# Multivariate Fractionally Integrated APARCH Modeling of Stock Market Volatility: A multi-country study

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**Abstract**

Tse (1998) proposes a model which combines the fractionally integrated GARCH formulation of Baillie, Bollerslev and Mikkelsen (1996) with the asymmetric power ARCH specification of Ding, Granger and Engle (1993). This paper analyzes the applicability of a multivariate constant conditional correlation version of the model to national stock market returns for eight countries. We find this multivariate specification to be generally applicable once power, leverage and long-memory effects are taken into consideration. In addition, we find that both the optimal fractional differencing parameter and power transformation are remarkably similar across countries. Out-of-sample evidence for the superior forecasting ability of the multivariate FIAPARCH framework is provided in terms of forecast error statistics and tests for equal forecast accuracy of the various models.

**Keywords**: Asymmetric Power ARCH, Fractional integration, Stock returns, Volatility forecast evaluation.

**JEL Classification**: C13, C22, C52.

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# 1 Introduction

A common finding in much of the empirical finance literature is that although the returns on speculative assets contain little serial correlation, the absolute returns and their power transformations are highly correlated (see, for example, Dacorogna et al. 1993, Granger and Ding, 1995a, 1995b and Breidt et al. 1998). In particular, Ding et al. (1993) investigate the autocorrelation structure of $|r_t|^\delta$, where $r_t$ is the daily S&P 500 stock market returns, and $\delta$ is a positive number. They found that $|r_t|$ has significant positive autocorrelations for long lags. Motivated by this empirical result they propose a new general class of ARCH models, which they call the Asymmetric Power ARCH (APARCH). In addition, they show that this formulation comprises seven other specifications in the literature.[1] Brooks et al. (2000) analyze the applicability of the PARCH models to national stock market returns for ten countries plus a world index. Bollerslev and Mikkelsen (1996) provide strong evidence that the conditional variance for the S&P 500 composite index is best modeled as a mean-reverting fractionally integrated process. Christensen and Nielsen (2007) analyze the impulse response function for future returns with respect to a unit shock in current volatility. They show that the interaction of a positive risk-return link, long-memory in volatility, and a strong financial leverage effect, yields a perhaps surprisingly low impact of volatility shocks on asset values. McCurdy and Michaud (1996) analyze the CRSP value-weighted index using a fractionally integrated APARCH (FIAPARCH) type of model. McCurdy and Michaud (1996) and Tse (1996, 1998) extend the asymmetric power formulation of the variance to incorporate fractional integration, as defined by Baillie et al. (1996).[2]

The FIAPARCH model increases the flexibility of the conditional variance specification by allowing (a) an asymmetric response of volatility to positive and negative shocks, (b) the data to determine the power of returns for which the predictable structure in the volatility pattern is the strongest, and (c) long-range volatility dependence. These three features in the volatility processes of asset returns have major implications for many paradigms in modern financial economics. Optimal portfolio decisions, the pricing of long-term options and optimal portfolio allocations must take into account all of these three findings. E.g., Giot and Laurent (2003) have shown that APARCH volatility forecasts outperform those obtained from the RiskMetrics model, which is equivalent to an integrated ARCH with pre-specified autoregressive parameter values. The fractionally integrated process may lead to further improvement,

[1] These models are: the ARCH (Engle, 1982), the GARCH (Bollerslev, 1986), the Taylor/Schwert GARCH in standard deviation (Taylor, 1986, and Schwert, 1990), the GJR GARCH (Glosten et al., 1993), the TARCH (Zakoian, 1994), the NARCH (Higgins and Bera, 1992) and the log-ARCH (Geweke, 1986, and Pantula, 1986).

[2] The FIGARCH model of Baillie et al. (1996) is closely related to the long-memory GARCH process (see Karanasos et al., 2003, and Conrad and Karanasos, 2006, and the references therein). We should also mention the Hyperbolic GARCH (HYGARCH) model of Davidson (2004) and the fact that Robinson (1991) was the first to consider the long-memory potential in volatility.

if its forecasts are more accurate than those obtained from the stable specification.

Another important advantage of having a FIAPARCH model is that it nests the formulation without power effects and the stable one as special cases. This provides an encompassing framework for these two broad classes of specifications and facilitates comparison between them. The main contribution of this paper is to enhance our understanding of whether and to what extent this type of model improves upon its simpler counterparts.

The evidence provided by Tse (1996, 1998) suggests that the FIAPARCH model is applicable to the yen-dollar exchange rate. More recently, Degiannakis (2004) and Ñíguez (2007) applied univariate FIAPARCH specifications to stock return data. So far, multivariate versions of the framework have rarely been used in the literature. Only Dark (2004) applies a bivariate error correction FIAPARCH model to examine the relationship between stock and future markets, and Kim et al. (2005) use a bivariate FIAPARCH-in-mean process to model the volume-volatility relationship. Therefore, an interesting research issue is to explore how generally applicable this formulation is to a wide range of financial data and whether multivariate specifications can outperform their univariate counterparts. In this paper we attempt to address this issue by estimating both univariate and multivariate versions of this framework for eight series of national stock market index returns. These countries are Canada, France, Germany, Hong Kong, Japan, Singapore, the United Kingdom and the United States. As the general multivariate specification adopted in this paper nests the various univariate formulations, the relative ranking of each of these models can be considered using the Wald testing procedures. In addition, standard information criteria can be used to provide a ranking of the specifications. Furthermore, the ability of the FIAPARCH formulation to forecast (out-of-sample) stock volatility is assessed by a variety of forecast error statistics. In order to verify whether the difference between the statistics from the different models is statistically significant we employ the tests of Diebold and Mariano (1995) and Harvey et al. (1997).

The remainder of the paper is structured as follows. In section 2 we detail the FIAPARCH model and discuss how various ARCH specifications are nested within it. Section 3 discusses the data and presents the empirical results. Maximum likelihood parameter estimates for the various specifications are presented, as are the results of the Wald testing procedures. The robustness of these results is assessed using four alternative information criteria. To test for the apparent similarity of the power and fractional differencing terms across countries pairwise Wald tests are performed. Section 4 evaluates the different specifications in terms of their out-of-sample forecast ability. For each country and each formulation three forecast error measures are calculated and evaluated against each other. Moreover, we test for equal forecast accuracy of the competing models by utilizing three test statistics. Section 5 discusses our results and Section 6 concludes the analysis.

# 2 FIAPARCH Model

## 2.1 Univariate Process

One of the most common models in finance and economics to describe a time series $r_t$ of stock returns is the AR(1) process

$$(1-\zeta L)r_t = c+\varepsilon_t, \quad t\in\mathbb{N}, \tag{2.1}$$

with

$$\varepsilon_t = e_t\sqrt{h_t},$$

where $c\in(0,\infty)$, $|\zeta|<1$ and $\{e_t\}$ are independently, identically distributed (*i.i.d.*) student-$t$ random variables with $\mathsf{E}(e_t)=\mathsf{E}(e_t^2-1)=0$. $h_t$ is positive with probability one and is a measurable function of $\Sigma_{t-1}$, which in turn is the sigma-algebra generated by $\{r_{t-1},r_{t-2},\dots\}$. That is $h_t$ denotes the conditional variance of the returns $\{r_t\}$ and $r_t|\Sigma_{t-1}\overset{i.i.d.}{\sim}(c+\zeta r_{t-1},h_t)$.

Tse (1998) examines the conditional heteroskedasticity of the yen-dollar exchange rate by employing the FIAPARCH$(1,d,1)$ model. Accordingly, we utilize the following process

$$(1-\beta L)(h_t^{\delta/2}-\omega)=[(1-\beta L)-(1-\phi L)(1-L)^d](1+\gamma s_t)|\varepsilon_t|^\delta, \tag{2.2}$$

where $\omega\in(0,\infty)$, $|\phi|<1$, $0\leq d\leq 1$,[3] $s_t=1$ if $\varepsilon_t<0$ and 0 otherwise, $\gamma$ is the leverage coefficient, and $\delta$ is the parameter for the power term that takes (finite) positive values.

When $d=0$, the process in equation (2.2) reduces to the APARCH(1,1) one which nests two major classes of ARCH models. Specifically, a Taylor/ Schwert type of formulation is specified when $\delta=1$, and a Bollerslev type is specified when $\delta=2$. There seems to be no obvious reason why one should assume that the conditional standard deviation is a linear function of lagged absolute returns or the conditional variance a linear function of lagged squared returns. As Brooks et al. (2000, p. 378) point out "The common use of a squared term in this role ($\delta=2$) is most likely to be a reflection of the normality assumption traditionally invoked regarding financial data. However, if we accept that (high frequency) data are very likely to have a non-normal error distribution, then the superiority of a squared term is lost and other power transformations may be more appropriate. Indeed, for non-normal data, by squaring

[3] The fractional differencing operator, $(1-L)^d$ is most conveniently expressed in terms of the hypergeometric function

$$(1-L)^d = F(-d,1;1;L)=\sum_{j=0}^{\infty}\frac{\Gamma(j-d)}{\Gamma(-d)\Gamma(j+1)}L^j=\sum_{j=0}^{\infty}\binom{d}{j}(-1)^jL^j,$$

where

$$F(a,b;c;z)=\sum_{j=0}^{\infty}\frac{(a)_j(b)_j}{(c)_j}\frac{z^j}{j!}$$

is the Gaussian hypergeometric series, $(b)_j$ is the shifted factorial defined as $(b)_j=\prod_{i=0}^{j-1}(b+i)$ (with $(b)_0=1$), and $\Gamma(\cdot)$ is the gamma function.

the returns one effectively imposes a structure on the data which may potentially furnish sub-optimal modeling and forecasting performance relative to other power terms".

