The role of liquidity constraints in the response of monetary policy to house prices

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Abstract

We analyse the optimal response of monetary policy to house price movements in a New Keynesian framework. A wealth effect from housing is derived from liquidity constrained consumers. Housing equity withdrawal allows them to convert an increase in housing value into consumption and we show that monetary policy should react to house price movements due to their effect on consumption by constrained agents. We also allow the share of liquidity constrained consumers to vary with house prices. With time-varying liquidity constraints the optimal weights on expected inflation, the output gap and house prices in the optimal interest rate rule are affected and vary over time.

Keywords: Optimal monetary policy, liquidity constraints, house prices

JEL Classification: E52, E58, E44

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1 Introduction

Empirically there is a strong wealth effect on consumption spending. Conventional wisdom is that the marginal propensity to consume out of total net wealth is 3-5 cents per dollar (Altissimo et al., 2005). Furthermore, various studies find a stronger wealth effect of housing than of stock wealth for the U.S. (e.g. Davis and Palumbo, 2001; Case, Quigley and Shiller, 2001; Carroll, Otsuka and Slacalek, 2006). The difference may be explained by the more even distribution of housing wealth than of stock wealth across households, with a owner-occupier rate of nearly 70% in the U.S. and housing representing a larger part of total household wealth than equities (Illing and Klüh, 2005).

From a theoretical perspective it is not straightforward to justify the wealth effect from housing[1]. Consider a representative infinitely lived agent who owns the house in which she lives. An exogenous rise in house prices at a constant interest rate just compensates for the higher present value of expected future imputed rents. In this case the change in net wealth is zero and shouldn't have an effect on consumption. Even if the agent moved to a cheaper place, if housing services in the future improved, if higher collateral value resulted in saving on interest payments, or if the agent owned a high-value house but lived in a cheap one, there needn't be a wealth effect. Since the agent lives forever any change in net wealth is spread out into the infinite future and shouldn't affect consumption today. However, if the agent is liquidity constrained an increase in the value of the house can serve as additional collateral to borrow against. Housing value serves as a means to bring forward consumption and helps to smooth it over time, even though net worth hasn't changed[2]. In this case an increase in house prices can

[1] Carroll (2004) provides a discussion of this issue.

[2] A wealth effect of housing could also arise with finitely lived agents who don't care about the utility of their descendents. However, the focus here is one the role of liquidity constraints.

lead to an effect on consumption. Some authors argue theoretically and empirically that the process of financial liberalisation since the mid 1980s has increased the proportion of housing collateral that can be used to borrow against (e.g. Attanasio and Weber, 1994; Lustig and van Nieuwerburgh, 2006; Muellbauer et al., 1990; Ortalo-Magné and Rady, 2006). Others stress the role of rising house prices for a given level of financial liberalisation (e.g. Campbell and Cocco, 2007; Carroll, 2004). In the long run the fraction of liquidity constrained homeowners should decrease as financial innovation and liberalisation proceed and increase e.g. the loan-to-value ratio. In the short run the fraction of liquidity constrained agents varies because the possibility to smooth consumption depends on the level of house prices for given financial instruments. A sufficient increase in house prices is necessary for homeowners to benefit from the possibility of housing equity withdrawal. Housing equity withdrawal is the difference between net lending secured on housing and households' gross investment in housing (Bank of England). This way homeowners can increase their mortgage, i.e. cash flow, by a fraction of the increase in the value of their house[3]. Therefore it is clear that the fraction of liquidity constrained agents is not constant over time.

Housing equity withdrawal in the US and the UK has indeed increased considerably in the early 2000s at the same time as house prices increased as documented in figures (1) and (2). The simple correlation coefficient of the two series for the US is 0.83, while the one for the UK is smaller at 0.35.

Furthermore, Hurst and Stafford (2004) have shown that households do indeed use housing equity to smooth consumption in the face of an adverse shock such as unemployment[4]. For an economically significant effect on consumption a sufficiently large fraction of households must be homeowners and liquidity constrained. Figures (3) and (4) show the distribution of liquid

[3]For the construction of housing equity withdrawal from the data, see Greenspan and Kennedy (2005, 2007).

[4]Another use of housing equity would be to reoptimise the financial portfolio and not to spend it on consumption.

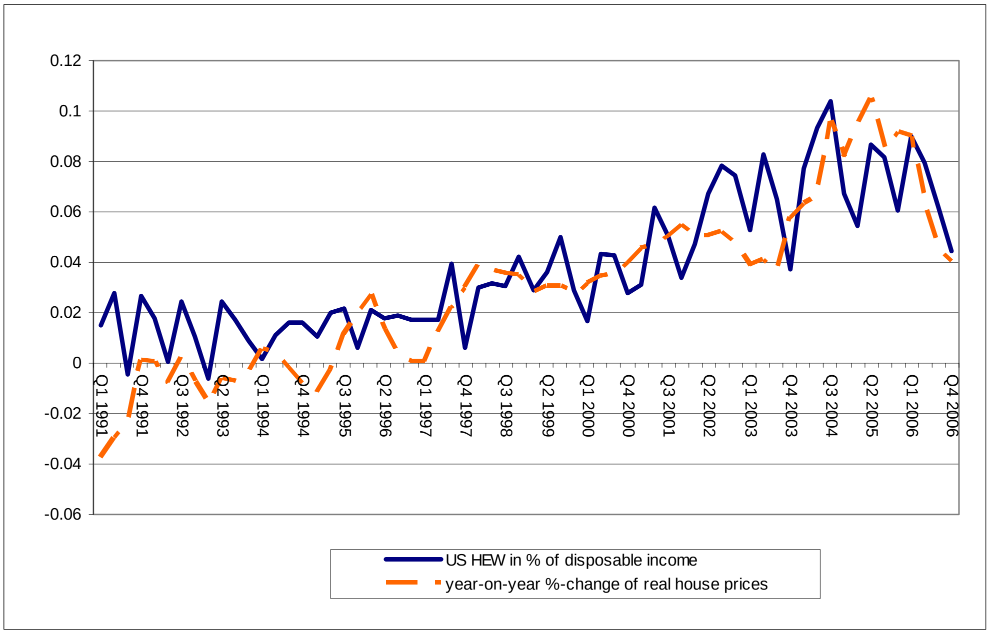

Figure 1: Housing equity withdrawal in % of disposable income (solid line) and the year-on-year real house price change (dashed line) in the US. House prices deflated by the CPI. Source: Greenspan and Kennedy (2005, 2007)

asset and income, respectively, across U.S. homeowners in 2003. Clearly, a non-negligible share of homeowners have liquid assets of at most \$1000 and earn at most \$30000 per year[5].

The objective of this paper is to derive the implications of time-varying liquidity constraints for the optimal conduct of monetary policy. In the long run financial innovation should reduce the volatility of consumption and output through an increase in the fraction of consumption smoothers in the economy. However, in the short run house prices are volatile and affect the capacity of constrained households to borrow and thereby smooth

[5] Of course, what also matters is the history of assets and income. The percentage of homeowners with liquid assets of at most \$1000 and an annual income of at most \$30000 is 0.12 in the sample. For cut-off values of \$6200 for liquid assets and \$58800 for income as chosen by Hurst and Stafford (2004) the number is 0.35.

