Nominal and Real Wage Rigidities. In Theory and in Europe

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February 2009

Abstract

In this paper I study the relation between real wage rigidity (RWR) and nominal price and wage rigidities. I show that in a standard DSGE model RWR is mainly affected by the two nominal rigidities and not by the other structural parameters. The degree of RWR, however, is influenced by the institutional assumption (e.g. Calvo vs. Taylor wage contracts and the clustering of contracts). I use empirical estimates for the duration of prices and wages for a number of European countries to calculate the degrees of RWR implied by the theoretical model. I then compare these values to existing cross-country evidence on measures that are related to the RWR.

Keywords: Inflation Persistence, Real Wage Rigidity, Nominal Wage Rigidity, DSGE models, Staggered Contracts,
JEL-Classification: E31, E32, E24, J51

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†The views expressed in this paper do not necessarily reflect those of the Oesterreichische Nationalbank.

1 Introduction

The simplest explanation for the existence of real wage rigidities sees them as a consequence of two nominal rigidities: a nominal price rigidity and a nominal wage rigidity. Although this type of real wage rigidity is a crucial element of the current generation of DSGE models (cf. Christiano et al., 2005; Smets and Wouters, 2003) it is usually not in the focus of these papers and has so far not been analyzed in any detail.[1] In this paper I want to fill this gap. In particular, I am going to study how the two nominal rigidities interact to create a real wage rigidity (RWR), how sensitive the RWR reacts to changes in the nominal rigidities and to what extent plausible assumptions about price and wage stickiness are able to produce reasonable degrees of RWR.

Besides offering the most parsimonious explanation for the ubiquitous phenomenon of real persistence, the employed concept of RWR is also useful for a number of additional purposes. First, it is a concise summary measure of DSGE models that can be considered as a complement to the more commonly used impulse response functions. Second, it can be employed to study and emphasize the consequences of different institutional designs (e.g. Calvo vs. Taylor wage contracts, symmetric vs. asymmetric sector sizes etc.). Third, it is a highly appropriate measure for cross-country comparisons, in particular since it can be related in various ways to the existing empirical literature. I deal with all of these issues in this paper. In the rest of the introduction I will give more details on the background of the analyses and I will offer a preview of the results.

The concept of nominal wage rigidity is commonly related to the speed with which nominal wages can be changed in reaction to economic shocks. There seems to exist less unanimity about the exact meaning of real wage rigidity.[2] The definition by Blanchard (2006) can serve here as a useful reference point: "'Real wage rigidities' [capture] the speed at which real wages [adjust] to changes in warranted real wages [...]. The slower the adjustment, the higher and the longer lasting the effects of adverse shocks on unemployment" (Blanchard, 2006, p. 16). In the benchmark labor market model with complete flexibility the "warranted real wage" is typically given by the marginal rate of substitution between consumption and leisure. In formal terms this flexprice labor market equilibrium

[1]Some discussions about this issue can be found in Woodford (2003, 231f.) and in Rabanal and Ramírez (2003, 2005).

[2]In addition there exists also the concept of "real rigidity" (without the epitheton "wage"; see e.g. Ball and Romer, 1990). This is normally related to the "aggressiveness" with which nominal variables react to real developments. "Real rigidity" is thus not related to stickiness but rather to the strength of the reaction. All agents are assumed to be able to adjust instantaneously, but they decide to do so only slowly or cautiously.

can thus be written as: $\omega_t = mrs_t$, where ω_t and mrs_t are the logarithms of the real wage and the marginal rate of substitution, respectively.

The recent years have shown an increased interest in the issue of RWR. This has to do with the fact that RWR has been identified as one plausible element that can be added to standard models in order to improve their explanatory power. Hall (2005) and Milgrom and Hall (2008), e.g., have shown that RWR offers a straightforward solution to the famous "Shimer puzzle" (i.e. the relatively large fluctuations of unemployment). Blanchard and Galí (2007), on the other hand, have forcefully argued that RWR is a reasonable way to break the "divine coincidence" of standard New Keynesian models, to reestablish more plausible effects of disinflations and more realistic trade-offs for monetary policy.

As far as the reasons behind the rigidities of real wages are concerned, however, there does not exist much agreement. Blanchard and Katz (1999), in an early contribution, present a model in which unemployment benefits and wages react differently to changes in productivity growth. Hall (2005), on the other hand, uses a model where RWR follows from the existence of social norms while Hall and Milgrom (2008) present an argument based on sequential (real) wage bargaining. Blanchard and Galí (2007), finally, simply assume that the real wage ω_t is rigid for whatever reason and can be written as: $\omega_t = \gamma\omega_{t-1} + (1-\gamma)mrs_t$, where γ is their measure of RWR. In an appendix they motivate this short-cut formulation by referring to a model with "real wage staggering" while in a footnote they state that a number of alternative theories (based on, e.g., efficiency wages, shirking, rule-of-thumb wage-setting etc.) might lead to a similar reduced form expression. Interestingly, however, none of these papers deals explicitly with the possibility that the RWR could simply be understood as the consequence of two nominal rigidities: a nominal price and a nominal wage rigidity. In this paper I take the latter mechanism as the most parsimonious starting point which does not require any complicated (and maybe context- and country specific) assumptions about the real structure of the economy.

The derivations in this paper are based on the model by Erceg, Henderson and Levin [EHL] (2000). This is the benchmark model in the DSGE literature where both nominal price and nominal wage rigidities are introduced via Calvo contracts (Calvo, 1983) with fixed period-probabilities of reoptimization of prices and wages. I use this framework as a benchmark to derive a number of results. First, the EHL model leads to a solution that is of a form similar to the short-cut formulation in Blanchard and Galí (2007). In particular, one can write $\omega_t = \delta^*\omega_{t-1} + f\,(outputgap, supplyshocks)$, where $f(\cdot)$ is a linear function of the stated variables. Since the output gap itself can be expressed as a function of the marginal rate of substitution this equation is in fact close (although not identical) to the short-cut relation in Blanchard and Galí (2007). The parameter δ^* is thus a natural measure of RWR in the EHL model. I show that the two nominal rigidities are in fact the main determinants of the degree of RWR and that δ^* reacts rather insensitive to changes in the other structural parameters. It even remains almost unchanged if one assumes that output is exogenously given rather than being determined by a forward-looking IS-curve and a monetary policy rule. The solutions of the forward-looking New Keynesian model can also be written in a form that is very similar to a backward-looking Phillips curve specification. In particular, the expression is closely related to the well-established "triangle" model (cf. Gordon, 1998) in which the current rate of inflation is written as a function of past inflation and of current and past levels of the output gap (or the "unemployment gap" as a measure of demand pull) and supply shocks (as a measure of cost push). The weight of past inflation in this version of the backward-looking Phillips curve is identical to the measure of RWR δ^*.

Survey data on wage-setting practices often suggest that the assumption of Calvo wage contracts is not in line with the evidence. This is, e.g., the case for the data from the Wage Dynamics Network (WDN) of the ECB (cf. Druant et al. 2009) that will be described later. In particular, the hazard rate of wage changes is not constant for all contracts. On the contrary, the majority of agreements seems to follow a predetermined pattern with given contract lengths of one to two years. Furthermore, the data from the WDN also suggest that in many countries one can observe a clustering of contracts in certain months (mostly in January). In order to account for these important institutional characteristics of actual wage-setting practices I also solve the EHL model under the assumption of Taylor wage contracts, i.e. contracts with a fixed and predetermined length (cf. Taylor, 1980). In addition, I also allow for the fact that the sectors might be of different size, i.e. that there might be a clustering of contracts. The solution to this model is somewhat more involved than the one for the model with Calvo wage contracts. It can, however, again be written in a way that contains a measure of RWR $\tilde{\delta}$. Comparing the different measures of RWR leads to two conclusions. First, for the same calibration of structural parameters and the same average duration of prices and wages, the model with Taylor wage contracts involves a considerable smaller degree of RWR than the model with Calvo contracts. The reason for this is that the potentially very long duration of new contracts in the Calvo case increases average rigidities. Second, asymmetries in the sector size reduce RWR in the model with Taylor contracts. In the case where one sector subsumes 10% of all contracts the RWR is only about half than in the symmetric case and it approaches zero as one sector starts to dominate the economy. This result is related to the analysis in Olivei and Tenreyro (2007) and I discuss the underlying logic with the help of impulse response functions.

In the second part of the paper I use the implications of the theoretical model in order to study whether the crucial mechanism of the EHL model corresponds to the available empirical evidence. For this purpose, I first use recent survey evidence from the WDN. In particular, I take the data on average durations of prices and wages and on the clustering of wage contracts from Durant et al. (2009). Using standard values for the other structural parameters, I can then calculate the measure for RWR that is implied by the theoretical model under different assumptions about the institutional structure of the economy (i.e. about Calvo vs. Taylor wage contracts and about the clustering of contracts). This leads to a ranking of countries with respect to their degree of RWR. For the moment the survey data are still being processed but I will add this section of the paper as soon as the results become available.

In the last section of the paper I turn to the question whether the measure of RWR implied by the theoretical models are in line with the existing empirical evidence. A natural starting point to answer this question would be country-specific evidence on the degree of RWR. Since I have not been able to find comparable international evidence on this magnitude I focus instead on three measures that are closely related to δ^* and $\tilde{\delta}$: the sensitivity of the RWR with respect to the output gap (or the unemployment rate), the speed of adjustment in an error-correction mechanism and a measure that is similar to the sacrifice ratio. I have been able to find comparable international evidence on all three measures. I show that the solution of the EHL model can be transformed in various ways such that it leads to expressions that closely resemble their empirical counterparts.

Since the data from the WDN are not yet available I have so far not been able to compare the rankings based on the theoretical model with the rankings based on the surrogate measures from the empirical literature. As a first take it is, however, possible to compare the empirical measures amongst each other. If the theoretical EHL model is an accurate description of reality and if the empirical measures have been derived from valid specifications then the rankings should be highly correlated. The evidence shows rather mixed results. The average sensitivity of RWR with respect to the unemployment rate among a group of 21 countries is significantly negatively related to the sacrifice ratio, as predicted by the theoretical model. The correlation of these two concepts with an error-correction term for a smaller group of 12 European countries is, however, weaker. The preliminary results suggest that the RWR inherent in the EHL model might capture one main transmission mechanism while at the same time missing other important elements of reality. Future research will have to investigate further what lays behind the discrepancy of the theoretical, model-based predictions and the empirical data. A careful analysis of the WDN data will here be a first important step in this direction.

The paper is structured as follows. In the next section I present the standard EHL model with Calvo price and wage contracts and I derive the measure of RWR. In section 3 I study how the introduction of Taylor wage contracts and of asymmetric sector sizes changes the results. In section 4 I discuss the evidence of the WDN and I calculate various measures of RWR that are implied by the theoretical model based on the average duration of prices and wages for a number of European countries. Section 5 compares these figures to the existing empirical evidence and section 6 concludes.

2 The basic model with nominal price and nominal wage rigidities à la Calvo

2.1 The set-up of the model

I use the standard model with sticky prices and wages by Erceg, Henderson and Levin [EHL] (2000). In order to facilitate the comparison with the existing literature I use the exact set-up and notation of the model that is used in chapter 6 of Galí (2008). I therefore do not want to derive the linearized solutions of the microfounded model explicitly but just use the equations that are presented in Galí (2008). The model assumes that there exists a continuum of monopolistically competitive firms that produce differentiated products where ε_p stands for the elasticity of substitution among the product varieties. There exists a Calvo constraint on price-setting and each period only a fraction $(1-\theta_p)$ can reset their price while a fraction θ_p leaves the price unchanged (and where the probability of reoptimizing is independent of the history of past price changes). The average duration of a price is thus given by $\frac{1}{1-\theta_p}$. Nominal wage rigidity is introduced in a similar fashion. In particular, it is assumed that each household is specialized in one particular type of labor for which he is the monopolistic supplier and where each firm needs all differentiated labor types to produce its differentiated product. Also households are subject to the Calvo constraint and in each period only a fraction $(1-\theta_w)$ can freely adjust the wage rate. The elasticity of substitution among the different types of labor is denoted by ε_w.

The production function for firm i is given by: $Y_t(i) = A_t N_t(i)^{1-\alpha}$, where $N_t(i)$ is an index of labor inputs used in the production of good $Y_t(i)$. The period utility function of a representative household is given by: $U(C,N) = \frac{C^{1-\sigma}}{1-\sigma} - \frac{N^{1+\varphi}}{1+\varphi}$. Galí (2008) shows that the dynamic equilibrium of the model can be summarized in 5 equations:[3]

$$\pi_t^p = \beta E_t \pi_{t+1}^p + \kappa_p \tilde{y}_t + \lambda_p \tilde{\omega}_t \tag{1}$$

$$\pi_t^w = \beta E_t \pi_{t+1}^w + \kappa_w \tilde{y}_t - \lambda_w \tilde{\omega}_t \tag{2}$$

$$\tilde{\omega}_t = \tilde{\omega}_{t-1} + \pi_t^w - \pi_t^p - \Delta \omega_t^n \tag{3}$$

$$\tilde{y}_t = -\frac{1}{\sigma}\left(i_t - E_t \pi_{t+1}^p - r_t^n\right) + E_t \tilde{y}_{t+1} \tag{4}$$

$$i_t = \rho + \phi_p \pi_t^p + \phi_w \pi_t^w + \phi_y \tilde{y}_t + \upsilon_t, \tag{5}$$

where $\pi_t^p = p_t - p_{t-1}$ and $\pi_t^w = w_t - w_{t-1}$ denote price and wage inflation, respectively, i_t is the nominal interest rate, $\omega_t \equiv w_t - p_t$ is the real wage, $\tilde{y}_t \equiv y_t - y_t^n$ is the output gap and $\tilde{\omega}_t \equiv \omega_t - \omega_t^n$ the real wage gap. The level of natural output y_t^n that is used in the definition of the output gap refers to the equilibrium level of output that would prevail in the absence of *both* price and wage rigidities. Similarly, the natural real wage ω_t^n and the natural real interest rate r_t^n correspond to the real wage rate and the real interest rate in the absence of both nominal rigidities. These natural levels can be derived as:

$$y_t^n = \psi_{ya}^n a_t \tag{6}$$

$$r_t^n = \rho + \sigma E_t y_{t+1}^n \tag{7}$$

$$\omega_t^n = \log(1-\alpha) - \mu^p + \psi_{\omega a}^n a_t = \bar{\omega}^n + \psi_{\omega a}^n a_t, \tag{8}$$

where $\mu^p \equiv \log\left(\frac{\varepsilon_p}{\varepsilon_p - 1}\right)$ is the log of the desired markup of firms and where $\bar{\omega}^n \equiv \log(1-\alpha) - \mu^p$ is defined as the real wage that would prevail in the absence of nominal rigidities *and* in the absence of technological shocks.

Equation (1) is a New Keynesian Phillips curve where inflation now also depends on the real wage gap. Equation (2) is a similar equation for wage inflation with the only difference that a positive wage gap will decrease wage inflation by moderating wage claims. (3) is an identity relating various measures of the real wage and inflation, (4) is the usual forward-looking IS curve and (5) is the monetary policy rule.

[3]These equations correspond to (15), (17), (18), (19) and (20) in chapter 6 of Galí (2008).

The various parameters in (1) to (8) are given as follows:

$$\lambda_p = \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p}\frac{1-\alpha}{1-\alpha+\alpha\varepsilon_p},\ \kappa_p = \frac{\alpha\lambda_p}{1-\alpha}$$

$$\lambda_w = \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\varepsilon_w\varphi)},\ \kappa_w = \lambda_w\left(\sigma+\frac{\varphi}{1-\alpha}\right)$$

$$\psi^n_{ya} = \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha},\ \psi^n_{\omega a} = \frac{1-\alpha\psi^n_{ya}}{1-\alpha},$$

and ϕ_p, ϕ_w and ϕ_y are non-negative coefficients that give the strength with which the central bank is assumed to adjust the nominal interest rate in response to price inflation, wage inflation and the output gap, respectively.[4] Furthermore, $\rho \equiv -\log\beta$ where β is the discount factor. The technology shocks a_t and the interest rate shock v_t are given by the AR(1) processes:

$$a_t = \rho_a a_{t-1} + \varepsilon^a_t \tag{9}$$

$$v_t = \rho_v v_{t-1} + \varepsilon^v_t, \tag{10}$$

where $\rho_a \in [0,1]$, $\rho_v \in [0,1]$ and ε^a_t and ε^v_t are uncorrelated zero mean white noise processes. For later reference I also want to state the equations for the marginal rate of substitution mrs_t.

$$mrs_t = \left(\sigma+\frac{\varphi}{1-\alpha}\right)y_t - \frac{\varphi}{1-\alpha}a_t \tag{11}$$

2.2 Measuring real wage rigidity

I am are interested in the implications of the model for real wage rigidity. For this purpose it is helpful to subtract (1) from (2). Defining real wage inflation as $\pi^\omega_t \equiv \pi^w_t - \pi^p_t$ it follows that:

$$\pi^\omega_t = \beta E_t\pi^\omega_{t+1} + (\kappa_w-\kappa_p)\tilde{y}_t - (\lambda_w+\lambda_p)\tilde{\omega}_t \tag{12}$$

Using the definitions for π^ω_t, $\tilde{\omega}_t$ and ω_t one can derive from (12) a second-order difference equation for the real wage ω_t:

$$\omega_t = \frac{1}{1+\beta+\lambda_w+\lambda_p}\left[\omega_{t-1} + \beta E_t\omega_{t+1} + (\kappa_w-\kappa_p)\tilde{y}_t + (\lambda_w+\lambda_p)\omega^n_t\right] \tag{13}$$

[4] See Galí (2008, p. 128) for conditions on ϕ_p, ϕ_w and ϕ_y for which one gets an unique equilibrium.