Since its introduction by Ding et al. (1993), the APARCH formulation has been frequently applied. It is worth noting that Fornari and Mele (1997) show the usefulness of this scheme in approximating models developed in continuous time as systems of stochastic differential equations. This feature has usually been overshadowed by its well-known role as simple econometric tool providing reliable estimates of unobserved conditional variances (Fornari and Mele, 2001). Hentschel (1995) defines a parametric family of asymmetric models that nests the APARCH one.[4]

When $\gamma = 0$ and $\delta = 2$ the process in equation (2.2) reduces to the FIGARCH$(1, d, 1)$ specification which includes Bollerslev's (1986) model (when $d = 0$) and the integrated specification (when $d = 1$) as special cases.[5] Baillie et al. (1996) point out that a striking empirical regularity that emerges from numerous studies of high-frequency, say daily, asset pricing data with ARCH-type models, concerns the apparent widespread finding of integrated behavior. This property has been found in stock returns, exchange rates, commodity prices and interest rates (see Bollerslev et al., 1992). Yet unlike I(1) processes for the mean, there is less theoretical motivation for truly integrated behavior in the conditional variance (see Baillie et al., 1996 and the references therein).[6]

Finally, as noted by Baillie et al. (1996) for the variance, being confined to only considering the extreme cases of stable and integrated specifications can be very misleading when long-memory (but eventually mean-reverting) processes are generating the observed data. They showed that data generated from a process exhibiting long-memory volatility may be easily mistaken for integrated behavior. Andersen and Bollerslev (1997) suggest that cross-sectional aggregation of a large number of volatility components or news information arrival processes with different degrees of persistence could lead to fractional integration. Kirman and Teyssiere (2001) use a microeconomic model to link herding and swing

[4]For applications of the APARCH model in economics see Campos and Karanasos (2008), Campos et al. (2008a, 2008b) and Karanasos and Schurer (2008).

[5]An excellent survey of major econometric work on long-memory processes and their applications in economics and finance is given by Baillie (1996). Karanasos et al. (2006) apply the FIAPARCH model to interest rates. For applications of the FIGARCH model to exchange rates see, among others, Conrad and Lamla (2007).

[6]In particular, the occurrence of a shock to the IGARCH volatility process will persist for an infinite prediction horizon. This extreme behavior of the IGARCH process may reduce its attractiveness for asset pricing purposes, where the IGARCH assumption could make the pricing functions for long-term contracts very sensitive to the initial conditions. This seems contrary to the perceived behavior of agents, who typically do not frequently and radically change their portfolio compositions. In addition, the IGARCH model is not compatible with the persistence observed after large shocks such as the Crash of October 1987. A further reason to doubt the empirical reasonableness of IGARCH models relates to the issue of temporal aggregation. A data generating process of IGARCH at high frequencies would also imply a properly defined weak IGARCH model at low frequencies of observation. However, this theoretical result seems at odds with reported empirical findings for most asset categories (abstracted from Baillie et al. 1996).

of opinion with long-memory in volatility. According to Beltratti and Morana (2006) volatility of output growth and, to a lesser extent, the volatility of the Fed fund rate and M1 growth affect both the persistent and non-persistent components of S&P 500 volatility (see Hyung et al., 2006).

### 2.2 Multivariate Formulation

In this section we discuss the multivariate time series model for the stock returns and discuss its merits and properties. Let us define the $N$-dimensional column vector of the returns $\mathbf{r}_t$ as $\mathbf{r}_t = [r_{it}]_{i=1,\ldots,N}$ and the corresponding residual vector $\boldsymbol{\varepsilon}_t$ as $\boldsymbol{\varepsilon}_t = [\varepsilon_{it}]_{i=1,\ldots,N}$. Regarding $\boldsymbol{\varepsilon}_t$ we assume that it is conditionally student-$t$ distributed with mean vector $\mathbf{0}$, variance vector $\mathbf{h}_t = [h_{1t}]_{i=1,\ldots,N}$ and constant conditional correlations (ccc), $\rho_{ij} = h_{ij,t}/\sqrt{h_{it}h_{jt}}$, $|\rho_{ij}| \leq 1$, $i, j = 1, \ldots, N$.

Next, the structure of the AR (1) mean equation is given by

$$\mathbf{Z}(L)\mathbf{r}_t = \mathbf{c} + \boldsymbol{\varepsilon}_t, \tag{2.3}$$

where $\mathbf{Z}(L) = \mathbf{I}_N \boldsymbol{\zeta}(L)$ with $\mathbf{I}_N$ being the $N \times N$ identity matrix and $\boldsymbol{\zeta}(L) = [1 - \zeta_i L]_{i=1,\ldots,N}$, $|\zeta_i| < 1$, and $\mathbf{c} = [c_i]_{i=1,\ldots,N}$ with $c_i \in (0, \infty)$.

Further, to establish terminology and notation, the multivariate FIAPARCH (M-FIAPARCH) process of order $(1, d, 1)$ is defined by

$$\mathbf{B}(L)(\mathbf{h}_t^{\wedge\frac{\delta}{2}} - \boldsymbol{\omega}) = [\mathbf{B}(L) - \boldsymbol{\Delta}(L)\boldsymbol{\Phi}(L)][\mathbf{I}_N + \boldsymbol{\Gamma}\mathbf{s}_t]|\boldsymbol{\varepsilon}_t|^{\wedge\delta}, \tag{2.4}$$

where $^\wedge$ denotes elementwise exponentiation and $|\boldsymbol{\varepsilon}_t|$ is the vector $\boldsymbol{\varepsilon}_t$ with elements stripped of negative values. Moreover, $\mathbf{B}(L) = \mathbf{I}_N \boldsymbol{\beta}(L)$ with $\boldsymbol{\beta}(L) = [1 - \beta_i L]_{i=1,\ldots,N}$, and $\boldsymbol{\Phi}(L) = \mathbf{I}_N \boldsymbol{\phi}(L)$ with $\boldsymbol{\phi}(L) = [1 - \phi_i L]_{i=1,\ldots,N}$, $|\phi_i| < 1$. In addition, $\boldsymbol{\omega} = [\boldsymbol{\omega}_i]_{i=1,\ldots,N}$ with $\omega_i \in (0, \infty)$ and $\boldsymbol{\Delta}(L) = \mathbf{I}_N \mathbf{d}(L)$ with $\mathbf{d}(L) = [(1-L)^{d_i}]_{i=1,\ldots N}$, $0 \leq d_i \leq 1$. Finally, $\boldsymbol{\Gamma} = \boldsymbol{\gamma}\mathbf{I}_N$ with $\boldsymbol{\gamma} = [\gamma_i]_{i=1,\ldots,N}$, and $\mathbf{s}_t = [s_{it}]_{i=1,\ldots,N}$ where $s_{it} = 1$ if $\varepsilon_{it} < 0$ and 0 otherwise.[7]

# 3 Empirical Analysis

### 3.1 Data

Daily stock price index data for eight countries were sourced from the Datastream database for the period 1st January 1988 to 22nd April 2004, giving a total of 4,255 observations. We will use the period 1st January 1988 to 16th July 2003 for the estimation, while we produce 200 out-of-sample forecasts for the period 17th July 2003 to 22nd April 2004. The eight countries and their respective price indices are: UK:

[7] $\mathbf{Z}(L)$, $\mathbf{B}(L)$, $\boldsymbol{\Phi}(L)$ and $\boldsymbol{\Delta}(L)$ are $N \times N$ diagonal polynomial matrices with diagonal elements $1 - \zeta_i L$, $1 - \beta_i L$, $1 - \phi_i L$ and $(1 - L)^{d_i}$ respectively. Further, $\boldsymbol{\Gamma}$ is a $N \times N$ diagonal matrix with diagonal elements $\gamma_i$.

FTSE 100 (F), US: S&P 500 (SP), Germany: DAX 30 (D), France: CAC 40 (C), Japan: Nikkei 225 (N), Singapore: Straits Times (S), Hong Kong: Hang Seng (H) and Canada: TSE 300 (T). For each national index, the continuously compounded return was estimated as $r_t = 100[\log(p_t) - \log(p_{t-1})]$ where $p_t$ is the price on day $t$.