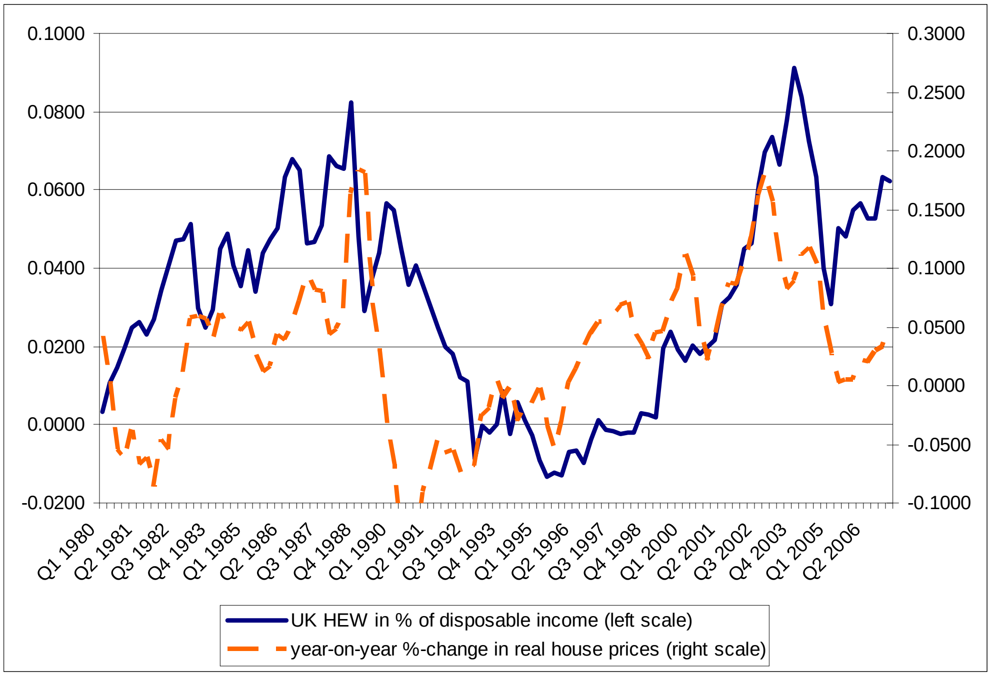

Figure 2: Housing equity withdrawal in % of disposable income (solid line) and the year-on-year real house price change (dashed line) in the UK. House prices deflated by the CPI. Source: Datastream, own calculations.

consumption. Rising house prices allow for higher equity withdrawal boosting consumption, while falling house prices may make debtor households bankrupt or at least liquidity constrained depressing consumption. The contribution of this paper is to take account of the fact that higher house prices temporarily reduce the fraction of constrained households, who become consumption smoothers, while falling house prices temporarily increase it. The question asked is how monetary policy should react to house prices and the corresponding time-varying liquidity constraints.

A wealth effect from housing is derived by assuming that young homeowners are liquidity constrained in the sense that they have high permanent income relative to current income as it is typical for the life-cycle pattern of

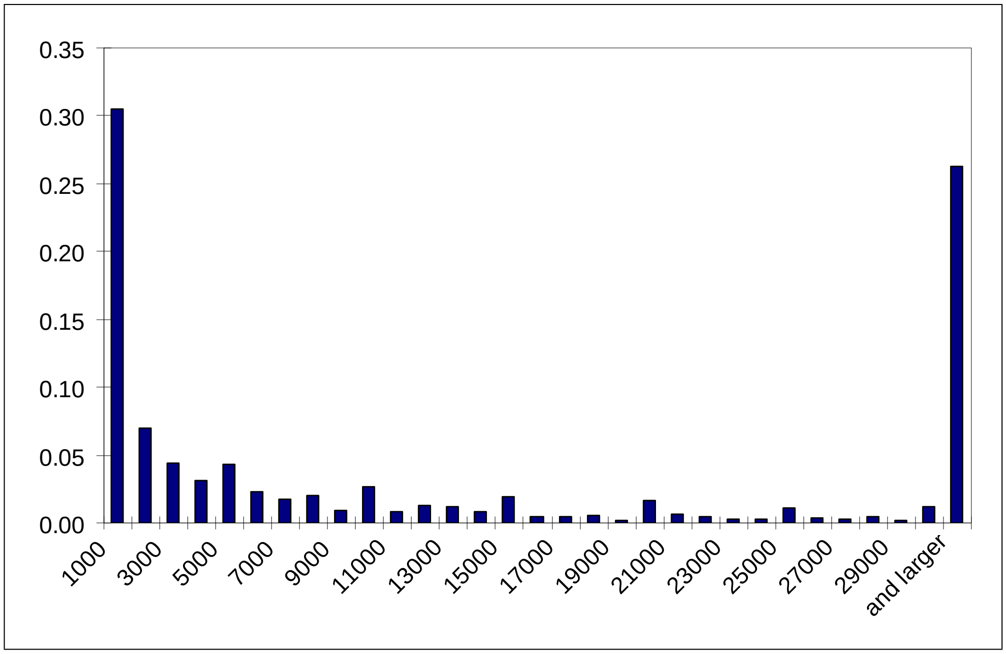

Figure 3: Distribution of liquid assets in 2003 $ across U.S. homeowners. Liquid assets are the sum of stocks, checking and savings accounts, money market funds, certificate of deposits, government savings bonds, treasury bills, bond funds and life insurances. Source: PSID, own calculations.

income. To the extent that they are owner-occupiers a rise in house prices enables them to extract the extra value and increase their consumption towards the optimal level as implied by the permanent income hypothesis. This way house prices increase aggregate demand and affect the output gap and inflation.

Our main results are that monetary policy should react to house price movements due to their effect on consumption by constrained agents. Moreover, with time-varying liquidity constraints, the optimal weights on expected inflation, the output gap and house price changes are affected. It is one of the main contributions of the paper to work out explicitly this mechanism. To the best of our knowledge this has not been looked at yet. Our results are

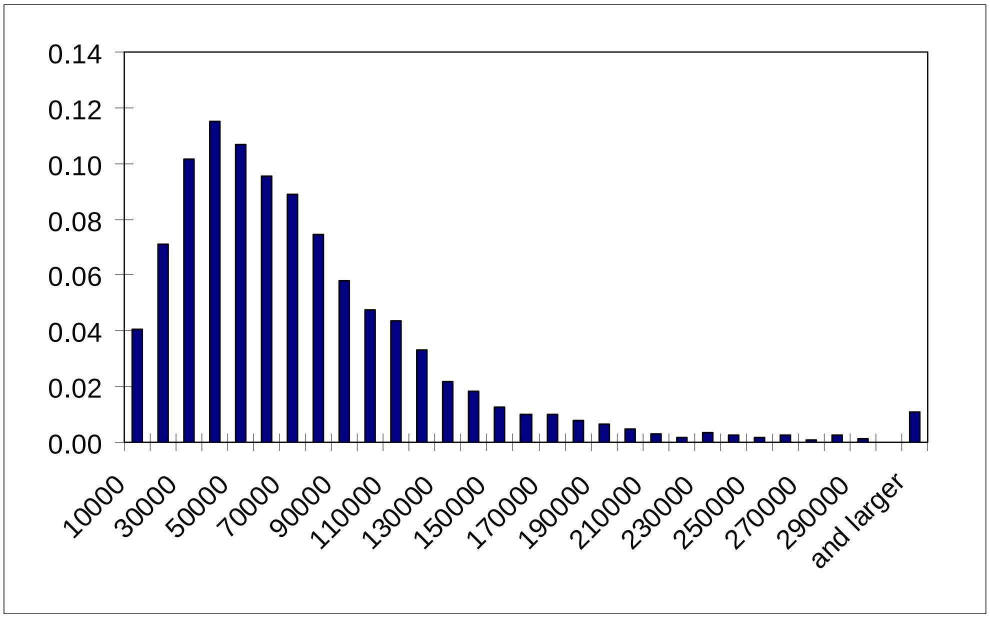

Figure 4: Distribution of annual income in 2002 $ across U.S. homeowners. Annual income is reported income in 2003 about tax year 2002. Source: PSID, own calculations.

of interest because they show that it is not only the house prices per se that matter but also their interaction with liquidity constraints and the associated effect on the weight on expected inflation and output in the optimal interest rate rule. This gives additional information to the policy maker about the strength of the optimal interest rate response to house prices. The optimal interest rate response crucially depends on the sensitivities of a change in the share of constrained agents with respect to house prices, expected inflation, the output gap and the interest rate.

The paper is structured as follows. Section 2 relates the paper to the literature. Section 3 sets up a life-cycle model of consumption and derives an IS curve with liquidity constraints. Section 4 derives the optimal monetary policy in a New Keynesian framework and a wealth effect from housing.

Section 5 analyses the optimal interest rate response when there are time-varying liquidity constraints. Section 6 discusses some robustness checks of the model and section 7 concludes.

2 Related literature

The present paper relates to a vast amount of papers analysing the relationship of monetary policy and asset prices. Typically they don't distinguish between different types of assets. Broadly speaking there are two main questions in the context of the optimal response of monetary policy to asset prices. The first is how should monetary policy react to asset prices over and above a conventional wealth effect from asset prices, especially bubbles. Two approaches can be found in the literature. One looks at demand effects from asset prices (Bernanke and Gertler, 1999, 2001; Cecchetti et al., 2000; Greenspan, 1999, 2004; Gruen, Plumb and Stone, 2005; Filardo, 2004; Kent and Lowe, 1997; Kontonikas and Montagnoli, 2006). In this approach a developing and consequently bursting bubble might lead to household and firm bankruptcies, thereby affecting the output gap and inflation. A sufficiently forward looking central bank might want to take these repercussions into account. This argument suggests adjusting the central bank's forecast horizon for expected inflation to include periods of possible asset bubble bursts. There is disagreement, however, about how to identify a bubble with certainty, about the timing, direction and strength of the warranted interest rate response. Also a pre-emptive restrictive monetary policy at the expense of current output might be hard to justify to the public.