Equation (13) can be solved to get:[5]

$$\omega_t = \delta\omega_{t-1} + \delta\left(\kappa_w - \kappa_p\right)\sum_{s=0}^{\infty}\left(\beta\delta\right)^s E_t\tilde{y}_{t+s} + \delta\left(\lambda_w + \lambda_p\right)\sum_{s=0}^{\infty}\left(\beta\delta\right)^s E_t\omega_{t+s}^n \tag{14}$$

The root δ is given by:

$$\delta = \frac{1 - \sqrt{1 - 4\beta\tilde{\lambda}^2}}{2\beta\tilde{\lambda}}, \tag{15}$$

where $\tilde{\lambda} \equiv \frac{1}{1+\beta+\lambda_w+\lambda_p}$. δ is a first approximate measure for the extent of real wage rigidity in the EHL model. It is, however, not the ultimate solution since also future variables that are present in (14) might depend on past levels of the real wage. In order to derive the general solution one has to use the complete model, in particular the assumptions about how the output gaps $\tilde{y}_{t+s}$ are determined.

2.3 Real wage rigidities under the assumption of exogenous output

Before turning to the solution that is implied by the complete EHL model (that includes the forward-looking IS-curve (4) and the monetary policy rule (5) to pin down the values of $\tilde{y}_{t+s}$) I want to start with the simple assumption that the output is exogenously given and always equal to its natural level y_t^n, i.e. $\tilde{y}_t = 0$, $\forall t$.[6] Using (8) and (9) in (14) it follows that:

$$\omega_t = \delta\omega_{t-1} + \frac{\delta\left(\lambda_w + \lambda_p\right)}{1 - \beta\delta}\bar{\omega}^n + \frac{\delta\left(\lambda_w + \lambda_p\right)\psi_{\omega a}^n}{1 - \beta\delta\rho_a}a_t \tag{16}$$

In the case of exogenous output the root δ captures the degree of real wage rigidity. One can use (15) to derive a straightforward and at the same time crucial result: The rigidity measure δ goes to zero if either λ_w or λ_p go to infinity or, equivalently, if either θ_p or θ_w are equal to zero (see Appendix A.1). As a consequence, real wages are flexible ($\delta = 0$) if either prices or wages are flexible. As one would have expected, only the combination of nominal price and nominal wage rigidity creates real wage rigidity.[7]

[5] For a difference equation of the form: $x_t = ax_{t-1} + bE_tx_{t+1} + cz_t$ the solution is given by: $x_t = \Upsilon x_{t-1} + \Upsilon\frac{c}{a}\sum_{s=0}^{\infty}\left(\frac{b}{a}\Upsilon\right)^s E_t z_{t+s}$, where $\Upsilon = \frac{1-\sqrt{1-4ab}}{2b}$.

[6] This model thus closely resembles a RBC model with fixed labor supply and a real wage rigidity (caused by the existence of two nominal rigidities).

[7] If the supply shock a_t is persistent ($\rho_a \neq 0$) then the estimation of the autocorrelation function of the real wage will of course also show a positive lag (cf. Rabanal and Ramírez, 2005). This persistence is, however, only a consequence of the persistence of the supply shock. In order to emphasize the genuine

In the next section I will show that this conclusion still holds for the more general case where the output gap is not assumed to be equal to zero but where it is given by (4) and (5). In fact, it will come out that the degree of real wage rigidity in the more general framework is also *quantitatively* similar to δ.

2.4 Real wage rigidities in the EHL model

For the full EHL model consisting of equations (1) to (8) it is not possible anymore to derive a closed form solution for the degree of RWR. One can use, however, standard methods to solve the model numerically. In particular, in appendix A.2 I show that the solution takes the form: $x_t = \Psi_0^x + \Psi_1^x \omega_{t-1} + \Psi_2^x a_t + \Psi_3^x v_t$, where $x_t \in \{\pi_t^p, \tilde{y}_t, \omega_t\}$ and Ψ_0^x to Ψ_3^x are coefficients.[8] The coefficient Ψ_1^ω in the expression for ω_t thus provides a measure for RWR in the EHL model. I prefer, however, to focus on a slightly different measure that uses the solution for $\tilde{y}_t$ to substitute out for the interest rate shock v_t. Appendix A.2 reports the resulting expressions for π_t^p and ω_t. In particular, the evolution of the real wage ω_t can now be written as:

$$\omega_t = \delta^* \omega_{t-1} + \Psi_4^\omega a_t + \Psi_5^\omega \tilde{y}_t + \Psi_6^\omega, \tag{17}$$

where δ^* and Ψ_4^ω to Ψ_6^ω are defined in appendix A.2. I choose the coefficient δ^* in (17) as the measure of real wage rigidity in the EHL model since it is closely related to the existing literature and allows for better comparisons among different specifications.

The degree of (annual) RWR is illustrated in Figure 1 that also includes — as a comparison — the (annual) RWR from the model with exogenous output (cf. (16)). For the illustrations I use the standard calibration of the parameters as in chapter 6 of Galí (2008). The only difference is that Galí (2008) defines a quarter as the basic time unit while I use a semester for this purpose. This is done to later alleviate comparisons to a model with two-period Taylor wage contracts (in particular when sector sizes are asymmetric).[9] In using a semester as the basic time unit one has to be careful in correctly calibrating the parameters that govern the degree of nominal rigidity. In particular, an average price duration of 3 quarters corresponds to $\theta_p = 1/3$ while an average wage duration of 4 quarters corresponds to $\theta_w = 1/2$. These are the baseline values for the

real wage rigidity I will focus in this paper mostly on the case where $\rho_a = \rho_v = 0$.

[8]It is also shown in appendix A.2 how to write these policy functions in an equivalent way in terms of deviations of the real wage from the steady state value $\bar{\omega}^n$. In particular for $x_t \in \{\pi_t^p, \tilde{y}_t, (\omega_t - \bar{\omega}^n)\}$ one gets: $x_t = \Psi_1^x(\omega_{t-1} - \bar{\omega}^n) + \Psi_2^x a_t + \Psi_3^x v_t$.

[9]In section 3.4 I discuss some issues related to the structure of timing more extensively.

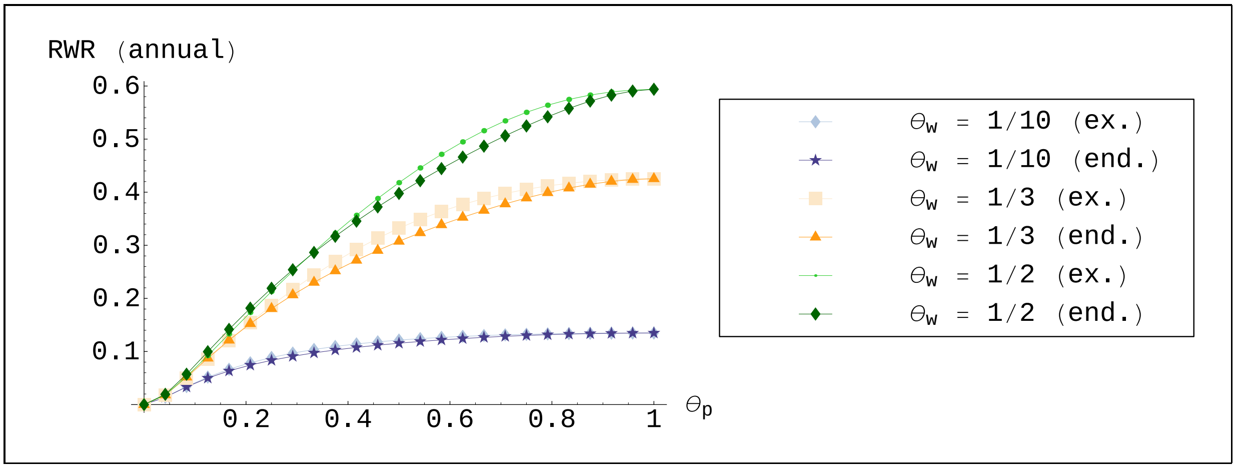

Figure 1: Comparison of the coefficients of annual real wage rigidity (δ^2 and $(\delta^*)^2$) in the specifications with exogenous and endogenous $\tilde{y}_t$, respectively.

duration of price and wage contracts used by Galí (2008) and I will refer to this in the following as the "baseline calibration".[10]

`Insert Figure 1 about here`

Figure 1 shows that in the absence of nominal price rigidity ($\theta_p = 0$) the real wage rigidity is zero. The same is true for the case of completely flexible wages ($\theta_w = 0$) where δ^* and δ also approach zero. For the baseline calibration one gets a sizable degree of annual RWR given by $(\delta^*)^2 = 0.288$ or $(\delta)^2 = 0.287$, respectively (which corresponds to a quarter-on-quarter RWR of 0.73).

As shown in the appendix, the measure of RWR is independent of the autocorrelation of shocks ρ_a and ρ_v. Furthermore, δ^* reacts only very weakly to changes in the parameters σ, φ, ϕ_π and ϕ_y. The largest effects one can observe for changes in α, ε_p and ε_w.[11] For the usual range of parameter values, however, the measure of annual RWR is fairly stable and stays between 0.25 and 0.3. Overall these robustness checks show that the main

[10]The rest of the parameters is calibrated as: $\alpha = 1/3$, $\beta = 0.98$ (corresponding to an annual real interest rate of roughly 4%), $\sigma = 1$, $\varphi = 1$, $\varepsilon_p = \varepsilon_w = 6$, $\phi_\pi = 1.5$ and $\phi_y = \phi_w = 0$. In the case of autocorrelated shocks I will use $\rho_a = 0.81$ and $\rho_v = 0.25$. Estimated versions of the EHL model typically arrive at somewhat different values (cf. Smets and Wouters, 2003; Rabanal and Ramírez, 2008).

[11]For σ between 0.5 and 5, δ^* decreases from 0.29 to 0.28, for φ between 1 and 5 it increases from 0.29 to 0.32 and for ϕ_π between 1.1 and 10 and ϕ_y between 0 and 1 it stays constant at $\delta^* = 0.29$. On the other hand, δ^* is 0.1 for $\alpha = 0$ and it is close to 0.55 as α approaches 1. For ε_p (ε_w) close to one gets values of $(\delta^*)^2 = 0.16$ ($(\delta^*)^2 = 0.21$) which increases to $(\delta^*)^2 = 0.34$ ($(\delta^*)^2 = 0.31$) for $\varepsilon_p = 10$ ($\varepsilon_w = 10$). The extreme values for α, ε_p and ε_w are, however, not typical for the calibration of DSGE models.

determinants of the degree of RWR are the two nominal rigidities. A related issue is whether the degree of RWR that is implied by the baseline calibration is reasonable and in line with the empirical evidence. I will come back to this question in section 4.

The main findings of the last two subsections can be summarized as follows:

Result 1 *The combination of nominal price and nominal wage rigidity can give rise to a considerable degree of real wage rigidity. The assumption of completely flexible prices ($\theta_p = 0$) or completely flexible wages ($\theta_w = 0$) implies zero real wage rigidity ($\delta = 0$ or $\delta^* = 0$).*

Result 2 *The degree of RWR is primarily determined by the extent of the two nominal rigidities. It is rather insensitive to changes in the other structural parameters and also to the specification of the monetary policy rule and the determination of output. In particular, the specifications with exogenous output and with endogenous output give rise to similar degrees of RWR.*

Result 1 emphasizes in a concentrated form the importance of complementarities (cf. Ascari, 2003; Huang and Liu, 2002). Nominal price rigidity without nominal wage rigidity as well as nominal wage rigidity without nominal price rigidity will result in completely flexible real wages ($\delta^* = 0$). Only the interplay between the two rigidities causes real wage rigidity. By the same token, Figure 1 also nicely illustrates that one class of stickiness increases the size of overall persistence holding the degree of the other stickiness constant.

Result 2 suggests that the dependence of the output gap on the interest rate and on monetary policy is only of secondary importance. What matters most for the magnitude of RWR is the extent of the nominal rigidities.

2.5 Inflation persistence and a backward-looking Phillips curve

The rational expectations solution to the EHL model can be transformed into an expression that resembles a traditional, backward-looking Phillips curve. This formulation is particularly useful for empirical analyses and also for the later comparisons between the models with Calvo and with Taylor wage contracts. In appendix A.2 it is shown that one can use the solution of the model to derive an equation of the form:

$$\pi_t^p = \delta^* \pi_{t-1}^p + f(\tilde{y}_t, \tilde{y}_{t-1}, a_t, a_{t-1}), \tag{18}$$

where $f(\cdot)$ is a linear function of the listed variables. Equation (18) is in fact fairly similar to traditional Phillips curve expressions (cf. Gordon, 1998) with the sole difference that inflation depends not only on current values of the output gap and the supply shock but also on past values $\tilde{y}_{t-1}$ and a_{t-1}. Interestingly, the coefficient on the lagged inflation term is identical to the degree of real wage rigidity in (17). An implication of this finding is stated as the following result.

Result 3 *The degree of intrinsic inflation persistence is the same as the degree of RWR. If there is no RWR than there will also be no intrinsic inflation persistence.*

A similar result has also been derived by Blanchard and Galí (2007, 51f.) who have shown that the presence of their (assumed) RWR leads to intrinsic inflation inertia.

2.6 Comparison to short-cut assumptions about RWR

One can use (11) to transform (17) into an expression of the form $\omega_t = \gamma_1\omega_{t-1} + \gamma_2 mrs_t + \gamma_3 a_t + const$. This is fairly close (but nevertheless not identical) to the short-cut formulation in Blanchard and Galí (2007) where they assume that $\omega_t = \gamma\omega_{t-1} + (1-\gamma)mrs_t$. In particular, the real wage ω_t cannot be written as a simple weighted average of the past real wage ω_{t-1} and the present marginal rate of substitution mrs_t since there is an additional variable a_t and since furthermore the weights on ω_{t-1} and mrs_t do not sum 1. For a standard calibration with $\theta_p = 1/3$, $\theta_w = 1/2$, $\rho_a = \rho_v = 0$ one gets, e.g.: $\omega_t = 0.54\omega_{t-1} + 0.003mrs_t + 0.22a_t - 0.27$.

Furthermore, also the other central equations of the model have to be adapted. This is in particular true for the New Keynesian Phillips curve where Blanchard and Galí (2007) use the normal formulation: $\pi_t^p = \beta E_t\pi_{t+1}^p + \kappa_p\tilde{y}_t$, while the EHL models implies a more complicated expression of the form $\pi_t^p = \xi_1 E_t\pi_{t+1}^p + \xi_2\tilde{y}_t + \xi_3 a_t$ (see appendix A.2.6). The first difference is that this expression includes again an additional term related to the supply shock. Even more important, however, is the fact that $\xi_1 \neq \beta$. For the baseline calibration it comes out as: $\pi_t^p = 2.53E_t\pi_{t+1}^p + 0.17\tilde{y}_t - 0.34a_t$. Note that the coefficient on expected inflation is considerably larger (!) than 1.

3 The model with nominal price rigidities à la Calvo and nominal wage rigidities à la Taylor

The EHL model is based on Calvo wage contracts and Calvo price contracts. This is the standard assumption that dominates the DSGE literature. In recent years, however, this assumption has also been criticized as being restrictive and implausible. In particular, it has been argued that a constant hazard rate for wage contracts is at odds with the empirical evidence (cf. Gottfries and Söderberg, 2008). Survey data by the WDN has documented, e.g., that most wage contracts have a clearly specified time length and that more than 60% are written for exactly one year (see also Knell and Stiglbauer, 2009). What is more, it has also been shown that wage changes are not spread uniformly over the year but are typically clustered in certain periods. 54% of the firms asked in the WDN survey have indicated that they carry out wage changes in a particular month (most of them in January).[12] In order to be able to take the empirical evidence on the fixed length of contracts and the clustering of wage agreements into account one has to move beyond the convenient but restictive framework of Calvo wage contracts. Accordingly, in this section I am going to present a model with Taylor wage contracts and with asymmetric sector sizes. This framework allows to study the impact of institutional details on the implied degree of RWR and it can be used for cross-country comparisons.

