# Exit Options in Incomplete Contracts with Asymmetric Information*

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November 2, 2008

**Abstract**

This paper analyzes bilateral contracting in an environment with contractual incompleteness and asymmetric information. One party (the seller) makes an unverifiable quality choice and the other party (the buyer) has private information about its valuation. A simple exit option contract, which allows the buyer to refuse trade, achieves the first–best in the benchmark cases where either quality is verifiable or the buyer's valuation is public information. But, when unverifiable and asymmetric information are combined, exit options induce inefficient pooling and lead to a particularly simple contract. Inefficient pooling is unavoidable also under the most general form of contracts, which make trade conditional on the exchange of messages between the parties. Indeed, simple exit option contracts are optimal if random mechanisms are ruled out.

*Keywords:* Incomplete Contracts, Asymmetric Information, Exit Options
*JEL Classification No.:* D82, D86, L15

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*We wish to thank Oliver Gürtler, Paul Heidhues, Timofiy Mylovanov, and Klaus Schmidt for their comments. Support by the German Science Foundation (DFG) through SFB/TR 15 is gratefully acknowledged.

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# 1 Introduction

This paper analyzes bilateral trade in environments with two potential contracting imperfections: one party has to take a decision which is publicly not verifiable, and the other party receives decision relevant private information. The environment is thus characterized by contractual incompleteness *and* asymmetric information. The parties' contracting problem is to provide incentives both for the informed party to reveal its private information and for the other party not to abuse its discretion that arises due to the lack of verifiability.

The existing literature provides core insights on what contracting can achieve if only one of the two imperfections, either non–verifiability or asymmetric information, prevails. The literature on implementation under complete information (Maskin (1977), Moore and Repullo (1988)) has studied the extent to which contracting can overcome problems caused by non-verifiable information, while the Revelation Principle (Myerson (1979)) represents the key tool to describe the set of implementable outcomes in the presence of asymmetric information. Yet little is known about how contracting is affected by the combination of unverifiable and asymmetric information. This paper presents a step in this direction.

To focus on the interaction between unverifiable and asymmetric information, we consider an environment in which efficiency can be attained in either of the two benchmark cases in which merely one of the imperfections is present. One of our main insights is that, unlike in the benchmark cases, efficient contracting is no longer possible under the joint constraints imposed by non–verifiability and asymmetric information. Therefore, it is the concurrence of contractual incompleteness and asymmetric information that causes inefficiencies. In addition to this general inefficiency result, we characterize the optimal second–best contract when we rule out contracts with random trading outcomes. We demonstrate that the optimal contract is an exit option contract which gives one party the right to exit the relation at pre–specified terms. The optimal contract is particularly simple, suggesting a rationale for why observed contracts are often simple. Finally, we characterize the optimal contract in terms of first–order conditions.

We consider a model with a seller who has to make a non–verifiable quality choice and a buyer whose valuation for quality is his private information. There is a continuum of buyer types and the efficient level of quality is a strictly increasing function of the buyer's type. Quality is publicly not verifiable (neither ex ante nor ex post), but we assume that it is observable by the buyer. Consequently, quality cannot be legally enforced and so the seller has only imperfect commitment.

Exit option contracts play a key role in our analysis.[1] An exit option gives the buyer the right, after having observed the seller's quality choice, to refuse or accept to trade at a pre–specified price. We assume that the buyer learns his private information only after contracting has been completed. Hence, contracting takes place under symmetric information so that the parties maximize their expected joint surplus of the relationship. This allows us to establish that the first–best can be achieved through exit option contracts in the two benchmark cases in which only one of the imperfections, either non–verifiability of quality or private information, is present.

We derive our inefficiency result for the most general form of contracting when the terms of trade can be made conditional on the exchange of messages between the parties. Yet, in a preliminary step we investigate how exit option contracts perform when asymmetric information and non–verifiability jointly prevail. We consider exit option contracts which require the buyer, after having privately observed his type, to send a verifiable message to the seller who then selects a quality level. Since quality is non–verifiable, we cannot appeal to the standard Revelation Principle and, instead, allow for general, not only direct, communication. We demonstrate that exit options can implement at most a single positive level of quality and can sort buyer types in at most two groups: low valuation types will not trade the good, and high buyer types will trade the same quality of the good. Thus, while efficiency calls for a perfect sorting of types, pooling of buyers is unavoidable under exit option contracts.

To understand the inefficiency result, it is useful to understand why first–best efficient exit options can be designed in our two benchmark cases. If the buyer's valuation is public information, the optimal exit option leaves the buyer indifferent between exit and trade at the efficient quality level. This induces the seller to choose the efficient quality since a downward deviation would trigger the buyer to exit, leaving the seller without sales. In contrast, when information is private and the seller can commit to quality, the standard revelation principle is applicable, and a contract specifies a quality contingent on (a report about) the buyer's type. Incentive compatibility then requires that higher buyer types obtain a higher utility *ex post* since otherwise they would have incentives to mimic lower types. This, in turn, implies

[1] It is well–known from the incomplete contracts literature that contracts with pre–specified default options can resolve obstacles that arise from non–verifiability. See, e.g., Chung (1991), Aghion, Dewatripont and Rey (1994), Nöldeke and Schmidt (1995, 1998), Edlin and Reichelstein (1996), Evans (2008). Option contracts are frequently observed in practice. For example, almost all labor contracts give the employee the right to quit. Also, certain financial contracts such as convertible bond securities can be interpreted as exit option contracts.

that higher buyer types must strictly prefer trade over exit for otherwise low types could achieve the same utility as high types by claiming to be a high type and then simply exiting.

Therefore, there is a tension between providing first–best incentives jointly for the seller and the buyer. While limited commitment by the seller requires all buyer types to be indifferent between trade and exit, incentive compatibility requires (almost all) buyer types to prefer trade over exit. Thus, the constraints that arise from limited commitment and private information cannot be met jointly by an exit option contract without violating efficiency.

We then turn to the question of whether efficiency can be achieved by a contract that has a more general form than an exit option. Indeed, exit options are not fully general, as they restrict the communication between the parties after the seller has chosen quality to a 'trade'– or 'exit'–message by the buyer. In addition, they restrict the probability of trade to be either one or zero. We therefore consider contracts that condition the possibly random trading outcome on arbitrary forms of verifiable communication between the seller and the buyer after the buyer has announced an initial message about his private information and has observed the seller's quality choice.

We show that even when we allow for the most general contracts, the main tenet of our analysis of exit options remains to be true: partial pooling of buyer types is unavoidable, and first–best efficiency cannot be attained. The driving force is similar to the exit option case. To induce the seller to choose first–best quality, the contract needs to endow the buyer with a credible threat that deters the seller not to deviate from the first–best quality. Efficiency requires that in equilibrium no buyer makes use of his threat. Moreover, for the threat to be credible the buyer must be indifferent between what he gets in equilibrium and what he would get did he enforce the threat. But similarly as in the case of exit option contracts, it would then become attractive for low buyer types to claim to be of a high type and then exert the threat.

In practice, contracts that prescribe trade to be random are questionable with regard to their legal enforceability. This raises the issue of what can be achieved by general mechanisms with deterministic trade. We demonstrate that if random trade is ruled out, then in fact allowing for more general message games does not generate an efficiency gain over the use of simple exit option contracts. The significance of this result is that it provides a rationale for why observed contracts are often simple. Notice that the efficient exit option contract of the benchmark cases is more complex than the exit option contract in the general environment to the extent that the former implements a continuum of qualities, each one fine–tuned to the buyer's valuation, whereas the latter implements only a single positive quality level. In this sense, as the contracting environment becomes more complex, the resulting contractual arrangement actually becomes simpler.

We characterize the optimal exit option contract. Since only a single quality level can be implemented under an exit option contract, the optimal contract can be derived from a straightforward maximization problem, which represents a substantial simplification of the seller's original mechanism design problem.

**Related Literature**

This paper contributes to the literature by combining implementation under complete and incomplete information which the existing literature largely treats as separate domains. The basic idea of implementation under complete information is that the information that the parties commonly observe can be reflected in verifiable messages to a third party.[2] A contract may therefore specify an outcome as a function of such messages and thus provide appropriate incentives for parties to select non–verifiable actions ex ante. Indeed, the efficient exit option mechanism of our first benchmark case in which the buyer's valuation is public information is an example of a sequential mechanism in the spirit of subgame perfect implementation (cf. Che and Hausch (1999), Proposition 1). However, in an environment in which there is not only non–verifiable but also asymmetric information at the communication stage, we cannot apply implementation results that rely on complete information. Instead, we study which trading outcomes can be implemented as a Bayesian Nash equilibrium after the seller has chosen quality. In the spirit of Maskin (1977), we require strong implementation and demonstrate that the combination of private and unverifiable information severely restricts the range of implementable outcomes. Importantly, since we assume contracting to take place under symmetric information, the first–best can be achieved in our other benchmark case in which quality is verifiable. Therefore, our inefficiency result does not originate simply in the buyer's power to extract information rents. It is the lack of verifiability in combination with asymmetric information that generates inefficiencies.

Reversely, the predominant focus of the literature on implementation under incomplete information has been how to elicit private information when contracts are complete. The

[2] See the seminal papers by Maskin (1977) and Moore and Repullo (1988). For a survey, see Moore (1992).

standard Revelation Principle (see e.g. Myerson (1979)) states that the range of implementable outcomes coincides with the set of outcomes that can be achieved through direct and truthful communication. Yet since our model displays contractual incompleteness, we cannot rely on this principle because it requires the contracting parties to write a complete contract in the sense that all message–dependent variables are specified as part of the mechanism.[3] As Bester and Strausz (2001, 2007) show, if this requirement is not satisfied, the optimal mechanism may use some form of noisy communication with only partial information revelation. Indeed, for our analysis of optimal exit options we can apply the framework of Bester and Strausz (2001), except for the technical problem that we do not consider a finite type space. In our context, noisy communication actually simplifies the optimal contract because it pools the continuum of buyer types into merely two groups: all types below a critical type do not trade, and all other types purchase the same quality.

Finally, our work is related to the large literature on the hold–up problem. The key difference is that in line with much of the literature on implementation, we assume that the parties can commit not to renegotiate ex post inefficient outcomes.[4] In contrast, the hold–up literature has studied what contracts can achieve in the absence of this commitment. Our setup can be seen as a hold–up problem where the seller's quality choice corresponds to a 'purely cooperative' ex ante investment that enhances the buyer's valuation, and the buyer does not invest. In the context of an exit option contract, our commitment assumption means that the parties can commit not to renegotiate the pre-specified terms of trade if the buyer exerts the exit option while gains from trade would exist.

While some authors argue that contract renegotiation leads to inefficient investments by substantially or even fully undermining the power of contracting (Hart and Moore (1988), Che and Hausch (1999), Edlin and Hermalin (2007)), others have identified contractual devices that induce first–best investments (Chung (1991), Aghion, Dewatripont and Rey (1994), Nöldeke and Schmidt (1995), Edlin and Reichelstein (1996), Evans (2006, 2008)). Our paper is complementary to this debate. It provides an inefficiency result which is not rooted in the parties' lack of commitment to enforce ex post inefficient default outcomes. Since the inefficiencies associated with unverifiable investments are important for providing explanations for different economic institutions (e.g. Grossman and Hart (1986), Hart and Moore (1990)), our analysis suggests that enriching the incomplete contracts paradigm by the consideration

[3] In our model, this would require that the seller's quality choice is contractually determined as a function of the buyer's report about his valuation.

[4] For implementation and renegotiation under complete information see Maskin and Moore (1999).

of asymmetric information may be a fruitful direction for the analysis of organizations.

This paper is organized as follows. Section 2 we describe the contracting environment. In Section 3 we consider exit option contracts in the benchmark cases, where either quality is verifiable or the buyer's valuation is public information. Section 4 studies the optimal exit option contract with private *and* unverifiable information. Section 5 extends the analysis by considering messages games. Section 6 provides concluding remarks. The proofs of all formal results are relegated to an appendix in Section 7.

# 2 The Model

We consider a buyer and a seller, who are both risk neutral. In the first stage $t = 0$ they can write a contract about the terms of trade, which occurs in some future stage $t = 3$. After a contract has been signed, the realization of a random variable $\theta$ determines the buyer's type in stage $t = 1$. In stage $t = 2$ the seller selects the quality $q \geq 0$ of an indivisible good. The buyer's valuation of consuming quality $q$ depends on his type $\theta$ and is given by $v(q, \theta)$. The seller's cost of producing quality $q$ is $c(q)$. In stage $t = 3$ the buyer observes the seller's quality choice. Figure 1 summarizes the sequence of events.

