{4}\beta^2 t^3 - \frac{1}{144}x^2 t = 0 \tag{25}$$

From equation (25) it follows with $t^* \geq 0$ that we obtain one possible Pareto-improving bargaining solution $t_1^* = 0$ such that the welfare function is maximized under free trade with a total welfare level of $\frac{1}{20736}\frac{x^2}{\beta^2}$ and each of the linked countries obtains a welfare gain from the bargaining tariffs of $Y_I(t^* = 0) - Y_I = \frac{x^2}{144 \cdot \beta^2}$.

Next we investigate which tariff will be negotiated when countries share a multilateral global link. Now the Nash welfare function is given by:

$$\begin{aligned}\max_{t^*} V =&(Y_I(t^*(\mathcal{L}^G)) - Y_I(\mathcal{L}^G))(Y_J(t^*(\mathcal{L}^G)) - Y_J(\mathcal{L}^G))(Y_K(t^*(\mathcal{L}^G)) - Y_K(\mathcal{L}^G)) \\ =& - \frac{1}{7077\,888}\frac{\left(64t^2\beta^2 - x^2\right)^3}{\beta^3}\end{aligned}$$

Maximizing the above expression with respect to t^* leads to the following first order condition:

$$-\frac{6}{27}\beta^3 t^5 + \frac{4}{576}x^2\beta t^3 - \frac{2}{36\,864}\frac{x^4}{\beta}t = 0 \tag{26}$$

We obtain with $t^* \geq 0$ two possible solutions $t_1^* = 0$ such that $Y_I(0) - Y_I(\mathcal{L}^G) = \frac{x^2}{192\beta} > 0$ and $t_2^* = \frac{x}{8\beta}$ such that $Y_I(t_2^*) - Y_I(\mathcal{L}^G) = 0$.

When countries are linked multilaterally and have no additional PTAs zero tariffs are an optimal solution for each country.

Proposition 4.1. *When countries are symmetric we obtain the following bargaining solution that maximize the total welfare function:*

- *When the linking structure is such that two countries are linked bilaterally the optimal tariff is given by $t^* = 0$.*

- *When all countries share a global multilateral link the optimal bargaining solution is given by $t^* = 0$ such that the total welfare is maximized.*

We can see that when countries are symmetric and all countries have the same number of PTAs free trade is always an optimal solution. Multilateral bargaining always selects the free trade solution as a Pareto-efficient bargaining outcome.

We can observe that whenever countries are not symmetrically linked such that some countries are strongly networked within the GATT and some other countries belong to very few PTAs this result will change. Above we have assumed that all negotiation parties have the same number of trade agreements. In the following we will assume asymmetries with respect to linking structure and investigate whether a country that has fewer PTAs has a higher incentive for multilateral liberalization since it will benefit more. With respect to income all countries are symmetric and we consider the situation where under MFN regime one country is not linked bilaterally and the other two countries share a PTA. Since countries are symmetric with respect to market size we can concentrate on the case in which country I and J belong to the same PTA.
We have the following maximization problem for $\mathcal{L} := \mathcal{L}^G \cup \{\{I, J\}\}$.

$$\begin{aligned}\max_t V =& (Y_I(t^*(\mathcal{L})) - Y_I(\mathcal{L}))(Y_J(t^*(\mathcal{L})) - Y_J(\mathcal{L}))(Y_K(t^*(\mathcal{L})) - Y_K(\mathcal{L})) \\ =& -\frac{1}{338\,550\,579\,265\,536} x\,(1936t\beta - 39x)\,\frac{\left(-7744xt\beta + 69\,696t^2\beta^2 - 303x^2\right)^2}{\beta^3},\end{aligned}$$

such that $Y_I(t^*(\mathcal{L})) - Y_I(\mathcal{L}) \geq 0 \; \forall I \in N$, because otherwise the bargaining solution results in a welfare level which lies below the threatpoint for at least one country and negotiations fail.
Maximizing the equation with respect to t^* yields:

$$-\frac{5}{36}x\beta^2t^4 + \frac{4 \cdot 4223}{627\,264}x^2\beta t^3 - \frac{3 \cdot 2549}{11\,290\,752}x^3t^2 - \frac{2 \cdot 60\,217}{2428\,766\,208}\frac{x^4}{\beta}t + \frac{101}{6476\,709\,888}\frac{x^5}{\beta^2} = 0$$

under the condition that $t^* > 0$ we obtain as a solution for the maximization problem of the social welfare function that $t^* = 3.131\,6 \times 10^{-4}\frac{x}{\beta}$ which increases each country's welfare. Calculations show the following result:

Proposition 4.2. *From multilateral bargaining under MFN the less bilaterally linked country K profits more from the multilateral bargaining solution than a bilaterally linked country.*

This result explains a bit more why countries like Brazil, which have very few RTAs, depend more on multilateral liberalization than strongly networked countries and regionalism serves as a substitute for multilateral cooperation.
Furthermore we observe that in asymmetric networks global free trade can not be achieved as an optimal bargaining solution.