Another approach looks at the supply effects of asset prices[6]. Bean (2004) sets up a model drawing on results from a study by Borio and Lowe (2002) where asset prices are correlated with the build-up of debt, which is used

[6] At the intersection of demand and supply effects is a paper by Smets (1997), who focuses on the informational content in asset prices for expected inflation.

to finance capital accumulation. An asset bubble crash leads to a credit crunch, which affects total factor productivity due to the lack of funds from intermediaries. The output gap suddenly widens with adverse effects on inflation. One way in which monetary policy can affect the probability of a credit crunch is to deter the debt build-up. In the model this can be achieved by a policy under committment where the central bank affects expectations of future output gaps. A higher interest rate leads to a lower expected future output gap, which in turn means slower capital accumulation today. Correspondingly, this limits the build-up of debt. Thus, an interest rate response over and above the one warranted by expected inflation and the current output gap is optimal. Bordo and Jeanne (2002) argue in a similar way that raising the interest rate today to bring down debt accumulation can be considered an insurance against negative future supply shocks when asset prices crash. In their model the real interest rate directly affects firm's demand for debt.

The second question is about the mechanism of the wealth effect, by which asset prices (stock or house prices) affect consumption and the appropriate policy reaction. When looking at the channel from asset prices to consumption it is important to distinguish different classes of assets. House and stock prices can have different effects on consumption, e.g. stock-ownership is much less widely spread than home-ownership in the U.S. Then again house price increases do not necessarily always represent increases in net wealth. Yet many papers commonly just append a variable for asset prices to the IS equation or directly to the interest rate rule. In contrast in this paper we focus on the role of house prices and explicitly derive a wealth effect from liquidity constrained consumers. We can show that the precise channel by which house prices affect consumption is important because the weights on inflation, output and house prices in the interest rate rule are affected. Our paper relates most closely to the papers by Iacoviello (2004, 2005) and Monacelli (2006) who also derive a wealth effect from asset prices from a microfounded model. Some home-owners are assumed to be impatient while others are patient. This determines who becomes borrower or lender. In Iacoviello (2005) borrowing capacity is limited by the expected future value of the house such that a house price increase results in higher consumption by borrowers. He analyses optimal monetary policy using a postulated interest rate rule, instead of deriving it from a loss function. In Monacelli (2006) borrowers are constrained by the value of their general assets. He analyses to which extent it might be optimal for a central bank, which maximises the weighted utility of borrowers and savers, to deviate from price stability when inflation erodes the real value of debt and relaxes borrowing constraints. In our paper liquidity constrained consumers are essentially defined by age and the value of their home, which is intuitive and corresponds well with the life-cycle pattern of income. It allows to let the share of constrained agents vary over time. In contrast, when constraints are defined by a fixed rate of time preference this is not possible. Moreover, we explicitly exclude the possibility of precautionary saving to be able to uniquely determine when liquidity constraints are binding and when not. Furthermore, we derive an interest rate rule from loss minimisation by the central bank.

Time-varying liquidity constraints have been considered e.g. by Deaton (1991), Ludvigson (1999) and Pesaran and Smith (1995). Commonly, constraints are a complex function of past income and net asset accumulation. This makes most models with time-varying liquidity constraints intractable. Therefore we aimed at finding a way to make liquidity constraints independent from past values of income and assets and only conditional on the actual value of the home, albeit at the expense of a more stylised setup.

To sum up, the contributions of our paper are first to derive an explicit wealth effect from house prices on consumption via relaxing liquidity constraints, and second to analyse optimal monetary policy when liquidity constraints vary over time with house prices.

3 A life-cycle model of consumption

Since the aim of the analysis is to evaluate monetary policy with time-varying liquidity constraints in a standard New Keynesian setup we first derive the IS curve from individual utility optimisation taking into account that a fraction of households is liquidity constrained and consumes out of current income and liquid assets. Together with a Phillips curve and the central bank's loss function we derive the optimal monetary policy under constant and under time-varying liquidity constraints.

3.1 Derivation of the IS curve

Typically, the IS curve in the New Keynesian model is derived from household utility maximisation using a standard utility function such as the CES utility. In this model we use a quadratic utility function because we want to separate precautionary saving from liquidity constraints as a source for the high correlation of current income and liquid assets with current consumption for the constrained agents. The marginal utility of a quadratic utility function is linear, which implies that the expected marginal utility of consumption equals the marginal utility of expected consumption. An increase in uncertainty about future consumption doesn't affect marginal utility, i.e. certainty equivalence holds. Therefore there is no effect on current consumption and saving[7]. The precautionary saving motive may result in consumption that follows current income closely and is observationally similar to the effect of liquidity constraints (Carroll, 1997). An agent may save little and consumption might follow current income closely either because the agent is liquidity constrained, or because the agent is not liquidity constrained, and would want to borrow as much as necessary to attain a smooth consumption path, but the percautionary saving motive counteracts the desire to borrow just so

[7]With quadratic utility, however, possibly binding liquidity constraints in the future may affect current consumption (see Romer's textbook, 2001). This will be ruled out by assumption.

that consumption and current income are closely correlated.

Moreover, to have borrowing in equilibrium some agents must be constrained and others not. Therefore we build a model with three types of agents: young, middle-aged and old. In every period all types coexist and all are owner-occupiers of their house.

The main challenge of the model is to avoid having to account for the history of assets and income in determining when an agent is constrained. To this end it is assumed that all agents face the same hump-shaped profile of life-time income and only the young agents can be constrained. In each period the three type of agents differ in the shares of aggregate income they receive as well as in their share of total consumption. Thus each agent's income is a fixed share of aggregate income and so is her consumption.

3.1.1 Unconstrained consumers

Without any borrowing constraints a young agent in period t maximises her life-time utility subject to her life-time budget constraint.

$$\max_{\{C_{1t},C_{2t+1},C_{3t+2}\}}$$

$$\begin{aligned} U\left(C_{1t}, C_{2t+1}, C_{3t+2}\right) = \left(C_{1t} - aC_{1t}^2\right) + \beta E_t\left(C_{2t+1} - aC_{2t+1}^2\right) \\ +\beta^2 E_t\left(C_{3t+2} - aC_{3t+2}^2\right) \end{aligned} \tag{1}$$

s.t.

$$\begin{aligned} C_{1t} + \frac{1+\pi_{t+1}}{1+i_t}C_{2t+1} + \frac{1+\pi_{t+1}}{1+i_t}\frac{1+\pi_{t+2}}{1+i_{t+1}}C_{3t+2} = \\ Y_{1t} + \frac{1+\pi_{t+1}}{1+i_t}Y_{2t+1} + \frac{1+\pi_{t+1}}{1+i_t}\frac{1+\pi_{t+2}}{1+i_{t+1}}Y_{3t+2} \end{aligned} \tag{2}$$

where $a > 0$, C_{jt} and Y_{jt} are consumption and income, respectively, of agent j in period t, and $j = \{1, 2, 3\}$ denotes young, middle-aged and old agents. a determines the curvature of the utility function and β is the discount factor.

i_t is the nominal interest rate during period t and π_{t+1} is the inflation rate from period t to $t+1$. The first-order conditions with respect to C_{1t}, C_{2t+1}, C_{3t+2} are

$$
\begin{aligned}
1 - 2aC_{1t} - \mu &= 0 && (3)\\
\beta E_t \left(1 - 2aC_{2t+1}\right) - \frac{1+\pi_{t+1}}{1+i_t}\mu &= 0 && (4)\\
\beta^2 E_t \left(1 - 2aC_{3t+2}\right) - \frac{1+\pi_{t+1}}{1+i_t}\frac{1+\pi_{t+2}}{1+i_{t+1}}\mu &= 0 && (5)
\end{aligned}
$$

with μ being the Lagrange multiplier. They can be written more compactly in form of two Euler equations for the two adjacent periods:

$$
\begin{aligned}
C_{1t} &= -\beta E_t \left(\frac{1+i_t}{1+\pi_{t+1}}\right)\left(\frac{1}{2a} - C_{2t+1}\right) + \frac{1}{2a} && (6)\\
C_{2t+1} &= -\beta E_{t+1} \left(\frac{1+i_{t+1}}{1+\pi_{t+2}}\right)\left(\frac{1}{2a} - C_{3t+2}\right) + \frac{1}{2a} && (7)
\end{aligned}
$$