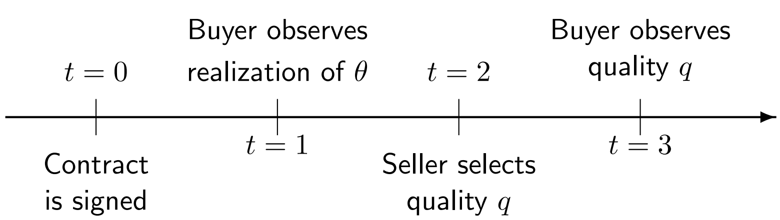


Figure 1: THE SEQUENCE OF EVENTS

In the first step of the analysis we study what the parties can achieve by using exit option contracts. In Section 5 we extend the analysis to more general contracts. An exit option contract allows the buyer in stage $t = 3$ to decide whether to accept delivery or to reject and exit. We assume that the buyer's decision is publicly observable. Thus at $t = 0$ it is possible to write a contract that specifies the buyer's payment $p = (p_T, p_N)$ contingent on whether trade takes place or not.[5] Note that we do not rule out payments from the seller to the buyer

[5] In principle, a contract could also require the buyer to make some down–payment $p_0$ in stage $t = 0$. But, it is easy to see that this would be equivalent to setting $p'_T = p_T + p_0$ and $p'_N = p_N + p_0$.

because $p_T$ and $p_N$ are not restricted to be non–negative. Also note that the buyer's exit option in stage $t = 3$ is endogenously determined by the contract. A contract can eliminate this option simply by specifying a sufficiently large payment $p_N$.[6]

The buyer's (gross) outside option value is zero, independently of his type $\theta$. Therefore, type $\theta$ accepts trade as long as $v(q, \theta) - p_T \geq -p_N$. We denote the buyer's decision behavior in the final stage by

$$h(q, p \,|\, \theta) = \begin{cases} 1 & \text{if} \quad v(q, \theta) - p_T \geq -p_N, \\ 0 & \text{if} \quad v(q, \theta) - p_T < -p_N. \end{cases} \tag{1}$$

Thus, the buyer type $\theta$'s payoff depends on $q$ and $p$ according to

$$\begin{aligned} U(q, p \,|\, \theta) &= h(q, p \,|\, \theta)[v(q, \theta) - p_T] - (1 - h(q, p \,|\, \theta))p_N \\ &= \max[v(q, \theta) - p_T, -p_N], \end{aligned} \tag{2}$$

The seller's profit is

$$\Pi(q, p \,|\, \theta) \;=\; h(q, p \,|\, \theta)p_T + (1 - h(q, p \,|\, \theta))p_N - c(q) \tag{3}$$

when he faces a buyer of type $\theta$.

The buyer's type $\theta$ is drawn from the interval $\Theta = [\underline{\theta}, \bar{\theta}] \subset \mathbb{R}$ according to the continuously differentiable cumulative distribution function $F(\cdot)$ with $F'(\theta) > 0$ for all $\theta \in \Theta$. Let $\mathcal{T}$ denote the Borel $\sigma$–algebra on $\Theta$. We make the following assumptions about $v(\cdot)$ and $c(\cdot)$:[7]

$$v(0, \theta) = 0,\; v_q(q, \theta) > 0,\; v_\theta(q, \theta) > 0,\; v_{qq}(q, \theta) \leq 0,\; v_{q\theta}(q, \theta) > 0, \tag{4}$$

$$c(0) = 0,\; c'(q) > 0,\; c''(q) > 0. \tag{5}$$

Finally, to avoid corner solutions, we assume that $v_q(0, \theta) > c'(0)$ and $v_q(\bar{q}, \theta) < c'(\bar{q})$ for $\bar{q}$ sufficiently large.

Our assumptions ensure that for any realization of $\theta \in \Theta$ the first–best quality, which maximizes the joint surplus,

$$\tilde{q}(\theta) \equiv \text{argmax}_{q \geq 0}\, v(q, \theta) - c(q) \tag{6}$$

[6] In contrast, Compte and Jehiel (2007) define *quitting rights* by requiring that transfers are zero in the disagreement case.

[7] Subscripts are used to denote partial derivatives.

is positive and unique. Also, by the last condition in (4), $\tilde{q}(\cdot)$ is strictly increasing in $\theta$. If, in addition to the transfers $p$, the buyer and the seller were able to contractually specify the quality-level $\tilde{q}(\theta)$ contingent upon the realization of $\theta$, this would maximize their ex ante expected total surplus in stage $t = 0$.

In what follows, however, we consider two limitations on the parties' contracting possibilities that prevent them from making $\tilde{q}(\theta)$ part of the contract. First, we assume that, although quality $q$ is perfectly observable by both parties, it is not verifiable to outsiders. Thus a contract that explicitly specifies some $q$ cannot be enforced by the courts. The buyer and the seller can only write an *incomplete contract* that leaves the selection of $q$ at the seller's discretion.

Second, we assume that the buyer is privately informed about his type $\theta$. This problem of *asymmetric information* makes it impossible to condition the variables of the contract directly upon the buyer's observation of $\theta$. But, a contract may specify a set $M$ of verifiable messages and require the buyer to select a message $m \in M$ after observing his type. An exit option contract $(M, p)$ thus consists of a message set $M$ and message contingent transfers $p\colon M \to \mathbb{R}^2$ such that, when in stage $t = 1$ the buyer reports $m \in M$, he has to pay $p_T(m)$ in stage $t = 3$ if accepting trade and $p_N(m)$ otherwise. Upon receiving the message $m$, the seller updates his beliefs about the buyer's type and chooses some quality $q(m)$ in stage $t = 2$.

The objective of our analysis is to characterize the contract that maximizes the seller's expected profit in $t = 0$ subject to the buyer's participation constraint and the restrictions imposed by contractual incompleteness and asymmetric information. But we relegate the derivation of the optimal exit option contract to Section 4. In the following section, we first consider two benchmark environments where either the quality $q$ is contractible *or* the buyer's type $\theta$ is publicly observable.

# 3 Two Benchmarks

To disentangle the implications of contractual incompleteness and asymmetric information, we consider two reference points in this section. We first derive the seller's optimal contract when quality is verifiable and contractible, but the buyer's type is private information. We then analyse the case where the buyer's type is publicly observable, but quality is not verifiable. It will turn out that in either situation the seller can appropriate the first–best surplus

$$\tilde{S} \equiv \int_{\underline{\theta}}^{\bar{\theta}} [v(\tilde{q}(\theta), \theta) - c(\tilde{q}(\theta))] \, dF \tag{7}$$

under a contract that induces the buyer of type $\theta$ to accept quality $\tilde{q}(\theta)$. This means that there is no efficiency loss as long as at least one of the variables $q$ and $\theta$ is publicly observable.

**Contractible $q$, asymmetric information about $\theta$**

Suppose quality $q$ is verifiable so that the seller can contractually commit to $q(m)$ after receiving the buyer's message $m \in M$. In this situation, the Revelation Principle (see e.g. Myerson (1979)) allows restricting the analysis to direct and truthful communication. Therefore, without loss of generality, the seller can use a contract with $M = \Theta$, $q\colon \Theta \to \mathbb{R}_+$ and $p\colon \Theta \to \mathbb{R}^2$. Further, the contract has to be incentive–compatible so that reporting truthfully is optimal for each type $\theta$ of the buyer.

The seller's problem is thus to maximize his expected profit subject to the incentive–compatibility conditions and the buyer's participation constraint:

$$\max_{\{q(\cdot),\, p(\cdot)\}} \int_{\underline{\theta}}^{\bar{\theta}} \Pi(q(\theta), p(\theta) \,|\, \theta) dF \tag{8}$$

subject to

$$U(q(\theta), p(\theta) \,|\, \theta) \geq U(q(\theta'), p(\theta') \,|\, \theta) \quad \text{for all } (\theta, \theta'), \tag{9}$$

$$\int_{\underline{\theta}}^{\bar{\theta}} U(q(\theta), p(\theta) \,|\, \theta) dF \geq 0. \tag{10}$$

The incentive compatibility constraints (9) ensure that no buyer has an incentive to misrepresent his type. Note that our incentive compatibility constraints are somewhat non–standard, compared e.g. to a standard price discrimination problem, because they also comprise that no buyer has an incentive to misreport his type and subsequently refuse to trade. The participation constraint (10) guarantees that the buyer's expected utility at the contracting stage, before he learns his type, is at least zero. The next proposition states that the first–best can be implemented.

**Proposition 1** *(a) There exists a* $p^*(\cdot)$ *such that* $\{\tilde{q}(\cdot), p^*(\cdot)\}$ *solves problem (8) – (10). Moreover,* $h(\tilde{q}(\theta), p^*(\theta) \,|\, \theta) = 1$ *for all* $\theta \in \Theta$ *and*

$$\int_{\underline{\theta}}^{\bar{\theta}} \Pi(\tilde{q}(\theta), p^*(\theta) \,|\, \theta) dF = \tilde{S}.$$

*(b) For any solution* $\{q^*(\cdot), p^*(\cdot)\}$ *of problem (8) – (10) it holds for almost all* $\theta \in \Theta$ *that*

$$v(q^*(\theta), \theta) - p_T^*(\theta) > -p_N^*(\theta).$$

The idea behind part (a) is to specify a large exit payment so that no buyer type wants to submit a report that leads to exit. This effectively eliminates the exit option and we are back in a standard price discrimination framework for which it is well known that the seller can fully extract the first–best surplus if the buyer learns his private information only ex post.

Part (b) is an implication of incentive compatibility for the buyer's trade incentives that any optimal contract has to satisfy. In light of (a), any optimal contract must extract all gains from trade and thus induce almost all buyer types to trade. Now if two buyer types trade, a straightforward implication of incentive compatibility is that the high valuation buyer must obtain a larger ex post utility $v - p_T$ than the low valuation buyer. It follows that almost any buyer type (except possibly the lowest) must strictly prefer trade over exit after reporting his type truthfully. Otherwise, if one buyer type $\theta$ was exactly indifferent, all smaller types $\theta' < \theta$ would be better off by pretending to be type $\theta$ in $t = 1$ and exiting in $t = 3$.

### Non–contractible $q$, public information about $\theta$

Suppose now that the buyer's type $\theta$ is public information and that quality $q$, though observable by both parties, is not contractible. In this situation, messages from the buyer about his type are redundant, and the seller can simply offer a contract $p : \Theta \to \mathbb{R}^2$ where the trade and exit transfers are $p(\theta)$, when the buyer's type is $\theta$. Since $q$ is not contractible, the seller will select $q$ ex post so as to maximize his profits given the transfers $p(\theta)$. In other words, the choice of $q$ is constrained by imperfect commitment on part of the seller.

The seller's problem is thus to maximize his expected profit subject to his *no–commitment constraint* and the buyer's participation constraint:

$$\max_{\{q(\cdot),\, p(\cdot)\}} \int_{\underline{\theta}}^{\bar{\theta}} \Pi(q(\theta), p(\theta) \,|\, \theta) dF \tag{11}$$

subject to

$$\Pi(q(\theta), p(\theta) \mid \theta) \geq \Pi(q', p(\theta) \mid \theta) \quad \text{for all } q', \theta, \tag{12}$$

$$\int_{\underline{\theta}}^{\bar{\theta}} U(q(\theta), p(\theta) \mid \theta) dF \geq 0. \tag{13}$$

The no–commitment constraint (12) describes the seller's choice of quality in $t = 2$. He selects $q$ to maximize his profit ex post, given the transfers $p$ and the buyer's type $\theta$. Thus, when designing the contract, the seller has to take into account his ex post incentives for selecting $q$. Even though quality cannot be contractually determined, the next proposition demonstrates that by the appropriate choice of exit options the seller can commit himself to choose the first–best quality $\tilde{q}$ ex post.

**Proposition 2** *(a) There exists a $p^*(\cdot)$ such that $\{\tilde{q}(\cdot), p^*(\cdot)\}$ solves problem (11) – (13). Moreover, $h(\tilde{q}(\theta), p^*(\theta) \mid \theta) = 1$ for all $\theta \in \Theta$ and*

$$\int_{\underline{\theta}}^{\bar{\theta}} \Pi(\tilde{q}(\theta), p^*(\theta) \mid \theta) dF = \tilde{S}.$$

*(b) For any solution $\{q^*(\cdot), p^*(\cdot)\}$ of problem (8) – (10) it holds for almost all $\theta \in \Theta$ that*

$$v(q^*(\theta), \theta) - p_T^*(\theta) = -p_N^*(\theta).$$

The basic idea behind part (a) is to contract an exit payment of zero and to specify the trade transfer in such a way that each buyer type is exactly indifferent between trade and exit when the seller offers the first–best quality. This contract commits the seller not to shirk ex post because otherwise the buyer would exit and leave the seller with a zero payment. Part (b) illuminates the implications of the no–commitment constraint for the buyer's trade incentives. Under any optimal contract the buyer needs to be indifferent between exit and trade when offered the first–best quality. Otherwise, incentives would arise for the seller to shade quality below the first–best.