3.1 Wage-setting in the model with Taylor wage contracts

I use a simple two period Taylor model where the basic time-unit is again one semester. The total workforce is divided into two sectors where sector A negotiates the wage in periods $t = 0, 2, 4, \ldots$ while sector B negotiates in periods $t = 1, 3, 5, \ldots$. All wage contracts are fixed for two periods. The relative size of sector A (B) is denoted by s_A ($s_B = 1 - s_A$). Furthermore, it is assumed that firms' price-setting decisions are still characterized by a Calvo structure and that all firms use labor from both sectors in proportion to their relative sizes s_A and s_B.

When compared to the model of section 2 one has to change two equations (see appendix A.3). The following wage-setting equation takes the place of (2):

$$w_t^i = \frac{1}{1+\varepsilon_w \varphi} \sum_{k=0}^{1} \frac{\beta^k}{1+\beta} E_{t+k} \left\{ (1+\varepsilon_w \varphi) w_{t+k} - \tilde{\omega}_{t+k} + \tilde{y}_{t+k} (\sigma + \frac{\varphi}{1-\alpha}) \right\}, \tag{19}$$

where $i = A$ for $t = 0, 2, 4, \ldots$ and $i = B$ for $t = 1, 3, 5, \ldots$. For the periods where sector $i \in \{A, B\}$ does not adjust wages it holds that $w_t^i = w_{t-1}^i$. Instead of (3) one has to use

[12] Cf. also Olivei and Tenreyro (2007). The WDN data show also a concentration of price changes in certain months. The degree of clustering is, however, less pronounced than in the area of wage-setting (only 35% of the firms indicate to follow such a strategy) and I will concentrate in the following on the clustering of wage changes.

the following definition of aggregate wages:

$$w_t = s^A w_t^A + s^B w_{t-1}^B \tag{20}$$

The complete model is now given by the five equations (1), (4), (5), (19) and (20).

3.2 Real wage rigidity in the model with Taylor wage contracts

One can again use standard methods to solve the model. In appendix A.3 I discuss the solution and I derive a number of useful transformations.

A direct comparison between the solutions and the degrees of RWR for the formulations with Calvo and with Taylor contracts is, however, not straightforward. First, even in the case with symmetric sector sizes (i.e. $s_A = s_B = 1/2$) the Taylor model does not lead to a formulation where the average real wage $\bar{\omega}_t$ depends just on $\bar{\omega}_{t-1}$, a_t and $\tilde{y}_t$ (as in (17)).[13] In particular, for the case with symmetric Taylor wage contracts $\bar{\omega}_t$ depends on $\bar{\omega}_{t-1}$, a_t, a_{t-1}, $\tilde{y}_t$, $\tilde{y}_{t-1}$, π_t^p and π_{t-1}^p.[14]

Second, for the case of asymmetric sector sizes the period-on-period RWR differs between the two subperiods and depends on the sector that sets the new wage. In order to deal with these difficulties and to allow for comparisons I use a year-on-year formulation. In appendix A.3 it is shown that the evolution of the average real wage can be written as:

$$\bar{\omega}_t^i = \tilde{\delta}\bar{\omega}_{t-2}^i + f^i(\tilde{y}_t^i, \tilde{y}_{t-1}^j, \tilde{y}_{t-2}^i, a_t, a_{t-1}, a_{t-2}, \pi_t^{p,i}, \pi_{t-2}^{p,i}) \tag{21}$$

The coefficient $\tilde{\delta}$ measures the (year-on-year) rigidity of the average real wage. It is the same in both sectors of the economy, independent of which sector sets the new wage. The reaction of $\bar{\omega}_t^i$ to supply shocks, output gaps and inflation rates is, however, different in the two subperiods (as indicated by the indexation of the function $f^i(\cdot)$). This year-on-year measure of RWR $\tilde{\delta}$ can be compared with the year-on-year measure of RWR in the Calvo model (given by $(\delta^*)^2$).

[13] I have to write $\bar{\omega}_t$ for the average real wage in order to distinguish it from the real wage of the two individual sectors. In particular: $\omega_t^A \equiv w_t^A - p_t$, $\omega_t^B \equiv w_t^B - p_t$, $\bar{\omega}_t^A \equiv w_t - p_t$ in periods when sector A is changing the wage while $\bar{\omega}_{t+1}^B \equiv w_{t+1} - p_{t+1}$ in periods when sector B is changing. For symmetric sector sizes the dynamics of $\bar{\omega}_t^A$ and $\bar{\omega}_{t+1}^B$ are described by the same equation and one can thus leave out the sectoral index.

[14] It is interesting to note that one gets a mirror-inverted result here. In the Taylor model $\bar{\omega}_t$ depends on $\bar{\omega}_{t-1}$, a_t, a_{t-1}, $\tilde{y}_t$, $\tilde{y}_{t-1}$, π_t^p and π_{t-1}^p while the newly set real wage ω_t^i can be written just as a function of ω_{t-1}^i, a_t and $\tilde{y}_t$. In the Calvo formulation, on the other hand, it is exactly the opposite. The average real wage is just a function of $\bar{\omega}_{t-1}$, a_t and $\tilde{y}_t$ while ω_t^i depends on ω_{t-1}^i, a_t, a_{t-1}, $\tilde{y}_t$, $\tilde{y}_{t-1}$, π_t^p and π_{t-1}^p.

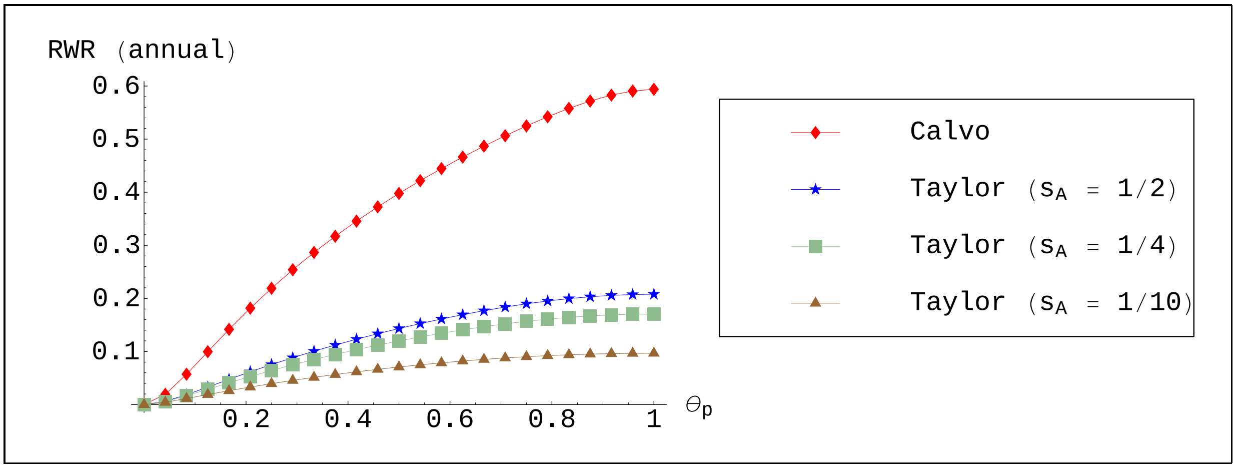

Figure 2: Comparison of annual RWR in the model with Calvo ($(\delta^*)^2$) and with Taylor ($\tilde{\delta}$) wage contract and for various values of s_A. The pictures show the extent of annual RWR based on a model where the basis time period is one semester and the average duration of wage contracts is one year. For the case of Calvo wage contracts this means that $\theta_w = 1/2$.

The correspondence between $\tilde{\delta}$ and $(\delta^*)^2$ is further emphasized if one again derives a backward-looking Phillips curve for the model with Taylor wage contracts. In appendix A.3 I show how it can be written as:

$$\pi_t^{p,i} = \tilde{\delta}\pi_{t-2}^{p,i} + f^i(\tilde{y}_t^i, \tilde{y}_{t-1}^j, \tilde{y}_{t-2}^i, a_t, a_{t-1}, a_{t-2}) \tag{22}$$

Using (18) one observes that for the model with Calvo contracts the expression for year-on-year inflation persistence has exactly the same form as (22): $\pi_t^p = (\delta^*)^2 \pi_{t-2}^p + f(\tilde{y}_t, \tilde{y}_{t-1}, \tilde{y}_{t-2}, a_t, a_{t-1}, a_{t-2})$.

In Figure 2 I plot $\tilde{\delta}$ and $(\delta^*)^2$ for the baseline calibration and for three assumptions about the sector size ($s_A = 1/2$, $s_A = 1/4$, $s_A = 1/10$).

`Insert Figure 2 about here`

The findings can be summarized in the following results.

Result 4 *For the same average durations of price and wage contracts, the assumption of Taylor wage contracts implies a considerably lower degree of real wage rigidity than the assumption of Calvo wage contracts.*

Result 5 *For the case of Taylor wage contracts the assumption of asymmetric sector sizes ($s_A < 1/2$) lowers the degree of real wage rigidity.*

As far as result 4 is concerned one gets for the baseline calibration (with $\theta_p = 1/3$) that the annual RWR implied by the model with Calvo contracts is given by $(\delta^*)^2 = 0.29$ while for Taylor contracts it is $\tilde{\delta} = 0.1$ (for $s_A = 1/2$). The reason for the considerably lower rigidity in the model with Taylor contracts is the fact that under the latter assumption there is an exactly given duration for every contract. In the Calvo framework, on the other hand, some contracts might last for a very long time span. This (unrealistic) feature considerably increases the extent of intrinsic persistence. This fact is known from the literature (cf. Dixon and Kara, 2006) although it is mostly ignored when calibrating the models.

Less well-known is the impact of asymmetries as stated in result 5. The RWR decreases considerably (from $\tilde{\delta} = 0.1$ (for $s_A = 1/2$) to $\tilde{\delta} = 0.085$ (for $s_A = 1/4$) and $\tilde{\delta} = 0.05$ (for $s_A = 1/10$)) when the share of wage-setting firms is not spread evenly over the year. Since such asymmetries are in fact characteristic for almost all European countries (see section 4) this should be taken into account when calibrating the model. Taken the two results together they suggest that the assumption of Calvo wage contracts does not seem to be innocuous. In particular, it might be highly misleading to simply translate the available information about the average duration of wage contracts into a parameter θ_w that is then used in a model with Calvo wage contracts. Institutional details about the wage-setting practices matter and they can have a considerable impact on the implied degree of persistence.

By comparing equations (21) and (22) one can make an additional observation. For empirical analyses (or cross-country comparisons) it seems to be preferable to use the model in terms of π_t^p instead of the model in terms of ω_t. In the latter case there are additional right-hand-side variables (involving present and past rates of inflation) and the estimations of the degree of RWR are less accurate. Furthermore, the model also suggests to use a specification that includes only one measure of inflation in the list of explanatory variables (in particular the annual lag of inflation) while using all available data on the output gap and the supply shocks over the last year.[15] I want to summarize this as follows:

Result 6 *The best way to estimate RWR from empirical data involves an estimation equation that regresses the inflation rate on the annual lag of inflation and all available*

[15]To be more specific, the empirical specification should be based on the frequency of the average wage contract duration (which is here one year or two subperiods, from t to $t-2$). If there exist contracts with different durations at the same time then it might again be optimal to include all lags of inflation up to the longest contract duration. In the appendix (especially in appendix B.1) one can find more on this issue.

intermediate measures of real activity (the output gap or an "unemployment gap") and supply shocks.

A specification as it is sketched in Result 6 will return the correct degree of RWR in a model with both Calvo and with Taylor wage contracts. Furthermore, it is interesting to note that this preferred specification is closely related to the more traditional "triangle" model (cf. Gordon, 1998) in which the current rate of inflation is written as a function of past inflation and of current and past levels of demand factors (output gap, cyclical unemployment) and supply factors (oil price shocks, import price shocks etc.).

3.3 Impulse response functions for different models

An alternative way to compare the properties of the different assumptions concerning wage contracts is to look at impulse response functions (IRFs). The study of IRFs is the most prominent tool of analysis in recent DSGE models. They are particularly useful to investigate the differential impact of economic shocks while the RWR is a reasonable summary measure for the inherent persistence in an ongoing economy. I show the IRFs of ω_t, π_t^p and $\tilde{y}_t$ to a one-unit shock in v_t in Figure 3. The pictures distinguish between the Calvo model, the symmetric Taylor model ($s_A = 1/2$) and the asymmetric Taylor model ($s_A = 1/10$). All IRFs are based on the baseline calibration with an average duration of wage contracts of one year.

`Insert Figure 3 about here`

The impulse response functions in Figure 3 show the expected result. In the model with symmetric Taylor contracts a monetary policy shock (an increase in v_t) leads to a larger decrease in the inflation rate and a smaller decrease in the output gap than in the Calvo model. For asymmetric Taylor contracts, however, one has to distinguish between the periods when the shocks hits the economy. If it is in the periods when many firms adjust their wages (i.e. when sector B sets the wage) then the real effects (the decrease in the output gap) are smaller than for the case of symmetric sector sizes. In particular, the inflation rate reacts more strongly to the increase in the interest rate which causes a sizable fall in the real wage. In the opposite case, however, only a small segment of the economy (only 10% in the example of Figure 3) can adjust the wages when the shock occurs. In this situation the average real wage actually *increases* and a larger part of the adjustment process involves a decrease in the output gap rather than a fall in the inflation

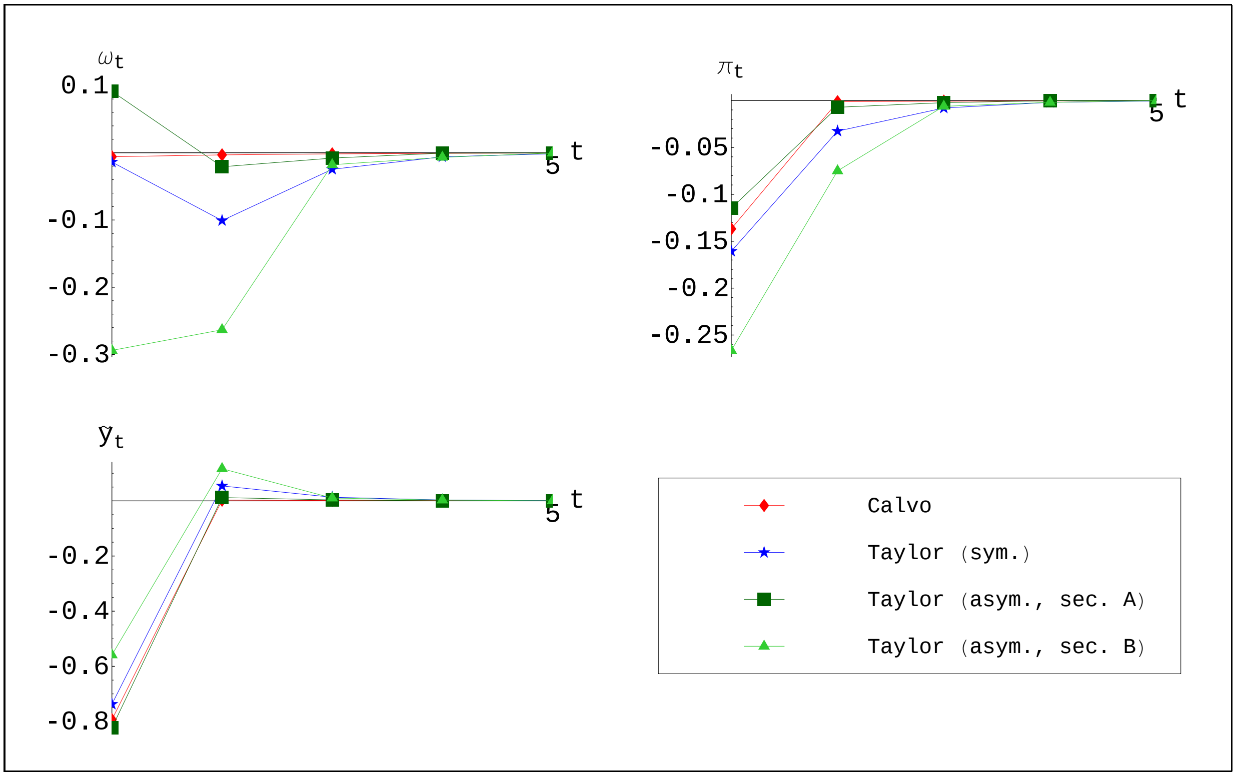

Figure 3: Comparison of the impulse response functions of ω_t, π_t^p and $\tilde{y}_t$ in reaction to a one-unit shock in v_t in period 1. The four lines correspond to the Calvo model (red), the symmetric ($s_A = 1/2$) Taylor model (blue) and the asymmetric ($s_A = 1/10$) Taylor model (green). The lines in dark (light) green show the reaction when the shock happens in a period when sector A (B) is setting the wage.

rate. The mechanism and the intuition behind these results for the asymmetric case are parallel to the analysis in Olivei and Tenreyro (2007) who look at a Calvo model with quarter-specific adjustment probabilities.[16]

3.4 Discussion of the time structure

Before turning to the empirical data I want to briefly deal with two issues that are related to the time structure of the model. First, how large is the bias introduced by working with a semester as the basic time-unit in both the Calvo and the Taylor model? Second, is it possible to stick to the standard Taylor structure of two-period-staggering while still allowing for an average wage duration that is longer or shorter than one year? The latter question is particularly important when trying to match the model with the empirical data.