Note that in the special case of a constant real interest rate of zero and a discount factor of one optimal consumption is equal across the three periods. In the general case, log-linearisation of (6) and (7) results in the following consumption equations

$$
\begin{aligned}
c_{1t} &= \phi_1 E_t c_{2t+1} - \phi_2 \left(i_t - E_t \pi_{t+1}\right) && (8)\\
c_{2t+1} &= \phi_3 E_{t+1} c_{3t+2} - \phi_4 \left(i_{t+1} - E_{t+1} \pi_{t+2}\right) && (9)
\end{aligned}
$$

From here on lower case letters denote percentage deviations from trend. ϕ_1, ϕ_2, ϕ_3, ϕ_4 are positive linearisation constants[8]. Given the finite lives of agents one needs to specify what happens to housing wealth at the end of the

[8] $\phi_1 = \frac{\beta\left(\frac{1+i_0}{1+\pi_0}\right)C_{20}}{C_{10}}$, $\phi_2 = \frac{\beta\left(\frac{1}{2a} - C_{20}\right)\frac{1+i_0}{1+\pi_0}}{C_{10}}$, $\phi_3 = \frac{\beta\left(\frac{1+i_0}{1+\pi_0}\right)C_{30}}{C_{20}}$, $\phi_4 = \frac{\beta\left(\frac{1}{2a} - C_{20}\right)\frac{1+i_0}{1+\pi_0}}{C_{20}}$, $U'_{C_{j,t+j-1}} = 1 - 2aC_{j,t+j-1} > 0$

third period of an agent's life. If there were no bequests a house price rise would have an effect on consumption of the old. Since they have only one last period to live they would consume all their remaining housing wealth. Since the focus of the paper is on the role of liquidity constraints as a housing wealth channel, we shut down the wealth effect from finite lives by implicitly assuming that agents care for their descendants and bequeath their total remaining housing wealth at the end of the third period to the middle-aged agents. This way housing wealth always either exactly compensates for future imputed rents or is spread into the infinite future such that the net change in housing wealth is always zero.

3.1.2 Who is constrained and why?

To work out the role of liquidity constraints in the transmission mechanism from house prices to consumption as simply as possible, it is assumed that only the young agents can be constrained. Japelli (1990) reports not having a credit history or the age of the loan applicant as the single most frequent reason given by lenders when they rejected loan applications. Constrained young agents just consume their current income plus liquid assets.

$$c_{1t}^{c} = \psi_1 y_{1t} + \psi_2 b_t \tag{10}$$

where ψ_1, ψ_2 are positive linearisation constants[9]. Suppose that liquid assets consist only of housing equity withdrawal, which in turn depends on the house price change q_t

$$b_t = bq_t \tag{11}$$

where b measures the extent to which an increase in house prices can be cashed in.

In the model the young are constrained if desired consumption according to utility optimisation and consumption smoothing c_{1t}^{u} is larger than current

[9] $\psi_1 = \frac{Y_{10}}{C_{10}} < 1, \psi_2 = \frac{B_0}{C_{10}} < 1$

income y_{1t} and liquid assets b_t, "cash-on-hand".

$$c_{1t}^{u} > \psi_1 y_{1t} + \psi_2 b q_t \tag{12}$$

It is in addition assumed that the middle-aged and the old are always unconstrained. Typically, life-time income is hump-shaped (Attanasio and Browning, 1995; Campbell and Cocco, 2007; Carroll, 1997; Gourinchas and Parker, 2002) and consumption smoothing implies borrowing from the middle-aged when young and paying off the debt to the old in the following period[10]. However, future income of young agents is not pledgeable, unless they use the value of their house as collateral.

3.1.3 Time-varying liquidity constraints

As explained in the introduction, the capacity of homeowners to withdraw equity from their houses varies over time as house prices vary. Therefore the share of constrained agents in the economy should vary too. Typically in existing models of monetary policy and house prices this aspect is not taken into account and the share of constrained agents is fixed (e.g. Iacoviello, 2005). We relax this assumption by making the share of constrained agents a function of the house price. While the total amount of income going to the young is fixed, we assume that the income going to an individual young agent k, denoted by y_{1t}^{k}, is distributed over all young agents according to some distribution function $h\left(y_{1t}^{k}\right)$, which is illustrated in figure 5.

For young agents with income below y_{1t}^{c} and a given amount of housing equity withdrawal liquid assets are insufficient to cover desired consumption and they are constrained. For young agents with income above y_{1t}^{c} and a given amount of housing equity withdrawal liquid asset are enough to cover desired consumption and they are unconstrained. y_{1t}^{c} is the income of a young agent that just makes her unconstrained, since her liquid assets just cover her

[10] The ability to borrow in equilibrium is the reason to have three types of agents.

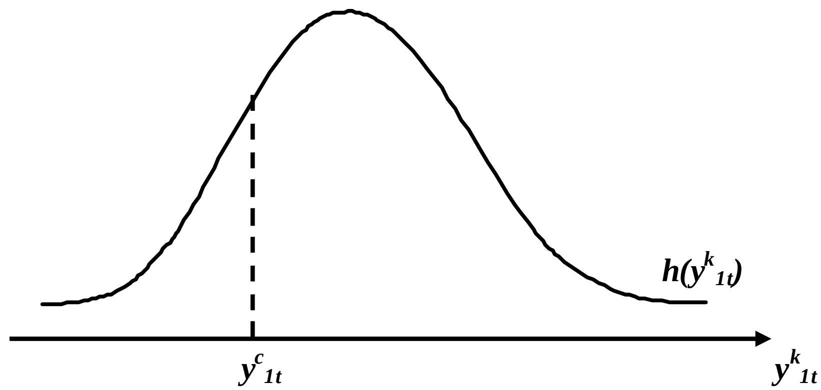

Figure 5: Distribution function of income going to the young agents

desired optimal consumption, $c_{1t}^u = \psi_1 y_{1t}^c + \psi_2 b q_t$, which can be rearranged and substituted in to

$$y_{1t}^c = \frac{\phi_1}{\psi_1} E_t c_{2t+1} - \frac{\phi_2}{\psi_1} \left(i_t - E_t \pi_{t+1} \right) - \frac{\psi_2 b}{\psi_1} q_t \tag{13}$$

The proportion of constrained agents is the share of young agents with income below that critical level.

$$\alpha_t = \int_0^{y_{1t}^c} h\left(y_{1t}^k\right) dy_{1t}^k = F\left(\frac{\phi_1}{\psi_1} E_t c_{2t+1} - \frac{\phi_2}{\psi_1} \left(i_t - E_t \pi_{t+1} \right) - \frac{\psi_2 b}{\psi_1} q_t \right) \tag{14}$$

The share of constrained agents depends on expected future consumption, the real interest rate and real house prices and not the entire history of income and assets. This is by construction to keep the model tractable. When expected future consumption rises, more agents are constrained ceteris paribus since optimal desired consumption rises. Similarly, when the nominal interest rate falls or expected inflation rises, the real interest rate falls ceteris paribus and optimal desired consumption increases making more agents constrained. Finally, note that the proportion of constrained agents falls ceteris paribus with higher house prices. This is because higher house prices allow to withdraw equity from the house, which can be used to finance consumption.

Note that the house price q_t possibly also depends on the interest rate i_t.

$$q_t = q_t\left(i_t\right) \tag{15}$$

There are, however, arguments for why monetary policy should not expect to be able to influence asset prices via interest rate changes in a boom phase. Even with higher interest rates expectations might be sufficiently optimistic to overcompensate the dampening effect of higher interest rates. As a start we will take house prices to be exogenous, while later on relaxing that assumption.