Proposition 2 (a) is closely related to an observation by Che and Hausch (1999) who show that the first–best can be implemented when the parties can commit themselves not to renegotiate the contract. They continue their analysis by establishing an inefficiency result if committing not to renegotiate the contract is impossible. In contrast, we maintain the assumption that contracts are not renegotiated. In the next section, we provide a different inefficiency result for the case where the buyer's type is private information. In this sense, our analysis is complementary to Che and Hausch (1999).

Our inefficiency result is inspired by the observation that part (b) of Propositions 1 and 2 are clearly incompatible: when the buyer's type is private information, each buyer type must strictly prefer trade over exit in order to prevent lower types from untruthfully reporting a high valuation and exiting subsequently. In contrast, when quality is non–contractible, each buyer type needs to be indifferent between trade and exit in order to prevent the seller from abusing his ex post discretion. Thus, there is a tension in providing appropriate incentives jointly for the buyer (incentive compatibility) and the seller (no–commitment). This indicates that the first–best cannot be implemented when quality is non–contractible *and* the buyer's type is private information.

# 4 Exit Options

We now turn to characterizing the optimal exit option contract when the seller cannot contractually commit to some quality $q$ and, at the same time, the buyer is privately informed about his type $\theta$. For this type of problem, it is well–known that it may not be optimal to use a direct communication mechanism that induces truthful revelation. Indeed, as shown in Bester and Strausz (2001), an indirect mechanism may support outcomes that cannot be replicated by a direct mechanism. Bester and Strausz (2001) also show, however, that when the set of types $\Theta$ is finite, any *incentive efficient* outcome can be replicated by an equilibrium of a direct mechanism. Unfortunately, their result does not apply to our environment since the set $\Theta$ represents a continuum of types. To overcome this problem, we first characterize the outcomes that can be supported as a Perfect Bayesian Equilibrium under some arbitrary message set $M$.[8] This allows us in a second step to derive the seller's optimal exit option contract.

[8]In a different context also Krishna and Morgan (2004) consider a contracting problem with imperfect commitment and a continuum of types.

**Perfect Bayesian Equilibrium**

Let the message set $M$ be an arbitrary metric space and let $\mathcal{M}$ denote the Borel $\sigma-$algebra on $M$. The contract between the seller and the buyer specifies the transfers $p\colon M \to \mathbb{R}^2$. Thus, when the buyer reports $m \in M$, he has to pay $p_T(m)$ if accepting trade, and $p_N(m)$ if he exits in the final stage. The functions $p_N(\cdot)$ and $p_T(\cdot)$ are taken to be measurable.

We denote the $\theta$–type buyer's *reporting strategy* by $r(\cdot|\theta) \in Q$, where $Q$ is the set of probability measures on $\mathcal{M}$. Thus, if $r(H \,|\, \theta) > 0$ for some $H \in \mathcal{M}$, this means the message chosen by the $\theta$–type buyer lies in $H$ with probability $r(H \,|\, \theta)$.

After receiving message $m$, the seller updates his *beliefs* about the buyer's type. We denote these beliefs as $\mu(T, m)$. Thus, upon observing message $m$, the seller believes that the buyer's true type is in the set $T \in \mathcal{T}$ with probability $\mu(T, m)$. Given his beliefs, the seller chooses $q(m)$ to maximize his expected payoff.

To constitute a *Perfect Bayesian Equilibrium*, the functions $(r, \mu, q)$ have to satisfy three conditions: First, the seller's choice of $q$ has to be optimal given his beliefs. This means that $q(\cdot)$ has to satisfy *the no–commitment constraint*

$$q(m) = \operatorname*{argmax}_q \int_{\underline{\theta}}^{\bar{\theta}} \Pi(q, p \,|\, \theta)\mu(\theta, m)d\theta \tag{14}$$

for all $m \in M$.

Second, as the buyer anticipates that message $m$ will induce the seller to select $q(m)$, he will select an optimal reporting strategy. The *set of optimal messages* for type $\theta$ is

$$M(\theta) \equiv \{m \in M \,|\, U(q(m), p(m) \,|\, \theta) \geq U(q(m'), p(m') \,|\, \theta) \text{ for all } m' \in M\}. \tag{15}$$

Let $R(\theta)$ denote the *support* of the $\theta$-type buyer's reporting strategy $r(\cdot|\theta)$. Then optimality of the buyer's reporting strategy requires that

$$R(\theta) \subseteq M(\theta) \text{ for all } \theta \in \Theta. \tag{16}$$

We refer to the constraint (16) as the buyer's *communication incentive constraint*.

Third, the seller's belief $\mu$ has to be consistent with Bayesian updating on the support of the buyer's reporting strategy. This means that $\mu(\cdot, m)$ is derived from Bayes' rule whenever $m \in R(\theta)$ for some $\theta \in \Theta$. Of course, the belief $\mu$ determines the seller's choice of $q$ also for messages that lie outside the support of the buyer's reporting strategy. Yet, there are no consistency restrictions on beliefs for such messages.

**Feasible contracts**

Our next aim is to characterize the equilibrium outcomes that can arise under an arbitrary contract $(M, p)$. We demonstrate that at most a single positive quality level can be implemented in equilibrium. Let us begin by introducing further notation. Consider a Perfect Bayesian Equilibrium under some arbitrary message set $M$. In equilibrium, each buyer type submits a message $m$ and will then be offered the quality $q(m)$. We say that *trade at a positive quality* takes place if $q(m) > 0$ and the buyer accepts to trade. We denote by $M^+(\theta) \subseteq M(\theta)$ the set of all messages that are optimal for the $\theta$–type buyer and lead to trade at a positive quality:

$$M^+(\theta) \equiv \{m \in M(\theta) \,|\, q(m) > 0 \text{ and } h(q(m), p(m) \,|\, \theta) = 1\}. \tag{17}$$

We denote by $R^+(\theta) \subseteq R(\theta)$ the set of all messages that are in the support of the $\theta$–type buyer and lead to trade at a positive quality:

$$R^+(\theta) \equiv R(\theta) \cap M^+(\theta). \tag{18}$$

If $m \in R^+(\theta)$, we refer to $m$ as a *positive trade message for buyer type* $\theta$. For a given message $m$, we collect all types for whom $m$ is a positive trade message in the set $T^+(m)$:

$$T^+(m) \equiv \{\theta \in \Theta \,|\, m \in R^+(\theta)\}. \tag{19}$$

Notice that $T^+(m) = \emptyset$ if and only if there is no buyer type for whom $m$ is a positive trade message, that is, $m$ is in no buyer type's support, or $q(m) = 0$, or each buyer who submits $m$ exits. Therefore, we refer to $m$ as a *positive trade message* if $T^+(m) \neq \emptyset$. For any positive trade message, we define

$$\theta_\ell(m) \equiv \inf T^+(m). \tag{20}$$

The next two lemmas state basic consequences of the no–commitment (14) and the communication incentive (16) constraints. Lemma 1 follows from (14).

**Lemma 1** *Let $m$ be a positive trade message, then the buyer type $\theta_\ell(m)$ is indifferent between trade and exit, i.e. $v(q(m), \theta_\ell(m)) - p_T(m) = -p_N(m)$ if $T^+(m) \neq \emptyset$.*

To see the intuition for Lemma 1, note that each type for whom $m$ is a positive trade message, weakly prefers trade over exit conditional on reporting $m$. Thus, by continuity, also the type $\theta_\ell(m)$ weakly prefers trade over exit when offered $q(m)$. The fact that he cannot strictly prefer trade over exit is a consequence of the seller's no–commitment constraint: when receiving message $m$, the seller infers that the buyer's type cannot be smaller than $\theta_\ell(m)$ because no type smaller than $\theta_\ell(m)$ sends message $m$ in equilibrium. Thus, if the $\theta_\ell(m)$–type strictly preferred trade over exit, the seller could slightly reduce the quality and the buyer would still accept to trade with probability 1.

The next lemma follows from Lemma 1 and the communication incentive constraint.

**Lemma 2** *The exit payments $p_N(m)$ and the types $\theta_\ell(m)$ are the same for all positive trade messages $m$, i.e. $p_N(m) = p_N(m')$ and $\theta_\ell(m) = \theta_\ell(m')$ if $T^+(m) \neq \emptyset$ and $T^+(m') \neq \emptyset$.*

To understand Lemma 2, observe first that continuity of $U$ in $\theta$ and the definition of the infimum imply that any positive trade message $m$ is an optimal message for the buyer type $\theta_\ell(m)$. Since $\theta_\ell(m)$ is indifferent between exit and trade when he sends message $m$, his utility from sending $m$ is simply $-p_N(m)$. Hence, if there was some other message $m'$ with $p_N(m') < p_N(m)$, message $m$ could not be optimal, as submitting $m'$ and exiting would yield the buyer a larger utility.

Further, the intuition for why $\theta_\ell(m) = \theta_\ell(m')$ is similar to the case in which $q$ is contractible. If two buyer types weakly prefer trade over exit upon sending some message, then the higher type must obtain a strictly larger utility $v - p_T$ in order for him not to have incentives to deviate to the message of the lower type. Hence, $\theta_\ell(m)$ must be the same as $\theta_\ell(m')$ because by Lemma 1 both types weakly prefer to trade and their utility $v - p_T$ is the same due to Lemma 1 and because $p_N(m') = p_N(m)$.

Lemma 2 allows us to define a *critical type* and constant exit payments for all positive trade messages $m$:[9]

$$\hat{\theta} \equiv \theta_\ell(m) \text{ and } \hat{p}_N \equiv p_N(m) \text{ for all } m \text{ with } T^+(m) \neq \emptyset. \tag{21}$$

From Lemmas 1 and 2 we deduce:

$$v(q(m), \hat{\theta}) - p_T(m) = -\hat{p}_N \text{ for all } m \text{ with } T^+(m) \neq \emptyset. \tag{22}$$

Condition (22) says that only such positive quality levels can be implemented as an equilibrium for which the critical type is indifferent between trade and exit. In fact, the no–commitment and communication incentive constraints together imply that only a single

[9] If there is no positive trade message, i.e. if $T^+(m) = \emptyset$ for all $m \in M$, we set $\hat{\theta} = \bar{\theta}$.

positive quality level can be implemented in equilibrium. This is stated in the following equilibrium characterization:

**Proposition 3** *In any Perfect Bayesian Equilibrium, there is a* $\hat{\theta}$ *and a* $\hat{q} > 0$ *such that:*

*(i) For all* $\theta > \hat{\theta}$ *and* $m \in R(\theta)$ *it holds that* $q(m) = \hat{q}$ *and* $h(q(m), p(m) \,|\, \theta) = 1$*.*

*(ii) For all* $\theta < \hat{\theta}$ *and* $m \in R(\theta)$ *it holds that* $q(m) = 0$ *or* $h(q(m), p(m) \,|\, \theta) = 0$*.*

The proposition says that in equilibrium only an imperfect sorting of types into two groups can occur and that at most one group can trade at a positive quality level. A finer sorting of types, say with two positive quality levels, is impossible because communication incentives would imply that the high quality traders must get a higher utility from trade than the low quality traders. At the same time, for high quality provision by the seller to be credible, the lowest high quality trader must be indifferent between trade and exit. But then a low quality trader can obtain the same utility as this high quality trader by asking for the high quality and then exiting.

An immediate corollary of Proposition 3 is that the first–best cannot be implemented. By Propositions 1 and 2, this inefficiency result is driven by the combined presence of private information and contractual incompleteness.

### Optimal Exit Options

We now derive the optimal exit option contract for the seller. Proposition 3 implies that the optimal contract can be found in the class of contracts that have only two messages, say $m_l, m_h$. Such a contract induces a Perfect Bayesian Equilibrium in which all 'high' types above a critical $\hat{\theta}$ report the message $m_h$ and trade the positive quality $q(m_h) = \hat{q}$, and all 'low' types below $\hat{\theta}$ report message $m_l$ and trade a zero quality $q(m_l) = 0$.