### 3.2 Univariate Models

We proceed with the estimation of the AR(1)-FIAPARCH$(1, d, 1)$ model[8] in equations (2.1) and (2.2) in order to take into account the serial correlation[9] and the GARCH effects observed in our time series data, and to capture the possible long-memory in volatility. We estimate the various specifications using the maximum likelihood estimation (MLE) method as implemented by Davidson (2008) in Time Series Modelling (TSM). The existence of outliers, particularly in daily data, causes the distribution of returns to exhibit excess kurtosis.[10] To accommodate the presence of such leptokurtosis, we estimate the models using student-$t$ distributed innovations. Hence, for the univariate models, the log-likelihood to be maximized is given by

$$\begin{aligned}\log \mathcal{L} \quad &= \quad T\left[\log\Gamma\left(\frac{\upsilon+1}{2}\right) - \log\Gamma\left(\frac{\upsilon}{2}\right) - \frac{1}{2}\log\pi(\upsilon-2)\right] \\ &\quad -\frac{1}{2}\sum_{t=1}^{T}\left\{\log h_t^2 + (\upsilon+1)\left[\log\left(1+\frac{\varepsilon_t^2}{h_t^2(\upsilon-2)}\right)\right]\right\},\end{aligned}$$

where $\Gamma(\cdot)$ denotes the gamma function. For more details, see, Davidson (2008).

Table 1 reports the estimation results.[11] In all countries the AR coefficient ($\zeta$) is highly significant. The estimate for the $\phi(\beta)$ parameter is insignificant only in one(two) out of the eight cases. In three countries the estimates of the leverage term ($\gamma$) are statistically significant, confirming the hypothesis that there is negative correlation between returns and volatility. For the other countries we reestimated the models without an asymmetry term. For all indices the estimates of the power term ($\delta$) and the fractional differencing parameter ($d$) are highly significant. Interestingly, the highest power terms are obtained for the two American indices, while the European ones are characterized by the highest degree of persistence. In all cases, the estimated degrees of freedom parameter ($\upsilon$) is highly significant and leads

[8]The only exceptions are the Canadian and Singaporean indices, where an AR(1)-FIAPARCH$(0, d, 1)$ model is used. For these two indices the AR(1)-FIAPARCH$(1, d, 1)$ estimates for $\beta$ were insignificant and the IC came out in favor of the $(0, d, 1)$ specification. In addition, for the Hang Seng index, the criteria favor the $(1, d, 0)$ formulation.

[9]The 12th order Ljung-Box Q-statistics on the squared return series indicate high serial correlation in the second moment for all indices.

[10]For all indices the Jarque-Bera statistic rejects the normality hypothesis at the 1% level. The estimated kurtosis coefficient is significantly above three for all indices but FTSE 100 and Nikkei 225.

[11]We do not present the estimates of the constants in the mean and the variance, which were significant in all cases but one.

to an estimate of the kurtosis which is different from three.[12]

Table 1: Univariate AR-FI(A)PARCH models (ML Estimation)

| | SP | T | C | D | F | H | N | S |
|---|---|---|---|---|---|---|---|---|
| $\zeta$ | $-0.05^*$ $(-3.28)$ | 0.17 (10.94) | 0.04 (2.34) | $0.03^*$ (2.31) | 0.04 (2.38) | 0.06 (3.71) | $-0.02$ $(-1.63)$ | 0.15 (9.20) |
| $\beta$ | 0.54 (5.81) | $-$ | 0.66 (6.94) | 0.56 (5.65) | 0.59 (5.32) | 0.08 (2.01) | 0.51 (4.83) | $-$ |
| $\phi$ | 0.27 (4.11) | $-0.11$ $(-2.95)$ | 0.20 (3.84) | 0.21 (4.82) | 0.19 (3.77) | $-$ | 0.14 (2.03) | $-0.07$ $(-2.23)$ |
| $\gamma$ | $-$ | $-$ | $-$ | 0.46 (3.73) | $-$ | 0.69 (3.65) | $-$ | 0.76 (3.90) |
| $\delta$ | 2.35 (23.50) | 2.42 (17.28) | 1.77 (12.64) | 1.24 (11.46) | 1.86 (14.31) | 1.28 (12.80) | 2.07 (18.81) | 1.40 (12.73) |
| $d$ | 0.30 (6.00) | 0.19 (6.33) | 0.52 (4.33) | 0.40 (4.34) | 0.46 (4.60) | 0.18 (4.50) | 0.42 (6.00) | 0.21 (5.25) |
| $\upsilon$ | 5.60 (10.77) | 5.38 (10.76) | 8.53 (6.56) | 6.83 (6.90) | 10.70 (6.04) | 4.56 (11.12) | 5.80 (10.54) | 4.86 (11.04) |
| $Q_{12}$ | 18.45 [0.10] | 9.52 [0.66] | 10.00 [0.61] | 13.18 [0.36] | 12.86 [0.38] | 22.85 [0.03] | 10.59 [0.56] | 18.50 [0.10] |
| $Q_{12}^2$ | 5.12 [0.95] | 19.47 [0.08] | 11.74 [0.47] | 8.13 [0.77] | 18.00 [0.12] | 33.24 [0.00] | 20.90 [0.05] | 2.20 [1.00] |

Notes: For each of the eight indices, Table 1 reports ML parameter estimates for the AR(1)-FI(A)PARCH model. The numbers in parentheses are $t$-statistics. $^*$For the S&P 500 and Dax 30 indices we estimate AR(3) and AR(4) models respectively. $Q_{12}$ and $Q_{12}^2$ are the 12th order Ljung-Box tests for serial correlation in the standardized and squared standardized residuals respectively. The numbers in brackets are $p$-values.

In all cases, the ARCH parameters satisfy the set of necessary conditions sufficient to guarantee the non-negativity of the conditional variance (see Conrad and Haag, 2006). According to the values of the Ljung-Box tests for serial correlation in the standardized and squared standardized residuals there is no statistically significant evidence of misspecification.

### 3.2.1 Tests of Fractional Differencing and Power Term Parameters

A large number of studies have documented the persistence of volatility in stock returns; see, e.g., Ding et al. (1993), Ding and Granger (1996), Engle and Lee (2000). Using daily data many of these studies have concluded that the volatility process is very persistent and appears to be well approximated by an IGARCH process. For the stable APARCH(1,1) model[13] the condition for the existence *of* the $\delta/2$ th moment of the conditional variance is $\mathsf{V} = \alpha\mathsf{E}(1+\gamma s)|e|^{\delta} + \beta < 1$ which depends on the density of $e$. For

[12]The kurtosis of a student-$t$ distributed random variable with $\upsilon$ degrees of freedom is $3\frac{\upsilon-2}{\upsilon-4}$.

[13]Restricting $d$ to be 0 in equation (2.2) leads to an APARCH(1,1) model with parameters $\beta$ and $\phi - \beta$.

a student-t distributed innovation with $\upsilon$ degrees of freedom we have $\frac{\mathsf{V}-\beta}{a} = \frac{(1+\frac{\gamma}{2})}{\sqrt{\pi}}(\upsilon-2)^{\frac{\delta}{2}}\frac{\Gamma(\frac{\delta+1}{2})\Gamma(\frac{\upsilon-\delta}{2})}{\Gamma(\frac{\upsilon}{2})}$. Notice that if $\gamma = 0$ the expression for the $\frac{\mathsf{V}-\beta}{\alpha}$ is the one for the symmetric PARCH model (see Paolella, 1997 and Karanasos and Kim, 2006). In addition, if $\gamma = 0$, $\delta = 2$, $\mathsf{V} = \alpha + \beta < 1$ reduces to the usual stationarity condition of the GARCH(1,1) model.

Thus, estimating a $\mathsf{V}$ which is close to one is suggestive of integrated APARCH behavior. Table 2 presents the estimates for $\mathsf{V}$ from the AR-APARCH$(1,1)$ model with student-t distributed innovations. For all indices $\mathsf{V}$ is close to 1, indicating that $h_t^{\frac{\delta}{2}}$ may be integrated.[14]

Table 2: Estimates of V for AR-APARCH$(1,1)$ models

| | SP | T | C | D | F | H | N | S |
|---|---|---|---|---|---|---|---|---|
| $V$ | 0.998 | 0.991 | 1.000 | 0.985 | 0.985 | 0.963 | 1.013 | 0.946 |

However, from the FI(A)PARCH estimates (reported in table 1), it appears that the long-run dynamics are better modeled by the fractional differencing parameter. To test for the persistence of the conditional heteroskedasticity models, we examine the Wald statistics for the linear constraints $d = 0$ (stable APARCH) and $d = 1$ (IAPARCH).[15] As seen in table 3 the W tests clearly reject both the stable and integrated null hypotheses against the FIAPARCH one.[16] Clearly, the results which emerged from table 2 were misleading, i.e. imposing the restriction $d = 0$ leads to parameter estimates which falsely suggest integrated behavior. Thus, purely from the perspective of searching for a model that best describes the volatility in the stock return series, the fractionally integrated one appears to be the most satisfactory representation.[17]

This result is an important finding because the time series behavior of volatility affects asset prices through the risk premium. Christensen and Nielsen (2007) establish theoretically and empirically the consequences of long-memory in volatility for asset prices. Using a model for expected returns to discount streams of expected future cash flows, they calculate asset prices. Within this context the risk-return trade-off and the serial correlation in volatility are the two most important determinants of asset values. Christensen and Nielsen (2007) derive the way in which these two ingredients jointly determine the level of stock prices. They also investigate the quantitative economic consequences of these changes in asset

[14]We do not report the estimated AR-APARCH$(1,1)$ coefficients for space considerations.