In section 3 we have observed that a global link with one PTA is a stable state, but so far we have not investigated whether this can still be achieved as a stable state under bargaining or whether in this setting global free trade is a unique stable network. Calvo-Armengol (2003) introduced bargaining during the formation of networks and characterized the stable and efficient bargaining structures. To fully characterize the effects of PTAs and multilateral tariff negotiations on the world trading system one should additionally check for stability of bargaining networks.

5 Many Country Extension

We want to investigate implications for stable networks when we increase the number of countries. In the following we allow an arbitrary number of countries to form bilateral links with all the other countries and the whole player set can form a multilateral link. We want to investigate whether MFN is essential for stability or whether we can achieve stability without MFN. The set of countries is given by N where $\mid N \mid= n$ and n different goods are traded among these countries such that country $I \in N$ is endowed with $\frac{x_I}{2}$ units of each good except for good $i \in M$ itself. Furthermore we want to investigate the effect on multilateral tariffs and multilateral liberalization when countries form more and more PTAs. In equation (15) we could observe that tariffs on third countries decrease when we increase a country's number of PTAs. Will these result still hold when we increase the number of players? In the following we assume that all countries are symmetric with $x_I = x \; \forall I$.

Without MFN welfare in country I under network $\mathcal{L}$ is given by:

$$
\begin{aligned}
Y_I(\mathcal{L}) = & \sum_{J\in N_I(\mathcal{L})} t_J^i(\frac{x}{2} - \frac{(\eta_I-1)x}{2\eta_I} + \frac{\beta\sum_{K\in N_I(\mathcal{L})\setminus\{I\}\cup\{J\}} t_K^i}{\eta_I} - \frac{(\eta_I-1)\beta t_J^i}{\eta_I}) \\
& +\frac{x}{2}[\sum_{J\in N_I(\mathcal{L})\setminus\{I\}}(\frac{\alpha}{\beta} - \frac{(\eta_J-1)x}{2\eta_J\beta} + \frac{\sum_{K\in N_J(\mathcal{L})\setminus\{J\}} t_K^j}{\eta_J} - t_I^j) + \sum_{J\notin N_I(\mathcal{L})}(\frac{\alpha}{\beta} - \frac{x}{2\beta})] \\
& +\frac{1}{2\beta}\sum_{J\notin N_I(\mathcal{L})}(\frac{x}{2})^2 + \frac{1}{2\beta}\sum_{J\in N_I(\mathcal{L})\setminus\{I\}}(\frac{(\eta_J-1)x}{2\eta_J} - \frac{\beta\sum_{K\in N_J(\mathcal{L})\setminus\{J\}} t_I^j}{\eta_J} + \beta t_I^j)^2 \\
& +\frac{1}{2\beta}(\frac{(\eta_I-1)x}{2\eta_I} - \frac{\beta\sum_{K\in N_I(\mathcal{L})\setminus\{I\}} t_K^i}{\eta_I}))^2
\end{aligned}
$$

Without the global link countries form bilateral links without MFN such that from equation (9) we obtain that :

$$
t_J^i(\mathcal{L}) = \frac{(4\eta_I(\mathcal{L})+2)\beta\sum_{K\in N_I(\mathcal{L})\setminus\{I\}\cup\{J\}} t_K^i(\mathcal{L}) + x}{(4\eta_I^2(\mathcal{L}) - 4\eta_I(\mathcal{L}) - 2)\beta}
$$

Since all countries are symmetric and the tariffs levied on each country depend only on the own tariffs and not on the tariffs of the others we obtain that:

$$
t^i(\mathcal{L}) = \frac{x}{\beta(2\eta_I(\mathcal{L})+2)} \qquad \forall I\in N \tag{27}
$$

Therefore the tariffs on other countries decrease with the number of bilateral trade agreements and even without MFN a country does not impose higher tariffs when it increases its number of trade agreements and the tariff complementary effect still holds.