In appendix B I deal with both questions and the interested reader is referred to this part of the paper. I show there that these are in fact nonnegligible issues that might affect the results. As far as the first issue is concerned, appendix B.1 illustrates, e.g., that the baseline calibration of a model with Calvo wage contracts and with semesters as the basic time units underestimates the RWR by 37% when compared to a model with quarters as the basic time unit. The underestimation is even larger when it is compared to a model with months (52%) or days (54%) as the basic time unit. The ranking of countries, however, is not affected by possible biases due to the choice of the basic time unit.

As far as the second timing issue is concerned I present in appendix B.2 a straightforward method to allow for average wage durations that are different from one year. The main idea behind the procedure is to redefine the length of the basic time unit as one half of the average wage duration. The time discount factor and the parameter capturing the degree of price stickiness have then to be adapted such as to conform to this new timing.

Overall, the analysis of these two issues indicates that the impact of the exact timing is primarily on the magnitude of the estimated RWR. The relative position of countries with different degrees of nominal rigidities and/or different institutional structures is left unchanged as long as prices are not too flexible and average price duration is not too much shorter than average wage duration. These two conditions are fulfilled by the empirical

[16]Note also, that after the first periods with differential reactions from period 4 onwards the decrease in the rate of inflation is governed by δ^* and by $\tilde{\delta}$, respectively. From results 4 and 5 it is known that the Calvo model shows more persistence than the symmetric Taylor model which again is more persistent than the asymmetric Taylor models.

of the WDN that are studied later. For international comparisons that are based on a ranking of countries one can thus be confident that the stylized time structure of the model does not affect the main conclusions.

4 Survey evidence on nominal rigidities and what they imply for real wage rigidity

A natural question is to what extent the standard EHL model is able to explain empirical regularities. The current literature mostly focuses on the ability of the microfounded model to produce an adjustment process that is sufficiently close to its real-world counterpart. For this purpose one commonly employs the study of impulse response functions. As argued above, the measure for RWR offers an alternative variable that can be used to evaluate whether the EHL model captures the essential features of real-world persistence and transmission channels. To this end I will in the following present survey evidence on price and wage stickiness for a number of European countries. Using this information on the degree of nominal rigidity together with the other baseline parameter values leads to estimates for the degree of real wage rigidity in these countries. These numbers (or at least their ranking) can then be compared with the findings of the existing empirical literature that has come up with estimations for real wage rigidities or with related concepts. This will reveal whether the EHL model and the parsimonious explanation for the existence of RWRs is in line with the data.

One can use the results from firms surveys that have been conducted in a number of European countries in the context of the ECB's Wage Dynamics Network (WDN). Aggregate data are contained in Druant et al. (2009). So far, the final results concerning the country-specific data on the average duration of price and of wage contracts and the asymmetry in wage-setting are not yet available. For the future I plan to use these survey results in an otherwise baseline calibration of the theoretical model in order to calculate and compare estimations of RWR based on a model with Calvo wage contracts ($(\delta^*)^2$) and on a model with symmetric ($\tilde{\delta}_{sym.}$) and with asymmetric ($\tilde{\delta}_{asym.}$) Taylor wage contracts.

5 Comparison to the existing literature

It would be interesting to compare the theory-based measures for RWR with existing empirical evidence. There does not seem to exist a long literature on this issue. Blanchard and Galí (2007), e.g., do not provide an estimate or a "reasonable" value for the (assumed) magnitude of RWR. In discussing the results they use illustrative values between 0.5 and 0.9 (for a quarter-on-quarter basis). Duval and Vogel (2007), on the other hand, use values for the quarterly RWR between 0.79 and 0.93 (referring to the paper by Arpaia and Pichelmann (2007)), which corresponds to an annual RWR between 0.39 and 0.75. These values are (slightly) above the value of $(\delta^*)^2 = 0.29$ that is implied by the EHL model for the baseline calibration.[17]

A cross-country comparison of the estimates for RWR would be even more interesting. Unfortunately, I have not been able to find a study that contains such comparable estimations for a larger number of European economies. One can use, however, various transformations and equivalent expressions of the EHL model to derive relations that correspond to empirical specifications for which cross-country results are available. In appendix C I show in detail how the theory-based measure of RWR is related to these alternative measures.[18] In particular, I refer to three measures: (i) the sensitivity of the real wage with respect to the output gag, (ii) the speed of adjustment in an error-correction mechanism and (iii) a measure that resembles a sacrifice ratio. In the appendix I show that each of these measures is perfectly related to δ^* when either θ_p or θ_w are held constant and still closely related if one allows for country-specific variations in θ_p and θ_w. In the following I briefly present these three measures and their relation to the degree of RWR δ^*.

5.1 The degree of RWR and the sensitivity to the output gap

There exists a voluminous literature on wage curve estimations for a large number of countries (Blanchflower and Oswald, 2005). A good part of this literature is summarized (via meta-analyses) in a paper by Clar et al. (2007). In particular, they report and analyze measures of the parameter ι which is defined as measuring the reaction of the real wage to changes in the unemployment rate, i.e. $\omega_t = \ldots + \iota u_t$. In Table 1, columns (1) and (2), I report the values for the European countries contained in Clar et al. (2007).

`Insert Table 1 about here`

[17]The use of shorter basic time units would increase this value to 0.39 (quarters), 0.44 (months) and 0.46 (days). See appendix B.1.

[18]For the ease of exposition these transformations are derived in the framework of the model with Calvo wage contracts. As shown above, the *ranking* of countries does not seem to be much affected by the assumption concerning wage contracts (whereas the exact *magnitude* of RWR is very much affected). Since the following comparisons are mainly based on country rankings this focus on the Calvo model is therefore justified.

The coefficient ι is related to Ψ_5^ω in (17), i.e. the sensitivity of the real wage with respect to the output gap. Both reflect how strongly the real wage reacts to changes in real activity. Furthermore, in appendix C.1 I show that Ψ_5^ω is closely related to the measure of RWR δ^*. The more rigid real wages the smaller also their reaction to changes in the output gap or in unemployment. The correlation is perfect for constant values of θ_p or θ_w and still fairly high for reasonable variations.

5.2 The speed of error-correction

The EHL model can also be used to derive an error-correction formulation that is similar to the specifications used in the empirical literature (cf. Blanchard and Katz, 1999; Bardsen et al., 2004). In appendix A.2 it is shown how one can use the solution for π_t^p together with (2) and (3) to get:

$$\pi_t^w = \beta E_t \pi_{t+1}^p - \Phi_1 \left(\omega_{t-1} - \bar{\omega}^n\right) + \Phi_2 a_t + \Phi_3 \tilde{y}_t \tag{23}$$

The change in nominal wages π_t^w depends on expected inflation $E_t \pi_{t+1}^p$, the output gap $\tilde{y}_t$, the size of the productivity shock a_t and the error-correction-term $(\omega_{t-1} - \bar{\omega}^n)$, where ω^n stands for the real wage in the absence of shocks and rigidities. The coefficient Φ_1 measures the importance of deviations of the past real wage from the equilibrium value for wage inflation. In appendix C.2 I show that Φ_1 is closely (positively) related to the RWR δ^*.

There exists an empirical literature that has estimated equations that resemble (23). Blanchard and Katz (1999), e.g., arrive at a fairly similar specification, although they derive it in a completely different model without nominal rigidities where the source of RWR (and the error-correction-term) is a direct influence of productivity on the wage and the reservation wage. They report that for annual data a value of $\Phi_1 = 0.25$ is characteristic for European countries while a value of $\Phi_1 = 0$ is typical for the US. Arpaia and Pichelmann (2007) use the modeling framework of Blanchard and Katz (1999) to estimate an error-correction model for a large number of European countries. They report an average (annual) value of $\Phi_1 = 0.4$ for EU12 with a large dispersion ranging from 0.76 (Germany) to values around 0 (for Portugal and Spain).

In Table 1, columns (3) and (4) I report the estimates of Φ_1 that are contained in Arpaia and Pichelmann (2007) and I also give the country ranking.

	CDR (2007)		AP (2007)		R (2000)	
	Sensitivity of RW wrt. Unemployment		Error-Correction Term		Sacrifice Ratio	
	Coeff. ($-\Psi_5^\omega$)	Rank	Coeff. (Φ_1)	Rank	Coeff.	Rank
	(1)	(2)	(3)	(4)	(5)	(6)
Australia (AUS)	-0.99	15	–	–	1	14
Austria (AUT)	-2.17	6	0.25	10	0.8	11
Belgium (BEL)	-1.07	12	0.32	6	0.2	1
Canada (CAN)	-0.59	19	–	–	0.9	12
Switzerland (CHE)	-7.5	1	–	–	0.5	5
Denmark (DEN)	-0.38	20	–	–	1.7	20
Germany (DEU)	-1.04	14	0.74	1	1	14
Spain (ESP)	-0.61	18	-0.18	12	0.2	1
Finland (FIN)	-1.41	8	0.54	3	1.2	18
France (FRA)	-0.94	16	0.3	8	1.1	17
United Kingdom (GBR)	-0.37	21	–	–	1	14
Greece (GRC)	-1.62	7	0.53	4	0.5	5
Ireland (IRL)	-1.11	11	0.31	7	0.9	12
Italy (ITA)	-1.12	10	0.67	2	0.7	9
Japan (JPN)	-7.44	2	–	–	0.4	4
Netherlands (NLD)	-0.74	17	0.28	9	1.9	21
Norway (NOR)	-2.68	4	–	–	0.5	5
New Zealand (NZL)	-0.17	23	–	–	0.5	5
Portugal (PRT)	-1.06	13	0.11	11	0.7	9
Sweden (SWE)	-2.67	5	–	–	0.2	1
United States (USA)	-0.36	22	–	–	1.5	19

Table 1: The values in the table come from Clar et al. [CDR] (2007, Table 2), Arpai and Pichelmann [AP] (2007, Table 4) and Richardson et al. [R] (2000, Table A2). These papers refer to different countries and thus not all of the cells could be filled. The rankings are specified in such a way that higher numbers correspond to "more rigid" countries.

5.3 The sacrifice ratio

It was shown above that the solution of the EHL model gives rise to an equation similar to a backward-looking Phillips curve (cf. (18) and (22)) where the measure of RWR also determines the degree of intrinsic inflation persistence. It thus seems tempting to use existing cross-country evidence on inflation persistence to validate the theory-based country ranking. This, however, is not straightforward since most of the prominent cross-country evidence uses univariate empirical specifications (cf. Cecchetti and Debelle, 2006) while the theoretical model implies an estimation equation that includes lagged inflation *plus* additional variables for the output gap and supply shocks. This difference is not a minor issue as indicated by regression results. In fact, using simulated data (that are based on different degrees of nominal wage and price stickiness) and estimating various forms of autoregressive inflation equations gives a rather disappointing result. The inflation persistence measures are not very robust across specifications and — what is more — they show only a very weak correlation with the theory-based measure of RWR and inflation persistence δ^*.

It is thus more promising to compare coefficients of lagged inflation that stem from Phillips curve estimations. This is, however, again not straightforward since the empirical estimations typically involve different specifications with different lag structures and additional variables. A reasonable compromise seems, however, to focus on cross-country evidence of the sacrifice ratio that "summarizes" the main properties and the different specifications in a single number. For the sake of comparisons, one can use the impulse response functions of the model (cf. section 3.3) to derive a parallel measure that resembles (somewhat loosely) a sacrifice ratio. This is discussed in appendix C.3. The measure is defined as $sr = \frac{\sum_{i=0}^{T} \tilde{y}_{t+i}^{IRF}}{\sum_{i=0}^{T} \pi_{t+i}^{IRF}}$, where $\tilde{y}^{IRF}$ and π^{IRF} stand for the response of the output gap and the rate of inflation to a one unit monetary shock in period 0 ($v_0 = 1$) and where $T = 40$.[19] In the appendix I show that sr and δ^* are highly (positively) correlated.

Richardson et al. (2000) contains estimations for the sacrifice ration for 21 OECD countries. The empirical specifications in the paper are in fact fairly similar to the theoretical equation (18) and they include an unemployment gap (comparable to $\tilde{y}_t$) and supply shocks (comparable to a_t). The difference is that the specifications involve variable lags for both inflation and the unemployment gap while the theoretical model clearly

[19]This can only be understood as an approximate measure. In particular, one has to be careful in referring to processes that involve a permanent disinflation in this model since its main equations are based on log-linearizations around a zero inflation steady state (cf. Ascari and Merkl, 2007; Ascari and Ropele, 2009).

determines the lag structure. It thus seems again appropriate to focus just on the ranking of countries and not so much on the precise numerical values.

In Table 1, columns (5) and (6) I report the values for the sacrifice ratio and the country rank contained in Richardson et al. (2000).

5.4 Comparing the empirical measures

All of the discussed measures are not perfect and the relation between the theoretical model and the most widely used empirical specifications is sometimes rather loose. Nevertheless, on the total they offer an interesting frame of reference in order to analyze the validity and relevance of the EHL model. Since for the moment data availability has prevented me from calculating theory-based measures of δ^* and $\tilde{\delta}$ I will focus here on a brief discussion of how the three empirical measures correlate amongst each other. This is done in Figure 4 where — as dicussed above — I focus on the country ranking.

`Insert Figure 4 about here`

The sensitivity of the real wage with respect to unemployment is significantly positively related to the sacrifice ratio (Spearman's rank correlation coefficient [SRCC]: 0.48 with a p-value of 0.027). The smaller the decrease in the real wage in reaction to an increase in the unemployment rate the higher the sacrifice ratio. This result is in line with intuition and also with the prediction of the theoretical model. The second panel in Figure 4 shows that countries with a small sensitivity of real wages also have a lower error-correction term Φ_1. This corresponds again to the theoretical prediction although the correlation is not statistically significant (SRCC: -0.41, p-value: 0.19). The correlation between the sacrifice ratio and the error-correction-term is the weakest (SRCC: -0.2) and least significant (p-values: 0.56). This might also have to do with the small number of observations for the latter two comparisons.

On the total the picture is rather mixed. The pairwise comparisons of the empirical measures go in fact into the direction that is predicted by the theoretical model. The correlations are, however, far from perfect and other factors seem to affect the cross-country performance in terms of real flexibility and adjustment. This might, e.g., have to do with differences in the importance of openness or with differences in wage-setting institutions (e.g. in the role of reference norms and wage leadership, see Knell and Stiglbauer, 2009). I plan to deal with these issues more extensively in the future.

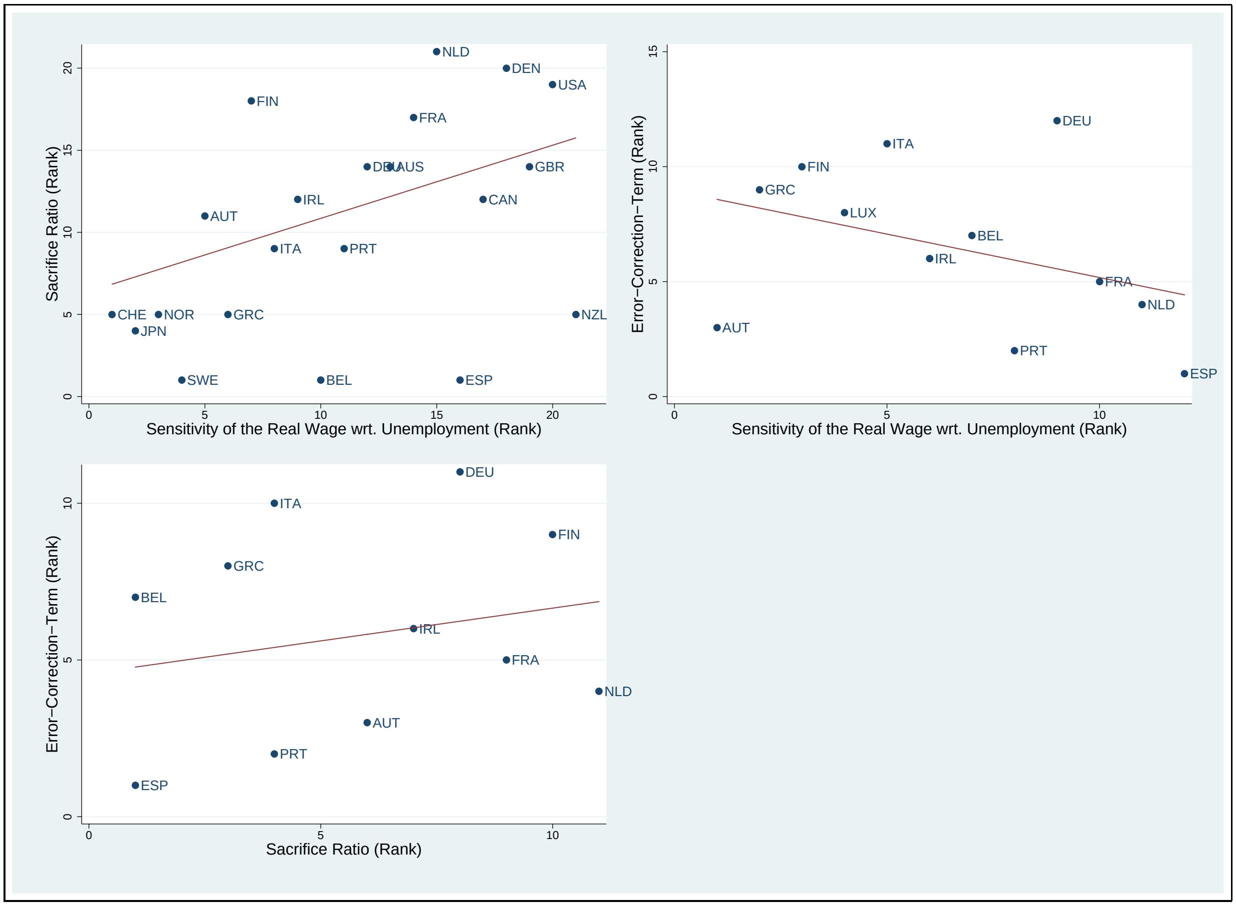

Figure 4: Pairwise comparisons of the country ranking of the sensitivity of the real wage with respect to unemployment (cf. Clar et al., 2007), the sacrifice ratio (cf. Richardson et al., 2000) and the error-correction-term (cf. Arpai and Pichelmann, 2007). The (rank) correlation coefficients are 0.48, -0.41 and 0.20, respectively. Only the first one is, however statistically significant at the 5% level.