[15]Restricting $d$ to be one leads to an IAPARCH(1,2) model with parameters $\beta$, $1+\phi-\beta$ and $-\phi$ (see equation (2.2)).

[16]Various tests for long-memory in volatility have been proposed in the literature (see, for details, Karanasos and Kartsaklas, 2008).

[17]It is worth mentioning the empirical results in Granger and Hyung (2004). They suggest that there is a possibility that, at least part of the long-memory may be caused by the presence of neglected breaks in the series. We look forward to clarifying this out in future work.

price elasticities.

Table 3: Tests for restrictions on fractional differencing and power term parameters

| $H_0$: | | $d=0$ | $d=1$ | | $\delta=1$ | $\delta=2$ |
|---|---|---|---|---|---|---|
| | $d$ | $W$ | $W$ | $\delta$ | $W$ | $W$ |
| S&P 500 | 0.30{0.05} | 33[0.00] | 173[0.00] | 2.35{0.10} | 178[0.00] | 9[0.00] |
| TSE 300 | 0.19{0.03} | 28[0.00] | 522[0.00] | 2.42{0.14} | 102[0.00] | 10[0.00] |
| CAC 40 | 0.52{0.12} | 18[0.00] | 15[0.00] | 1.77{0.14} | 31[0.00] | 3[0.09] |
| DAX 30 | 0.40{0.09} | 18[0.00] | 39[0.00] | 1.24{0.11} | 15[0.00] | 52[0.00] |
| FTSE 100 | 0.46{0.10} | 21[0.00] | 29[0.00] | 1.86{0.13} | 37[0.00] | 1[0.30] |
| Hang Seng | 0.18{0.04} | 16 [0.00] | 322[0.00] | 1.28{0.10} | 8[0.00] | 72[0.00] |
| Nikkei 225 | 0.42{0.07} | 35[0.00] | 67[0.00] | 2.07{0.11} | 114[0.00] | 0.50[0.54] |
| Straits Times | 0.21{0.04} | 32[0.00] | 444[0.00] | 1.40{0.11} | 16[0.00] | 36[0.00] |

Notes: For each of the eight indices, table 3 reports the value of the Wald (W) statistics for the unrestricted FI(A)PARCH and restricted ($d = 0, 1; \delta = 1, 2$) models respectively. The numbers in $\{\cdot\}$ are standard errors. The numbers in $[\cdot]$ are $p$ values.

Following the work of Ding et al. (1993), Hentschel (1995), Tse (1998) and Brooks et al. (2000) among others, the Wald test can be used for model selection. Alternatively, the Akaike, Schwarz, Hannan-Quinn or Shibata information criteria (AIC, SIC, HQIC, SHIC respectively) can be applied to rank the various ARCH type of models.[18] These model selection criteria check the robustness of the Wald testing results discussed above.[19] Specifically, according to the AIC, HQIC and SHIC, the optimal specification (i.e., FIAPARCH, APARCH or IAPARCH) for all indices was the FIAPARCH one.[20] The SIC results largely concur with the AIC, HQIC or SHIC results.[21]

Next, recall that the two common values of the power term imposed throughout much of the GARCH literature are the values of two (Bollerslev's model) and unity (the Taylor/Schwert specification). The invalid imposition of a particular value for the power term may lead to sub-optimal modeling and forecasting performance (Brooks et al., 2000). Accordingly, we test whether the estimated power terms are

[18]As a general rule, the information criteria approaches suggest selecting the model which produces the lowest AIC, SIC, HQIC or SHIC values.

[19]The use of the information criteria techniques for comparing models has the advantage of being relatively less onerous compared to Wald testing procedures, which only allow formal pairwise testing of nested models (Brooks et al., 2000).

[20]Caporin (2003) performs a Monte Carlo simulation study and verifies that information criteria clearly distinguish the presence of long- memory in volatility.

[21]We do not report the AIC, SIC, HQIC or SHIC values for space considerations.

significantly different from unity or two using Wald tests. As reported in table 3, all eight estimated power coefficients are significantly different from unity (see column six). Further, with the exception of the CAC 40, FTSE 100 and Nikkei 225 indices, each of the power terms are significantly different from two (see the last column of table 3). Hence, on the basis of these results, in the majority of cases support is found for the (asymmetric) power fractionally integrated model, which allows an optimal power transformation term to be estimated. The evidence obtained from the Wald tests is reinforced by the model ranking provided by the four model selection criteria.[22] This is a noteworthy result since He and Teräsvirta (1998) emphasized that if the standard Bollerlsev type of model is augmented by the 'heteroscedasticity' parameter, the estimates of the ARCH and GARCH coefficients almost certainly change. More importantly, Karanasos and Schurer (2008) show that in the univariate GARCH-in-mean level formulation the significance of the in-mean effect is sensitive to the choice of the power term.

## 3.3 Multivariate Models

The analysis above suggests that the FIAPARCH formulation describes the conditional variances of the eight stock indices well. However, financial volatilities move together over time across assets and markets. Recognizing this commonality through a multivariate modeling framework can lead to obvious gains in efficiency and to more relevant financial decision making than can be obtained when working with separate univariate specifications (Bauwens and Laurent, 2005). Therefore, multivariate GARCH models are essential for enhancing our understanding of the relationships between the (co)volatilities of economic and financial time series. For recent surveys on multivariate specifications and their practical importance in various areas such as asset pricing, portfolio selection and risk management see e.g., Bauwens et al., (2006) and Silvennoinen and Teräsvirta (2007). Thus in this section, within the framework of the multivariate ccc model, we will analyze the dynamic adjustments of the variances for the various indices. Overall we estimate seven bivariate specifications; three for the European countries: CAC 40-DAX 30 (C-D), CAC 40-FTSE 100 (C-F) and DAX 30-FTSE 100 (D-F); three for the Asian countries: Hang Seng-Nikkei 225 (H-N), Hang Seng-Straits Times (H-S) and Nikkei 225-Straits Times (N-S); one for the S&P 500 and TSE 300 indices (SP-T). Moreover, we estimate two trivariate models: one for the three European countries (C-D-F) and one for the three Asian countries (H-N-S).

For the multivariate models, the log-likelihood to be maximized is given by

$$\begin{aligned}\log\mathcal{L} &= T\left[\log\Gamma\left(\frac{\upsilon+1}{2}\right)-\log\Gamma\left(\frac{\upsilon}{2}\right)-\frac{1}{2}\log\pi(\upsilon-2)\right]\\ &\quad -\frac{1}{2}\sum_{t=1}^{T}\left\{\log\det\mathbf{H}_t+\log\det\boldsymbol{\rho}+(\upsilon+1)\left[\log\left(1+\frac{\boldsymbol{\varepsilon}_t'\mathbf{H}_t^{-1/2}\boldsymbol{\rho}^{-1}\mathbf{H}_t^{-1/2}\boldsymbol{\varepsilon}_t}{(\upsilon-2)}\right)\right]\right\},\end{aligned}$$

[22] We do not report the AIC, SIC, HQIC or SHIC values for space considerations.

where $\Gamma(\cdot)$ denotes again the gamma function, $H_t = diag(h_t)$ and $\rho$ is the $2 \times 2$ $(3 \times 3)$ correlation matrix with unit diagonal elements and off-diagonal entries $\rho_{ij}$. Note, that the degrees of freedom are constrained to be equal for all equations. For more details, see, Davidson (2008).

#### 3.3.1 Bivariate Processes

The best fitting bivariate specification is chosen according to likelihood ratio results and the minimum value of the information criteria (not reported). In the majority of the models the AR coefficients are significant at the 5% level or better. In almost all cases a $(1, d, 1)$ order is chosen for the FIAPARCH formulation. Only for the H-S and N-S models do we choose $(0, d, 1)$ order for the Straits Times index, and $(1, d, 0)$ order for the Hang Seng index. Note that this in line with our findings for the univariate models where the $\beta$ parameter was insignificant for Straits Times, while the $\phi$ parameter was insignificant for Hang Seng. In six out of the fourteen models the leverage term $(\gamma)$ is significant.

As in the univariate case, it is significant in both indices for the H-S case and in the DAX 30 index for the D-F case. In addition, in the bivariate case it is also significant in the Tse 300 index for the SP-T model and in the Nikkei 225 for the N-S one. In almost all cases the power term $(\delta)$ and the fractional differencing parameter $(d)$ are highly significant. In the D-F, H-S and N-S models the two countries generated very similar power terms: (1.28, 1.36), (1.42, 1.47) and (1.70, 1.62) respectively. In four out of the seven bivariate formulations the two countries generated very similar fractional parameters. These are the SP-T, the C-F, the H-N and the H-S models. The corresponding pairs of values are: (0.22, 0.21), (0.24, 0.29), (0.36, 0.35) and (0.16, 0.13). Interestingly, in the majority of the cases the estimated power and fractional differencing parameters of the bivariate models take lower values than those of the corresponding univariate models. In all cases the estimated ccc $(\rho)$ is highly significant. Interestingly, it is rather high among the American and European indices, and rather low among the Asian indices. Finally, the degrees of freedom $(\upsilon)$ parameters are highly significant and the ARCH parameters satisfy the set of necessary conditions sufficient to guarantee the non-negativity of the conditional variances (see, Conrad and Haag, 2006). In the majority of the cases the hypothesis of uncorrelated standardized and squared standardized residuals is well supported (see the last two rows of table 4).