Furthermore under MFN all countries are linked multilaterally and each country imposes the same tariffs on imports of good i on each other country, except on those that belong to a PTA with country I. We denote the set of countries that have a PTA with country I in network $\mathcal{L}$ by $\tilde{N}_I(\mathcal{L})$ with $I\in\tilde{N}_I(\mathcal{L})$ and its cardinality by $\mid \tilde{N}_I(\mathcal{L}) \mid = \tilde{\eta}_I(\mathcal{L})$.

Under MFN we can calculate country I's welfare in network $\mathcal{L}$ with

$$
\begin{aligned}
Y_I(\mathcal{L}) = & \frac{1}{2\beta}\sum_{J\notin\tilde{N}_I(\mathcal{L})}(\frac{(n-1)x}{2n} + \frac{\beta t^j\tilde{\eta}_J(\mathcal{L})}{n})^2 + \frac{1}{2\beta}\sum_{J\in\tilde{N}_I(\mathcal{L})}(\frac{(n-1)x}{2n} - \frac{\beta t^j(n-\tilde{\eta}_J(\mathcal{L}))}{n})^2 \\
& +t^i(n-\tilde{\eta}_I(\mathcal{L}))(\frac{x}{2n} - \frac{\beta\tilde{\eta}_I(\mathcal{L})t^i}{n}) \\
& +\frac{x}{2}[\sum_{J\notin\tilde{N}_I(\mathcal{L})}(\frac{\alpha}{\beta} - \frac{(n-1)x}{2n\beta} - \frac{\tilde{\eta}_J(\mathcal{L})t^j}{n}) + \sum_{J\in\tilde{N}_I(\mathcal{L})\setminus\{I\}}(\frac{\alpha}{\beta} - \frac{(n-1)x}{2n\beta} + \frac{(n-\tilde{\eta}_J(\mathcal{L}))t^j}{n})]
\end{aligned}
$$

Country i's optimal tariff under MFN is given by:

$$
t^i(\mathcal{L}) = \frac{(n^2-n-1)x_J - (n+1)\sum_{K\in N\setminus\{I\}\cup\{J\}} x_K + (4n^2+2)\beta\sum_{K\in N\setminus\tilde{N}_I(\mathcal{L})\setminus\{J\}} t_K^i(\mathcal{L})}{(4n^2-4n-2)\beta}
$$

When all countries are symmetric we can calculate a country's optimal tariff by:

$$t^i(\mathcal{L}) = \frac{x}{2\beta(2n\tilde{\eta}_I(\mathcal{L}) - n + \tilde{\eta}_I(\mathcal{L}))}, \tag{28}$$

for all J and $K \in N \backslash \tilde{N}_I(\mathcal{L})$. This shows that with an increasing number of PTAs the MFN-tariffs on the other countries decreases and therefore in this model PTAs enhance the incentives for multilateral liberalization.
First we want to investigate the change in welfare from an additional PTA with respect to the number of PTAs of the foreign market. It can be shown that:

Lemma 5.1. *Under MFN country I's welfare change from an additional PTA with country J is lower, the more PTAs country J already has, and higher the more PTAs country I belongs to. Therefore a country's incentive to sign a PTA with country J is highest if the foreign market does not belong to any PTA and country I has a PTA with every other country.*

A country's incentive to sign a PTA depends on the linking structure of the foreign market. The more linked a foreign market is, the less market I wants to form a trade agreement with the foreign market. This result strengthens the result of Proposition 3.2 which states that a global link with a PTA between one pair of countries is stable since the country without any PTAs has no incentives to sign an additional trade agreement since the other two markets are already linked bilaterally. The complete proof of the result is shown in the appendix.
To get an idea of the actual value of the welfare change in country I we use a simulation. In the following we set $n = 100$, $x = 200$ and $\beta = 1$. Figure 3 depicts the relation between the number of PTAs of country J on the welfare change of country I from an additional PTA with country J for two different values of $\tilde{N}_I$.
It is shown that the welfarechange is decreasing with the number of PTAs of country J. In Figure 4 we can observe the influence of country I's linking structure on its own welfare change from a PTA. The more linked country I already is, the higher the welfare gains from an additional PTA.
Now we seek for results on stability and investigate whether global free trade can still be achieved as a stable trading structure even if we allow an arbitrary number of countries.