6 Conclusion

In this paper I have used a standard DSGE model to show that real wage rigidity can be simply explained by the synchronous presence of a nominal price and a nominal wage rigidity. There exists a notable complementarity between the two nominal rigidities and RWR requires the presence of both. Furthermore, I have shown that the institutional details of wage determination can have a considerable impact on the extent of RWR. If wages are assumed to be set for a fixed length of time (Taylor wage contracts) then the resulting real wages are much less rigid than in the case of Calvo wage contracts with an identical average duration. The phenomenon of clustering of wage agreements further diminishes RWR. Since wage contracts of a predetermined length and wage clustering are a prevalent feature of European wage-setting institutions it seems imperative to include these elements into the set-up and the calibration of reasonable DSGE models.

In the second part of the paper I plan to use recent survey evidence on price and wage durations for a number of European countries in order to study whether the predictions of the parsimonious theoretical model are in line with the existing empirical evidence. So far I have shown that existing empirical measures that are related to the concept of RWR do in fact possess the cross-country correlations that are predicted by the theoretical model. It remains to be shown whether and to what extent the measures of RWR based on the survey evidence will fit into this picture.

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Appendices

A Derivations and Proofs

A.1 The model with exogenous output (section 2.3)

From the definitions of λ_p and λ_w it follows that: $\lim_{\theta_p \to 0} \lambda_p = \infty$ and $\lim_{\theta_w \to 0} \lambda_w = \infty$. So using the definition $\tilde{\lambda} \equiv \frac{1}{1+\beta+\lambda_w+\lambda_p}$ it follows that $\lim_{\theta_p \to 0} \tilde{\lambda} = 0$ and $\lim_{\theta_w \to 0} \tilde{\lambda} = 0$. From the definition of δ (equation (15)) one thus gets that: $\lim_{\tilde{\lambda} \to 0} \delta = 0/0$. Using l'Hospital's rule one can conclude that $\lim_{\tilde{\lambda} \to 0} \delta = \lim_{\tilde{\lambda} \to 0} 2\tilde{\lambda} \left(1 - 4\beta\tilde{\lambda}^2\right)^{-\frac{1}{2}} = 0$.

In a similar vein one can calculate that $\lim_{\tilde{\lambda} \to 0} \frac{\delta(\lambda_w+\lambda_p)}{1-\beta\delta}\bar{\omega}^n = \bar{\omega}^n$ and $\lim_{\tilde{\lambda} \to 0} \frac{\delta(\lambda_w+\lambda_p)\psi^n_{\omega a}}{1-\beta\delta\rho_a} = \psi^n_{\omega a}$. Therefore for $\theta_p \to 0$ or $\theta_w \to 0$ one can write that $\omega_t = \bar{\omega}^n + \psi^n_{\omega a} a_t$ or(using the definitions of mrs_t, $\psi^n_{\omega a}$ and ψ^n_{ya}) $\omega_t - \bar{\omega}^n = mrs_t$. This is in fact the expected result for the situation with completely flexible prices.

For complete persistence ($\theta_p = 1$, $\theta_w = 1$), on the other hand, one gets that $\lambda_p = 0$, $\lambda_w = 0$ and $\tilde{\lambda} = \frac{1}{1+\beta}$ and thus $\delta = 1$. In this case it thus holds that $\omega_t = \omega_{t-1}$.

A.2 Solution to the model with Calvo wage contracts (section 2)

A.2.1 Basic Solution

One can use the methods of undetermined coefficients to derive a solution of the following form:

$$\pi_t^p = \Psi_0^p + \Psi_1^p \omega_{t-1} + \Psi_2^p a_t + \Psi_3^p v_t \tag{24}$$

$$\tilde{y}_t = \Psi_0^y + \Psi_1^y \omega_{t-1} + \Psi_2^y a_t + \Psi_3^y v_t \tag{25}$$

$$\omega_t = \Psi_0^\omega + \Psi_1^\omega \omega_{t-1} + \Psi_2^\omega a_t + \Psi_3^\omega v_t \tag{26}$$

The coefficient Ψ_1^ω thus gives the degree of real wage rigidity in a specification where one corrects for the realization of the two shocks a_t and v_t. It holds that $\Psi_0^p = -\Psi_1^p \bar{\omega}^n$, $\Psi_0^y = -\Psi_1^y \bar{\omega}^n$ and $\Psi_0^\omega = (1 - \Psi_1^\omega)\bar{\omega}^n$ and thus the system (24) to (26) could as well be written in terms of deviation of ω_t from its flexible-prices-no-shocks value $\bar{\omega}^n$: $\pi_t^p = \Psi_1^p (\omega_{t-1} - \bar{\omega}^n) + \Psi_2^p a_t + \Psi_3^p v_t$, $\tilde{y}_t = \Psi_1^y (\omega_{t-1} - \bar{\omega}^n) + \Psi_2^y a_t + \Psi_3^y v_t$ and $(\omega_t - \bar{\omega}^n) = \Psi_1^\omega (\omega_{t-1} - \bar{\omega}^n) + \Psi_2^\omega a_t + \Psi_3^\omega v_t$.[20]

A.2.2 Solution in terms of ω_{t-1}, a_t and $\tilde{y}_t$

Using (25) v_t can be expressed in terms of $\tilde{y}_t$: $v_t = \frac{1}{\Psi_3^y}(\tilde{y}_t - \Psi_0^y - \Psi_1^y \omega_{t-1} - \Psi_2^y a_t)$. Inserting this in (24) and (26) equilibrium inflation and the equilibrium real wage can be written just in terms of the past real wage (ω_{t-1}), the output gap ($\tilde{y}_t$) and a supply shock (a_t) which corresponds loosely to a specification one can frequently find in the empirical literature:

$$\pi_t^p = \left(\Psi_0^p - \Psi_0^y \frac{\Psi_3^p}{\Psi_3^y}\right) + \left(\Psi_1^p - \Psi_1^y \frac{\Psi_3^p}{\Psi_3^y}\right)\omega_{t-1} + \left(\Psi_2^p - \Psi_2^y \frac{\Psi_3^p}{\Psi_3^y}\right) a_t + \frac{\Psi_3^p}{\Psi_3^y}\tilde{y}_t \tag{27}$$

$$\omega_t = \left(\Psi_0^\omega - \Psi_0^y \frac{\Psi_3^\omega}{\Psi_3^y}\right) + \left(\Psi_1^\omega - \Psi_1^y \frac{\Psi_3^\omega}{\Psi_3^y}\right)\omega_{t-1} + \left(\Psi_2^\omega - \Psi_2^y \frac{\Psi_3^\omega}{\Psi_3^y}\right) a_t + \frac{\Psi_3^\omega}{\Psi_3^y}\tilde{y}_t \tag{28}$$

The degree of RWR used in the text is defined as $\delta^* \equiv \left(\Psi_1^\omega - \Psi_1^y \frac{\Psi_3^\omega}{\Psi_3^y}\right)$. The other coefficients used in equation (17) are: $\Psi_4^\omega \equiv \left(\Psi_2^\omega - \Psi_2^y \frac{\Psi_3^\omega}{\Psi_3^y}\right)$, $\Psi_5^\omega \equiv \frac{\Psi_3^\omega}{\Psi_3^y}$ and $\Psi_6^\omega \equiv \left(\Psi_0^\omega - \Psi_0^y \frac{\Psi_3^\omega}{\Psi_3^y}\right)$.

[20] It can be easily seen that only if the restrictions on Ψ_0^p, Ψ_0^y and Ψ_0^ω are fulfilled it holds that in a steady state $\pi_t^p = 0$, $\tilde{y}_t = 0$ and $\omega_t = \bar{\omega}^n$.

A.2.3 Solution in terms of ω_{t-1}, a_t and mrs_t

Using the expressions for the mrs_t (27) and (28) can also be easily transformed into expressions ins terms of ω_{t-1}, a_t and mrs_t. They come out as:

$$\begin{aligned}\pi_t^p \;=\; & \left(\Psi_0^p - \Psi_0^y \frac{\Psi_3^p}{\Psi_3^y}\right) + \left(\Psi_1^p - \Psi_1^y \frac{\Psi_3^p}{\Psi_3^y}\right)\omega_{t-1} + \\ & \left(\Psi_2^p - \Psi_2^y \frac{\Psi_3^p}{\Psi_3^y} - \frac{\left[\,\psi_{ya}^n\left(\sigma(1-\alpha)+\varphi\right)+\varphi\right]\Psi_3^p}{\left(\sigma(1-\alpha)+\varphi\right)\Psi_3^y}\right) a_t + \frac{(1-\alpha)\Psi_3^p}{\left(\sigma(1-\alpha)+\varphi\right)\Psi_3^y} mrs_t\end{aligned} \tag{29}$$

$$\begin{aligned}\omega_t \;=\; & \left(\Psi_0^\omega - \Psi_0^y \frac{\Psi_3^\omega}{\Psi_3^y}\right) + \left(\Psi_1^\omega - \Psi_1^y \frac{\Psi_3^\omega}{\Psi_3^y}\right)\omega_{t-1} + \\ & \left(\Psi_2^\omega - \Psi_2^y \frac{\Psi_3^\omega}{\Psi_3^y} - \frac{\left[\,\psi_{ya}^n\left(\sigma(1-\alpha)+\varphi\right)+\varphi\right]\Psi_3^\omega}{\left(\sigma(1-\alpha)+\varphi\right)\Psi_3^y}\right) a_t + \frac{(1-\alpha)\Psi_3^\omega}{\left(\sigma(1-\alpha)+\varphi\right)\Psi_3^y} mrs_t\end{aligned} \tag{30}$$

Note that the coefficient that captures the degree of real wage rigidity is the same as in (28) and thus again given by δ^*.[21]

A.2.4 Error Correction Form

One can also transform the solution to derive an error-correction form that is similar to various specifications that can be found in the theoretical and empirical literature. As a starting point, note that (3) can be rewritten (using (26)) as:

$$\pi_t^w = \pi_t^p - \left(1-\Psi_1^\omega\right)\left(\omega_{t-1} - \bar{\omega}^n\right) + \Psi_2^p a_t + \Psi_3^p \upsilon_t$$

From this one can derive an expression for $E_t \pi_{t+1}^w$. Using this together with (9) and (10) in (2) it follows:

$$\begin{aligned}\pi_t^w \;=\; & \beta E_t \pi_{t+1}^p - \Psi_1^\omega\left[\beta\left(1-\Psi_1^\omega\right)+\lambda_w\right]\left(\omega_{t-1}-\bar{\omega}^n\right) + \\ & \left[\lambda_w\left(\psi_{ya}^n - \Psi_2^\omega\right) - \beta\Psi_2^\omega\left(1-\rho_a-\Psi_1^\omega\right)\right] a_t - \\ & \Psi_3^\omega\left[\lambda_w + \beta\left(1-\rho_\upsilon-\Psi_1^\omega\right)\right]\upsilon_t + \kappa_w \tilde{y}_t\end{aligned} \tag{31}$$

Wage inflation thus depends on expected future price inflation ($E_t \pi_{t+1}^p$), on the output gap ($\tilde{y}_t$), on the technology and interest rate shocks (a_t and υ_t) and on the error-correction

[21] Note that all equations (27) to (30) could again also be formulated in terms of the real wage deviation: $(\omega_{t-1} - \bar{\omega}^n)$.

term $(\omega_{t-1} - \bar{\omega}^n)$, i.e. the deviation of the past real wage ω_{t-1} from the flexible-prices-no-shocks value $\bar{\omega}^n$. Substituting in (31) for v_t one can again derive an error-correction form just in terms of $E_t \pi_{t+1}^p$, a_t, $\tilde{y}_t$ and $(\omega_{t-1} - \bar{\omega}^n)$:

$$\begin{aligned} \pi_t^w \;=\;& \beta E_t \pi_{t+1}^p - \left\{ \Psi_1^\omega \left[\beta \left(1 - \Psi_1^\omega\right) + \lambda_w \right] - \Psi_1^y \frac{\Psi_3^\omega}{\Psi_3^y} \left[\lambda_w + \beta \left(1 - \rho_v - \Psi_1^\omega\right) \right] \right\} (\omega_{t-1} - \bar{\omega}^n) + \\ & \left\{ \lambda_w \left(\psi_{ya}^n - \Psi_2^\omega \right) - \beta \Psi_2^\omega \left(1 - \rho_a - \Psi_1^\omega\right) + \Psi_2^y \frac{\Psi_3^\omega}{\Psi_3^y} \left[\lambda_w + \beta \left(1 - \rho_v - \Psi_1^\omega\right) \right] \right\} a_t + \\ & \left\{ \kappa_w - \frac{\Psi_3^\omega}{\Psi_3^y} \left[\lambda_w + \beta \left(1 - \rho_v - \Psi_1^\omega\right) \right] \right\} \tilde{y}_t \end{aligned} \tag{32}$$

In the text equation (32) is used as equation (23) where the coefficients Φ_1, Φ_2 and Φ_3 are defined accordingly. Equation (32) is a form that corresponds closely to the empirical counterparts. In fact this equation is parallel to equation (6)[22] in Blanchard and Katz (1999). The coefficient of the error-correction-term depends in a rather complicated way on all parameters of the model. Note that one can write, however:

$$\begin{aligned} &\left\{ \Psi_1^\omega \left[\beta \left(1 - \Psi_1^\omega\right) + \lambda_w \right] - \Psi_1^y \tfrac{\Psi_3^\omega}{\Psi_3^y} \left[\lambda_w + \beta \left(1 - \rho_v - \Psi_1^\omega\right) \right] \right\} = \\ &\qquad \left\{ \delta^* \left[\lambda_w + \beta \left(1 - \Psi_1^\omega\right) \right] + \rho_v \Psi_1^y \tfrac{\Psi_3^\omega}{\Psi_3^y} \right\} \end{aligned} \tag{33}$$

In a similar vein one could express also the other coefficients in terms of Ψ_4^ω and Ψ_5^ω.

A.2.5 A Phillips curve (π_t^p depending on π_{t-1}^p)

One can also derive an equation that is fairly close to the traditional Phillips curve formulation. First, lag (24) by one period and then use $(\omega_{t-1} - \bar{\omega}^n) = \Psi_1^\omega (\omega_{t-2} - \bar{\omega}^n) + \Psi_2^\omega a_{t-1} + \Psi_3^\omega v_{t-1}$ to substitute for $(\omega_{t-2} - \bar{\omega}^n)$. One gets:

$$\pi_{t-1}^p = \frac{\Psi_1^p}{\Psi_1^\omega} (\omega_{t-1} - \bar{\omega}^n) + \left(\Psi_2^p - \Psi_2^\omega \frac{\Psi_1^p}{\Psi_1^\omega} \right) a_{t-1} + \left(\Psi_3^p - \Psi_3^\omega \frac{\Psi_1^p}{\Psi_1^\omega} \right) v_{t-1}$$

This can be used to find an expression for $(\omega_{t-1} - \bar{\omega}^n)$ which can then plugged into (24) to get:

$$\pi_t^p = \Psi_1^\omega \pi_{t-1}^p + \Psi_2^p a_t + \Psi_3^p v_t - \left(\Psi_2^p \Psi_1^\omega - \Psi_2^\omega \Psi_1^p \right) a_{t-1} - \left(\Psi_3^p \Psi_1^\omega - \Psi_3^\omega \Psi_1^p \right) v_{t-1} \tag{34}$$

[22] While equation (5) in Blanchard and Katz (1999) is more similar to (26).