Table 4: Bivariate AR-FI(A)PARCH models (ML Estimation)

| | SP-T | | C-D | | C-F | | D-F | | H-N | | H-S | | N-S | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | SP | T | C | D | C | F | D | F | H | N | H | S | N | S |
| $\zeta_i$ | $-0.05^*$ $(-4.51)$ | 0.17 (13.86) | $-0.03$ $(-2.63)$ | $0.02^*$ (1.53) | 0.05 (3.88) | 0.04 (2.70) | $0.01^*$ (0.37) | $-0.03$ $(-2.14)$ | 0.05 (3.44) | $-0.02$ $(-1.07)$ | 0.03 (1.79) | 0.14 (9.16) | $-0.03$ $(-2.08)$ | 0.15 (9.33) |
| $\beta_i$ | 0.46 (4.78) | 0.33 (2.27) | 0.50 (3.94) | 0.62 (9.00) | 0.35 (1.55) | 0.45 (1.48) | 0.55 (5.51) | 0.42 (1.93) | 0.57 (3.70) | 0.46 (3.82) | 0.08 (2.85) | | 0.43 (3.41) | |
| $\phi_i$ | 0.26 (3.73) | 0.18 (1.52) | 0.26 (4.30) | 0.24 (5.60) | 0.16 (1.24) | 0.20 (1.48) | 0.20 (4.96) | 0.17 (1.74) | 0.33 (3.94) | 0.15 (2.11) | | $-0.02$ (0.87) | 0.14 (1.76) | $-0.07$ (1.78) |
| $\gamma_i$ | | 0.34 (2.46) | | | | | 0.14 (1.68) | | | 0.10 (1.74) | 0.11 (1.73) | 0.47 (3.16) | 0.11 (2.02) | |
| $\delta_i$ | 1.85 (8.81) | 1.59 (8.37) | 1.55 (9.12) | 1.23 (9.84) | 1.76 (7.65) | 1.55 (5.54) | 1.28 (11.64) | 1.36 (8.00) | 1.49 (17.71) | 1.69 (13.75) | 1.42 (12.07) | 1.47 (12.01) | 1.70 (13.75) | 1.62 (15.71) |
| $d_i$ | 0.22 (5.50) | 0.21 (5.25) | 0.30 (3.00) | 0.44 (6.28) | 0.24 (2.18) | 0.29 (1.61) | 0.40 (4.44) | 0.28 (2.15) | 0.36 (3.18) | 0.35 (5.04) | 0.16 (7.58) | 0.13 (5.79) | 0.33 (5.15) | 0.23 (6.64) |
| $\rho$ | 0.65 (21.33) | | 0.65 (20.54) | | 0.67 (20.90) | | 0.54 (19.48) | | 0.33 (11.03) | | 0.43 (17.02) | | 0.26 (12.32) | |
| $\upsilon$ | 13.69 (9.85) | | 16.69 (6.76) | | 18.96 (6.94) | | 18.13 (6.06) | | 12.62 (11.03) | | 11.31 (11.44) | | 12.42 (10.47) | |
| $Q_{12}$ | 18.08 [0.11] | 10.74 [0.55] | 34.92 [0.00] | 10.17 [0.60] | 10.33 [0.59] | 15.80 [0.20] | 12.48 [0.41] | 36.27 [0.00] | 22.30 [0.03] | 9.78 [0.63] | 39.18 [0.00] | 20.69 [0.06] | 12.93 [0.37] | 16.36 [0.17] |
| $Q^2_{12}$ | 2.77 [0.99] | 2.81 [0.99] | 20.28 [0.06] | 5.17 [0.95] | 24.51 [0.02] | 40.18 [0.00] | 3.31 [0.99] | 17.44 [0.13] | 35.79 [0.00] | 54.52 [0.00] | 108.29 [0.00] | 5.03 [0.96] | 58.83 [0.00] | 1.50 [1.00] |

Notes: For each of the seven pairs of indices, table 4 reports ML parameter estimates for the bivariate AR-FI(A)PARCH model. SP-T denotes the bivariate process for the S&P 500 and TSE 300 indices. C-D, C-F and D-F indicate the three bivariate models for the European indices. H-N, H-S and N-S stand for the three bivariate specifications for the Asian indices. * For the S&P 500 and DAX 30 indices we estimate AR models of order 3 and 4 respectively. The numbers in parentheses are t-statistics. $Q_{12}$ and $Q^2_{12}$ are the 12th order Ljung-Box tests for serial correlation in the standardized and squared standardized residuals respectively. The numbers in brackets are $p$-values.

Next we examine the Wald statistics for the linear constraints $d = 0$ (stable APARCH) and $d = 1$ (IAPARCH). As seen in table 5 the W tests clearly reject both the stable and integrated null hypotheses against the FIAPARCH one. We should emphasize that in the presence of long-memory in volatility Christensen and Nielsen (2007) reassess the relation between the risk-return trade-off, serial dependence in volatility, and the elasticity of asset values with respect to volatility. They show that the elasticity is smaller in magnitude than earlier estimates, and much more stable under variations in the long-memory parameter than in the short-memory case. Thus, they point out that the high elasticities reported earlier should be interpreted with considerable caution. They also highlight the fact that the way in which volatility enters in the asset evaluation model is crucial and should be considered carefully. This is due to the fact that the memory properties of the volatility process carry over to the stock return process through the risk premium link.

We also test whether the estimated power terms are significantly different from unity or two using Wald tests. The eight estimated power coefficients are significantly different from either unity or two (see the last two columns of table 5).

Table 5: Tests for restrictions on fractional differencing and power term parameters

| $H_0$: | | $d$'s= 0 | $d$'s= 1 | | $\delta$'s= 1 | $\delta$'s= 2 |
|---|---|---|---|---|---|---|
| | $d$'s | $W$ | $W$ | $\delta$'s | $W$ | $W$ |
| SP-T | 0.22 {0.04}-0.21 {0.04} | 37[0.00] | 432[0.00] | 1.85 {0.21}-1.59 {0.19} | 141[0.00] | 141[0.00] |
| C-D | 0.30 {0.10}-0.44 {0.07} | 39[0.00] | 241[0.00] | 1.55 {0.17}-1.23 {0.12} | 97[0.00] | 124[0.00] |
| C-F | 0.24 {0.11}-0.29 {0.18} | 5[0.10] | 112[0.00] | 1.76 {0.23}-1.55 {0.28} | 30[0.00] | 81[0.00] |
| D-F | 0.40 {0.09}-0.28 {0.13} | 25[0.00] | 279[0.00] | 1.29 {0.11}-1.36 {0.17} | 130[0.00] | 155[0.00] |
| H-N | 0.36 {0.11}-0.35 {0.07} | 36[0.00] | 65[0.00] | 1.49 {0.08}-1.69 {0.12} | 319[0.00] | 318[0.00] |
| H-S | 0.16 {0.02}-0.13 {0.02} | 33[0.00] | 255[0.00] | 1.42 {0.12}-1.47 {0.12} | 228[0.00] | 247[0.00] |
| N-S | 0.33 {0.06}-0.23 {0.03} | 77[0.00] | 158[0.00] | 1.70 {0.12}-1.62 {0.10} | 341[0.00] | 284[0.00] |

Notes: For each of the seven pairs of indices, table 5 reports the values of the Wald (W) statistics of the unrestricted bivariate FI(A)PARCH and restricted ($d$'s= $0, 1$; $\delta$'s= $1, 2$) models respectively. SP-T denotes the bivariate model for the S&P 500 and TSE 300 indices. C-D, C-F and D-F indicate the three bivariate models for the European indices. H-N, H-S and N-S stand for the three bivariate models for the Asian indices. The numbers in $\{\cdot\}$ are standard errors. The numbers in $[\cdot]$ are p values.

### 3.3.2 Trivariate Specifications

Table 6 reports the parameters of interest for the two trivariate FI(A)PARCH(1,1) models. In two out of the three Asian countries the leverage term ($\gamma$) is weakly significant. In all cases the power term ($\delta$) and the fractional differencing parameter ($d$) are highly significant. Similarly, in all cases the estimated ccc ($\rho$) and degrees of freedom ($\upsilon$) parameters are highly significant and the ARCH parameters satisfy the set of necessary conditions sufficient to guarantee the non-negativity of the conditional variances (see, Conrad and Haag, 2006). In particular, the estimates of $\rho$ confirm the results from the bivariate models, i.e. the conditional correlation between the European indices is considerably stronger than between the Asian indices.