Proposition 5.1. *Global Free trade is a stable state. There exists no stable network without MFN.*

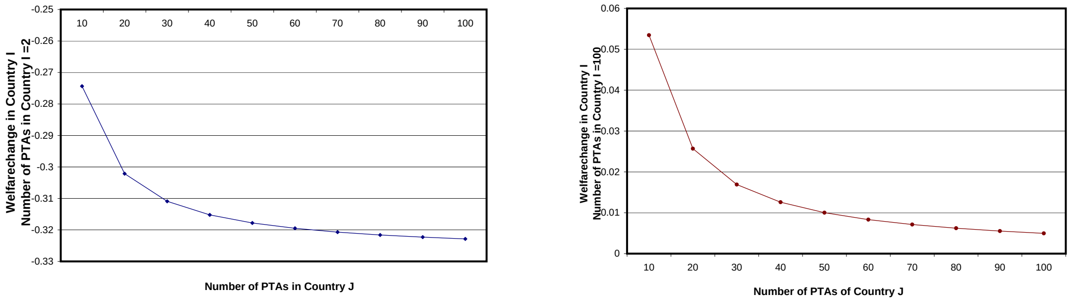

Figure 3: The effect of an increasing number of PTAs of the foreign market on country I's welfare

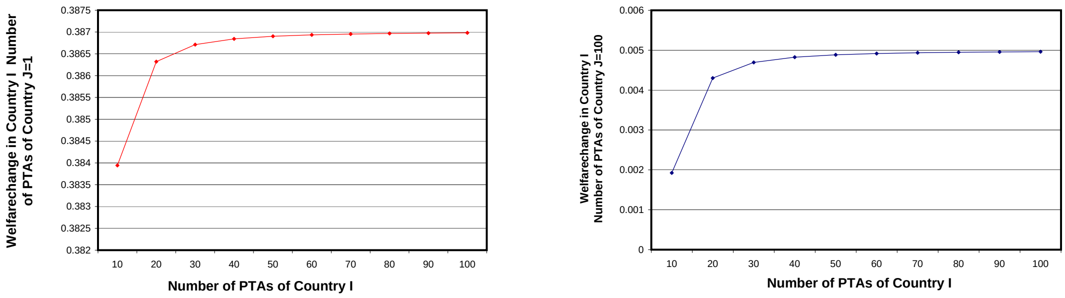

Figure 4: The effect of an increasing number of PTAs of the home market on its welfare

The second result shows that we cannot have stability without the MFN-clause, since without MFN all players form as many bilateral links as possible. The MFN-clause has positive welfare effects on member countries and GATT stabilizes the trading network.[6]
The proof is shown in the appendix and it proceeds in the way that I first show that no player has an incentive to delete any of his bilateral links and later that no player has an incentive to delete the global link. For the second part I show that under each trading network without MFN each country improves by forming an additional bilateral link and

[6]There is another strand of literature that concentrates on the question of how MFN effects the prospects of multilateral tariff cooperation and whether MFN faciliates multilateral cooperation (see e.g. Bagwell and Staiger (1998) and Saggi (2009)).

therefore the linking structure converges to the complete network. Since with part one we have already proved that the complete network is not stable and this completes the proof.

6 Conclusion

One characteristic of global trade is the occurrence of PTAs. There is hardly any country that is not involved in at least one PTA and the tendency is increasing that the world of trade agreements gets more and more complex.
This paper helps to understand why such trading structures form and assumes that trading structure as endogenously arising. We allow players to form bilateral and multilateral links and investigate whether bilateralism facilitates or hinders global free trade. One main result is that without MFN we cannot achieve stability.
We show that global free trade is a stable state and that multilateral tariffs on third countries decrease with the number of PTAs that a country forms. We allow for heterogenous players and show that free trade can still be stable, whenever countries' income level is relatively similar.
When countries can negotiate tariffs to multilaterally reduce tariffs on each other after the network is in place we find that whenever each country has the same number of trade agreements, the unique efficient bargaining solution is zero tariffs and free trade is welfare improving for all countries. Whenever countries are linked asymmetrically such that some countries have more PTAs than others, the less bilaterally linked countries free trade is not a bargaining solution and less linked countries can profit more from multilateral tariff reduction.
In section 4 we assumed that network structure is exogenously given and calculated Nash-bargaining tariffs under different trading regimes. Calvo-Armengol (2003) introduced bargaining during the formation of network structures and characterized the stable and efficient bargaining structures. For further research we suggest investigating the nature of stable bargaining networks in an international trade framework.