3.1.4 Aggregation and equilibrium

Aggregate consumption c_t is the sum of the weighted consumption of the young, the consumption of the middle-aged and the old.

$$c_t = \left(1-\alpha_t\right) c_{1t}^u + \alpha_t c_{1t}^c + c_{2t} + c_{3t} \tag{16}$$

As in the standard model of aggregate consumption we use the Euler equations (6) and (7) to determine each agent's consumption at t and aggregate, which yields

$$\begin{aligned} c_t \;=\; & \left(1-\alpha_t\right)\left[\phi_1 E_t c_{2t+1} - \phi_2\left(i_t - E_t \pi_{t+1}\right)\right] + \alpha_t\left[\psi_1 y_{1t} + \psi_2 b q_t\right] \\ & +\phi_3 E_t c_{3t+1} - \phi_4\left(i_t - E_t\pi_{t+1}\right) \\ & +\phi_5 E_t c_{2t+1} - \phi_6\left(i_t - E_t\pi_{t+1}\right) \end{aligned} \tag{17}$$

The first line is the weighted average of constrained and unconstrained young agents, the second line the consumption of the middle-aged and the third line is the consumption of the old. Consumption of the old is a usual Euler equation under the assumption that the old care about consumption of their descendants, who are middle-aged in the following period. This assumption is innocuous with regard to the qualitative results of the model and follows the assumption above about housing bequests[11]. It is justified by the focus of the paper on housing as a means to bring forward consumption in time, as opposed to a wealth effect from housing from finite lives.

In equilibrium $c_t = y_t$ must hold. In addition, as stated above, each agent faces the same life-time pattern of income and receives a fixed fraction s_j of aggregate income. In particular the income of the young $y_{1t} = s_1 y_t$. This assumption does not mean that income is predetermined. Rather as in the standard New Keynesian model it is demand determined. Also note that while the share of income going to the young is a fixed fraction of aggregate income, it is is distributed over the young agents as specified above. Moreover, each agent consumes a fixed fraction x_j of aggregate income[12].

$$\begin{aligned} c_{1t} &= x_1 y_t \quad (18) \\ c_{2t} &= x_2 y_t \quad (19) \\ c_{3t} &= x_3 y_t \quad (20) \end{aligned}$$

Using these assumptions results in the following IS curve.

$$\begin{aligned} y_t = {} & (1-\alpha_t)\left[\phi_1 x_2 E_t y_{t+1} - \phi_2 \left(i_t - E_t \pi_{t+1}\right)\right] + \alpha_t \left[\psi_1 s_1 y_t + \psi_2 b q_t\right] \quad (21) \\ & + \phi_3 x_3 E_t y_{t+1} - \phi_4 \left(i_t - E_t \pi_{t+1}\right) \\ & + \phi_5 x_2 E_t y_{t+1} - \phi_6 \left(i_t - E_t \pi_{t+1}\right) \end{aligned}$$

As usual the IS curve is increasing in the expected future output gap, decreasing in the real interest rate. In addition and in contrast to the representative

[11] Consumption of the old could alternatively be set to their permanent income. This would, however, also involve past values of income, the nominal interest rate and inflation rate, as well as the current inflation rate. This would make no qualitative difference, while decreasing tractability of the model.

[12] Note that as long as $x_1 \neq x_2 \neq x_3$, $c_{10} \neq c_{20} \neq c_{30}$.

infinitely lived agent, there is an explicit wealth effect from housing through housing equity withdrawal by the constrained young agent. Moreover, aggregate consumption and income depend on the share of constrained agents, on their current income and on current inflation due to consumption of the old. The share of constrained agents is now

$$\alpha_t = F\left(\frac{\phi_1 x_2}{\psi_1} E_t y_{t+1} - \frac{\phi_2}{\psi_1}\left(i_t - E_t \pi_{t+1}\right) - \frac{b\psi_2}{\psi_1} q_t\right) \tag{22}$$

4 Optimal monetary policy

In the long-run financial liberalisation such as the introduction of housing equity withdrawal or gradually rising loan-to-value ratios (see Ortalo-Magné and Rady, 1999) alleviate borrowing constraints on consumers if they have permanently better access to credit. This could in principle help consumers to better smooth consumption and therefore make output and inflation less variable. Monetary policy makers would welcome it provided financial liberalisation doesn't increase financial instability. In this paper, however, we are concerned with the short-run implications of financial liberalisation. In particular, how should monetary policy react to house price movements when, combined with financial innovations such as housing equity withdrawal, these result in variation of the share of liquidity constrained consumers in the economy? When house prices rise constrained consumers are able to expand their consumption, which leads to a wealth effect from house prices in the model above. However, at the same time some previously constrained consumers become unconstrained, which reduces the share of constrained agents in the economy. From (21) it is then not immediately clear anymore how the output gap is affected and how monetary policy should respond.

To analyse optimal monetary policy we use the standard New Keynesian framework as e.g. in Walsh (2003). The key innovation in the paper is, however, the modified IS curve (21), which is reproduced here for convenience.

$$\begin{aligned} y_t \;=\; & (1-\alpha_t)\left[\phi_1 x_2 E_t y_{t+1} - \phi_2\left(i_t - E_t\pi_{t+1}\right)\right] + \alpha_t\left[\psi_1 s_1 y_t + \psi_2 b q_t\right] \\ & +\phi_3 x_3 E_t y_{t+1} - \phi_4\left(i_t - E_t\pi_{t+1}\right) \\ & +\phi_5 x_2 E_t y_{t+1} - \phi_6\left(i_t - E_t\pi_{t+1}\right) \end{aligned} \tag{23}$$

Furthermore, there is a forward looking Phillipscurve

$$\pi_t = \beta E_t\pi_{t+1} + \kappa y_t + e_t \tag{24}$$

where π_t is inflation from period $t-1$ to t, β is the discount factor, E_t the expectations operator as of period t, κ is the impact of the output gap on inflation and e_t is a cost push shock, which obeys

$$e_t = \rho e_{t-1} + \widehat{e}_t \tag{25}$$

with $0 < \rho \leq 1$, and $\hat{e}_t$ is an i.i.d. random variable with zero mean and constant finite variance. Finally, the central bank's loss function is specified as

$$L_t = \frac{1}{2} E_t \sum_{i=0}^{\infty} \beta^i \left(\pi_{t+i}^2 + \lambda y_{t+i}^2\right) \tag{26}$$

where λ is the weight the central bank puts on deviations of the output gap from target. Moreover, since the focus of the paper is on house prices and time-varying liquidity constraints we keep it as simple as possible and derive the optimal policy under discretion. To eliminate an inflation bias under discretion we assume a target for the output gap of zero. The monetary policy maker minimises in every period the loss function L_t subject to the Phillipscurve using the Lagrangean Λ_t.

$$\Lambda_t = \frac{1}{2} E_t \sum_{i=0}^{\infty} \beta^i \left(\pi_{t+i}^2 + \lambda y_{t+i}^2\right) + \theta_t\left(\pi_t - \beta E_t\pi_{t+1} - \kappa y_t - e_t\right) \tag{27}$$

where θ_t is the Lagrange multiplier on the Phillipscurve. The IS curve is no constraint on monetary policy as long as it can costlessly vary the nominal interest rate[13]. The first order conditions for optimal monetary policy are

$$\text{w.r.t. } \pi_t \quad : \quad 2\pi_t + \theta_t = 0 \tag{28}$$

$$\text{w.r.t. } y_t \quad : \quad 2\lambda y_t - \theta_t \kappa = 0 \tag{29}$$

which can be written more compactly as

$$\pi_t = -\frac{\lambda}{\kappa} y_t \tag{30}$$

The optimality condition states that the marginal cost in terms of higher inflation must be equal to the marginal benefit of a larger output gap. The central bank trades off inflation against the output gap taking into account its preferences and the Phillipscurve. Using the optimality condition (30), the AR(1) process of the cost push shock (25) and the Phillipscurve (24) in the IS curve (21) yields the interest rate rule as a function of the optimal inflation rate and output gap as well as house prices[14]

$$i_t = f_\pi E_t \pi_{t+1} + f_y y_t + f_q q_t \tag{31}$$

$$\begin{aligned}
f_\pi &= 1 + \frac{(1 - \alpha_t \psi_1 s_1)\,\kappa}{\rho\lambda\left(\phi_6 + \phi_4 + \phi_2\left(1 - \alpha_t\right)\right)} > 1 \\
f_y &= \frac{\left(\left(1 - \alpha_t\right)\phi_1 x_2 + \phi_3 x_3 + \phi_5 x_2\right)\rho}{\phi_6 + \phi_4 + \phi_2\left(1 - \alpha_t\right)} > 0 \\
f_q &= \frac{b\psi_2 \alpha_t}{\phi_6 + \phi_4 + \phi_2\left(1 - \alpha_t\right)} > 0
\end{aligned}$$

[13] Formally including the IS curve in the optimisation problem leads to a Lagrange multiplier of zero for the IS curve constraint. Modifications of the setup that only affect the IS curve don't change the first order conditions of the standard setup under discretion.

[14] Details in the appendix.