Table 6: Trivariate AR-FI(A)PARCH$(1, d, 1)$ models (ML Estimation)

| | C-D-F | | | H-N-S* | | |
|---|---|---|---|---|---|---|
| | C | D | F | H | N | S |
| $\beta_i$ | 0.19 (1.40) | 0.43 (4.61) | 0.22 (1.15) | 0.39 (1.92) | 0.38 (2.50) | 0.78 (18.85) |
| $\phi_i$ | 0.11 (0.90) | 0.22 (3.35) | 0.09 (0.61) | 0.28 (1.56) | 0.15 (1.53) | 0.81 (22.08) |
| $\gamma_i$ | - | - | - | 0.02 (1.46) | 0.07 (1.60) | – |
| $\delta_i$ | 1.83 (10.95) | 1.25 (9.52) | 1.56 (7.12) | 1.47 (13.36) | 1.53 (10.20) | 1.88 (11.75) |
| $d_i$ | 0.11 (4.16) | 0.25 (5.43) | 0.15 (3.27) | 0.18 (4.50) | 0.26 (0.07) | 0.08 (4.39) |
| | C-D | D-F | C-F | H-N | N-S | H-S |
| $\rho$ | 0.66 (21.07) | 0.56 (19.86) | 0.68 (21.70) | 0.32 (14.84) | 0.25 (12.19) | 0.43 (16.92) |
| $\upsilon$ | | 9.60 (17.36) | | | 8.42 (20.54) | |

Notes: Table 6 reports ML parameter estimates for the two trivariate (white noise) FI(A)PARCH$(1, d, 1)$ models. C-D-F and H-N-S denote the models for the European and Asian countries respectively. *For the Nikkei 225 and Straits Times indices we estimate AR(1) models. The numbers in parentheses are $t$-statistics.

### 3.4 On the Similarity of the Fractional/Power Parameters

We test for the apparent similarity of the optimal fractional differencing and power term parameters for each of the eight country indices using pairwise Wald tests:

$$\mathrm{W}_d = \frac{(d_1 - d_2)^2}{\mathsf{Var}(d_1) + \mathsf{Var}(d_2) - 2\mathsf{Cov}(d_1, d_2)}, \quad \mathrm{W}_\delta = \frac{(\delta_1 - \delta_2)^2}{\mathsf{Var}(\delta_1) + \mathsf{Var}(\delta_2) - 2\mathsf{Cov}(\delta_1, \delta_2)},$$

where $d_i$ ($\delta_i$), $i = 1, 2$, is the fractional differencing (power term) parameter from the bivariate FIAPARCH model estimated for the national stock market index for country $i$, $\mathsf{Var}(d_i)$, $\mathsf{Var}(\delta_i)$ are the corresponding variances, and $\mathsf{Cov}(d_1, d_2)$, $\mathsf{Cov}(\delta_1, \delta_2)$ are the corresponding covariances. The above Wald statistics test whether the fractional differencing (power term) parameters of the two countries are equal $d_1 = d_2$ ($\delta_1 = \delta_2$), and are distributed as $\chi^2_{(1)}$.

The following table presents the results of this pairwise testing procedure for the various bivariate models.[23] Several findings emerge from this table. The estimated long-memory parameters for the various (a)symmetric specifications are in the range $0.20(0.13) \leq d \leq 0.48(0.36)$ while the estimated power terms are in the range $1.19(1.18) \leq \delta \leq 2.00(1.86)$. In all cases for the American and Asian indices (and in the majority of the cases for the European countries) the values of the two coefficients ($d_i$, $\delta_i$) for the asymmetric models (see columns $\mathrm{B}^a$) are lower than the corresponding values for the symmetric formulations (see columns $\mathrm{B}^s$). The values of the Wald tests in the table support the null hypothesis that the two estimated fractional parameters and the two power term coefficients are not significantly different from one another.

All specifications generated very similar long-memory coefficients between countries. For example, in the asymmetric SP-T and H-N models, which generated very similar fractional parameters ($0.22, 0.23$ and $0.36, 0.35$ respectively), the two coefficients were, as expected, not significantly different ($W = 0.04, 0.02$ respectively). The null hypothesis of equal long-memory coefficients is rejected at the 5% level only for the symmetric C-D and the asymmetric D-F models. Both include the DAX 30 index with a relatively high persistence parameter. As regards the power term, the two models for CAC 40 and DAX 30 indices are those with the highest differences: $1.59 - 1.18 = 0.41$ and $1.55 - 1.19 = 0.36$ respectively. For these two cases the values of the Wald tests ($W = 6.57$, $3.85$ respectively) are significant at the 5% level. For all other models, but one, the equality of the power terms cannot be rejected. For example, in models which generated very similar power terms, such as the symmetric D-F one ($1.35, 1.40$) or the asymmetric H-S ($1.42, 1.47$) the two coefficients were, as expected, not significantly different ($W = 0.10$ in both cases). Finally, it is noteworthy that in the majority of cases the values of the coefficients $d_i$ and $\delta_i$ for the univariate (a)symmetric formulations (not reported) are higher than the corresponding values for the

[23] For reasons of comparability, in all the various bivariate models for both indices we estimated AR(1)-FI(A)PARCH(1,1) processes. That is, the parameter values for $d$ and $\delta$ presented in table 7 are not necessarily the same as the ones in table 4.

Table 7: Tests for similarity of fractional and power terms (Bivariate Models)

| | Symmetric Models | | | | | | | Asymmetric Models | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | SP-T | C-D | C-F | D-F | H-N | H-S | N-S | SP-T | C-D | C-F | D-F | H-N | H-S | N-S |
| | | | | | | | | $d$ | | | | | | |
| $d_1$ | 0.24 | 0.30 | 0.24 | 0.48 | 0.38 | 0.26 | 0.34 | 0.22 | 0.19 | 0.26 | 0.36 | 0.36 | 0.16 | 0.32 |
| $d_2$ | 0.27 | 0.45 | 0.29 | 0.41 | 0.36 | 0.20 | 0.24 | 0.23 | 0.29 | 0.33 | 0.23 | 0.35 | 0.13 | 0.22 |
| $W$ | 0.25 | 4.16 | 0.26 | 0.75 | 0.04 | 0.85 | 1.46 | 0.04 | 1.24 | 0.05 | 6.00 | 0.02 | 1.61 | 1.62 |
| | | | | | | | | $\delta$ | | | | | | |
| $\delta_1$ | 2.00 | 1.55 | 1.76 | 1.35 | 1.50 | 1.49 | 1.80 | 1.86 | 1.59 | 1.74 | 1.27 | 1.49 | 1.42 | 1.66 |
| $\delta_2$ | 1.68 | 1.19 | 1.55 | 1.40 | 1.79 | 1.68 | 1.68 | 1.51 | 1.18 | 1.51 | 1.39 | 1.70 | 1.47 | 1.58 |
| $W$ | 2.38 | 3.85 | 1.08 | 0.10 | 4.43 | 1.59 | 0.60 | 2.59 | 6.57 | 1.55 | 0.24 | 1.66 | 0.10 | 0.19 |

Notes: SP-T denotes the bivariate model for the S&P 500 and TSE 300 indices respectively. C-D, C-F and D-F indicate the three bivariate models for the European indices. H-N, H-S and N-S stands for the three bivariate models for the Asian indices. The $W$ rows report the corresponding Wald statistics. The 5% and 1% critical values are 3.84 and 6.63 respectively.

(a)symmetric bivariate and trivariate (not reported) models.[24]

# 4 Forecasting Methodology

## 4.1 Evaluation Criteria

Financial market volatility is one of the most important attributes that affect the day-to-day operation of the Finance industry. It is a key driver in investment analysis and risk management. More recently, there is an increasing interest in trading on volatility itself as evidence by the volatility option contracts launched by the CBOE (Chicago Board of Option Exchange) in March 2006 (Hyung, Poon and Granger, 2006).

As Poon and Granger (2003) point out volatility forecasting is an important task in financial markets, and it has held the attention of academics and practitioners over the last two decades.[25] Elliot and Timmermann (2008) review various issues concerning economic forecasts. Since the publication of Ding et. al. (1993) there has been a lot of research investigating if the fractional integrated models could help to

[24]We do not report the results from the univariate and trivariate models for reasons of brevity.

[25]Several empirical studies examine the forecast performance of various GARCH models. The survey by Poon and Granger (2003) provides, among other things, an interesting and extensive synopsis of them.

make better volatility forecasts. Hyung et al. (2006) compare the out-of-sample forecasting performance of various short and long-memory volatility models. They find that for volatility forecasts of 10 days and beyond, the FIGARCH specification is the dominant one. In this section we examine the ability of the various univariate/multivariate fractionally integrated and power asymmetric ARCH models to forecast stock return volatility.[26]

Our full sample consists of 4,255 trading days and each model is estimated over the first 4,055 observations of the full sample, i.e. over the period 1st January 1988 to 16th July 2003. As a result the out-of-sample period is from 17th July 2003 to 22nd April 2004 providing 200 daily observations. The parameter estimates obtained with the data from the in-sample period are inserted in the relevant forecasting formulas and volatility forecasts $\widehat{h}_{t+1}$ calculated given the information available at time $t = T(= 4,055), \ldots, T + 199(= 4,254)$, i.e. 200 one-step ahead forecasts are calculated.