References

[1] Bagwell, K., and Staiger, R. W. (1998), Regionalism and Multilateral Tariff Cooperation, in J. Piggott and A. Woodland, eds., International Trade Policy and the Pacific Rim (Macmillan, London, 1998).

[2] Bhagwati, J., and Panagariya, A. (1996), The Theory of Preferential Trade Agreements: Historical Evolution and Current Trends, American Economic Review Papers and Proceedings, 86, 2, 82-87.

[3] Calvo-Armengol, A. (2003), Stable and Efficient Bargaining Networks, Review of Economic Design, 7, 411-428.

[4] Durieu, J., Haller, H. and Solal, P. (2005), Interaction on Hypergraphs, Mimeo, Sonderforschungsbereich 504, University of Mannheim.

[5] Freund, C. (2000), Multilateralism and the Endogenous Formation of Preferential Trade Agreements, Journal of International Economics, 52, 359-376.

[6] Furusawa, T., and Konishi, H. (2007), Free Trade Networks, Journal of International Economics, 72, 310-335.

[7] Goyal, S., and Joshi, S. (2006), Bilateralism and Free Trade, International Economic Review, 47, 3, 749-778.

[8] Jackson, M.O., and Wolinsky, A. (1996), A Strategic Model of Social and Economic Networks, Journal of Economic Theory, 71, 44-74.

[9] Krishna, P. (1998), Regionalism and Multilateralism: A Political Economy Approach, Quarterly Journal of Economics, 113, 1, 227-251.

[10] Krugman, P. (1992), Is Bilateralism Bad?, in E. Helpman and A. Razin, eds., International Trade and Trade Policy (Cambridge, MA: MIT Press, 1992).

[11] Panagariya, A. (2000), Preferential Trade Liberalization: The Traditional Theory and New Developments, Journal of Economic Literature, 38, 2, 287-331.

[12] Saggi, K. (2009), The MFN Clause, Welfare, and Multilateral Cooperation between Countries of Unequal Size, Journal of Development Economics, 88, 1, 132-143.

[13] van den Nouweland, A., Borm, P., Tijs, S. (1992), Allocation Rules for Hypergraph Communication Situations, International Journal of Game Theory, 20, 255-268.

Appendix

Proof of Proposition 3.4

$$
\begin{aligned}
&Y_I(\mathcal{L}^{G\cup N}) - Y_I(\mathcal{L}^{G\cup N}\backslash\{I,J\})\\
=&\frac{x_I}{2}(\frac{\alpha}{\beta} - \frac{x_I+x_K}{6\beta}) + \frac{x_I}{2}(\frac{\alpha}{\beta} - \frac{x_I+x_J}{6\beta}) + \frac{1}{2\beta}(\frac{x_J+x_K}{6})^2 + \frac{1}{2\beta}(\frac{x_I+x_K}{6})^2 + \frac{1}{2\beta}(\frac{x_J+x_I}{6})^2\\
-&\frac{1}{2\beta}(\frac{x_J+x_K}{6} - \frac{(5x_J-4x_K)}{66})^2 - \frac{1}{2\beta}(\frac{x_I+x_K}{6} + \frac{2(5x_I-4x_K)}{66})^2 - \frac{1}{2\beta}(\frac{x_J+x_I}{6})^2\\
-&\frac{x_I}{2}(\frac{\alpha}{\beta} - \frac{x_I+x_J}{6\beta}) - \frac{x_I}{2}(\frac{\alpha}{\beta} - \frac{x_I+x_K}{6\beta} + \frac{5x_I-4x_K}{66\beta} - \frac{3(5x_J-4x_K)}{66\beta})\\
-&\frac{5x_K-4x_Z}{22\beta}(\frac{x_J}{3} - \frac{x_K}{6} - \frac{3(5x_J-4x_K)}{66} + \frac{(5x_J-4x_K)}{66})\\
=&-\frac{1}{8712}\frac{-340x_I^2+412x_Ix_K-440x_Jx_K+275x_J^2+64x_K^2}{\beta}
\end{aligned}
$$

$$
\begin{aligned}
&Y_I(\mathcal{L}^{G\cup N}) - Y_I(\mathcal{L}^{N})\\
=&\frac{1}{2\beta}(\frac{x_J+x_K}{6})^2 + \frac{1}{2\beta}(\frac{x_I+x_K}{6})^2 + \frac{1}{2\beta}(\frac{x_J+x_I}{6})^2\\
-&\frac{1}{2\beta}(\frac{5x_I+x_K}{16})^2 - \frac{1}{2\beta}(\frac{x_J+x_K}{8})^2 - \frac{1}{2\beta}(\frac{5x_I+x_J}{16})^2\\
-&(\frac{3x_J-x_Kx_J}{16\beta})(\frac{3x_J-x_K}{16}) - (\frac{3x_K-x_J}{16\beta})(\frac{3x_K-x_J}{16}) + \frac{x_I}{2}(\frac{14x_I-5x_K-5x_J}{48\beta})\\
=&\frac{1}{4608}\frac{350x_I^2-202x_Ix_K-97x_K^2-97x_J^2+272x_Jx_K-202x_Ix_J}{\beta}
\end{aligned}
$$