The seller's problem is to choose transfers $p = \{p_N(m_l), p_T(m_l), p_N(m_h), p_T(m_h)\}$, a quality $\hat{q}$, and a critical type $\hat{\theta}$ that maximize his ex ante profit subject to the participation constraint and the constraint that $(\hat{q}, \hat{\theta})$ can be supported as a Perfect Bayesian Equilibrium given the transfers $p$. Without loss of generality, we set $p_N(m_l) = p_T(m_l) = p_N(m_h)$ and define $p_N = p_N(m_h)$, and $p_T = p_T(m_h)$ with $p_T > p_N$.[10] Formally, the seller's problem is:

$$\max_{p_N, p_T, \hat{q}, \hat{\theta}} F(\hat{\theta})p_N + (1 - F(\hat{\theta}))(p_T - c(\hat{q})) \tag{23}$$

subject to

$$\hat{q} \in \operatorname*{argmax}_q \int_{\hat{\theta}}^{\bar{\theta}} \frac{\Pi(q, p \mid \theta)}{1 - F(\hat{\theta})} \, dF(\theta), \tag{24}$$

$$v(\hat{q}, \hat{\theta}) = p_T - p_N, \tag{25}$$

$$-F(\hat{\theta})p_N + \int_{\hat{\theta}}^{\bar{\theta}} [v(\hat{q}, \theta) - p_T] \, dF(\theta) \geq 0. \tag{26}$$

The seller's objective (23) consists of two parts. The first part is the expected profit that he extracts from the types who announce message $m_l$ and pay the transfer $p_N$. Since the quality traded is zero, no production costs accrue to the seller in this case. The second part is the expected profit that the seller extracts from the types who announce message $m_h$ and pay the transfer $p_T$. Since all these types trade quality $\hat{q}$, the seller has costs $c(\hat{q})$ in this case.

Constraints (24) and (25) require that $(\hat{q}, \hat{\theta})$ constitutes an equilibrium. By the no–commitment constraint (24), if the seller receives message $m_h$, he infers that the buyer type is larger than $\hat{\theta}$, and his belief that he faces a type $\theta$ is given by the conditional distribution $dF(\theta)/(1 - F(\hat{\theta}))$. Given these beliefs, $\hat{q}$ has to be the optimal quality selection. Condition (25) is the equilibrium requirement from Proposition 3 that the critical type $\hat{\theta}$ be indifferent between exit and trade at transfers $p$ and quality level $\hat{q}$. Finally, (26) is the buyer's ex ante participation constraint.

We proceed by making the seller's problem more tractable. Observe first that the participation constraint must obviously be binding at the optimum. Combining this with the seller's objective, the seller's problem becomes to maximize the total surplus

$$S(\hat{q}, \hat{\theta}) = \int_{\hat{\theta}}^{\bar{\theta}} (v(\hat{q}, \theta) - c(\hat{q})) \, dF(\theta) \tag{27}$$

subject to (24) and (25).

[10] By Proposition 3, all types who send message $m_l$ trade quality 0 and exit. Hence, we need that $v(0, \theta) - p_T(m_l) \leq -p_N(m_l)$, and that the seller optimally set $q = 0$ if he receives message $m_l$. Any transfers with $p_N(m_l) = p_T(m_l)$ satisfy these two requirements. Further, equating $p_T(m_l)$ and $p_N(m_h)$ is a normalization. Finally, $p_T > p_N$ because by Proposition 3, we must have: $v(\hat{q}, \hat{\theta}) - p_T = -p_N$. Since $\hat{q} > 0$, this implies that $p_T > p_N$.

Next, we reformulate the constraints (24) and (25). As explained above, these constraints embody the two requirements that the seller's choice be optimal given his beliefs, and that the seller's beliefs be consistent with the buyer's reporting strategy. To describe equilibrium, we first consider the seller's optimal quality choice (his 'best response') against arbitrary beliefs. Suppose the seller has received message $m_h$ and holds the belief that all types larger than an *arbitrary* type $\hat{\theta}$ have submitted $m_h$. Then choosing a relatively high quality $q$ with $v(q, \hat{\theta}) - p_T > -p_N$ is clearly suboptimal for him, because all types $\theta \geq \hat{\theta}$ have a strict incentive to trade and so the seller could gain by slightly lowering quality. Therefore, the seller must optimally choose a quality level $q'$ such that $v(q', \hat{\theta}) - p_T \leq -p_N$. By setting such a quality $q'$, the seller effectively chooses a type $\theta' \in [\hat{\theta}, \bar{\theta}]$ who is indifferent between trade and exit because $v(q', \theta') = p_T - p_N$. All types $\theta \geq \theta'$ accept quality $q'$, whereas all types $\theta \in [\hat{\theta}, \theta']$ exit. Thus, the seller anticipates that quality $q'$ will be rejected with probability $(F(\theta') - F(\hat{\theta}))/(1 - F(\hat{\theta}))$ and accepted with probability $(1 - F(\theta'))/(1 - F(\hat{\theta}))$.

Thus, given transfers $p$ and given the belief that all types larger than type $\hat{\theta}$ have submitted $m_h$, the seller's optimal behavior is defined by the pair

$$(q^*(\hat{\theta}, p), \theta^*(\hat{\theta}, p)) \qquad \equiv \underset{q', \theta'}{\operatorname{argmax}} \frac{F(\theta') - F(\hat{\theta})}{1 - F(\hat{\theta})} p_N + \frac{1 - F(\theta')}{1 - F(\hat{\theta})} p_T - c(q') \tag{28}$$

$$\text{subject to} \quad v(q', \theta') = p_T - p_N \quad \text{and} \quad \theta' \geq \hat{\theta}. \tag{29}$$

While (28) describes the seller's best response against arbitrary beliefs, in equilibrium the seller's beliefs are consistent with the buyer's actual behavior. This is made explicit in the next lemma which provides an alternative characterization of the equilibrium conditions (24) and (25).

**Lemma 3** *Let $p$ be given. Then $(\hat{q}, \hat{\theta})$ satisfies (24) and (25) if and only if $(\hat{q}, \hat{\theta})$ solves the following fixed–point problem:*

$$\hat{q} = q^*(\hat{\theta}, p) \quad \textit{and} \quad \hat{\theta} = \theta^*(\hat{\theta}, p). \tag{30}$$

Since the conditions (30) include the optimality conditions for the seller, $(\hat{q}, \hat{\theta})$ has to satisfy the necessary first–order conditions for optimality of problem (28). We now impose conditions on $F$ and $v$ such that the first–order conditions are actually sufficient for optimality. This allows us to state Lemma 3 in terms of first–order conditions.

**Lemma 4** *Let $F(\cdot)$ be convex and $v(\cdot)$ be quasi–concave.*[11] *For given $p$, $(\hat{q}, \hat{\theta})$ then solves the fixed–point problem (30) if and only if*

$$-\frac{F'(\hat{\theta})}{1-F(\hat{\theta})}(p_T - p_N) + c'(\hat{q})\frac{v_\theta(\hat{q},\hat{\theta})}{v_q(\hat{q},\hat{\theta})} \leq 0, \tag{31}$$

$$v(\hat{q},\hat{\theta}) = p_T - p_N. \tag{32}$$

Finally, we can eliminate transfers from the seller's problem. To see this, note that for any $(\hat{q}, \hat{\theta})$, transfers can be found such that (32) holds. Hence, we can insert (32) in (31) and obtain a single constraint that is independent of transfers. Since the objective $S(\hat{q}, \hat{\theta})$ is also independent of transfers, the seller's problem reduces to a maximization problem just over $(\hat{q}, \hat{\theta})$. The next proposition summarizes our findings.

**Proposition 4** *Let $F(\cdot)$ be convex and $v(\cdot)$ be quasi–concave. Then the seller's problem is*

$$\max_{\hat{q},\hat{\theta}} \int_{\hat{\theta}}^{\bar{\theta}} [v(\hat{q},\theta) - c(\hat{q})]\, dF(\theta) \tag{33}$$

*subject to*

$$-\frac{F'(\hat{\theta})}{1-F(\hat{\theta})} v(\hat{q},\hat{\theta}) + c'(\hat{q})\frac{v_\theta(\hat{q},\hat{\theta})}{v_q(\hat{q},\hat{\theta})} \leq 0. \tag{34}$$

In other words, the feasible set of $(\hat{q}, \hat{\theta})$–combinations which jointly satisfy the seller's no–commitment and the buyer's incentive communication incentive constraints reduces to the simple inequality constraint (34).[12] This is a rather remarkable simplification of the problem that we started out with.

Using Proposition 4, it is straightforward to compute the optimal contract. Let, for example, $\theta$ be uniformly distributed on $[0, 1]$, $v(q, \theta) = q\theta$, and $c(q) = cq^2$. Then it is easily verified that $\hat{q} = 0.3492/c$ and $\hat{\theta} = 0.5565$ solve problem (33)–(34). By (25) and (26) the optimal contract specifies the trade payment $p_T = 0.2286/c$ and the exit payment $p_N = 0.0343/c$.

[11] A sufficient condition for $v(\cdot)$ to be quasi–concave is that $v_{\theta\theta} \leq 0$ in addition to the assumptions in (4).
[12] The corresponding transfers $p_N$ and $p_T$ are determined by (25) and (26).

# 5 Message Games

In the two benchmark situations considered in Section 3, a simple exit option contract implements the first–best already. Therefore the seller cannot increase his profit by using a more complicated mechanism. Yet, as our analysis in the previous section has shown, the first–best cannot be achieved by such simple contracts when the buyers' type is private information *and* quality is not verifiable. In this section we extend this observation by showing that the first–best cannot be implemented even under the most general form of contracting.

From a general contracting perspective, exit option contracts are restrictive in two ways. First, the trade outcome described by (1) is deterministic. If a publicly verifiable randomisation device is available, a contract can more generally specify a *probability of trade*. Second, an exit option contract limits communication to a simple message of the buyer whether he accepts or refuses trade. Given that both parties are informed about the seller's quality choice, trade can more generally be made contingent on the outcome of a *message game* in which the parties use their information to exchange verifiable messages.

To remove these restrictions of the exit option contract, we modify stage $t = 3$ of the environment described in Section 2. In $t = 3$, the seller and the buyer now become engaged in a message game, where they simultaneously select messages $z_S \in Z_S$ and $z_B \in Z_B$, respectively. Even though we describe the exchange of messages as a static game, this description may be thought of as the normal form representation of a dynamic game involving many stages of communication. The messages selected in $t = 3$ are verifiable so that the terms of trade can be contractually specified as a function of the buyer's message $m \in M$ in stage $t = 1$ and the outcome $z = (z_S, z_B) \in Z \equiv Z_S \times Z_B$ of the message game in stage $t = 3$. Thus, in addition to the message sets $(M, Z)$, a contract in most general form determines a probability of trade $x(m, z)$ and an *expected* payment $p(m, z)$ from the buyer to the seller. More formally, a contract is now a combination $(M, Z, x, p)$, where $x\colon M \times Z \to [0, 1]$ and $p\colon M \times Z \to \mathbb{R}$.[13] The buyer's and the seller's expected payoffs are defined as

$$U(q, m, z|\theta) \equiv x(m, z)v(q, \theta) - p(m, z), \quad \Pi(q, m, z) \equiv p(m, z) - c(q). \tag{35}$$

It is easy to see that this environment entails the exit option contract as a special case, where $x$ and $p$ do not depend on the seller's message and the buyer has only two messages, with $x = 1$ for one message and $x = 0$ for the other.

[13]In principle, $Z$ may depend on $m \in M$. In what follows, we ignore this possibility because it does not affect our results.