Again one can use (25) (and a lagged version of (25)) to express v_t and v_{t-1} in terms of the other variables. It follows:

$$
\begin{aligned}
\pi_t^p \quad = \quad & \delta^* \pi_{t-1}^p + \left(\Psi_2^p - \Psi_2^y \frac{\Psi_3^p}{\Psi_3^y} \right) a_t + \frac{\Psi_3^p}{\Psi_3^y} \tilde{y}_t + \frac{1}{\Psi_3^y} \left(\Psi_1^p \Psi_3^\omega - \Psi_1^\omega \Psi_3^p \right) \tilde{y}_{t-1} + \\
& \frac{1}{\Psi_3^y} \left[\Psi_1^\omega \left(\Psi_3^p \Psi_2^y - \Psi_2^p \Psi_3^y \right) + \Psi_2^\omega \left(\Psi_1^p \Psi_3^y - \Psi_3^p \Psi_1^y \right) + \Psi_3^\omega \left(\Psi_2^p \Psi_1^y - \Psi_1^p \Psi_2^y \right) \right] a_{t-1}
\end{aligned}
\tag{35}
$$

This corresponds to equation (18) in the text.

A.2.6 The New Keynesian Phillips Curve

Expressions (24) and (25) can also be used in (1) to get an equations that resembles a New Keynesian Phillips curve. The only difference is that one gets an additional dependence on a_t.

$$
\begin{aligned}
\pi_t^p \quad = \quad & \frac{\beta \left(\Psi_1^y \Psi_3^p - \Psi_1^p \Psi_3^y \right)}{\lambda_p (\Psi_3^y \Psi_1^\omega - \Psi_1^y \Psi_3^\omega) - \Psi_3^y \ \Psi_1^p + \Psi_3^p \Psi_1^y} E_t \pi_{t+1}^p + \\
& \frac{\lambda_p (\Psi_3^p \Psi_1^\omega - \Psi_3^\omega \Psi_1^p) + \kappa_p (\Psi_3^p \Psi_1^y - \Psi_3^y \Psi_1^p)}{\lambda_p (\Psi_3^y \Psi_1^\omega - \Psi_1^y \Psi_3^\omega) - \Psi_3^y \ \Psi_1^p + \Psi_3^p \Psi_1^y} \tilde{y}_t + \\
& \frac{\lambda_p \left(\psi_{\omega a}^n \left(\Psi_3^y \Psi_1^p - \Psi_3^p \Psi_1^y \right) + \Psi_1^\omega \left(\Psi_3^y \Psi_2^p - \Psi_3^p \Psi_2^y \right) + \Psi_1^y \left(\Psi_2^\omega \Psi_3^p - \Psi_3^\omega \Psi_2^p \right) + \Psi_1^p \left(\Psi_3^\omega \Psi_2^y - \Psi_2^\omega \Psi_3^y \right) \right)}{\lambda_p (\Psi_3^y \Psi_1^\omega - \Psi_1^y \Psi_3^\omega) - \Psi_3^y \ \Psi_1^p + \Psi_3^p \Psi_1^y} a_t
\end{aligned}
\tag{36}
$$

The coefficient on the expected inflation is, however, no longer simply given by β as in the usual NKPC with only price stickiness. In this case it is in fact the case that the coefficient of $E_t \pi_{t+1}^p$ can be much larger than 1 (for the standard calibration with $\theta_p = \frac{1}{3}$ and $\theta_w = \frac{1}{2}$) one gets, e.g., that it is 2.53.

A.3 Solution to the model with Taylor wage contracts (section 3)

A.3.1 The wage-setting equation

There is no explicit treatment of the model with Taylor contracts in Galí (2008).[23] It is, however, straightforward to derive a wage-setting equation following analogous steps as in Galí (2008, chap. 6.1.2.1). Instead of equation (10) in chapter 6 the two-period Taylor model implies the following optimal wage-setting equation (for sector $i \in \{A, B\}$ that is allowed to choose a new wage in period t):

$$w_t^i = \frac{1}{1+\varepsilon_w\varphi}\sum_{k=0}^{1}\frac{\beta^k}{1+\beta}E_{t+k}\left\{\mu_w + mrs_{t+k} + \varepsilon_w\varphi w_{t+k} + p_{t+k}\right\}$$

One can also follow Galí (2008) and define $\hat{\mu}_t^w \equiv \mu_t^w - \mu^w$ as the deviation of the economy's (log) average wage markup $\mu_t^w \equiv (w_t - p_t) - mrs_t$ from its steady state level $\mu^w \equiv \log \mu^w = \log\left(\frac{\varepsilon_w}{\varepsilon_w - 1}\right)$. The marginal rate of substitution is given by (11) and thus one can also write:

$$w_t^i = \frac{1}{1+\varepsilon_w\varphi}\sum_{k=0}^{1}\frac{\beta^k}{1+\beta}E_{t+k}\left\{(1+\varepsilon_w\varphi)\,w_{t+k} - \hat{\mu}_{t+k}^w\right\}$$

Using other definitions and transformations one can also derive that $\hat{\mu}_t^w = \tilde{\omega}_t - \tilde{y}_t\left(\sigma + \frac{\varphi}{1-\alpha}\right)$. Thus another equivalent expression is:

$$w_t^i = \frac{1}{1+\varepsilon_w\varphi}\sum_{k=0}^{1}\frac{\beta^k}{1+\beta}E_{t+k}\left\{(1+\varepsilon_w\varphi)\,w_{t+k} - \tilde{\omega}_{t+k} + \tilde{y}_{t+k}\left(\sigma + \frac{\varphi}{1-\alpha}\right)\right\}$$

This formulation is used in the text as equation (19).

The average wage in period t is the weighted average of the two sectoral wages, i.e.:

$$w_t = s_A w_t^A + s_B w_t^B$$

The rest of the model is the same as in the standard case.

[23] The only exception is the end-of-chapter exercise 3.5 that deals with optimal price-setting in the Taylor model.

A.3.2 Basic Solution

One can again use the method of undetermined coefficients to derive a solution of the form:

$$w_t^i = \Gamma_0^{wi} + \Gamma_1^{wi} w_{t-1}^j + \Gamma_2^{wi} p_{t-1}^j + \Gamma_3^{wi} a_t + \Gamma_4^{wi} v_t \tag{37}$$

$$p_t^i = \Gamma_0^{pi} + \Gamma_1^{pi} w_{t-1}^j + \Gamma_2^{pi} p_{t-1}^j + \Gamma_3^{pi} a_t + \Gamma_4^{pi} v_t \tag{38}$$

$$\tilde{y}_t^i = \Gamma_0^{yi} + \Gamma_1^{yi} w_{t-1}^j + \Gamma_2^{yi} p_{t-1}^j + \Gamma_3^{yi} a_t + \Gamma_4^{yi} v_t \tag{39}$$

For $t = 0, 2, 4, \ldots$ one has that $i = A$ and $j = B$ while for $t = 1, 3, 5, \ldots$it holds that $i = B$ and $j = A$. For the symmetric case with $s^A = s^B = \frac{1}{2}$ the various coefficients are the same across sectors, i.e. $\Gamma_0^{wA} = \Gamma_0^{wB}$, $\ldots$, $\Gamma_4^{yA} = \Gamma_4^{yB}$. This, however, is no longer true for the asymmetric case with $s^A \neq s^B$ where it is thus important to distinguish between the determination of the key variables in sectors A and B. The numerical calculations have shown a useful result:[24]

$$\Gamma_1^{wi} + \Gamma_2^{wi} = 1,\ \Gamma_1^{pi} + \Gamma_2^{pi} = 1,\ \Gamma_1^{yi} + \Gamma_2^{yi} = 0 \tag{40}$$

A.3.3 The period-on-period and year-on-year RWR in terms of v_t

The solutions can be used to derive an expression for the real wage $\omega_t^i \equiv w_t^i - p_t^i$ in sector i:

$$\omega_t^i = \left(\Gamma_0^{wi} - \Gamma_0^{pi}\right) + \left(\Gamma_1^{wi} - \Gamma_1^{pi}\right) \omega_{t-1}^j + \left(\Gamma_3^{wi} - \Gamma_3^{pi}\right) a_t + \left(\Gamma_4^{wi} - \Gamma_4^{pi}\right) v_t \tag{41}$$

Since the coefficients might be different across the two sectors also the reaction of ω_t^A to ω_{t-1}^B might be different from the reaction of ω_{t+1}^B to ω_t^A etc. For this reason (and in order to make the cases with different weights s_A and s_B better comparable) it is useful to write the real wage ω_t^i in sector i as a function of its own last optimally set real wage wage ω_{t-2}^i. This comes out as:

$$\omega_t^i = \Gamma_0^{\omega i} + \Gamma_1^{\omega i} \omega_{t-2}^i + \Gamma_2^{\omega i} a_t + \Gamma_3^{\omega i} a_{t-1} + \Gamma_4^{\omega i} v_t + \Gamma_5^{\omega i} v_{t-1}, \tag{42}$$

where $\Gamma_0^{\omega i} = \left(\Gamma_0^{wi} - \Gamma_0^{pi}\right) + \left(\Gamma_1^{wi} - \Gamma_1^{pi}\right)\left(\Gamma_0^{wj} - \Gamma_0^{pj}\right)$, $\Gamma_1^{\omega i} = \left(\Gamma_1^{wi} - \Gamma_1^{pi}\right)\left(\Gamma_1^{wj} - \Gamma_1^{pj}\right)$, $\Gamma_2^{\omega i} = \left(\Gamma_3^{wi} - \Gamma_3^{pi}\right)$, $\Gamma_3^{\omega i} = \left(\Gamma_1^{wi} - \Gamma_1^{pi}\right)\left(\Gamma_3^{wj} - \Gamma_3^{pj}\right)$, $\Gamma_4^{\omega i} = \left(\Gamma_4^{wi} - \Gamma_4^{pi}\right)$, $\Gamma_5^{\omega i} = \left(\Gamma_1^{wi} - \Gamma_1^{pi}\right)\left(\Gamma_4^{wj} - \Gamma_4^{pj}\right)$.

[24] Intuitively, these relationships follow from the fact that the "order" of normal variables on each side of equations (37) to (39) has to be the same. Unfortunately, I have not been able to show analytically that these relationships have to hold. Nevertheless, they have been confirmed for all numerical cases that have been scrutinized.

Note that the coefficient $\Gamma_1^{\omega i}$ that determines the extent of (year-on-year) rigidity of the sectoral real wage is the same in both sectors:

$$\Gamma_1^{\omega} = \Gamma_1^{\omega A} = \Gamma_1^{\omega B} = \left(\Gamma_1^{wA} - \Gamma_1^{pA}\right)\left(\Gamma_1^{wB} - \Gamma_1^{pB}\right) \tag{43}$$

For comparisons across models and across countries etc. one is, however, not so much interested in the rigidity of the sectoral real wage but in the rigidity of the average real wage given by:

$$\bar{\omega}_t^A \equiv w_t - p_t^A = \left(s_A w_t^A + s_B w_{t-1}^B\right) - p_t^A = s_A \omega_t^A + s_B \omega_{t-1}^B - s_B\left(p_t^A - p_{t-1}^B\right) \tag{44}$$

$$\bar{\omega}_{t+1}^B \equiv w_{t+1} - p_{t+1}^B = \left(s_A w_t^A + s_B w_{t+1}^B\right) - p_{t+1}^B = s_A \omega_t^A + s_B \omega_{t+1}^B - s_A\left(p_{t+1}^B - p_t^A\right) \tag{45}$$

I use here $\bar{\omega}_t^A$ and $\bar{\omega}_{t+1}^B$ to distinguish clearly between the average real wage in periods when sector A sets the new wage and when sector B does so. One can take the expressions for ω_t^A and ω_{t-1}^B (from (42)) and insert them into (44). Noting that $s_A\omega_{t-2}^A + s_B\omega_{t-3}^B = \bar{\omega}_{t-2}^A + s_B\left(p_{t-2}^A - p_{t-3}^B\right)$ one can derive:

$$\begin{aligned} \bar{\omega}_t^A \quad = \quad & \left(s_A\Gamma_0^{\omega A} + s_B\Gamma_0^{\omega B}\right) + \left(\Gamma_1^{\omega}\right)\bar{\omega}_{t-2}^A + \\ & \left(s_A\Gamma_2^{\omega A}\right)a_t + \left(s_A\Gamma_3^{\omega A} + s_B\Gamma_2^{\omega B}\right)a_{t-1} + \left(s_B\Gamma_3^{\omega B}\right)a_{t-2} + \\ & \left(s_A\Gamma_4^{\omega A}\right)v_t + \left(s_A\Gamma_5^{\omega A} + s_B\Gamma_4^{\omega B}\right)v_{t-1} + \left(s_B\Gamma_5^{\omega B}\right)v_{t-2} - \\ & s_B\left(p_t^A - p_{t-1}^B\right) + \Gamma_1^{\omega}s_B\left(p_{t-2}^A - p_{t-3}^B\right) \end{aligned} \tag{46}$$

Following similar steps one can derive a parallel expression for $\bar{\omega}_{t+1}^B$:

$$\begin{aligned} \bar{\omega}_{t+1}^B \quad = \quad & \left(s_B\Gamma_0^{\omega B} + s_A\Gamma_0^{\omega A}\right) + \left(\Gamma_1^{\omega}\right)\bar{\omega}_{t-1}^B + \\ & \left(s_B\Gamma_2^{\omega B}\right)a_{t+1} + \left(s_B\Gamma_3^{\omega B} + s_A\Gamma_2^{\omega A}\right)a_t + \left(s_A\Gamma_3^{\omega A}\right)a_{t-1} + \\ & \left(s_B\Gamma_4^{\omega B}\right)v_{t+1} + \left(s_B\Gamma_5^{\omega B} + s_A\Gamma_4^{\omega A}\right)v_t + \left(s_A\Gamma_5^{\omega A}\right)v_{t-1} - \\ & s_A\left(p_{t+1}^B - p_t^A\right) + \Gamma_1^{\omega}s_A\left(p_{t-1}^B - p_{t-2}^A\right) \end{aligned} \tag{47}$$

Note that the (year-on-year) rigidity of the average real wage is the same in both periods, independent of which sector sets the new wages. It is given by Γ_1^{ω}. Note, however, that the reaction of the average real wage to supply shocks and monetary policy shocks is different in the two subperiods. And note also that the average real wage also depends on current and past inflation rates.[25]

A.3.4 A Phillips curve (in terms of v_t)

One can again also derive a expression that is similar to a backward-looking Phillips curve. Using $\pi_t^A \equiv p_t^A - p_{t-1}^B$, $\pi_{t-1}^B \equiv p_{t-1}^B - p_{t-2}^A$ and equations (37) and (38) for w_t^A, w_{t-1}^B, p_t^A and p_{t-1}^B one can solve (using (40)) for π_t^A as a function of π_{t-1}^B, a_t, a_{t-1}, v_t and v_{t-1}. Similarly, π_{t-1}^B can be written as a function of π_{t-2}^A, a_{t-1}, a_{t-2}, v_{t-1} and v_{t-2}. Taking these two together one can thus write π_t^A as a function of last year's inflation π_{t-2}^A and present and past levels of a_t and v_t. Doing this, the term on lagged inflation comes out as Γ_1^ω.[26] This is the same result as in the case of the standard model where the coefficient of RWR (26) is the same as the one of past inflation in the backward looking Phillips curve (see (34)).

A.3.5 Real wage rigidity and a Phillips curve (in terms of $\tilde{y}_t$)

For the sake of comparison it is sometimes better to express all relations in terms of $\tilde{y}_t$ instead of v_t. One can use (39) to write $v_t = \frac{1}{\Gamma_4^{yi}}\left(\tilde{y}_t^i - \Gamma_0^{yi} - \Gamma_1^{yi} w_{t-1}^j - \Gamma_2^{yi} p_{t-1}^j - \Gamma_3^{yi} a_t\right)$. From this it follows that:

$$\begin{aligned} \omega_t^i \;=\; & \left(\Gamma_0^{wi} - \Gamma_0^{pi} - \frac{\left(\Gamma_4^{wi} - \Gamma_4^{pi}\right)\Gamma_0^{yi}}{\Gamma_4^{yi}}\right) + \left(\Gamma_1^{wi} - \Gamma_1^{pi} - \frac{\left(\Gamma_4^{wi} - \Gamma_4^{pi}\right)\Gamma_1^{yi}}{\Gamma_4^{yi}}\right)\omega_{t-1}^j + \\ & \left(\Gamma_3^{wi} - \Gamma_3^{pi} - \frac{\left(\Gamma_4^{wi} - \Gamma_4^{pi}\right)\Gamma_3^{yi}}{\Gamma_4^{yi}}\right)a_t + \left(\frac{\left(\Gamma_4^{wi} - \Gamma_4^{pi}\right)}{\Gamma_4^{yi}}\right)\tilde{y}_t^i \end{aligned} \tag{48}$$

This equation corresponds to (41), the only difference is that it is specified in terms of ω_{t-1}^j, a_t and $\tilde{y}_t^i$ (instead of v_t). One can then follow the same steps as above and write first ω_t^i as a function of ω_{t-2}^i, a_t, a_{t-1}, $\tilde{y}_t^i$ and $\tilde{y}_{t-1}^j$. The coefficient of ω_{t-2}^i is now given by

[25] In fact, it can be shown that there exist expressions that are similar to (46) and (47) and where $\bar{\omega}_t^A$ and $\bar{\omega}_{t+1}^B$ depend on exactly the same variables while there are different restrictions on the coefficients. Also the measures of RWR will differ in these specifications. I stick to the expressions in (46) and (47) since Γ_1^ω reappears in the Phillips curve formulation (see below).