Proof of Proposition 4.2

Under $\mathcal{L} = \mathcal{L}^G \cup \{\{I,J\}\}$ we obtain for player I and player J with $x_I = x_J = x_K$

$$
Y_I(t) - Y_I = Y_J(t) - Y_J = -\frac{1}{139\,392}\frac{-7744t\beta x + 69\,696t^2\beta^2 - 303x^2}{\beta} \tag{29}
$$

Furthermore

$$
Y_K(t) - Y_K = -\frac{1}{17\,424}x\frac{1936t\beta - 1047x}{\beta} \tag{30}
$$

When maximizing the Nash-product we obtain three possible solutions for t^* with $t > 0$.

$$
\begin{aligned}
t_1^* &= \frac{1}{88}\left(\frac{44}{9} + \frac{1}{9}\sqrt{4663}\right)\frac{x}{\beta}\\
t_2^* &= \frac{1}{8}\left(\frac{601}{1815} + \frac{2}{1815}\sqrt{88\,939}\right)\frac{x}{\beta}\\
t_3^* &= \frac{1}{8}\left(\frac{601}{1815} - \frac{2}{1815}\sqrt{88\,939}\right)\frac{x}{\beta}
\end{aligned}
$$

Since t_1^* will result in a negative welfare change for country I and J with $Y_I(t_1^*) - Y_I = Y_J(t_1^*) - Y_J = -\frac{1}{139\,392}\frac{-7744t_1^*\beta x + 69\,696(t_1^*)^2\beta^2 - 303x^2}{\beta} = -3.9164 \times 10^{-7}\frac{x^2}{\beta} < 0$ and t_2^* results in a loss for country K with $Y_K(t_2^*) - Y_K = -\frac{1}{17\,424}x\frac{1936(8.2469\times 10^{-2}\frac{x}{\beta})\beta - 39x}{\beta} = -6.9249 \times 10^{-3}\frac{x^2}{\beta} < 0$ we can exclude t_1^* and t_2^*.
The only possible solution is given by t_3^* with
$Y_I(t_3^*) - Y_I(t_3^*) = -\frac{1}{139\,392}\frac{-303x^2 - 7744x(3.1316\times 10^{-4}\frac{x}{\beta})\beta + 69\,696(3.1316\times 10^{-4}\frac{x}{\beta})^2\beta^2}{\beta} = 2.1911 \times 10^{-3}\frac{x^2}{\beta} > 0$ and $Y_K(t_3^*) - Y_K(t_3^*) = 2.2035 \times 10^{-3}\frac{x^2}{\beta} > 0$. It can easily be verified that country K's benefits more from the multilateral bargaining solution.

Proof of Lemma 5.1

First we calculate the change in welfare under MFN from an additional PTA and then we show that this welfare change is higher, the lower the number of PTAs of country J. Substituting equation (28) in the social welfare components we obtain for country I's welfare:

$$
\begin{aligned}
& Y_I(\mathcal{L}) - Y_I(\mathcal{L}\backslash\{I,J\}) \\
=& \frac{1}{2\beta}(\frac{(n-1)x}{2n} - \frac{\beta t^j(\mathcal{L})(n-\tilde{\eta_J})}{n})^2 + \frac{1}{2\beta}(\frac{(n-1)x}{2n} - \frac{\beta(n-\tilde{\eta_I})t^i(\mathcal{L})}{n})^2 \\
-& \frac{1}{2\beta}(\frac{(n-1)x}{2n} + \frac{\beta t^j(\mathcal{L}\backslash\{I,J\})(\tilde{\eta_J}-1)}{n})^2 - \frac{1}{2\beta}(\frac{(n-1)x}{2n} - \frac{\beta(n-(\tilde{\eta_I}-1))t^i(\mathcal{L}\backslash\{I,J\})}{n})^2 \\
+& \frac{x}{2}(-\frac{(n-1)x}{2n\beta} + \frac{(n-\tilde{\eta_J})t^j(\mathcal{L})}{n} + \frac{(n-1)x}{2n\beta} + \frac{(\tilde{\eta_J}-1)t^j(\mathcal{L}\backslash\{I,J\}}{n}) \\
+& t^i(\mathcal{L})(n-\tilde{\eta_I})(\frac{(n-1)x}{2n} - \frac{(n-2)x}{2n} + \frac{\beta(n-\tilde{\eta_I})t^i(\mathcal{L})}{n} - \beta t^i(\mathcal{L})) \\
-& t^i(\mathcal{L}\backslash\{I,J\})(n-(\tilde{\eta_I}-1))(\frac{(n-1)x}{2n} - \frac{(n-2)x}{2n} + \frac{\beta(n-(\tilde{\eta_I}-1))t^i(\mathcal{L}\backslash\{I,J\})}{n} - \beta t^i(\mathcal{L}\backslash\{I,J\}))
\end{aligned}
$$

we obtain:

$$
\begin{aligned}
& \frac{\partial \triangle Y_I(\mathcal{L})}{\partial \tilde{\eta_J}} \\
=& -\frac{400\,000\,000}{201\,(201(\tilde{\eta_J})-100)^3} - \frac{32\,482\,404\,000\,000}{(8120\,601(\tilde{\eta_J})-4040\,100)^2} \\
+& \frac{10\,000}{201\,(201(\tilde{\eta_J})-301)^3} - \frac{162\,412\,020\,000}{(8120\,601(\tilde{\eta_J})-12\,160\,701)^2} < 0,
\end{aligned}
$$

whenever $\tilde{\eta_J} \geq 2$ and

$$
\frac{\partial \triangle Y_I(\mathcal{L})}{\partial \tilde{\eta_I}} = \frac{2010\,000}{(201(\tilde{\eta_I})-301)^2\,(201(\tilde{\eta_I})-100)} + \frac{2010\,000}{(201(\tilde{\eta_I})-301)\,(201(\tilde{\eta_I})-100)^2} > 0
$$

whenever $\tilde{\eta}_I \geq 2$.

Proof of Proposition 5.1

Condition (i) of Definition 2.2 is trivially satisfied since no additional link can be added. Under free trade we know that tariffs on good i are $t^i* = 0$ for all $I \in N$ such that total welfare reduces to:

$$Y_I(\mathcal{L}^N \cup \mathcal{L}^G) = \frac{n}{2\beta}(\frac{(n-1)x}{2n})^2 + \frac{x}{2}(\frac{\alpha}{\beta} - \frac{(n-1)x}{2n\beta})(n-1)$$

When one player I deletes one of his bilateral links $\{I, J\}$ we will have $t^i* = t^j* = T$ and I's welfare will be:

$$\begin{aligned}Y_I(\mathcal{L}^N \cup \mathcal{L}^G \backslash \{\{I,J\}\}) =& \frac{(n-2)}{2\beta}(\frac{(n-1)x}{2n})^2 + \frac{1}{2\beta}(\frac{(n-1)x}{2n} + \frac{\beta T(n-1)}{n})^2 \\ &+ \frac{1}{2\beta}(\frac{(N-1)x}{2N} - \frac{\beta T}{N})^2 + \frac{x}{2}(\frac{\alpha}{\beta} - \frac{(N-1)x}{2N\beta})(N-2) \\ &+ \frac{x}{2}(\frac{\alpha}{\beta} - \frac{(n-1)x}{2n\beta} - \frac{(n-1)T}{n}) - T(\frac{x}{2n} - \frac{\beta(n-1)T}{n})\end{aligned}$$

Therefore we otain:

$$Y_I(\mathcal{L}^N \cup \mathcal{L}^G) - Y_I(\mathcal{L}^N \cup \mathcal{L}^G \backslash \{\{I,J\}\}) = \frac{1}{8}x^2 \frac{2 - 11n^2 + 6n + 4n^3}{\beta\left(2n^2 - 2n - 1\right)^2 n^2} > 0$$