The coefficient on expected inflation is positive and larger than 1, the coefficient on the output gap is positive and the coefficient on the house price is positive too. Moreover, using the optimality condition (30) in the defintion of the share of constrained agents results in

$$\alpha_t = F\left(\frac{\phi_1 x_2}{\psi_1}\rho y_t - \frac{\phi_2}{\psi_1} i_t + \frac{\phi_2}{\psi_1} E_t \pi_{t+1} - \frac{\psi_2 b}{\psi_1} q_t\right) \tag{32}$$

Note that the cumulative distribution function F has the following characteristics.

$$\frac{\partial F}{\partial i_t} \equiv \alpha_i' < 0 \tag{33}$$

$$\frac{\partial F}{\partial \left(E_t \pi_{t+1}\right)} \equiv \alpha_\pi' > 0 \tag{34}$$

$$\frac{\partial F}{\partial y_t} \equiv \alpha_y' > 0 \tag{35}$$

$$\frac{\partial F}{\partial q_t} \equiv \alpha_q' < 0 \tag{36}$$

5 The role of liquidity constraints

Having derived an interest rate rule for monetary policy in (31) we are now able to analyse the role of house prices and the associated time-varying liquidity constraints in the conduct of monetary policy. From (31) it is clear that the optimal policy implies an interest rate response to expected inflation, output and to house prices. Moreover, however, the weights on each variable depend on the share of constrained agents α_t, which in turn varies with y_t, i_t, $E_t\pi_{t+1}$ and q_t.

5.1 Constant liquidity constraints

Consider, as a benchmark, the simple case in which liquidity constraints are constant, $\alpha_t = \alpha$. Monetary policy should react to the house price shock with a weight given by

$$\left.\frac{di_t}{dq_t}\right|_{\alpha\text{ const.}} = f_q = \frac{b\psi_2\alpha}{\phi_6 + \phi_4 + \phi_2(1-\alpha)} > 0 \tag{37}$$

Monetary policy should thus respond to rising house prices by increasing the interest rate. It has been shown that optimal monetary policy in the New Keynesian model should respond to a wealth effect from asset prices only to the extent that they affect the output gap and inflation expectations (Bean, 2004). This means the policy-maker needn't worry about asset prices themselves if they have only little information about their movements. Instead it is enough to observe the output gap and respond accordingly[15]. The same result holds here when liquidity constraints are constant. The extent of an interest rate reaction to house price movements depends on the degree to which liquidity constrained consumers are able to convert the increased value of their home into cash and ultimately into consumption, as denoted by the parameter b. Furthermore, the optimal weights on the output gap and expected inflation are given by (31) with $\alpha_t = \alpha$. While an increase in house prices or other factors affecting consumption, e.g. an increase in expected future income and consumption, could in principle be judged only by their impact on the output gap the separation of the two effects in this model allows to get information about the strength of the appropriate response since the coefficients on aggregate consumption, i.e. in equilibrium the output gap, and house prices differ.

For illustration, let's look at the extreme case where monetary policy has

[15] If asset prices conveyed better infomation about the underlying state of the economy than the output gap, it might theoretically be better to respond to them and not the output gap. This is, however, unlikely.

to deal either with the young agents all constrained or all unconstrained. If α was equal to 1 the optimal rule suggests reacting to the house price shock with a weight

$$f_q|_{\alpha=1} = \frac{b\psi_2}{\phi_6 + \phi_4} > 0$$

The weights on expected inflation and the output gap are then given by

$$\begin{aligned} f_\pi|_{\alpha=1} &= 1 + \frac{(1-\psi_1 s_1)\,\kappa}{\rho\lambda\,(\phi_6+\phi_4)} > 1 \\ f_y|_{\alpha=1} &= \frac{(\phi_3 x_3 + \phi_5 x_2)\,\rho}{\phi_6+\phi_4} > 0 \end{aligned}$$

If α turned out to be 0 we are back in the standard scenario without a wealth effect from house prices. Consequently there is no separate response to house prices required over and above the one to expected inflation and the output gap since there are only unconstrained agents, whose consumption doesn't react to house prices. The weights on expected inflation and the output gap are

$$\begin{aligned} f_\pi|_{\alpha=0} &= 1 + \frac{\kappa}{\rho\lambda\,(\phi_6+\phi_4+\phi_2)} > 1 \\ f_y|_{\alpha=0} &= \frac{(\phi_1 x_2 + \phi_3 x_3 + \phi_5 x_2)\,\rho}{\phi_6+\phi_4+\phi_2} > 0 \end{aligned}$$

The model shows that the weights on expected inflation and the output gap differ in the two cases. Whether the response to expected inflation is smaller when all young agents are constrained, $\alpha = 1$, compared to all agents being unconstrained, $\alpha = 0$, depends on the share of income s_1 going to the constrained. The fact that the constrained don't react to the interest rate would by itself call for a stronger interest rate response to bring down inflation by a given amount. However, to bring down inflation requires a fall in the outgap, i.e. income. Since the constrained consume out of current income, their consumption falls and with it the pressure on the output gap.

This compensates for a stronger interest rate response. In particular, the weight on expected inflation is smaller when all young agents are constrained if the share of income going to the young s_1 is large enough, as defined by $s_1 > \frac{\phi_2}{\psi_1(\phi_2+\phi_4+\phi_6)}$.

The weight on the output gap f_y is larger when $\alpha = 1$ than when $\alpha = 0$ if $x_2 < \frac{\phi_2\phi_3 x_3}{\phi_1(\phi_6+\phi_4)-\phi_2\phi_5}$[16]. Key to understanding the effect here is that a given shock spreads to future expected output via the autocorrelated cost-push shock. An increase in the expected output gap increases the share of constrained agents because optimal consumption increases (see (32)). On the one hand the constrained don't respond to interest rate changes, which requires a stronger interest rate response. On the other hand they don't respond to future expected output anymore, so that pressure on the output gap is partly relieved. The pressure relieved is small if the share of consumption in income of the middle-aged is small. Then the first effect dominates and the weight on the output gap increases.

In addition, as will be discussed in the next section, if liquidity constraints are time-varying, house price movements have an impact on the weights in the optimal interest rate rule.

5.2 Time-varying liquidity constraints

Let now the share of liquidity constrained consumers be determined by (32). While the rule still suggests increasing the interest rate in the face of an increase in expected inflation, the ouput gap or house prices, the weights on these variables now vary with the share of constrained agents. The share of constrained agents is positively related to the output gap and expected inflation and negatively related to the nominal interest rate and house prices. Consequently, each weight is a function of expected inflation, the output gap, the interest rate and house prices. We will discuss each weight in turn.

[16] Since $x_2 \geqq 0$, it must hold that $\phi_1 (\phi_4 + \phi_6) > \phi_2\phi_5$.

5.2.1 The optimal weight on expected inflation

When liquidity constraints are time-varying the optimal rule (31) not only suggests responding to house prices, but also that the optimal weights on the arguments in the rule change with the house price shock. Consider how the optimal weight on expected inflation changes with house price movements

$$\frac{df_\pi}{dq_t} = \frac{\alpha_q' \kappa \rho \lambda \left(\phi_2 - \psi_1 s_1 \left(\phi_2 + \phi_4 + \phi_6\right)\right)}{\left[\rho \lambda \left(\phi_6 + \phi_4 + \phi_2 \left(1 - \alpha_t\right)\right)\right]^2} \tag{38}$$

Since $\alpha_q' < 0$ the weight on expected inflation decreases with house prices if the share of income going to the young s_1 is small, $s_1 < \frac{\phi_2}{\psi_1(\phi_2+\phi_4+\phi_6)}$. Higher house prices decrease the share of constrained agents because they allow to extract equity from the house to finance consumption. Intuitively, the same two effects as above with constant α are at work. On the one hand, since an agent doesn't react to changes in the interest rate when constrained, but does so when unconstrained, a weaker interest rate response is required with more agents unconstrained to bring down expected inflation by a given amount. On the other hand, the constrained consume out of current income and liquid assets, while the unconstrained don't. An interest rate increase depresses current income and lowers consumption by the constrained, which helps to bring down inflation through an indirect channel. When this channel is partially shut down because more agents are unconstrained, it must be compensated by a stronger direct interest rate channel. However, the smaller the share of income s_1 going to the constrained, the weaker is the indirect effect that must be compensated. If $s_1 < \frac{\phi_2}{\psi_1(\phi_2+\phi_4+\phi_6)}$ the direct effect more than compensates the indirect effect and a weaker response to expected inflation is warranted.