Our description of Perfect Bayesian Equilibrium readily extends to the present context. The only novelty is that, after $m$ and $q$ have been selected in the previous stages, in $t = 3$ now a continuation game $\Gamma(m, q)$ starts in which the seller has imperfect information about the buyer's type. After having observed the buyer's message $m$, the seller enters $\Gamma(m, q)$ with the belief that the buyer's true type is in the set $T \in \mathcal{T}$ with probability $\mu(T, m)$. The game $\Gamma(m, q)$ is thus a (static) Bayesian game, and as part of the Perfect Bayesian Equilibrium of the overall game, the players' message strategies have to constitute a Bayesian Nash Equilibrium. This means that $(\hat{z}_S, \hat{z}_B(\cdot))$, with $\hat{z}_S \in Z_S$ and $\hat{z}_B \colon \Theta \to Z_B$, is an equilibrium of $\Gamma(m, q)$ if the seller's message $\hat{z}_S$ satisfies

$$\int_{\underline{\theta}}^{\bar{\theta}} p(m, \hat{z}_S, \hat{z}_B(\theta))\mu(\theta, m)d\theta \geq \int_{\underline{\theta}}^{\bar{\theta}} p(m, z_S, \hat{z}_B(\theta))\mu(\theta, m)d\theta \quad \text{for all } z_S \in Z_S, \tag{36}$$

and each buyer type $\theta$ with $m \in R(\theta)$ selects a message $\hat{z}_B(\theta)$ such that

$$U(q, m, \hat{z}_S, \hat{z}_B(\theta)|\theta) \geq U(q, m, \hat{z}_S, z_B|\theta) \quad \text{for all } z_B \in Z_B. \tag{37}$$

Notice that in $t = 3$ the seller's production costs are already sunk so that in (36) he only cares about expected payments when choosing his message $\hat{z}_S$. In what follows we denote by $E(m, q)$ the set of Bayesian Nash Equilibria of the game $\Gamma(m, q)$.

As is well–known, message games typically admit a multiplicity of equilibria. While some of these equilibria may implement the desired outcome, others may induce unintended outcomes. To resolve this problem, we will apply the usual concept of *strong implementation*, which requires that *all* equilibria in $E(m, q)$ have identical outcomes. More specifically, we restrict the set of admissible contracts by imposing the following condition on all continuation games $\Gamma(m, q)$:

**Condition 1** If $(\hat{z}_S, \hat{z}_B(\cdot)) \in E(m, q)$ and $(\tilde{z}_S, \tilde{z}_B(\cdot)) \in E(m, q)$, then

$$x(m, \hat{z}_S, \hat{z}_B(\theta)) = x(m, \tilde{z}_S, \tilde{z}_B(\theta)) \quad \text{and} \quad p(m, \hat{z}_S, \hat{z}_B(\theta)) = p(m, \tilde{z}_S, \tilde{z}_B(\theta)) \tag{38}$$

for almost all $\theta$ such that $m \in R(\theta)$.

Thus, if the buyer type $\theta$ has reported $m \in M$ in stage $t = 1$ and the seller has produced quality $q$ in stage $t = 2$, Condition 1 implies that the probability of trade $x$ and the payment $p$ are uniquely determined by the outcome of the subsequent message game in $t = 3$, even when this game has multiple equilibria.

After a contract has been signed, the *path* of a Perfect Bayesian Equilibrium induces for each buyer type $\theta$ some message $m^*(\theta)$ in stage $t = 1$. Given his equilibrium beliefs $\mu^*(\cdot, m^*(\theta))$, the seller then chooses some quality $q^*(\theta)$ in $t = 2$. Finally, in $t = 3$ the equilibrium outcome $(z_S^*(\theta), z_B^*(\theta))$ of the message game $\Gamma(m^*(\theta), q^*(\theta))$ determines a probability of trade $x^*(\theta)$ and a payment $p^*(\theta)$. We say that a contract *implements* $(q^*, x^*)$, with $q^* : \Theta \to \mathbb{R}_+$ and $x^* : \Theta \to [0, 1]$, if there is a Perfect Bayesian Equilibrium such that, for each type $\theta$, equilibrium play results in trade of quality $q^*(\theta)$ with probability $x^*(\theta)$.[14]

In the remainder of this section, we show that even in the most general contracting environment the first–best outcome cannot be implemented as a Perfect Bayesian Equilibrium. Indeed, similarly to the exit option contract studied in the previous section, partial pooling of different buyer types is unavoidable in any equilibrium outcome. We begin with the following lemma.

**Lemma 5** *Suppose that $(q^*, x^*)$ can be implemented. If there is an interval $I = [\theta_1, \theta_2] \subseteq \Theta$ such that $q^*(\cdot)$ is strictly increasing on $I$ and $x^*(\cdot) = 1$ on $I$, then for each $\theta \in (\theta_1, \theta_2)$ the buyer's message $m^*(\theta)$ reveals his type to the seller so that $\mu^*(\theta, m^*(\theta)) = 1$.*

To see the intuition behind Lemma 5, observe that since $q^*(\cdot)$ is strictly increasing on $I$, the seller chooses distinct quality levels for each type in $I$. Therefore, each type in $I$ has to use a distinct message at stage 1. Moreover, the single–crossing property $v_{q\theta} > 0$ implies that types outside of $I$ have a stronger incentive to imitate a type at the boundary than in the interior of $I$. Hence, each buyer type in $(\theta_1, \theta_2)$ sends a message that is not used by any other type and therefore reveals his type to the seller. The next lemma is key to our analysis.

**Lemma 6** *Suppose that Condition 1 holds and that $(q^*, x^*)$ can be implemented. If there is an interval $I = [\theta_1, \theta_2] \subseteq \Theta$ such that $q^*(\cdot)$ is strictly increasing on $I$ and $x^*(\cdot) = 1$ on $I$, then for each $\theta \in (\theta_1, \theta_2)$ there exists a message $z'_B \neq z_B^*(\theta)$ such that $x(m^*(\theta), z_S^*(\theta), z'_B) < 1$ and*

$$U(q^*(\theta), m^*(\theta), z_S^*(\theta), z_B^*(\theta)|\theta) = U(q^*(\theta), m^*(\theta), z_S^*(\theta), z'_B|\theta). \tag{39}$$

[14]In principle, $q$ and $x$ could be lotteries since the buyer could use a mixed strategy in $t = 1$. Yet we restrict ourselves to non–random outcomes of $q$ and $x$ as the first–best, for which we want to establish an impossibility result, is non–random.

Recall from the analysis of exit options that to implement a positive quality level, by (22) the smallest buyer type who trades this quality level has to be indifferent between trade and exit. Lemma 6 extends this insight to the general contracting environment. Equation (39) says that for any buyer type $\theta$ in the interior of $I$ there has to be some message $z'_B$ such that the buyer is indifferent between trading $q^*(\theta)$ with probability 1, and trading $q^*(\theta)$ with probability $x(m^*(\theta), z^*_S(\theta), z'_B)$ at a lower price. As in the case of exit options, the underlying reason is the seller's limited commitment which makes it necessary to deter the seller from lowering the desired quality ex post.

To see this more clearly, observe that under the assumptions of Lemma 6, a buyer type $\theta$ in the interior of $I$ reveals himself by Lemma 5. Now suppose the seller deviated to a quality $q'$ slightly below $q^*(\theta)$ in stage 2 and announced his original equilibrium message $z^*_S(\theta)$ in stage 3. If there was no message $z'_B$ that left the buyer indifferent as in (39), then his best response would still be to announce $z^*_B(\theta)$ and thus trade $q'$ with probability 1 at the same price. In other words, the messages $(z^*_B(\theta), z^*_S(\theta))$ would remain to be an equilibrium of the continuation game that starts after a deviation of the seller to $q'$. Since $q'$ is less costly to produce than $q^*(\theta)$, the seller would benefit from such a deviation provided $(z^*_B(\theta), z^*_S(\theta))$ would indeed be played in stage 3. Under Condition 1, however, it does not matter which equilibrium is played since any equilibrium yields the same payment to the seller. Consequently, the seller could indeed gain from deviating to a lower quality.

In other words, the message $z'_B$ serves a similar function as an exit option in restraining the seller's limited commitment: it creates a credible threat for the buyer that deters the seller to lower quality ex post. In the previous sections, we have seen that designing exit options in a way that would give the seller incentives to choose first–best quality is incompatible with the buyer possessing private information, because incentives would arise for low valuation buyers to pretend a high valuation and then exit. The same force undermines efficient contracting in the general environment. In fact, pooling of different buyer types is unavoidable because no strictly increasing quality schedule can be implemented:

**Proposition 5** *Suppose that Condition 1 holds and that* $(q^*, x^*)$ *can be implemented. Then there is no interval* $I \subseteq \Theta$ *such that* $q^*(\cdot)$ *is strictly increasing and* $x^*(\cdot) = 1$ *on* $I$*. Thus, if* $q^*(\cdot)$ *is positive and continuous and* $x^*(\cdot) = 1$ *on some interval* $I$*, then* $q^*(\cdot)$ *is constant on* $I$*.*

The intuition for Proposition 5 is the same as for the simple exit option contract studied in the previous sections. To implement a strictly increasing quality schedule and to prevent high valuation buyers from mimicking low valuation buyers, high valuation buyers have to get a higher (ex post) utility than low valuation buyers. But Lemma 6 then implies that it becomes profitable for a type $\theta' < \theta$, who is sufficiently close to type $\theta$, to mimic type $\theta$ in stage 1 and then announce message $z'_B$ in stage 3.

The second part of Proposition 5 says that if for positive equilibrium qualities trade takes place with probability one, then only quality schedules can be implemented that have the shape of a step function. An exit option contract is a special case with only a single step. In the exit option case, the lowest buyer type who just trades a positive quality is indifferent between trading this quality and refusing to trade. In the general case, more than one positive quality levels can be potentially implemented. The basic difference between a simple exit option and a general mechanism is that in the general mechanism the buyer's off–equilibrium threat, which deters the seller from deviating to a lower quality, can involve random trade. In other words, the general mechanism allows for stochastic exit options. In this way, the buyer's expected value from 'exiting' can to some degree be made responsive to his type, whereas by (22) the value of a deterministic exit option must be the same for any buyer type. This mitigates the incentive constraint that low types must not announce a high type and then simply exit, and so permits a finer sorting of types than when the exit option is deterministic.

This reasoning also implies that once we rule out random trade the general mechanism essentially collapses to a simple exit option contract. This is the object of the next proposition.

**Proposition 6** *Suppose that Condition 1 holds and that $(q^*, x^*)$ can be implemented. If trade is deterministic so that $x : M \times Z \to \{0, 1\}$, then the following holds:*

*(i) At most a single positive quality level can be implemented. That is, there is $\hat{\theta}$ and a $\hat{q} > 0$ such that $q^*(\theta) = \hat{q}$ and $x^*(\theta) = 1$ for all $\theta > \hat{\theta}$ and $q^*(\theta) = 0$ or $x^*(\theta) = 0$ for all $\theta < \hat{\theta}$.*

*(ii) $(q^*, x^*)$ can be implemented by an exit option contract.*

Random mechanisms are frequently impossible to implement because of enforceability problems. Proposition 6 says that in such circumstances no efficiency gains are possible by using a more complex mechanism than a simple exit option contract. Even though we have shown in Section 2 that exit option contracts fail to achieve full efficiency, they are second–best efficient. Proposition 6 establishes a central role for exit option contracts in overcoming problems caused by contractual incompleteness and asymmetric information.

Note that the exit option contract described in Propositions 3 and 6 is simpler than the efficient contracts of the benchmark cases in the sense that the former supports only a single quality $\hat{q}$ rather than a schedule of type dependent qualities. Thus, as the contracting environment becomes more complex, the resulting contractual arrangement actually becomes simpler. Complex environments may therefore be consistent with the widespread use of relatively simple contracts in reality.

# 6 Conclusion

We have studied bilateral contracting in an environment which is characterized by both contractual incompleteness and asymmetric information. We demonstrate that even under the most general form of contracts, the joint occurrence of these imperfections necessarily upsets first–best efficient contracting. Moreover, when random contracts are precluded, general contracts cannot improve upon simple exit option contracts.

Our inefficiency result suggests that incomplete contracts with asymmetric information may be useful for studying institutional design, even in the absence of contract renegotiation. This is so because the allocation of property rights or decision rights may matter for the extent to which efficiency can be achieved. Imagine, for example, that the non–verifiable action is more broadly interpreted as some decision that an organization has to take. Suppose further that the right to take this decision can be conferred to one of its members. This assignment of authority may be enforced by the ownership of assets and resources that are necessary to implement a decision. In such an environment, the optimal institutional arrangement can be determined by comparing the efficiency implications of different allocations of property and decision rights.