In order to evaluate the forecast performance of the different model specifications we need (a) to obtain a valid proxy for the true but unobservable underlying volatility and (b) to specify certain loss functions.[27] A natural candidate for the proxy are the squared returns which are an unbiased estimator for the unobserved conditional variance. However, compared to realized volatility the squared returns are a noise proxy and as shown in Patton (2007) distortions in the rankings of competing forecasts can arise when using noisy proxies. Whether such distortions arise depends on the choice of the loss function. Patton (2007) provides necessary and sufficient conditions on the functional form of the loss function to ensure that the ranking is the same whether it is based on the true conditional variance or some conditionally unbiased volatility proxy. Two loss functions which satisfy these condition are the mean square error (MSE) statistic and the QLIKE statistic.[28] Consequently, we will employ the MSE which is, of course, one of the most commonly employed criteria in the existing literature (see, e.g., Andersen et al., 1999). In addition, we employ the QLIKE statistic, which corresponds to the loss implied by a Gaussian likelihood, is extensively discussed in Bollerslev et al. (1994) and applied in, e.g., Hansen and

[26]For the literature in the forecasting performance of univariate fractionally integrated and power ARCH models see, among others, Degiannakis (2004), Hansen and Lunde (2006) and Ñíguez (2007). In addition, Angelidis and Degiannakis (2005) examine whether a simple GARCH specification or a complex FIAPARCH model generates the most accurate forecasts in three areas: option pricing, risk management and volatility forecasting.

[27]As Andersen et al. (1999) point out, it is generally impossible to specify a forecast evaluation criterion that is universally acceptable (see also, e.g., Diebold et al., 1998). This problem is particularly acute in the context of nonlinear volatility forecasting. Accordingly, there is a wide range of evaluation criteria used in the literature. Following Andersen et al. (1999) we shall not use any of the complex economically motivated criteria but instead we will report summary statistics based directly on the deviation between forecasts and realizations. Three out-of-sample forecast performance measures will be used to evaluate and compare the various models.

[28]Similarly, Awartani and Corradi (2005) point out that in comparing the relative predictive accuracy of various models, if the loss function is quadratic, the use of squared returns ensures that we actually obtain the correct ranking of models.

Lunde (2005). Finally, in addition to those robust loss functions we make use of an error statistic which is applied by Peters (2001). This is the adjusted mean absolute percentage error (AMAPE) (see table 8 below). In contrast to the simple mean absolute percentage error the AMAPE corrects for the problem of asymmetry between the actual and forecast values.

Table 8: Forecast evaluation criteria

| | |
|---|---|
| MSE: | $k^{-1} \sum\limits_{t=T+1}^{T+k} (\widehat{h}_t - r_t^2)^2$ |
| QLIKE: | $k^{-1} \sum\limits_{t=T+1}^{T+k} [\ln(\widehat{h}_t) + r_t^2/\widehat{h}_t]$ |
| AMAPE: | $k^{-1} \sum_{t=T+1}^{T+k} \left\|(\widehat{h}_t - r_t^2)/(\widehat{h}_t + r_t^2)\right\|$ |

Notes: $k$ is the number of steps ahead, $T$ is the sample size, $\widehat{h}_t$ is the forecasted variance and $r_t^2$ are the squared returns.

On the basis of several model selection techniques the superior fitting specification was the FIAPARCH one (see section 3). While such model fitting investigations provide useful insights into volatility, the specifications are usually selected on the basis of full sample information. For practical forecasting purposes, the predictive ability of these models needs to be examined out-of-sample. The aim of this section is to examine the relative ability of the various long-memory and power formulations to forecast daily stock return volatility. For each index we calculated the three forecast error statistics for the specifications APARCH, IAPARCH, FIAPARCH($\delta = 1$), FIAPARCH($\delta = 2$) and FIAPARCH in the univariate, bivariate and (where possible) trivariate version. Hence, overall fifteen values of each forecast error statistic are available for each index. Instead of presenting all the figures, we decided to present in table 9 only the best and the worst specification for each index as identified by the forecast error statistic. In addition, we tested whether the values of the forecast error statistics from the best and the worst model are statistically significant using the Diebold and Mariano (1995) test. Table 9 contains the corresponding $p$-values (see the next section).

An examination of table 9 reveals that either a multivariate or a fractionally integrated (FI) or a power (P) or an asymmetric (A) process is clearly superior. That is, there is strong evidence that the restrictive univariate (U), stable, symmetric Bollerlsev's type of process is inferior to one of the more flexible specifications. The results can be summarized as follows. Only in three cases is the best ranked model, as assessed by the forecasting criteria, the univariate one. Both MSE and AMAPE loss functions uniformly favor either bivariate or trivariate specifications (see the second and fourth column of table 9).

Table 9: Best versus worst ranked models

| | MSE | QLIKE | AMAPE |
|---|---|---|---|
| S&P 500 | B-FIAP vs. U-FIAP | B-IAP vs. U-FIAP | B-AP vs. U-FIAP |
| | [0.00] | [0.03] | [0.02] |
| TSE 300 | B-FIAP vs. U-IAP | U-FIP vs. U-IAP | B-AP vs. U-IAP |
| | [0.14] | [0.00] | [0.00] |
| CAC 40 | T-P vs. $\mathrm{B_F}$-FIA($\delta = 2$) | T-IP vs. $\mathrm{B_F}$-FIA($\delta = 2$) | T-IP vs. $\mathrm{B_F}$-FIA($\delta = 2$) |
| | [0.00] | [0.15] | [0.00] |
| DAX 30 | $\mathrm{B_F}$-AP vs. U-FIAP | U-FIA($\delta = 1$) vs. $\mathrm{B_C}$-FIA($\delta = 2$) | $\mathrm{B_F}$-AP vs. $\mathrm{B_F}$-FIA($\delta = 2$) |
| | [0.00] | [0.08] | [0.17] |
| FTSE 100 | T-P vs. $\mathrm{B_C}$-FIA($\delta = 2$) | T-P vs. $\mathrm{B_C}$-FIA($\delta = 2$) | $\mathrm{B_D}$-AP vs. $\mathrm{B_C}$-FIA($\delta = 2$) |
| | [0.00] | [0.01] | [0.00] |
| Hang Seng | $\mathrm{B_S}$-FIA vs. U-AP | $\mathrm{B_N}$-AP vs. T-FIAP | T-FIA($\delta = 2$) vs. U-FIA($\delta = 2$) |
| | [0.00] | [0.02] | [0.26] |
| Nikkei 225 | $\mathrm{B_S}$-FIA($\delta = 1$) vs. U-FIAP | U-FI($\delta = 1$) vs. T-AP | T-FIA($\delta = 2$) vs. U-AP |
| | [0.12] | [0.03] | [0.67] |
| Straits Times | $\mathrm{B_H}$-FIAP vs. $\mathrm{B_N}$-IAP | $\mathrm{B_H}$-FIA($\delta = 2$) vs. U-AP | T-FIAP vs. U-AP |
| | [0.00] | [0.01] | [0.00] |

Notes: U, B and T stand for univariate, bivariate and trivariate specifications respectively. (F)I, A and P indicate (fractionally) integrated, asymmetric and power models respectively. The subscripts refer to the jointly estimated index of the bivariate model, e.g., the subscript $F$ indicates that the bivariate model is estimated with the FTSE 100 index. The numbers in brackets are the $p$-values from the Diebold and Mariano (1995) test.

For the two American indices in five out of the six cases a bivariate model is selected as being best (see the first two rows of table 9). The results for the European countries show the close connection between the three volatilities. In five cases a trivariate specification is the best performing model and in three cases a bivariate one. Similarly, for the Asian indices in only one case do the statistics rank the univariate formulation first (see the last three rows of table 9). Overall, the multivariate formulation has the best statistics for twenty one out of the twenty four cases.

Moreover, in the Asian countries the (fractionally) integrated model is favored in all but one case. Similarly, for the S&P 500 and the TSE 300 indices the statistics indicate the superiority of the fractionally integrated specification. The power formulation is the dominant one in the European and American countries. In particular, for the European indices the restriction that $\delta = 2$ characterizes with one exception the worst performing specification. In summary, the best formulations as ranked by the forecast error statistics are multivariate models. For the American and Asian indices the long-memory property appears to be important for the forecast performance, while for the European and American indices power specifications are dominant.

## 4.2 Tests of Equal Forecast Accuracy

In the previous Section in some cases the statistics do not allow for a clear distinction between the ranking models, which is evidenced by the marginal difference in relative accuracy which separates the three models (results not reported).[29] Thus next we move to the pairwise comparison of the best and the worst specifications.