Moreover, the weight on expected inflation is a function of expected inflation itself, via its effect on α_t.

$$\frac{df_\pi}{d\left(E_t\pi_{t+1}\right)} = \frac{\alpha'_\pi \kappa\rho\lambda\left(\phi_2 - \psi_1 s_1\left(\phi_2+\phi_4+\phi_6\right)\right)}{\left[\rho\lambda\left(\phi_6+\phi_4+\phi_2\left(1-\alpha_t\right)\right)\right]^2} \tag{39}$$

which is positive if the share of income going to the young s_1 is small, $s_1 < \frac{\phi_2}{\psi_1(\phi_2+\phi_4+\phi_6)}$. To recapitulate the intuition, consider now an increase in the share of constrained agents. Higher expected inflation increases the share of constrained agents by raising the optimal level of consumption. Since the constrained don't react to changes in the interest rate, a stronger interest rate response is required. On the other hand, the newly constrained consume out of current income and liquid assets. An interest rate increase depresses their income and as such lowers consumption by the constrained. This effect however is smaller the smaller the share of income s_1 going to the constrained, calling for a stronger rate increase.

Similarly, the weight on expected inflation also increases with the output gap if $s_1 < \frac{\phi_2}{\psi_1(\phi_2+\phi_4+\phi_6)}$:

$$\frac{df_\pi}{dy_t} = \frac{\alpha'_y \kappa\rho\lambda\left(\phi_2 - \psi_1 s_1\left(\phi_2+\phi_4+\phi_6\right)\right)}{\left[\rho\lambda\left(\phi_6+\phi_4+\phi_2\left(1-\alpha_t\right)\right)\right]^2} \tag{40}$$

Again with more agents constrained there are two effects: On the one hand a stronger interest rate increase is necessary to bring down expected inflation since fewer agents react to interest rate changes. On the other hand, the constrained react indirectly to the interest rate change as far as it affects aggregate income. If the indirect effect is small, because a small share of aggregate income goes to the constrained, then the first effect outweighs the second one and a stronger interest rate response is warranted.

Finally the optimal weight on expected inflation decreases with the interest rate if $s_1 < \frac{\phi_2}{\psi_1(\phi_2+\phi_4+\phi_6)}$.

$$\frac{df_\pi}{di_t} = \frac{\alpha_i' \kappa \rho \lambda \left(\phi_2 - \psi_1 s_1 \left(\phi_2 + \phi_4 + \phi_6\right)\right)}{\left[\rho \lambda \left(\phi_6 + \phi_4 + \phi_2 \left(1 - \alpha_t\right)\right)\right]^2} \tag{41}$$

The intuition is again the same as in the previous cases. Note that the interest rate effect and the house price effect tend to offset the other two, since $\alpha_\pi' > 0$, $\alpha_y' > 0$ and $\alpha_i' < 0$, $\alpha_q' < 0$, and the strength of each effect crucially depends on the sensitivities α_π', α_y', α_i' and α_i'.

5.2.2 The optimal weight on the output gap

House price changes have an impact on the share of constrained agents and therefore on the optimal weight on the output gap.

$$\frac{df_y}{dq_t} = \frac{\alpha_q' \rho \left[\phi_2 \phi_3 x_3 - x_2 \left(\phi_1 \left(\phi_4 + \phi_6\right) - \phi_2 \phi_5\right)\right]}{\left[\phi_6 + \phi_4 + \phi_2 \left(1 - \alpha_t\right)\right]^2} \tag{42}$$

The intuition is analogous to the case where either all the young are constrained or no one is constrained. The weight on output decreases with the house price if the share of consumption in aggregate income when middle-aged is small, as defined by $x_2 < \frac{\phi_2 \phi_3 x_3}{\phi_1(\phi_6+\phi_4)-\phi_2\phi_5}$[17]. Higher house prices reduce the proportion of constrained agents. On the one hand these newly unconstrained respond to interest rate changes, which allows to achieve a given reduction in the output gap with a smaller interest rate increase. On the other hand, the newly unconstrained also react to their expected future consumption, which increases with x_2. This effect calls for a stronger interest rate increase to bring down the output gap by a given amount. However, if this effect is small, the first effect dominates and a weaker interest rate response is required.

The optimal weight on the output gap reacts to an increase in the output

[17] Remember that since $x_2 \geqq 0$, it must hold that $\phi_1 \left(\phi_4 + \phi_6\right) > \phi_2 \phi_5$.

gap as follows.

$$\frac{df_y}{dy_t} = \frac{\alpha_y' \rho \left[\phi_2 \phi_3 x_3 - x_2 \left(\phi_1 \left(\phi_4 + \phi_6\right) - \phi_2 \phi_5\right)\right]}{\left[\phi_6 + \phi_4 + \phi_2 \left(1 - \alpha_t\right)\right]^2} \tag{43}$$

which is positive if $x_2 < \frac{\phi_2 \phi_3 x_3}{\phi_1(\phi_6 + \phi_4) - \phi_2 \phi_5}$. The intuition is the same as above.

The optimal weight on output changes with respect to expected inflation and the interest rate in an analogous manner.

$$\frac{df_y}{d\left(E_t \pi_{t+1}\right)} = \frac{\alpha_\pi' \rho \left[\phi_2 \phi_3 x_3 - x_2 \left(\phi_1 \left(\phi_4 + \phi_6\right) - \phi_2 \phi_5\right)\right]}{\left[\phi_6 + \phi_4 + \phi_2 \left(1 - \alpha_t\right)\right]^2} \tag{44}$$

$$\frac{df_y}{di_t} = \frac{\alpha_i' \rho \left[\phi_2 \phi_3 x_3 - x_2 \left(\phi_1 \left(\phi_4 + \phi_6\right) - \phi_2 \phi_5\right)\right]}{\left[\phi_6 + \phi_4 + \phi_2 \left(1 - \alpha_t\right)\right]^2} \tag{45}$$

5.2.3 The optimal weight on house prices

The optimal weight on house prices is a function of house prices themselves.

$$\frac{df_q}{dq_t} = \frac{\alpha_q' \psi_2 b \left(\phi_2 + \phi_4 + \phi_6\right)}{\left[\phi_6 + \phi_4 + \phi_2 \left(1 - \alpha_t\right)\right]^2} < 0 \tag{46}$$

This is because with rising house prices fewer agents are constrained, who don't react to house prices. In this case consumption and the output do increase. After all, becoming unconstrained means that consumption of the young has increased up to or beyond the optimal level of consumption. However, this increase in consumption is now captured by the increase in the output gap. Therefore a separate response to house prices is not warranted. The difference lies in the coefficients on house prices and the output gap. Pressure on the output gap due to a wealth effect from house prices requires a slightly different response than pressure due to an increase in expected future income and consumption.

Furthermore, the optimal weight on house prices increases with expected inflation.

$$\frac{df_q}{d\left(E_t\pi_{t+1}\right)} = \frac{\alpha'_\pi \psi_2 b\left(\phi_2+\phi_4+\phi_6\right)}{\left[\phi_6+\phi_4+\phi_2\left(1-\alpha_t\right)\right]^2} > 0$$

This is because more agents are constrained, who react to house price increases. The optimal weight also increases with the output gap

$$\frac{df_q}{dy_t} = \frac{\alpha'_y \psi_2 b\left(\phi_2+\phi_4+\phi_6\right)}{\left[\phi_6+\phi_4+\phi_2\left(1-\alpha_t\right)\right]^2} > 0$$

and decreases with the interest rate. A higher interest rate reduces the proportion of constrained agents, who react to changes in house prices.

$$\frac{df_q}{di_t} = \frac{\alpha'_i \psi_2 b\left(\phi_2+\phi_4+\phi_6\right)}{\left[\phi_6+\phi_4+\phi_2\left(1-\alpha_t\right)\right]^2} < 0$$

6 Discussion

6.1 House prices are affected by the interest rate

A standard present-value model for house prices would predict that the current house price is a function of the real interest rate[18].

$$q_t = \psi_3 E_t q_{t+1} - \psi_4\left(i_t - E_t\pi_{t+1}\right) + \eta_t \tag{47}$$

Then the fall in the share of constrained agents following an interest rate increase is smaller because in addition to the effect of a lower optimal level of consumption house prices fall reducing liquid assets. Conversely, an interest rate decrease leads to a smaller increase in the share of constrained agents because higher house prices compensate for the increased desired consumption level.

$$\alpha'_i|_{\psi_4=0} < \alpha'_i|_{\psi_4>0} \tag{48}$$

Furthermore, for any given inflation expectation, output gap or lagged

[18] Assume for simplicity that the expected future change in rents is zero .

house price changes the optimal interest rate rule requires a smaller response because any house price increase is immediately dampened by an interest rate increase.