[26] The complete equation is rather long and π_t^A now depends on π_{t-2}^A, a_t, a_{t-1}, a_{t-2}, v_t, v_{t-1} and v_{t-2}. It has been calculated in a *Mathematica* file which is available upon request. Note that this is also the coefficient of π_{t-1}^B that one gets if π_{t+1}^B is expressed as a function of π_{t-1}^B etc. So Γ_1^ω is the relevant persistence term in both periods and sectors.

$\tilde{\Gamma}_1^\omega$ where:

$$\tilde{\Gamma}_1^\omega \equiv \frac{\left[\Gamma_1^{yA}\left(\Gamma_4^{wA}-\Gamma_4^{pA}\right)-\Gamma_4^{yA}\left(\Gamma_1^{wA}-\Gamma_1^{pA}\right)\right]\left[\Gamma_1^{yB}\left(\Gamma_4^{wB}-\Gamma_4^{pB}\right)-\Gamma_4^{yB}\left(\Gamma_1^{wB}-\Gamma_1^{pB}\right)\right]}{\Gamma_4^{yA}\Gamma_4^{yB}} \tag{49}$$

It is again the case that $\tilde{\Gamma}_1^\omega$ is the same for both periods and sectors. It thus is the measure of (year-on-year) real wage rigidity. This is the magnitude I focus in the text when I compare different models and specifications ($\tilde{\delta} \equiv \tilde{\Gamma}_1^\omega$). In a next step one can again write $\bar{\omega}_t^A$ as a function of $\bar{\omega}_{t-2}^A$, a_t, a_{t-1}, a_{t-2}, $\tilde{y}_t^A$, $\tilde{y}_{t-1}^B$, $\tilde{y}_{t-2}^A$, π_t^A and π_{t-2}^A and $\bar{\omega}_{t+1}^B$ as a function of $\bar{\omega}_{t-1}^B$, a_{t+1}, a_t, a_{t-1}, $\tilde{y}_{t+1}^B$, $\tilde{y}_t^A$, $\tilde{y}_{t-1}^B$, π_{t+1}^B and π_{t-1}^B. The coefficient of $\bar{\omega}_{t-2}^A$ and $\bar{\omega}_{t-1}^B$ is again given by $\tilde{\Gamma}_1^\omega$. This is also the coefficient on the backward-looking Phillips curve, again in both sectors (periods).[27] This is the same result as in the case of the standard model where the coefficient of RWR δ^* in (28) is the same as in the backward looking Phillips curve (see (35)). So $\tilde{\Gamma}_1^\omega$ in the model with (symmetric or asymmetric) Taylor wage contracts corresponds to $(\delta^*)^2$ in the model with Calvo wage contracts.

[27]For this one can again use $\pi_t^A = p_t^A - p_{t-1}^B$, $\pi_{t-1}^B = p_{t-1}^B - p_{t-2}^A$ and equations (37), (38) and (39) for w_t^A, w_{t-1}^B, p_t^A, p_{t-1}^B, $\tilde{y}_t^A$ and $\tilde{y}_{t-1}^B$ to solve (using (40)) for π_t^A as a function of π_{t-1}^B, a_t, a_{t-1}, $\tilde{y}_t^A$ and $\tilde{y}_{t-1}^B$. Similarily, π_{t-1}^B can be written as a function of π_{t-2}^A, a_{t-1}, a_{t-2}, $\tilde{y}_{t-1}^B$ and $\tilde{y}_{t-2}^A$. Taking these two together one can thus write π_t^A as a function of last year's inflation π_{t-2}^A and present and past levels of a_t and $\tilde{y}_t$. Following these steps one gets the term on lagged inflation as $\tilde{\Gamma}_1^\omega$. Details can again be found in a *Mathematica* file.

B Notes on the time structure of the models

B.1 The impact of the choice of the basic time unit on RWR

The standard models with Calvo and with Taylor wage contracts is based on a structure where the basic time unit corresponds to one semester. In the two-period Taylor model this implies that a wage contract lasts for one year (=two semesters). This assumption has been primarily made for convenience and in order to be able to deal with the case of asymmetric sector sizes in a coherent and comprehensible way. The assumption differs, however, from the related literature where the basic time unit is normally defined as one quarter (which is in line with the frequency of the available macroeconomic data). When calibrating the model I had to be careful to choose the correct parameter values. E.g., in the baseline case I have used a discount rate of $\beta = 0.98$ and a price adjustment probability of $(1-\theta_p) = 1/3$ which implies an average price duration of $\frac{1}{1-\frac{1}{3}}$=1.5 semesters (or 270 days) which is a common value in the related literature (cf. Galí, 2008).

Despite the identical average duration of price and wage contracts it is nevertheless clear that the choice of the basic time unit has an effect on the dynamic properties of the model. In particular, a system where changes of prices and wages are *always* possible will imply higher persistence (for the same average contract duration) than a system where changes are only allowed on a quarterly or semiannually frequency. In order to study the extent of this effect I have solved the basic models under the assumption of shorter basic time units.

For the model with Calvo wage contracts this has been straightforward since it only involves some reparameterizations. In particular, if $frequ$ denotes the length of the basic time unit (measured in days), the structural parameter that corresponds to an average contract duration of x days is given by: $\theta_p = 1 - \frac{frequ}{x}$, where $x \geq frequ$. The time discount rate is given by $\beta = 0.96^{\frac{frequ}{360}}$. Following the same steps as sketched in appendix A.2 one gets an estimation for δ^*_{frequ}. This can be transformed into an annual measure of RWR by calculating $\delta^{*,annual}_{frequ} = \left(\delta^*_{frequ}\right)^{\frac{360}{frequ}}$. The results of this exercise are illustrated in Figure 5 where I have used the baseline calibration and held the average length of wage contracts constant at 360 days.

`Insert Figure 5 about here`

One observes that the choice of the basic time units has a nonneglible effect on the estimated degree of RWR. The shorter the basic time unit, the higher the RWR. For an

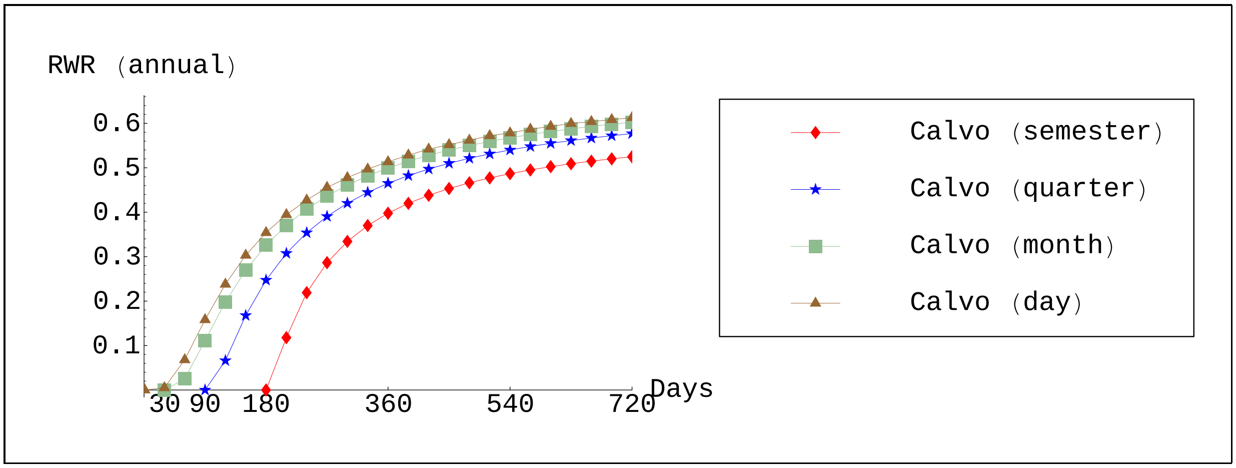

Figure 5: Comparison of the model with Calvo wage contracts with different durations of the basic time period. The figures are based on the parameters of the baseline calibration. The average duration of wage contracts is set equal to 360 days for all specifications while the average duration of price contracts is shown on the y-axis.

average price duration of 270 days, e.g., the annual RWR is given by 0.29 (semester), 0.39 (quarter), 0.44 (month) and 0.46 (day). The intuition behind this result is clear. For the case of quarterly frequencies of price changes an average duration of 90 days is the most flexible situation one can imagine (and $\theta_p = 0$ in this case). If one takes into account, however, that prices can in general be changed more frequently then an average duration of 90 days looks already rather sticky. Assuming a day as the correct basic time unit, the "true RWR" is higher than indicated by the values based on longer time units: by 59% (semester), 17% (quarter) and 5% (month). The larger the average price duration, the smaller the bias gets. For a price duration of 360 days, e.g., the corresponding percentages are reduced to: 29% (semester), 10% (quarter) and 3% (month).

The same exercise can also be performed for the (symmetric) Taylor model, even though in this case the calculations are less straightforward. For the Calvo model the degree of annual RWR $\delta^{*,annual}_{frequ}$ can be directly derived from the solution of the period model (i.e. from δ^*_{frequ}). This is not possible (or at least intractable) for the model with Taylor wage contracts. In fact, already for the two-period structure is has been rather difficult to derive an equation of the form (22) (see also appendix A.3). For the cases with shorter basic time units such an explicit derivation of $\tilde{\delta}_{frequ}$ is no longer feasible. Therefore I have chosen an alternative strategy to come up with comparable measures of RWR for different timing assumptions. In particular, the derivations of the two-period

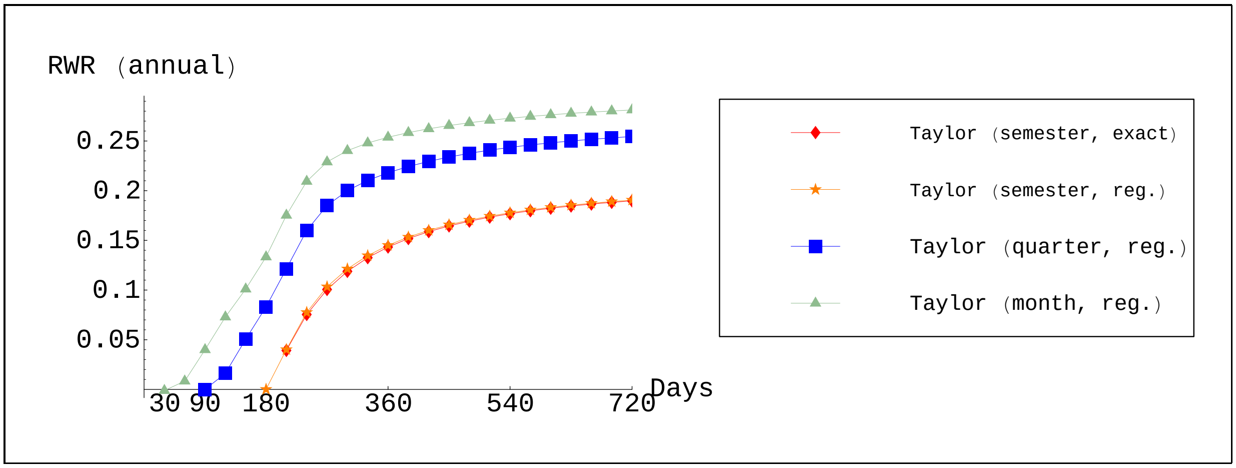

Figure 6: Comparison of the model with Taylor wage contracts with different durations of the basic time period. The figures are again based on the parameters of the baseline calibration, the average duration of wage contracts is set equal to 360 days for all specifications and the average duration of price contracts is shown on the y-axis. The red line is based on the exact measure $\tilde{\delta}$ as stated in (49) while the different-colored lines are based on regressions of current inflation π_t on the annual lag of inflation and on other explanatory variables as described in the text.

model have suggested that the extent of RWR can be accurately inferred from an empirical estimation where the rate of inflation π_t is regressed on the year-on-year lagged inflation π_{t-2} and measures for the output gap and the supply shocks for all intermediate periods (i.e. from a_t to a_{t-2} and from $\tilde{y}_t$ to $\tilde{y}_{t-2}$). As shown in (48) the coefficient on π_{t-2} is equal to $\tilde{\delta}$ and the estimated regression coefficient on π_{t-2} should thus give an accurate estimation of RWR. I have simulated 50.000 data points (assuming $\rho_a = 0 = \rho_v = 0$ and $\sigma_a = \sigma_v = 1$) and ran a regression like that. The result is plotted as the orange line in Figure 6, together with the exact (i.e. analytically derived) measure (red line) given by $\tilde{\delta}$ (as given in (49)). The two lines are indistinguishable.

`Insert Figure 6 about here`

Taking the regression results for the two-period model as a suggestive starting point I have also solved the Taylor model with 4 and with 12 subperiods.[28] I have then again simulated a large number of datapoints and I have run regressions that allow me to infer

[28] The case of daily basic time units (360 subperiods) was too cumbersome to analyze, as was the case with asymmetric sector sizes.

the degree of RWR. In particular, these regressions are of the form:

$$\pi_t = \tilde{\delta}_{quart}\pi_{t-4} + f(a_t, \ldots, a_{t-4}, \tilde{y}_t, \ldots, \tilde{y}_{t-4}) \tag{50}$$

and

$$\pi_t = \tilde{\delta}_{month}\pi_{t-12} + f(a_t, \ldots, a_{t-12}, \tilde{y}_t, \ldots, \tilde{y}_{t-12}) \tag{51}$$

The results are plotted in Figure 6. They are qualitatively similar to the case of Calvo wage contracts, although now the underestimation of RWR due to a longer basic time unit is somewhat larger. Compared to the RWR in the case of a monthly frequency of potential price changes it is 78% (semester) and 17% (quarter). The bias again decreases for larger price durations.

B.2 Accounting for different durations of wage contracts in the two-period Taylor model

The standard two-period Taylor model fixes the average duration of wage contracts at two semesters=one year. In order to be able to use the simple two-period framework also for cross-country comparisons and to allow for longer or shorter average wage durations it is necessary to make some adaptions. I use a straightforward method to make these adjustments that is based on the idea to take the average duration of wage contracts *dur* from the data and define $frequ = \frac{dur}{2}$ as the length of the basic time unit. Due to this change in units one has to respecify the discount rate and the parameter that captures price stickiness. In particular, $\beta = 0.96^{\frac{frequ}{360}}$ and $\theta_p = 1 - \frac{frequ}{x}$, where x stands for the average duration of price contracts (in days) and $x \geq frequ$. Using these values one can then follow the same steps as in chapter A.3 to calculate a value $\breve{\delta}$ as a measure of period-to-period RWR. The annual RWR can then be derived from $\tilde{\delta}^{annual}_{frequ} = \left(\breve{\delta}\right)^{\frac{360}{frequ}}$.[29]

The results for some alternative assumptions about the average duration of wage contracts are shown in Figure 7.

`Insert Figure 7 about here`

Intuition (and the experience from working with Calvo wage contracts) suggests that a longer average duration of wages should be associated — ceteris paribus — with a higher RWR. As one can observe from Figure 7 this requirement is in fact borne out by the

[29] Note that for the symmetric standard model one gets that $\tilde{\delta} = \breve{\delta}^2$ (cf. (49)).