# 7 Appendix

**Proof of Proposition 1**: (a) Let

$$k \equiv \int_{\underline{\theta}}^{\bar{\theta}} \int_{\underline{\theta}}^{\theta} v_\theta(\tilde{q}(x), x)\, dx\, dF(\theta) > 0, \tag{40}$$

and define

$$p_T^*(\theta) \equiv v(\tilde{q}(\theta), \theta) - \int_{\underline{\theta}}^{\theta} v_\theta(\tilde{q}(x), x)\, dx + k, \quad p_N^*(\theta) \equiv k. \tag{41}$$

We first show that the mechanism $(\tilde{q}, p^*)$ satisfies incentive compatibility (9), that is, for all $\theta, \theta'$:

$$v(\tilde{q}(\theta), \theta) - p_T^*(\theta) \;\; \geq \;\; -p_N^*(\theta'), \tag{42}$$

$$v(\tilde{q}(\theta), \theta) - p_T^*(\theta) \;\; \geq \;\; v(\tilde{q}(\theta'), \theta) - p_T^*(\theta'). \tag{43}$$

Inequality (42) is immediate by the definition of $p^*$. To see (43), let $\theta' > \theta$. By definition of $p_T^*$, the difference between the left and the right hand side of (43) is

$$-\int_{\theta}^{\theta'} v_\theta(\tilde{q}(x), x)\, dx - v(\tilde{q}(\theta'), \theta) + v(\tilde{q}(\theta'), \theta'). \tag{44}$$

Since $\tilde{q}(\cdot)$ is increasing and $v_{q\theta} > 0$, and since $\theta' > \theta$, we have

$$-\int_{\theta}^{\theta'} v_\theta(\tilde{q}(x), x)\, dx \geq -\int_{\theta}^{\theta'} v_\theta(\tilde{q}(\theta'), x)\, dx = -v(\tilde{q}(\theta'), \theta') + v(\tilde{q}(\theta'), \theta). \tag{45}$$

Thus, expression (44) is larger than 0, and this proves (43) for $\theta' > \theta$. The argument for $\theta' < \theta$ is analogous.

Next, we verify the participation constraint (10). We note in passing that (41) implies that $h(\tilde{q}(\theta), p^*(\theta) \mid \theta) = 1$ for all $\theta \in \Theta$. Hence, $U(\tilde{q}(\theta), p^*(\theta) \mid \theta) = v(\tilde{q}(\theta), \theta) - p_T^*(\theta)$. Thus,

$$\int_{\underline{\theta}}^{\bar{\theta}} U(q(\theta), p(\theta) \mid \theta)\, dF(\theta) \;\; = \;\; \int_{\underline{\theta}}^{\bar{\theta}} [v(\tilde{q}(\theta), \theta) - p_T^*(\theta)]\, dF(\theta) \tag{46}$$

$$= \;\; \int_{\underline{\theta}}^{\bar{\theta}} \int_{\underline{\theta}}^{\theta} v_\theta(\tilde{q}(x), x) dx\, dF(\theta) - k = 0, \tag{47}$$

where the last equality follows from the definition of $k$.

To complete the proof of (a), notice that $(\tilde{q}, p^*)$ is optimal, because the seller fully extracts the first–best surplus

$$\int_{\underline{\theta}}^{\bar{\theta}} \Pi(\tilde{q}(\theta), p^*(\theta) \,|\, \theta)\, dF = \int_{\underline{\theta}}^{\bar{\theta}} [p_T^*(\theta) - c(\tilde{q}(\theta))]\, dF \tag{48}$$

$$= \int_{\underline{\theta}}^{\bar{\theta}} [v(\tilde{q}(\theta), \theta) - c(\tilde{q}(\theta))]\, dF - \int_{\underline{\theta}}^{\bar{\theta}} \int_{\underline{\theta}}^{\theta} v_\theta(\tilde{q}(x), x) dx\, dF + k \tag{49}$$

$$= \tilde{S} - 0. \tag{50}$$

Thus, the seller can clearly not do better than this.

(b) It follows from (a) that under any optimal contract $(q^*, p^*)$, the seller must fully extract the first–best surplus $\tilde{S}$. This implies that $q^*(\theta) = \tilde{q}(\theta)$ for almost all $\theta \in \Theta$. Since $q^* = \tilde{q}$ is strictly increasing in $\theta$ almost everywhere, incentive compatibility implies by a standard argument that $U(q^*(\theta), p^*(\theta) \,|\, \theta)$ is strictly increasing in $\theta$ almost everywhere. Moreover, full surplus extraction also implies that $h(q^*(\theta), p^*(\theta) \,|\, \theta) = 1$ almost everywhere. Therefore, there is a $\Theta' \in \mathcal{T}$ with $\int_{\Theta'} dF = 1$ such that $U(q^*(\theta), p^*(\theta) \,|\, \theta)$ is strictly increasing on $\Theta'$ and

$$U(q^*(\theta), p^*(\theta) \,|\, \theta) = v(q^*(\theta), \theta) - p_T^*(\theta) \geq -p_N^*(\theta) \quad \text{for all } \theta \in \Theta'. \tag{51}$$

We now prove the claim by showing that $v(q^*(\theta), \theta) - p_T^*(\theta) > -p_N^*(\theta)$ for all $\theta \in \Theta' \setminus \{\underline{\theta}\}$. Indeed, suppose to the contrary that there is a $\theta \in \Theta' \setminus \{\underline{\theta}\}$ with $v(q^*(\theta), \theta) - p_T^*(\theta) = -p_N^*(\theta)$. Because $\Theta'$ has mass 1 and $\theta > \underline{\theta}$, we can find a $\theta' \in \Theta'$ with $\theta > \theta'$. Thus, we have

$$U(q^*(\theta'), p^*(\theta') \,|\, \theta') < U(q^*(\theta), p^*(\theta) \,|\, \theta) = v(q^*(\theta), \theta) - p_T^*(\theta) = -p_N(\theta). \tag{52}$$

But this is a contradiction because incentive compatibility requires that $U(q^*(\theta'), p^*(\theta') \,|\, \theta') \geq -p_N(\theta)$ for type $\theta'$. Q.E.D.

**Proof of Proposition 2**: (a) For given $\theta \in \Theta$, define

$$p_T^*(\theta) \equiv v(\tilde{q}(\theta), \theta), \qquad p_N^*(\theta) \equiv 0. \tag{53}$$

We first verify that $\tilde{q}(\cdot)$ satisfies the seller's no–commitment constraint (12). We show that the seller optimally selects $\tilde{q}(\cdot)$ given $p^*$. It follows from the definition of transfers in (53) that when the seller chooses $\tilde{q}(\theta)$, then the buyer is just willing to trade, and the seller's profit is $\Pi(\tilde{q}(\theta), p^*(\theta) \,|\, \theta) = p_T^*(\theta) - c(\tilde{q}(\theta))$. Now consider a quality $q < \tilde{q}(\theta)$. Then the buyer exits and the seller's profit is

$$\Pi(q, p^*(\theta) \,|\, \theta) = p_N^*(\theta) - c(q) \leq 0 \leq v(\tilde{q}(\theta), \theta) - c(\tilde{q}(\theta)) = \Pi(\tilde{q}(\theta), p^*(\theta) \,|\, \theta), \tag{54}$$

where the second inequality follows from the assumption that the surplus from the first–best quality is positive. Consider next a quality $q > \tilde{q}(\theta)$. Then the buyer accepts to trade and the seller's profit is

$$\Pi(q, p^*(\theta)\,|\,\theta) = p_T^*(\theta) - c(q) < p_T^*(\theta) - c(\tilde{q}(\theta)) = \Pi(\tilde{q}(\theta), p^*(\theta)\,|\,\theta), \tag{55}$$

where the inequality follows because costs are strictly monotone in $q$. Thus, (54) and (55) establish that $\tilde{q}(\cdot)$ satisfies the no–commitment constraint (12).

Next, we verify the participation constraint (13). We note in passing that (53) implies that $h(\tilde{q}(\theta), p^*(\theta)\,|\,\theta) = 1$ for all $\theta \in \Theta$. Hence, $U(\tilde{q}(\theta), p^*(\theta)\,|\,\theta) = v(\tilde{q}(\theta), \theta) - p_T^*(\theta) = 0$. Thus, the participation constraint is, in fact, binding.

To complete the proof of (a), notice that $(\tilde{q}, p^*)$ is optimal, because the participation constraint is binding so that the seller fully extracts the first–best surplus $\tilde{S}$.

(b) It follows from (a) that under any optimal contract $(q^*, p^*)$, the seller must fully extract the first–best surplus $\tilde{S}$. Therefore, it must hold that $h(q^*(\theta), p^*(\theta)\,|\,\theta) = 1$ for almost all $\theta \in \Theta$. That is, there is $\Theta' \in \mathcal{T}$ with $\int_{\Theta'} dF = 1$ and $v(q^*(\theta), \theta) - p_T^*(\theta) \geq -p_N^*(\theta)$ for all $\theta \in \Theta'$. Now suppose that there is a $\theta' \in \Theta'$ such that $v(q^*(\theta'), \theta') - p_T^*(\theta') > -p_N^*(\theta')$. Then, when faced with buyer type $\theta'$, the seller could increase his profit by slightly reducing $q^*(\theta')$, a contradiction to the no–commitment constraint (12). Thus, we must have that $v(q^*(\theta), \theta) - p_T^*(\theta) = -p_N^*(\theta)$ for all $\theta \in \Theta'$, and this completes the proof. Q.E.D.

**Proof of Lemma 1**: By definition of $M^+(\theta)$ in (17) one has $h(q(m), p(m)\,|\,\theta) = 1$ for all $\theta \in T^+(m)$. Therefore, by (1), $v(q(m), \theta) - p_T(m) \geq -p_N(m)$ for all $\theta \in T^+(m)$. By continuity of $v(q, \cdot)$ this implies

$$v(q(m), \theta_\ell(m)) - p_T(m) \geq -p_N(m). \tag{56}$$

Now suppose that $v(q(m), \theta_\ell(m)) - p_T(m) > -p_N(m)$. Since $v_\theta(q, \theta) > 0$, this implies

$$v(q(m), \theta) - p_T(m) > -p_N(m) \quad \text{for all } \theta \in T^+(m). \tag{57}$$

But this means that all buyer types who buy quality $q(m)$ after reporting $m$ would also purchase a quality slightly below $q(m)$. Hence the seller could gain by reducing $q(m)$, a contradiction to (14). Q.E.D.

**Proof of Lemma 2**: We first prove that $p_N(m) = p_N(m')$ if $T^+(m) \neq \emptyset$ and $T^+(m') \neq \emptyset$. By Lemma 1 there is a $\theta_\ell(m)$ and a $\theta_\ell(m')$ such that

$$\begin{aligned} U(q(m), p(m)\,|\,\theta_\ell(m)) &= v(q(m), \theta_\ell(m)) - p_T(m) = -p_N(m), \\ U(q(m'), p(m')\,|\,\theta_\ell(m')) &= v(q(m'), \theta_\ell(m')) - p_T(m') = -p_N(m'). \end{aligned} \tag{58}$$

Since $m \in M^+(\theta)$ for all $\theta \in T^+(m)$, one has $U(q(m), p(m) \,|\, \theta) \geq U(q(m'), p(m') \,|\, \theta)$ for all $\theta \in T^+(m)$ and $U(q(m'), p(m') \,|\, \theta) \geq U(q(m), p(m) \,|\, \theta)$ for all $\theta \in T^+(m')$. By continuity of $U(q, p \,|\, \cdot)$ this implies

$$\begin{aligned} U(q(m), p(m) \,|\, \theta_\ell(m)) &\geq U(q(m'), p(m') \,|\, \theta_\ell(m)), \\ U(q(m'), p(m') \,|\, \theta_\ell(m')) &\geq U(q(m), p(m) \,|\, \theta_\ell(m')). \end{aligned} \tag{59}$$

Further, by (2)

$$U(q(m'), p(m') \,|\, \theta_\ell(m)) \geq -p_N(m'), \quad U(q(m), p(m) \,|\, \theta_\ell(m')) \geq -p_N(m). \tag{60}$$

By (58)–(60), we have $-p_N(m) \geq -p_N(m')$ and $-p_N(m') \geq -p_N(m)$. Therefore $p_N(m) = p_N(m')$.