$\forall n \geq 3$. Therefore no player wants to delete any of his bilateral links. To show that no player wants to delete the global link we calculate:

$$\begin{aligned}&Y_I(\mathcal{L}^N \cup \mathcal{L}^G) - Y_I(\mathcal{L}^N) \\ =&\frac{1}{2\beta}(\frac{(n-1)x}{2n} - \frac{\beta T(n-1)}{n})^2 + \frac{1}{2\beta}(n-1)(\frac{(n-1)x}{2n} - \frac{\beta(n-1)T}{n} + \beta T)^2 \\ &+(n-1)\frac{x}{2}(\frac{\alpha}{\beta} - \frac{(n-1)x}{2n\beta} + \frac{(n-1)T}{n} - T) \\ &+(n-1)T(\frac{x}{2} - \frac{(n-1)x}{2n} + \frac{\beta(n-2)T}{n} - \frac{(n-1)\beta T}{n})\end{aligned}$$

Since

$$Y_I(\mathcal{L}^N \cup \mathcal{L}^G) - Y_I(\mathcal{L}^N) = \frac{1}{2}\frac{n-1}{n}T^2\beta > 0,$$

no player has an incentive to delete the global link. Thus condition (ii) is satisfied and this completes the proof of the first part. Since we have already shown that the complete

network results in the global free trade network, it is enough to show for the second part of Proposition 5.1. that in each arbitrary network without MFN each player will increase his welfare level when he forms more and more bilateral links. Since with the first part the complete network cannot be stable either, this will complete the proof.
Consider any arbitrary network $\mathcal{L}$ with $\eta_J(\mathcal{L})$ and $\eta_I(\mathcal{L})$ and any link $\{I,J\} \in \mathcal{L}$:

$$\Pi_I(\mathcal{L}) - \Pi_I(\mathcal{L}\backslash\{I,J\})$$
$$=\frac{x}{2}(\frac{\alpha}{\beta} - \frac{(\eta_J-1)x}{2\eta_J\beta} + \frac{(\eta_J-2)x}{\eta_J\beta(2\eta_J+2)} - \frac{(\eta_J-1)x}{\eta_J\beta(2\eta_J\beta(2\eta_J+2)}) - \frac{x}{2}(\frac{\alpha}{\beta} - \frac{x}{2\beta}) = \frac{x}{2}(\frac{x}{\beta(2\eta_J+2)})$$
$$CS_I^i(\mathcal{L}) - CS_I^i(\mathcal{L}\backslash\{I,J\})$$
$$=\frac{1}{2\beta}(\frac{(\eta_I-1)x}{2\eta_I} - \frac{(\eta_I-1)x}{\eta_I(2\eta_I+2)})^2 - \frac{1}{2\beta}(\frac{(\eta_I-2)x}{2(\eta_I-1)} - \frac{(\eta_I-2)x}{(\eta_I-1)2\eta_I})^2 = \frac{x^2}{2\beta}(\frac{4\eta_I^2 4\eta_I - 4)}{4(\eta_I+1)^2\eta_I^2})\eta_I$$

For the loss in tariff revenue we obtain:

$$TR_I(\mathcal{L}) - TR_I(\mathcal{L}\backslash\{I,J\})$$
$$=\frac{(\eta_I-1)x}{\beta(2\eta_I+2)}(\frac{x}{2\eta_I} - \frac{x}{\eta_I(2\eta_I+2)}) - \frac{(\eta_I-2)x}{2\beta\eta_I}(\frac{x}{2(\eta_I-1)} - \frac{x}{(\eta_I-1)2\eta_I}) = \frac{x^2}{2\beta}(\frac{(-\eta_I^2+3\eta_I+2)}{2(\eta_I+1)^2\eta_I^2})$$

For the change in consumer surplus from good j we obtain:

$$CS_I^j(\mathcal{L}) - CS_I^j(\mathcal{L}\backslash\{I,J\})$$
$$=\frac{1}{2\beta}(\frac{(\eta_J-1)x}{2\eta_J} - \frac{(\eta_J-2)x}{\eta_J(2\eta_J+2)} + \frac{(\eta_J-1)x}{(2\eta_J+2)\eta_J})^2 - \frac{1}{2\beta}(\frac{x}{2})^2 = \frac{x^2}{2\beta}(\frac{-2\eta_J-1)}{4(\eta_J+1)^2})$$

For the total change in welfare we obtain:

$$Y_I(\mathcal{L}) - Y_I(\mathcal{L}\backslash\{I,J\}) = \frac{x^2}{4\beta(\eta_I+1)^2\eta_I^2}(\eta_I+\eta_I^2) + \frac{x^2}{8\beta(\eta_J+1)^2} > 0$$

Starting from any arbitrary network each player improves by forming a bilateral link. This completes the proof.