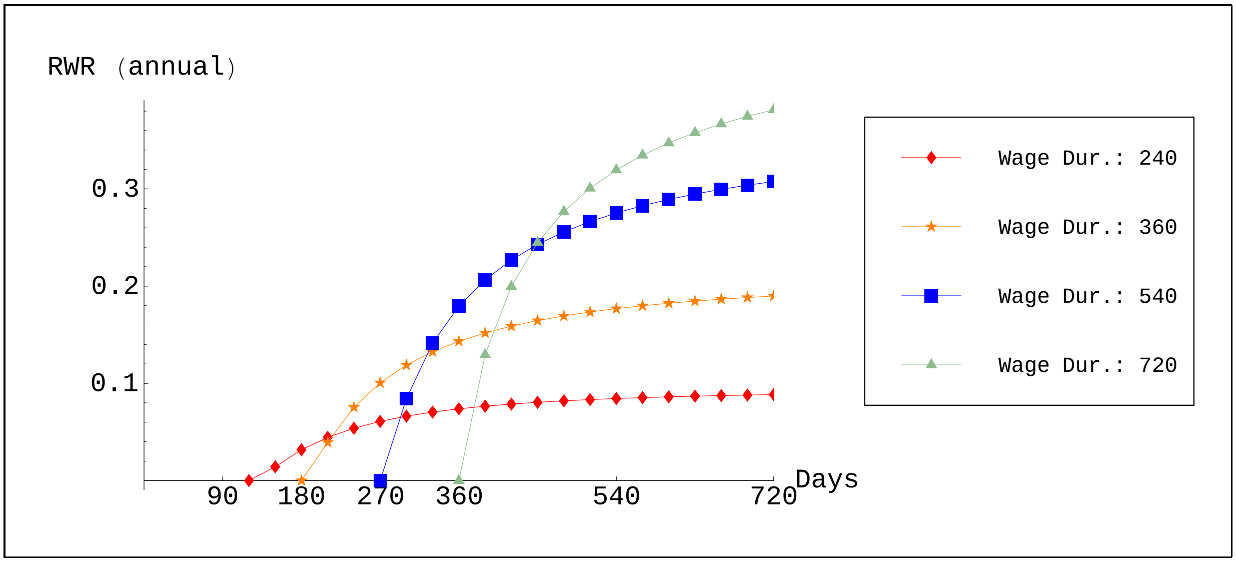

Figure 7: Comparison of the two-period Taylor model with different durations of the average wage contract (in days). The figures are based on the parameters of the baseline calibration. The average duration of price contracts is shown on the y-axis.

method described above as long as the average price duration is not too much shorter than the average wage duration. Assuming, e.g., that the two nominal variables are characterized by the same average duration one gets RWRs equal to 0.05, 0.14, 0.28 and 0.38 when the duration is 240, 360, 540 and 720 days, respectively. On the other hand, one sees that if this condition about the relative duration is not fulfilled then one might get erroneous results. If, e.g., wage agreements are written for two years while prices last on average for less than one year then the approximate method would imply a RWR of zero, which is obviously wrong. The data used for the cross-country comparisons, however, do not show such vast discrepancies in relative duration. I am thus quite confident that the employed strategy to deal with different wage durations does not lead to overly distorted results.[30]

[30]Needless to say that a model based on monthly basic time units would allow for even more accurate results. Such a finer timing structure is, however, computationally rather involved, especially in as far as asymmetric sector sizes are concerned.

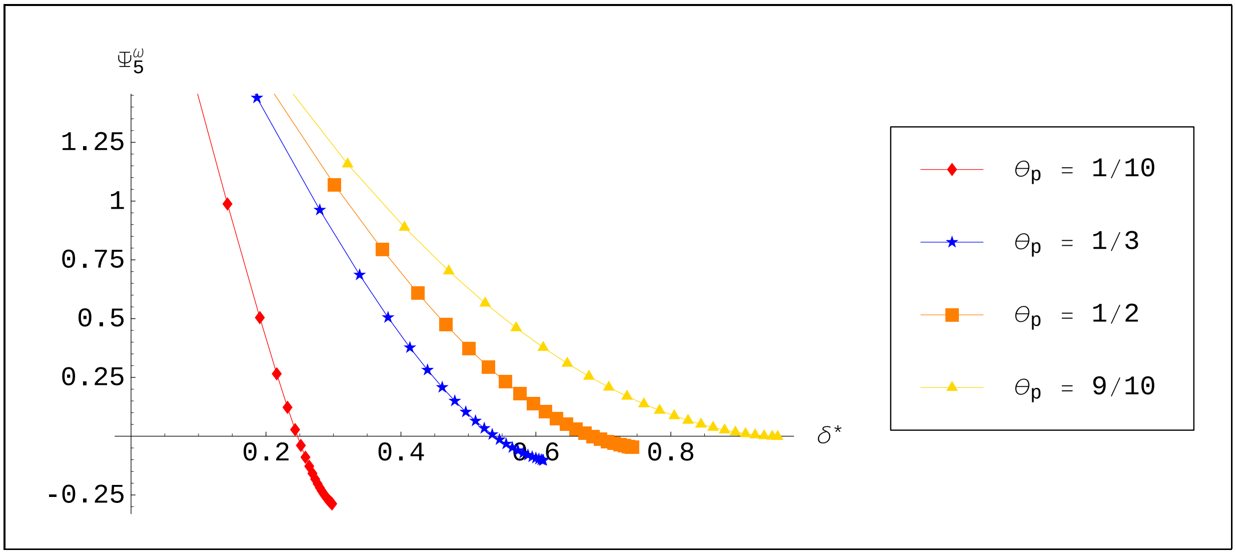

Figure 8: Scatterplot of RWR (δ^*) vs. the sensitivity of the output gap (Ψ_5^ω).

C Relation between the RWR and measures used in the empirical literature

C.1 The degree of RWR and the sensitivity to the output gap

As stated in the text, Clar et al. (2007) provide cross-country evidence on the parameter ι which is defined as measuring the reaction of the real wage to changes in the unemployment rate, i.e. $\omega_t = \ldots + \iota u_t$. One can compare the estimates of ι in Clar et al. (2007) and estimates of Ψ_5^ω if two conditions are fulfilled. First, $\tilde{y}_t$ and u_t are correlated and second Ψ_5^ω and δ^* are related. The first point is derived in Blanchard and Galí (2007) who show how to substitute a measure of unemployment for $\tilde{y}_t$. The second point is illustrated in Figure 8.

`Insert Figure 8 about here`

A higher degree of real wage rigidity δ^* is associated with a more sluggish reaction to the output gap. The negative correlation is perfect for an identical degree of price stickiness θ_p. The correlation is, however, still clearly negative even if one allows for different degrees of price rigidities. I have calculated δ^* and Ψ_5^ω for all $\theta_p \in [0, 1]$ and $\theta_w \in [0, 1]$ in steps of 0.02. The correlation among all values is -0.32 and it is even higher (-0.56) if one only focuses on more reasonable degrees of nominal price rigidity where

$\theta_p \in [0.1, 0.9]$.[31] It is interesting to note that for the baseline case with $\delta^* = 0.54$ the value for Ψ_5^ω is almost zero. This corresponds to the literature on the NKPC where it is often found that the coefficient of real marginal costs (or the output gap) is very small. Figure 8 illustrates, however, that this need not be the case and for different values of θ_p and/or θ_w one would get significantly higher values.

Figure 8 is, however, based on a specification that includes ω_{t-1} as an explanatory variable. A good number of estimations included in Clar et al. (2007) do not include a lagged real wage and it is a priori unclear how this exclusion might effect the estimation of the sensitivity with respect to the output gap or to unemployment. I have used simulated data to analyze the extent of the bias that might be introduced by such an omission. In particular, I have simulated 50.000 data point for a EHL model based on the baseline calibration and I have then estimated the parameter $\tilde{\Psi}_5^\omega$ in a regression of the form: $\omega_t = \tilde{\Psi}_5^\omega \tilde{y}_t + \tilde{\Psi}_4^\omega a_t + \tilde{\Psi}_6^\omega$. I then studied again the relation between δ^* and this biased measure $\tilde{\Psi}_5^\omega$. The correlation is still significantly negative although smaller (-0.2 for $\theta_p, \theta_w \in [0, 1]$). The Spearman rank correlation coefficient is, however, only -0.03 and not significantly different from zero.

C.2 The degree of RWR and the speed of adjustment

Equation (23) and (32) (in the appendix) show the error-correction form that follows from the EHL model.[32] The size of Φ_1 is illustrated in Figure 9.

`Insert Figure 9 about here`

Φ_1 decreases in θ_w. For flexible wages ($\theta_w = 0$) one gets that $\Phi_1 = 1$. For the baseline case (with $\theta_p = 1/3$, $\theta_w = 1/2$) the coefficient of the error-correction-term is $\Phi_1 = 0.28$. Furthermore, there exists a negative relation between the measure of real wage rigidity δ^* and the error-correction term Φ_1. This is illustrated in Figure 10.

`Insert Figure 10 about here`

[31] Using the Spearman rank correlation coefficient changes the values to -0.08 (for the large interval) and -0.30 (for the small interval).

[32] In empirical specifications one can alternatively find a measure for the error-correction-term given by $\omega_{t-1} - \omega_t^n$. A formulation like this does not follow directly from the EHL model. It can be estimated, however, using simulated data. The results (not reported) are similar to the ones that use equation (23).

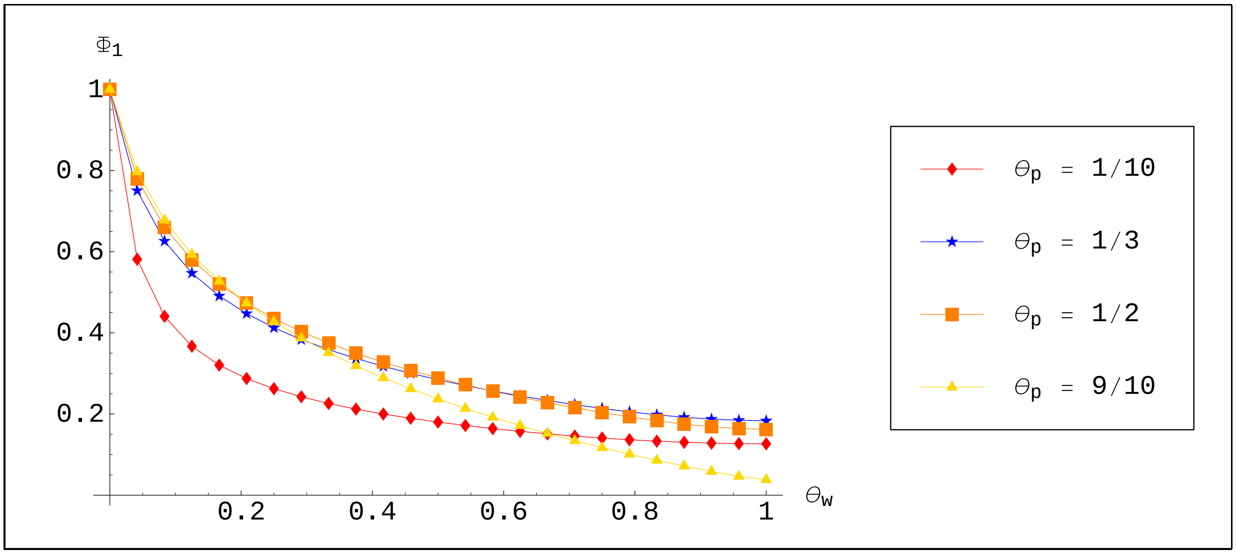

Figure 9: Coefficient of the error-correction-term (Φ_1) as specified in (32).

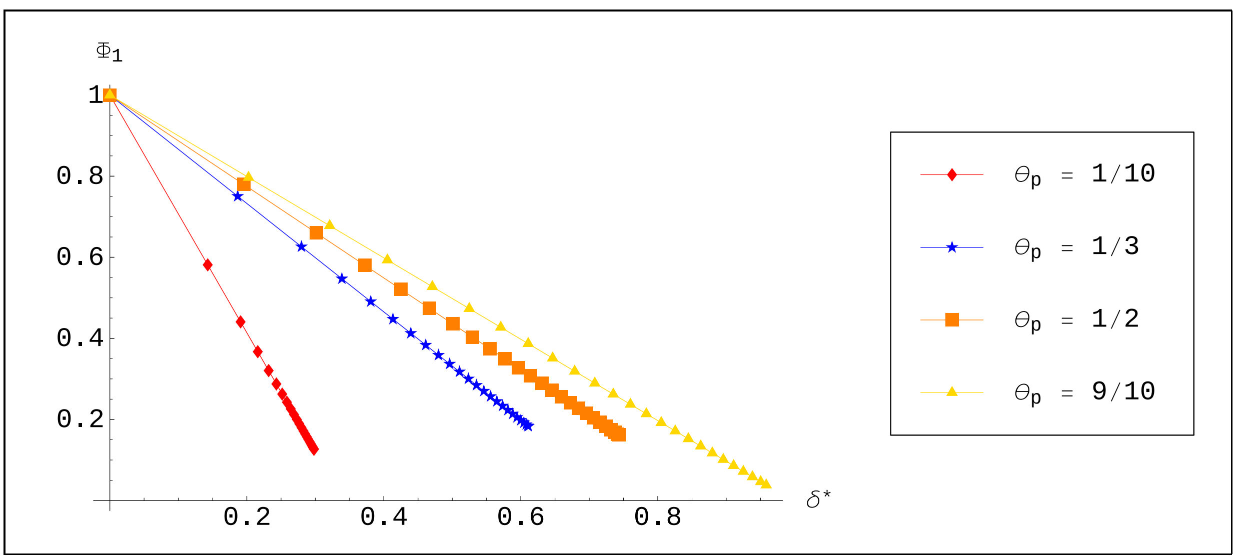

Figure 10: Scatterplot of RWR (δ^*) vs. the coefficient of the error-correction-term (Φ_1).

The negative correlation between Φ_1 and δ^* is perfect for identical θ_p or θ_w. The correlation is -0.47 (for $\theta_p \in [0,1]$) and -0.72 ($\theta_p \in [0.1, 0.9]$).[33] Note that for $\delta^* = 0$ one gets that $\Phi_1 = 1$.

A comparison of the exact magnitudes of Φ_1 in Figures 9 and 10 to the existing empirical literature is again problematic. Not only, because the empirical estimations are based on different specifications but also because the coefficient of the ECM refer specifically to the precise time structure of the model (semesters in the present case).

C.3 The degree of RWR and the sacrifice ratio

A precise measure for the sacrifice ratio cannot be derived in the framework of the basic model since a disinflation involves a new steady state and for this one cannot use the log-linearized system (cf. Ascari and Ropele, 2009). In order to see how different degrees of real rigidity affect the costs of adjustment to macroeconomic shocks I use a measure that is somewhat related to the sacrifice ratio. For this, I calculate the ratio of the sum of the deviations of the output gap to the sum of the deviations of the inflation rate after a one-period (monetary policy) shock. In particular, I denote by $\tilde{y}^{IRF}$ the impulse response to the output gap in period $t+i$ after a one-unit shock to v in period t. The variable π^{IRF} is defined in a parallel way. Examples for IRFs of this kind are presented in Figure 3. The sacrifice ratio for the cumulative impulse responses up to period $t+T$ is then defined as: $sr = \frac{\sum_{i=0}^{T} \tilde{y}_{t+i}^{IRF}}{\sum_{i=0}^{T} \pi_{t+i}^{IRF}}$.

A scatterplot in Figure 11 shows the positive correlation for various degrees of nominal price rigidity.

`Insert Figure 11 about here`

The correlation between this (somewhat sloppy) sacrifice ratio and δ^* is 0.27 (for $\theta_p \in [0,1]$).[34]

[33]The Spearman rank correlation coefficients are -0.42 and -0.66, respectively.
[34]The Spearman rank correlation coefficients is 0.67.

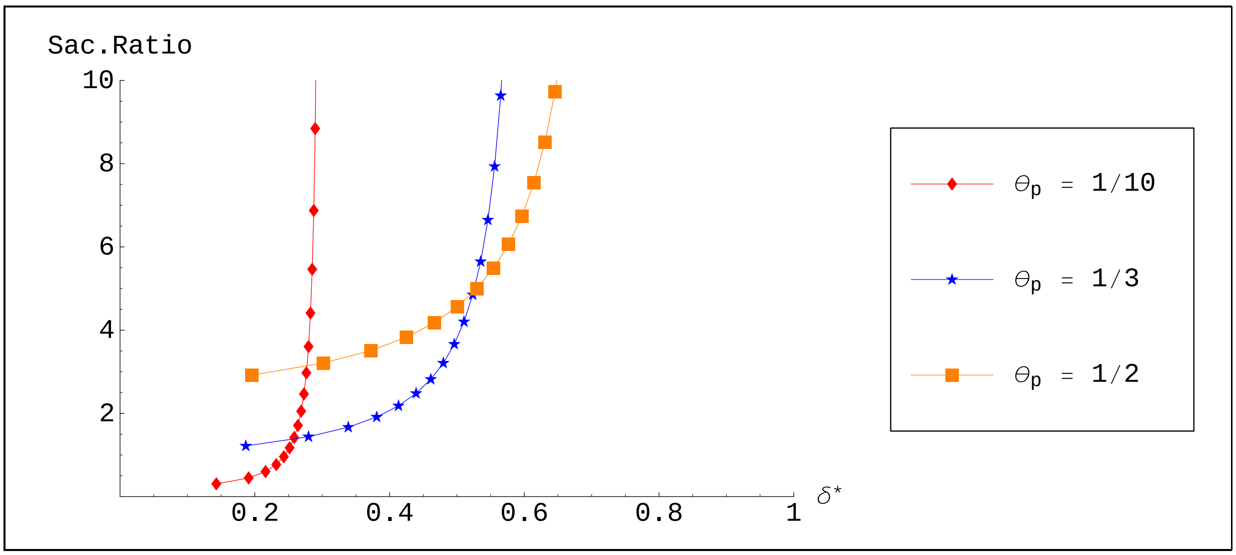

Figure 11: Scatterplot of RWR (δ^*) vs. the coefficient of the sacrifice ratio (after a shock to v_t).