In this section we utilize the tests proposed by Diebold and Mariano (1995) and Harvey et al. (1997). Before moving to the two tests some notation is needed. First, we denote the 1-step ahead loss functions for the best and worst models as $L_{bt}^{(i)}(r_t^2, \widehat{h}_{bt})$ and $L_{wt}^{(i)}(r_t^2, \widehat{h}_{wt})$ $(t = T+1, \ldots, T+k)$, where $i \in$ {MSE, QLIKE, AMAPE}, respectively. Forecasts of the squared returns are generated using the fixed forecasting scheme (described in West and McCracken, 1998, p. 819). Next, let $\Delta_t = L_{bt}^{(i)} - L_{wt}^{(i)}$ and $\overline{\Delta}$ denote its sample mean, i.e. $\overline{\Delta} = k^{-1} \sum\limits_{t=T+1}^{T+k} \Delta_t$. The test proposed by Diebold and Mariano (1995) is formed as

$$S = [\widehat{\mathsf{Var}}(\overline{\Delta})]^{-1/2} \overline{\Delta},$$

with

$$\widehat{\mathsf{Var}}(\overline{\Delta}) = \frac{2\pi \widehat{f}_\Delta(0)}{k},$$

where $\widehat{f}_\Delta(0)$ is a consistent estimate of the spectral density function of $\Delta$ at frequency zero. Under the null hypothesis $S$ has an asymptotic standard normal distribution.[30]

As seen in table 9 the evidence obtained from the loss functions is reinforced by the Diebold-Mariano test. Clearly the test discriminates between the best and the worst model. That is, in the majority of the cases (eighteen out of twenty four) the test indicates the superiority of the best formulation over the worst one. In particular, for the USA and Canada, in four out of the five cases the worst model (univariate) is rejected in favor of the best (multivariate) one. For the Asian indices, the Diebold-Mariano test indicates the superiority of the best (fractionally integrated) specification over the worst (stable) one in four out of the five cases. The long-memory characteristic has important implications for volatility forecasting and option pricing. Option pricing in a stochastic volatility setting requires a risk premium for the unhedgeable volatility risk. The fractionally integrated series lead to volatility forecasts larger than those from short-memory models which immediately translates into higher option prices. This could be an explanation for the better pricing performance of FIGARCH in this case (Hyung et al., 2006).

[29] In addition, in some cases the ranking of the models varies depending upon the choice of the error statistic. Hence, as Brailsford and Faff (1996) point out, caution should be exercised in the interpretation of the obtained rankings.

[30] Harvey et al. (1997) proposed a small sample correction for the Diebold and Mariano (1995) statistic. Their modified test statistic is $t$-distributed with $k-1$ degrees of freedom. The results from this statistic are qualitatively similar to the original Diebold and Mariano (1995) statistic and, hence, are not reported.

Further, for the European countries, in five out of the seven cases the power (best) formulation outperforms the Bollerslev (worst) one. Finally, it is noteworthy that in the majority of the cases both the best and the worst formulation is an asymmetric one.[31]

# 5 Discussion

## 5.1 The Empirical Evidence

Brooks et al. (2000) analyzed the applicability of the stable APARCH model to national stock market returns for various industrialized countries. However, as in all cases the estimated values of the persistence coefficients were quite close to one, there was a need to examine closely the possibility of long-memory persistence in the conditional volatility.

In our paper, strong evidence has been put forward suggesting that the conditional volatility for eight national stock indices is best modeled as a FIAPARCH process. On the basis of Wald tests and information criteria the fractionally integrated model provides statistically significant improvement over its integrated counterpart. One can also reject the more restrictive stable process, and consequently all the existing specifications (see Ding et al. 1993) nested by it in favor of the fractionally integrated parameterization. Hence, our analysis has shown that the FIAPARCH formulation is preferred to both the stable and the integrated ones. In other words, the fractionally integrated process appeared to have superior ability to differentiate between stable specifications and their integrated alternatives.

The Bollerslev formulation is nested within the power specification. Brooks et al. (2000) applied the likelihood ratio test to this nested pair. The results of this test were mixed as far as supporting the presence of power effects is concerned. For the German and French indices there was strong evidence of power effects. For a further two countries (US and Japan) there was mild evidence and for Hong Kong there was only weak evidence in support of the power specification. In contrast, United Kingdom, Canada and Singapore show no evidence of power effects as the Bollerslev formulation could not be rejected in favor of the power one.

Moreover, the Taylor/Schwert specification is nested within the power model. For all countries tested, with the exceptions of Hong Kong and Singapore, the test statistics indicated a preference for the Taylor/Schwert formulation over the power specification. Accordingly, Brooks et al. (2000) concluded that allowing the power term to take on values other than unity did not significantly enhance the model. In

[31] We also utilize two encompassing tests proposed by Ericsson (1992) and Harvey et al. (1998). We do not report the results for reasons of brevity. For example, we find that for the FTSE 100 index, in the univariate and bivariate F-C models, the FIAPARCH formulation outperforms the restricted Taylor/Schwert and Bollerlsev specifications, and the stable/integrated ones as well.

other words there was a lack of evidence to suggest the need for power effects in the absence of long-range volatility dependence, as the likelihood ratio tests produced insignificant calculated values, indicating an inability to reject the Taylor/Schwert formulation over the power specification for eight of the national indices tested.

The results for the more general FIAPARCH model are in stark contrast. According to our analysis all eight countries show strong evidence (both the likelihood ratio and Wald tests produce significant calculated values) of power effects when long-memory persistence in the conditional volatility has been taken into account, as both the Bollerslev and Taylor/Schwert specifications were rejected in favor of the power formulation. Further, comparing the pairwise testing results of the log-likelihood procedures to the relative model rankings provided by the four alternative criteria we observed that the findings were generally robust. That is, where the log-likelihood results provided unanimous support for the FIAPARCH specification over either the Bollerslev or Taylor/Schwert (asymmetric) FIGARCH formulations, the model selection criteria concurred without exception. Thus, the inclusion of a power term and a fractional unit root in the conditional variance equation appear to augment the model in a worthwhile fashion.

Finally, we should also emphasize that the above results were robust to the dimension of the process. That is, the evidence obtained from the univariate models on the superiority of the FIAPARCH specification was reinforced by the multivariate processes. It is noteworthy that the results are not qualitatively altered by changes in the dimension of the model.

## 5.2 Possible Extensions

The main goal of this paper was to explore the issue of how generally applicable the ccc M-FIAPARCH formulation is to a wide range of national stock market returns. Possible extensions of this article can go in different directions. Kim et al. (2005) use a bivariate ccc FIAPARCH-in-mean process to model the volume-volatility relationship. In the context of our analysis, incorporating volumes either in the mean or in the variance specification or in both could be at work. We look forward to clarifyng this out in future work. He and Teräsvirta (1999) emphasize that if the standard Bollerslev type of model is augmented by the power term, the estimates of the other variance coefficients almost certainly change. More importantly, Karanasos and Schurer (2008) find that the relationship between the level of the process and its conditional variance, as captured by the in-mean parameter, is sensitive to changes in the values of the power term (see also Conrad and Karanasos, 2008b). Therefore, one promising avenue would be to adapt the multivariate model in a way that incorporates in-mean effects.

Moreover, Conrad and Karanasos (2008a) consider a formulation of the extended constant or time varying conditional correlation M-GARCH specification which allows for volatility feedback of either sign, i.e., positive or negative. We have not been able, in such a short space, to deal with the unrestricted extended (and/or time varying conditional correlation) version of the M-FIAPARCH model. We should also emphasize that the most commonly used measures of stock volatility apart from the conditional variance from an ARCH type of process is the realized volatility (see Andersen et al., 2003, and Conrad and Lamla, 2007) and the range-based intraday estimator (see Karanasos and Kartsaklas, 2008). In addition, Bai and Chen (2008) consider testing distributional assumptions in M-GARCH formulations based on empirical processes. To highlight the importance of using alternative measures of volatility and multivariate distributions in order to model the national stock market returns (and forecast their variances) we should have to go into greater detail than space in this paper permits.

In addition, one can estimate multivariate versions of the Hyperbolic APARCH and Hyberbolic FIAPARCH models (see, Schoffer, 2003 and Conrad, 2007 and the references therein). Further, Baillie and Morana (2007) introduce a new long-memory volatility specification, denoted by Adaptive FIGARCH, which is designed to account for both long-memory and structural change in the conditional variance process. One could provide an enrichment of the M-FIAPARCH by allowing the intercepts of the two means and variances to follow a slowly varying function as in Baillie and Morana (2007). This is undoubtedly a challenging yet worthwhile task. Finally, Pesaran and Timmermann (2002) suggest an estimation strategy that takes into account breaks and provides gains in forecasting ability. Pesaran et al. (2006) provide a new approach to forecasting time series that are subject to discrete structural breaks. Their results suggest several avenues for further research.

# 6 Conclusion

The purpose of the current paper was to consider the applicability of the multivariate fractionally integrated asymmetric power ARCH model to the national stock market returns for eight countries. It was found that the M-FIAPARCH formulation captures the temporal pattern of volatility for observable returns better than previous parameterizations. It also improves forecasts for volatility and thus is useful for financial decisions which utilize such forecasts.

We have provided an interesting comparison to the stable and integrated specifications. The results reject both the stable and integrated null hypotheses. This is consistent with the conditional volatility profiles in Gallant et al. (1993), which suggest that shocks to the variance are very slowly damped, but do die out. Moreover, all eight countries show strong evidence of power effects when asymmetries and/or long-memory persistence in the conditional volatility have been taken into account, as both the Bollerslev and Taylor/Schwert formulations were rejected in favor of the power specification. As convincingly argued by Brooks et al. (2000), for high frequency data which have a non-normal error distribution the presumption of an obvious superiority of a squared power term is lost. Other power transformations are more appropriate. Finally, the apparent similarity of the fractional differencing and power terms suggest that the M-FIAPARCH model has a quite general empirical validity across many different markets.

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