6.2 House prices follow an autoregressive process

So far, we have not specified a time-series process for house prices. However, empirically the growth rate of house prices has been found to be fairly strongly autocorrelated (see Englund and Ioannides, 1997; Case and Shiller, 1989, 1990; Meese and Wallace, 1994). We take account of this empirical regularity by also considering the following process for house prices

$$q_t = \tau q_{t-1} + \eta_t \tag{49}$$

where q_{t-1} is the the lagged percentage deviation from steady-state, τ is the autocorrelation coefficient and η_t is the house price shock. Then the share of constrained agents becomes

$$\alpha_t^{AR} = F\left(\frac{\phi_1 x_2}{\psi_1}\rho y_t - \frac{\phi_2}{\psi_1}\left(i_t - E_t\pi_{t+1}\right) - \frac{\psi_2 \tau b}{\psi_1} q_{t-1} - \frac{\psi_2 b}{\psi_1}\eta_t\right)$$

The analysis proceeds as in the case of a random walk for house prices above. New is however that lagged house price changes now appear in the interest rate rule and in the defintion of the share of constrained agents α_t^{AR}. The optimal rule is now

$$i_t = f_\pi E_t \pi_{t+1} + f_y y_t + f_q\left(\tau q_{t-1} + \eta_t\right) \tag{50}$$

Clearly all results derived in the case of a random walk continue to hold with the addition that monetary policy should also react to lagged asset prices with a weight τf_q. The strength of the response to lagged changes in house prices increases with the autoregressive parameter τ. Moreover the weight on past house prices varies with α_t^{AR}, i.e. with expected inflation, the output gap, the interest rate, lagged house prices themselves and the current house price shock. In particular,

$$\frac{df_q}{d\left(q_{t-1}\right)} = \frac{\alpha'_{q_{t-1}}\psi_2 b\left(\phi_2+\phi_4+\phi_6\right)}{\left[\phi_6+\phi_4+\phi_2\left(1-\alpha_t\right)\right]^2} < 0$$

The higher lagged house prices the smaller is the required interest rate response to them. The reason is that the higher are lagged house prices the fewer agents are constrained for a given current house price shock. Unconstrained agents don't react to house prices anymore. Moreover, only those who are unconstrained react to interest rate changes. Constrained agents keep on consuming out of their liquid assets. With a higher share of unconstrained agents, a smaller interest rate change is needed to offset the wealth effect from house prices. The effects of expected inflation, output gap and interest rate on the optimal weight on past asset prices follow analogously.

Furthermore, the optimal weights on expected inflation and the output gap are now affected by the presence of lagged house prices.

$$\frac{df_\pi}{dq_{t-1}} = \frac{\alpha'_{q_{t-1}}\tau\kappa\rho\lambda\left(\phi_2-\psi_1 s_1\left(\phi_2+\phi_4+\phi_6\right)\right)}{\left[\rho\lambda\left(\phi_6+\phi_4+\phi_2\left(1-\alpha_t\right)\right)\right]^2} \tag{51}$$

The weight on expected inflation decreases with lagged house prices if $s_1 < \frac{\phi_2}{\psi_1(\phi_2+\phi_4+\phi_6)}$, where the intuition is the same as above. The weight on the output gap is affected in an analogous manner to the case where house price follow a random walk.

6.3 Discussion of some model assumptions

The role of bequests In the model it is assumed that the old generation bequeath their houses to their middle-aged descendants. This assumption rules out a wealth effect on consumption of the old. Abolishing housing bequests from the old would introduce another wealth channel into the model. This, however, would be separate from the wealth effect from house prices through relaxing liquidity constraints of the young and would not affect the results derived with regard to the optimal weights on the output gap and expected inflation. However, the weight on house prices themselves would probably increase since more agents would respond to an increase in house prices by expanding consumption. Furthermore, abolishing the bequest motive in terms of the old caring for consumption of their middle-aged descendants would call for specifying consumption of the old in a different way. For example the old could consume their permanent income, which, however would introduce lags of the interest rate, inflation and output. This wouldn't change the results qualitatively while rendering the model more complicated.

Possibility of default of middle-aged While carrying out the analysis above we have maintained the assumption that the middle-aged and the old are always unconstrained. In particular, the assumption was that their respective income is always more than enough to cover desired consumption and desired lending to the young. In addition there is no default on debt. Both assumptions allow to focus solely on the role of house prices as collateral in relaxing liquidity constraints. Default on the part of the middle-aged would have an effect if the repayment was used to finance consumption of the old. Then, a fall in house prices below the contracted loan-to-value ratio would depress consumption of the old in addition to the reduction in consumption by the constrained young. To connect the possibility of default to house prices one could introduce a fourth generation between the middle-aged and the old. The income of the middle-aged might not be sufficient to cover both consumption and repayment of the loan. They would have to roll over their loan by borrowing from the additional generation again with their housing value as collateral, the same mechanism as for borrowing by the young. If house prices fall, the middle-aged wouldn't be able to cover their repayment by a new loan and would default. However, even without appealing to these arguments we have shown that house prices do matter in the optimal conduct of monetary policy.

Distinction between bubble and fundamental price change So far we haven't made any assumption about the source of a house price increase. It could be fundamentally justified or it could be driven by non-fundamental factors. Whether this matters for the model depends on the expectations of the young borrowers and the middle-aged lenders about the persistence of the house price boom. If both expect it to last at least until the next period borrowers and lenders are happy to accept the value of the house as collateral even though at some point in time it might fall considerably. This again results from the role of housing as collateral, which allows to bring forward consumption from later periods. Under this view it doesn't matter whether consumers believe house prices are driven by fundamental or non-fundamental factors.

7 Conclusion

In this paper we have derived a wealth effect from house prices through their role as collateral to bring finance consumption. Housing value serves as a means to bring forward consumption in time without being of intrinsic value. Since house prices vary, so does the value of collateral and therefore the proportion of constrained agents varies too. Furthermore, the share of constrained agents depends on house prices, expected inflation, the output gap and the interest rate. Since constrained and unconstrained agents react differently to house price changes, expected future output gap, expected inflation and interest rate changes, the actual share of constrained agents is important for the weights monetary policy should put on each of these factors when setting interest rates. In sum, the analysis shows that the optimal weights on expected inflation, the output gap and house price changes vary over time, in turn depending on the values for expected inflation, the output gap and house price changes. Therefore house prices do seem to play a role in the optimal response of monetary policy to house prices over and above their effect on aggregate demand and the output gap. This result has been derived without appealing to supply side effects from defaults on debt or the informational content in asset prices about future productivity or inflation.

The model has also demonstrated that it is important where a wealth effect comes from. If it results from relaxed liquidity constraints there is the additional effect on the optimal weights on inflation, output and house prices in the interest rate rule. Therefore we have worked out another factor that is relevant for an appropriate interest rate response in the face of changes in expected inflation, the output gap and house prices.

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8 Appendix

8.A Derivation of optimal interest rate rule

The solution for optimal monetary policy are expressions for the output gap and inflation in only the state variables. Under discretion e_t is the only relevant state variable such that a conjectured solution is of the form

$$y_t = \delta e_t \tag{52}$$

From the optimality condition (30)

$$\pi_t = -\frac{\lambda}{\kappa} y_t \tag{53}$$

it follows

$$\pi_t = -\frac{\lambda}{\kappa} \delta y_t \tag{54}$$

Plugging this into the Phillipscurve (24) yields

$$y_t = \frac{\beta\delta\rho - \frac{\kappa}{\lambda}}{1 + \frac{\kappa^2}{\lambda}} e_t \tag{55}$$

Equating coefficients from (52) and (55) results in

$$\delta = \frac{-\kappa}{\kappa^2 + \lambda (1 - \rho\beta)} \tag{56}$$

Consequently,

$$y_t = \frac{-\kappa}{\kappa^2 + \lambda (1 - \rho\beta)} e_t \tag{57}$$

and

$$\pi_t = \frac{\lambda}{\kappa^2 + \lambda (1 - \rho\beta)} e_t \tag{58}$$

To arrive at the optimal interest rule use (57) and (58) together with the AR(1) process for the cost push shock in the IS curve (21).