It remains to show that $\theta_\ell(m) = \theta_\ell(m')$ if $T^+(m) \neq \emptyset$ and $T^+(m') \neq \emptyset$. Suppose the contrary, i.e. $\theta_\ell(m) \neq \theta_\ell(m')$. Without loss of generality, let $\theta_\ell(m) < \theta_\ell(m')$. Note that $q(m) > 0$ and $q(m') > 0$ because $m \in M^+(\theta)$ for all $\theta \in T^+(m)$ and $m' \in M^+(\theta)$ for all $\theta \in T^+(m')$. Since each type reports optimally, it must be the case that

$$v(q(m'), \theta) - p_T(m') \geq v(q(m), \theta) - p_T(m) \quad \text{for all } \theta \in T^+(m'), \tag{61}$$

so that, by continuity of $v(q, \cdot)$,

$$v(q(m'), \theta_\ell(m')) - p_T(m') \geq v(q(m), \theta_\ell(m')) - p_T(m). \tag{62}$$

Since $\theta_\ell(m) < \theta_\ell(m')$ and $v_\theta(q, \theta) > 0$,

$$v(q(m), \theta_\ell(m')) - p_T(m) > v(q(m), \theta_\ell(m)) - p_T(m). \tag{63}$$

By (62) and (63),

$$v(q(m'), \theta_\ell(m')) - p_T(m') > v(q(m), \theta_\ell(m)) - p_T(m). \tag{64}$$

Because $p_N(m) = p_N(m')$ this yields a contradiction to Lemma 1, which implies that

$$v(q(m'), \theta_\ell(m')) - p_T(m') = -p_N(m') = -p_N(m) = v(q(m), \theta_\ell(m)) - p_T(m). \tag{65}$$

Q.E.D.

**Proof of Proposition 3**: (i) We first show that $\theta > \hat{\theta}$ implies $R(\theta) \subseteq M^+(\theta)$, i.e. all types $\theta > \hat{\theta}$ only select positive trade messages. If $T^+(m) = \emptyset$ for all $m \in M$, then $\hat{\theta} = \bar{\theta}$, and the claim trivially holds. So suppose there is an $m \in M$ such that $T^+(m) \neq \emptyset$. Contrary to the claim, suppose there is a $\theta > \hat{\theta}$ and an $m' \in M(\theta) \setminus M^+(\theta)$. Since reporting $m'$ is optimal for type $\theta$, we have:

$$\begin{aligned} U(q(m'), p(m') \mid \theta) &= \max[-p_T(m'), -p_N(m')] \\ &\geq v(q(m), \theta) - p_T(m) \\ &> v(q(m), \hat{\theta}) - p_T(m) \end{aligned} \tag{66}$$

The first line follows since $q(m')h(q(m'), p(m')|\theta) = 0$ and $v(0, \theta) = 0$, the second line follows since $m'$ is optimal for type $\theta$, and the third line follows since $\theta > \hat{\theta}$ and $v_\theta(q, \theta) > 0$. By definition of $\hat{\theta}$, we have $m \in M(\hat{\theta})$, i.e. reporting $m$ is optimal for type $\hat{\theta}$. By (22) this implies

$$\begin{aligned} U(q(m), p(m) \mid \hat{\theta}) &= v(q(m), \hat{\theta}) - p_T(m) \geq U(q(m'), p(m') \mid \hat{\theta}) \\ &= \max[v(q(m'), \hat{\theta}) - p_T(m'), -p_N(m')] \\ &\geq \max[-p_T(m'), -p_N(m')] \end{aligned} \tag{67}$$

Thus, $v(q(m), \hat{\theta}) - p_T(m) \geq \max[-p_T(m'), -p_N(m')]$, which yields a contradiction to (66). This proves that $\theta > \hat{\theta}$ implies $R(\theta) \subseteq M^+(\theta)$.

To complete the proof of (i), it remains to show that there is a $\hat{q}$ such that $q(m) = \hat{q}$ for all $m \in R(\theta)$ with $\theta > \hat{\theta}$. Suppose the contrary. Then there is a $\theta > \hat{\theta}$ and a $\theta' > \hat{\theta}$ such that $m \in R(\theta)$ and $m' \in R(\theta')$ with $q(m) < q(m')$. Since we have shown above that $R(\theta) \subseteq M^+(\theta)$ and $R(\theta') \subseteq M^+(\theta')$, we know that $T^+(m) \neq \emptyset$ and $T^+(m') \neq \emptyset$. Hence, by (22)

$$v(q(m), \hat{\theta}) - p_T(m) = v(q(m'), \hat{\theta}) - p_T(m'). \tag{68}$$

Therefore, $q(m') > q(m)$, $\theta > \hat{\theta}$, and $v_{q\theta}(q, \theta) > 0$ implies

$$v(q(m'), \theta) - v(q(m), \theta) > v(q(m'), \hat{\theta}) - v(q(m), \hat{\theta}) = p_T(m') - p_T(m). \tag{69}$$

But this is a contradiction because $v(q(m), \theta) - p_T(m) \geq v(q(m'), \theta) - p_T(m')$ as $m \in R(\theta)$.

(ii) Let $\theta < \hat{\theta}$ and suppose to the contrary that there is an $m \in R(\theta)$ such that $q(m) > 0$ and $h(q(m), p(m)|\theta) = 1$. This implies

$$v(q(m), \theta) - p_T(m) \geq -p_N(m). \tag{70}$$

Since $m \in R^+(\theta)$, it follows by definition that $T^+(m) \neq \emptyset$. Hence (22) holds for $m$ and $p_N(m) = \hat{p}_N$. However, since $\theta < \hat{\theta}$ and $v_\theta(q, \theta) > 0$, (70) and (22) contradict each other. Q.E.D.

**Proof of Lemma 3**: For arbitrary $(q', \theta')$ with $v(q', \theta') - p_T = -p_N$, define

$$\phi(q', \theta'; \hat{\theta}) = \int_{\hat{\theta}}^{\bar{\theta}} \frac{\Pi(q', t \mid \theta)}{1 - F(\hat{\theta})} dF(\theta). \tag{71}$$

With the definition of $\Pi$ in (3) we have

$$\phi(q', \theta'; \hat{\theta}) = \begin{cases} p_T - c(q') & \text{if} \quad v(q', \hat{\theta}) - p_T > -p_N, \\ \frac{F(\theta') - F(\hat{\theta})}{1 - F(\hat{\theta})} p_N + \frac{1 - F(\theta')}{1 - F(\hat{\theta})} p_T - c(q') & \text{if} \quad v(q', \hat{\theta}) - p_T \leq -p_N. \end{cases} \tag{72}$$

Hence, $\phi$ is strictly decreasing in $q'$ if $v(q', \hat{\theta}) > p_T - p_N$, or equivalently if $\theta' < \hat{\theta}$. Therefore, any maximizer $(q, \theta)$ of $\phi(q', \theta'; \hat{\theta})$ satisfies $v(q, \theta) - p_T = -p_N$ and $\theta \geq \hat{\theta}$, which are the constraints in (29). Hence, since the bottom line on the right hand side in (72) coincides with the objective in (28):

$$(q, \theta) \in \operatorname*{argmax}_{(q', \theta')} \phi(q', \theta'; \hat{\theta}) \quad \Leftrightarrow \quad (q, \theta) = (q^*(\hat{\theta}, p), \theta^*(\hat{\theta}, p)). \tag{73}$$

Consequently: $(\hat{q}, \hat{\theta})$ satisfies (24) and (25) $\Leftrightarrow$ $(\hat{q}, \hat{\theta}) \in \operatorname{argmax}_{(q', \theta')} \phi(q', \theta'; \hat{\theta})$ $\Leftrightarrow$ $(\hat{q}, \hat{\theta}) = (q^*(\hat{\theta}, p), \theta^*(\hat{\theta}, p))$ $\Leftrightarrow$ $(\hat{q}, \hat{\theta})$ satisfies (30). Q.E.D.

**Proof of Lemma 4**: For given $\hat{\theta}$, and for all $(q', \theta')$ denote the objective in (28) by

$$\psi(q', \theta'; \hat{\theta}) = \frac{F(\theta') - F(\hat{\theta})}{1 - F(\hat{\theta})} p_N + \frac{1 - F(\theta')}{1 - F(\hat{\theta})} p_T - c(q'). \tag{74}$$

Below, we show that under convexity of $F$, $\psi$ is concave in $(q', \theta')$ for all $\theta$. Moreover, since $p_T > p_N$, it is evident that $\psi$ is decreasing in $q'$ and $\theta'$. Further, since $v$ is quasi–concave by assumption, the constraint $v(q', \theta') = p_T - p_N$ describes a convex curve in $(q, \theta)$-space. These three observations imply that the necessary first–order conditions for problem (28) are already sufficient to deliver a maximum. That is, $(\hat{q}, \hat{\theta}) = (q^*(\hat{\theta}, p), \theta^*(\hat{\theta}, p))$ if and only if the following Kuhn-Tucker conditions hold:

$$\psi_{q'}(\hat{q}, \hat{\theta}; \hat{\theta}) - \lambda v_q(\hat{q}, \hat{\theta}) = 0, \tag{75}$$

$$\psi_{\theta'}(\hat{q}, \hat{\theta}; \hat{\theta}) - \lambda v_\theta(\hat{q}, \hat{\theta}) \leq 0, \tag{76}$$

$$v(\hat{q}, \hat{\theta}) = p_T - p_N, \tag{77}$$

for some $\lambda \in \mathbb{R}$. It is easy to see that these conditions are equivalent to (31) and (32). This establishes the equivalence between (30) on the one and (31) and (32) on the other hand.

It remains to show that $\psi$ is concave in $(q', \theta')$. Observe first that the cross-partials $\psi_{q'\theta'}$ are zero. Thus, $\psi$ is concave if and only if $\psi_{q'q'}$ and $\psi_{\theta'\theta'}$ are each negative. We have:

$$\psi_{q'q'}(q', \theta'; \theta) = -c''(q') \quad \text{and} \quad \psi_{\theta'\theta'}(q', \theta'; \theta) = -\frac{F''(\theta')}{1 - F(\theta)}(p_T - p_N). \tag{78}$$

Since $c'' > 0$ by assumption, the left expression is negative. Further, since $p_T - p_N > 0$, a sufficient condition for the right expression to be negative is that $F'' \geq 0$, and this completes the proof. Q.E.D.

**Proof of Proposition 4**: Insert $p_T - p_N$ from (32) in (31), and observe that the seller's problem becomes thus independent of transfers. This yields the claim. Q.E.D.

**Proof of Lemma 5**: Consider a type $\theta' \in (\theta_1, \theta_2)$ and suppose he chooses message $m^*(\theta')$ in equilibrium. We show that type $\theta'$ is the only type who chooses message $m^*(\theta')$, implying full revelation of this type. Indeed, since $q^*(\cdot)$ is strictly increasing on $I = [\theta_1, \theta_2]$ by assumption, the seller chooses for each $\theta \in I$, $\theta \neq \theta'$, a distinct quality $q^*(\theta) \neq q^*(\theta')$. This can only be if each buyer type $\theta \in I$, $\theta \neq \theta'$, chooses a message which is different from $m^*(\theta')$. To see that a type $\theta < \theta_1$ does not choose message $m^*(\theta')$, note that since $x^*(\cdot) = 1$ on $I$, the type $\theta_1$ prefers quality $q^*(\theta_1)$ over the quality $q^*(\theta')$. Thus, the single–crossing property $v_{q\theta} > 0$ implies that $\theta < \theta_1$ strictly prefers $q^*(\theta_1)$ over $q^*(\theta')$. Hence, such a type does not choose the message $m^*(\theta')$ which would induce quality $q^*(\theta')$. An identical argument shows that also a type $\theta > \theta_2$ does not choose the message $m^*(\theta')$. Q.E.D.

**Proof of Lemma 6**: Let $\theta \in (\theta_1, \theta_2)$, and consider an equilibrium path $(m^*, q^*, z_S^*, z_B^*) = (m^*(\theta), q^*(\theta), z_S^*(\theta), z_B^*(\theta))$. Since $q^*(\cdot)$ is strictly increasing on $I$ we have that $q^* > 0$. We have to show that there is a message $z_B' \in Z_B$ with $x(m^*, z_S^*, s_B') < 1$ such that (39) holds.

To see this, let $(\hat{z}_S, \hat{z}_B(\cdot))$ be the Bayesian Nash Equilibrium (BNE) of the continuation game $\Gamma(m^*, q^*)$ whose outcome is $\hat{z}_S = z_S^*, \hat{z}_B(\theta) = z_B^*$. Since the seller knows the buyer's type by Lemma 5, the equilibrium conditions write:

$$p(m^*, z_S^*, z_B^*) \geq p(m^*, z_S', z_B^*) \quad \text{for all} \quad z_S' \in Z_S, \tag{79}$$

$$x(m^*, z_S^*, z_B^*)v(q^*, \theta) - p(m^*, z_S^*, z_B^*) \geq x(m^*, z_S^*, z_B')v(q^*, \theta) - p(m^*, z_S^*, z_B') \tag{80}$$

for all $z_B' \in Z_B$. We next show that there is a message $z_B' \in Z_B$ such that $x(m^*, z_S^*, z_B') < 1$. If the contrary were true, the buyer's equilibrium message in $\Gamma(m^*, q^*)$ would satisfy:

$$z_B^* \in \operatorname*{argmax}_{z_B} v(q^*, \theta) - p(m^*, z_S^*, z_B) \tag{81}$$

$$= \underset{z_B}{\text{argmax}} \quad -p(m^*, z_S^*, z_B). \tag{82}$$

(79) and (82) imply that $(\hat{z}_S, \hat{z}_B(\cdot))$ is also a BNE of the continuation game $\Gamma(m^*, 0)$ that starts after the seller has chosen the quality $q = 0$. Hence, Condition 1 implies that for all BNE $(\tilde{z}_S, \tilde{z}_B(\cdot)) \in E(m^*, 0)$ it holds that

$$p(m^*, z_S^*, z_B^*) = p(m^*, \tilde{z}_S, \tilde{z}_B(\theta)). \tag{83}$$

Thus, the seller gets the same payment in $\Gamma(m^*, 0)$ as he gets in $\Gamma(m^*, q^*)$. Since the quality zero is less costly than $q^* > 0$, the seller would therefore be better off by choosing zero quality at stage 2, but this contradicts the condition that in equilibrium $q^*$ maximizes the seller's profit.

Accordingly, the set of messages $z'_B$ for which $x(m^*, z_S^*, z'_B) < 1$ is non–empty. Denote this set by $Z'_B$. We now show that there is a $z'_B \in Z'_B$ such that (39) holds. Suppose to the contrary that for all $z'_B \in Z'_B$ the equality (39) does not hold. By (80), this implies that for all $z'_B \in Z'_B$

$$x(m^*, z_S^*, z_B^*)v(q^*, \theta) - p(m^*, z_S^*, z_B^*) > x(m^*, z_S^*, z'_B)v(q^*, \theta) - p(m^*, z_S^*, z'_B). \tag{84}$$

This inequality is also true for a quality $q'$ slightly below $q^*$ so that for all $z'_B \in Z'_B$

$$x(m^*, z_S^*, z_B^*)v(q', \theta) - p(m^*, z_S^*, z_B^*) > x(m^*, z_S^*, z'_B)v(q', \theta) - p(m^*, z_S^*, z'_B). \tag{85}$$

Now, (79) and (85) imply that $(\hat{z}_S, \hat{z}_B(\cdot))$ is also a BNE of the continuation game $\Gamma(m^*, q')$. Hence, Condition 1 implies that for all $(\tilde{z}_S, \tilde{z}_B(\cdot)) \in E(m^*, q')$ it holds that

$$p(m^*, z_S^*, z_B^*) = p(m^*, \tilde{z}_S, \tilde{z}_B(\theta)). \tag{86}$$

Thus, the seller gets the same payment in $\Gamma(m^*, q')$ as he gets in $\Gamma(m^*, q^*)$. Since the quality $q'$ is less costly than $q^*$, this contradicts the condition that in equilibrium $q^*$ maximizes the seller's profit. This establishes Lemma 6 Q.E.D.

**Proof of Proposition 5**: Consider an arbitrary interval $I$ with $x^*(\cdot) = 1$ on $I$. For each $\theta \in I$, let an equilibrium path be denoted by $(m^*(\theta), q^*(\theta), z_S^*(\theta), z_B^*(\theta))$. We have to show that $q^*(\cdot)$ is not strictly increasing on $I$. We begin with an auxiliary observation:

$$q^*(\cdot) \text{ is (weakly) increasing on } I. \text{ Therefore it is continuous a.e. on } I. \tag{87}$$

Since $x^*(\cdot) = 1$ on $I$, (87) follows from an incentive compatibility argument which is standard and therefore omitted. The rest of the proof is by contradiction. Suppose that $q^*(\cdot)$ is strictly increasing on $I$. By (87), we can choose a $\theta \in I$ so that $q^*(\cdot)$ is continuous at $\theta$. By Lemma 6, there is a message $z'_B$ with $x \equiv x(m^*(\theta), z^*_S(\theta), z'_B) < 1$ such that

$$v(q^*(\theta), \theta) - p(m^*(\theta), z^*_S(\theta), z^*_B(\theta)) = x \cdot v(q^*(\theta), \theta) - p(m^*(\theta), z^*_S(\theta), z'_B), \tag{88}$$

where on the left hand side we have used that $x(m^*(\theta), z^*_S(\theta), z^*_B(\theta)) = x^*(\theta) = 1$. Let

$$\varepsilon = (1 - x) v_\theta(q^*(\theta), \theta). \tag{89}$$

$v_\theta(q, \theta)$ is continuous in $q$, and since $\Theta$ is compact, $v_\theta(q, \cdot)$ is uniformly continuous on $\Theta$. Hence, there are $\delta_1, \delta_2 > 0$ such that for all $q', \tau, \tau'$ with $|q^*(\theta) - q'| < \delta_1$ and $|\tau - \tau'| < \delta_2$, we have:

$$|v_\theta(q^*(\theta), \tau) - v_\theta(q', \tau')| < \varepsilon. \tag{90}$$

Since $q^*(\cdot)$ is continuous at $\theta$, we can find a $\theta' \in I$, $\theta' < \theta$ such that $q^*(\theta') > 0$ and

$$|q^*(\theta) - q^*(\theta')| < \delta_1 \quad \text{and} \quad |\theta - \theta'| < \delta_2. \tag{91}$$

With these preparations, we will now derive a contradiction. In equilibrium, type $\theta'$ does not gain by deviating to message $m^*(\theta)$ in period 1 and message $z'_B$ in period 3. Since $x(m^*(\theta'), z^*_S(\theta'), z^*_B(\theta')) = x^*(\theta') = 1$, we therefore have:

$$v(q^*(\theta'), \theta') - p(m^*(\theta'), z^*_S(\theta'), z^*_B(\theta')) \geq x \cdot v(q^*(\theta), \theta') - p(m^*(\theta), z^*_S(\theta), z'_B). \tag{92}$$

Further, type $\theta$ does not gain by imitating the equilibrium strategy of type $\theta'$. Thus,

$$v(q^*(\theta), \theta) - p(m^*(\theta), z^*_S(\theta), z^*_B(\theta)) \geq v(q^*(\theta'), \theta) - p(m^*(\theta'), z^*_S(\theta'), z^*_B(\theta')). \tag{93}$$

Combining (88) and (93) gives:

$$x \cdot v(q^*(\theta), \theta) - p(m^*(\theta), z^*_S(\theta), z'_B) \geq v(q^*(\theta'), \theta) - p(m^*(\theta'), z^*_S(\theta'), z^*_B(\theta')). \tag{94}$$

By (92) and (94), we have:

$$v(q^*(\theta'), \theta) - v(q^*(\theta'), \theta') \leq x \cdot [v(q^*(\theta), \theta) - v(q^*(\theta), \theta')]. \tag{95}$$

By the mean value theorem, there are $\tau, \tau' \in [\theta', \theta]$ such that the previous inequality writes

$$v_\theta(q^*(\theta'), \tau')(\theta - \theta') \leq x \cdot v_\theta(q^*(\theta), \tau)(\theta - \theta'). \tag{96}$$

Thus, since $\theta - \theta' > 0$ and $v_\theta(q^*(\theta), \tau) > 0$, we obtain:

$$\begin{aligned} x &\geq \frac{v_\theta(q^*(\theta'), \tau') - v_\theta(q^*(\theta), \tau)}{v_\theta(q^*(\theta), \tau)} + 1 && (97) \\ &> \frac{-\varepsilon}{v_\theta(q^*(\theta), \tau)} + 1 && (98) \\ &\geq \frac{-\varepsilon}{v_\theta(q^*(\theta), \theta)} + 1 && (99) \\ &= x, && (100) \end{aligned}$$

where the second inequality follows from (90) and (91), the third inequality follows because $\tau \leq \theta$ implies that $v_\theta(q^*(\theta), \tau) \leq v_\theta(q^*(\theta), \theta)$, and the final inequality follows from (89). Thus, we have arrived at a contradiction, and this proves the proposition. Q.E.D.

**Proof of Proposition 6** (i) Consider qualities $0 < q_1^* < q_2^*$ and define the sets

$$\Theta_i = \{\theta \in \Theta \mid q^*(\theta) = q_i^* \wedge x^*(\theta) = 1\}, \quad i = 1, 2. \tag{101}$$

We first proof the following:

$$\Theta_2 \neq \emptyset \quad \Rightarrow \quad \Theta_1 \text{ has measure zero.} \tag{102}$$

Suppose to the contrary that $\Theta_2 \neq \emptyset$ and $\Theta_1$ has positive measure. Let $(\hat{z}_S(m^*(\theta)), \hat{z}_B(m^*(\theta), \cdot))$ be a BNE of the game $\Gamma(m^*(\theta), q^*(\theta))$. Note that for all $\theta \in \Theta_i$, the equilibrium payments have to be the same: $p(m^*(\theta), \hat{z}_S(m^*(\theta)), \hat{z}_B(m^*(\theta), \theta)) = p_i^*$. For if there were two types in $\Theta_i$ with different equilibrium payments, then since the chosen quality is constant on $\Theta_i$, the type with the higher payment could gain by sending the messages $m$ and $z_B$ of the type with the lower payment. Therefore, since no type in $\Theta_i$ has an incentive to mimic the equilibrium strategy of a type in $\Theta_j$, we have for all $\theta_i \in \Theta_i$:

$$v(q_i^*, \theta_i) - p_i^* \geq v(q_j^*, \theta_i) - p_j^*. \tag{103}$$

Next, since $\Theta_1$ has positive measure, $\Theta_1$ contains at least two types $\theta_1' < \theta_1''$. Let $m_1 = m^*(\theta_1')$ with $q(m_1) = q_1^*$ be the message used by the type $\theta_1'$. Moreover, let $m_2$ be some message with $q(m_2) = q_2^*$ and $m_2 = m^*(\theta_2)$ for some $\theta_2 \in \Theta_2$. Further, define by $\theta_{\ell i} = \inf\{\theta \in \Theta_i \mid m^*(\theta) = m_i\}$ the lowest type in $\Theta_i$ who uses the message $m_i$. We make use of the following claims.

*Claim 1*: There is a $z_B^i \in Z_B$ such that $v(q_i^*, \theta_{\ell i}) - p_i^* = -p(m_i, \hat{z}_S(m_i), z_B^i)$.

*Claim 2*: $p(m_1, \hat{z}_S(m_1), z_B^1) = p(m_2, \hat{z}_S(m_2), z_B^2)$.

Claims 1 and 2 follow from arguments which are similar to the arguments used in the proofs of Lemma 2 and Lemma 6. We omit the details. Claims 1 and 2 imply that

$$v(q_1^*, \theta_{\ell 1}) - p_1^* = v(q_2^*, \theta_{\ell 2}) - p_2^*. \tag{104}$$

Now, by continuity of $v(q, \cdot)$, the inequality (103) carries over to the types $\theta_{\ell i}$:

$$v(q_1^*, \theta_{\ell 1}) - p_1^* \geq v(q_2^*, \theta_{\ell 1}) - p_2^*, \qquad v(q_2^*, \theta_{\ell 2}) - p_2^* \geq v(q_1^*, \theta_{\ell 2}) - p_1^*. \tag{105}$$

Hence, (104) implies that $v(q_2^*, \theta_{\ell 2}) \geq v(q_2^*, \theta_{\ell 1})$ and $v(q_1^*, \theta_{\ell 1}) \geq v(q_1^*, \theta_{\ell 2})$. Consequently: $\theta_{\ell 1} = \theta_{\ell 2}$. Thus, since $q_1^* < q_2^*$, (104) together with $v_{q\theta} > 0$ implies that for the type $\theta_1'' > \theta_1' \geq \theta_{\ell 1}$, we have

$$v(q_2^*, \theta_1'') - v(q_1^*, \theta_1'') > p_2^* - p_1^*, \tag{106}$$

a contradiction to (103). This establishes (102).

We can now demonstrate (i). If $q^*(\cdot) = 0$ almost everywhere on $\Theta$, the claim is true for $\hat{\theta} = \bar{\theta}$. So suppose there is a positive measure set $\Theta' \subseteq \Theta$ with $q^*(\cdot) > 0$ and $x^*(\cdot) = 1$ on $\Theta'$. By incentive compatibility $q^*(\cdot)$ is increasing on $\Theta'$. It is well–known that this implies that $q^*(\cdot)$ is continuous almost everywhere on $\Theta'$. Thus, Proposition 5 entails that $q^*(\cdot)$ is piece–wise constant on $\Theta'$. Therefore, it follows from (102) that $q^*(\cdot)$ is in fact constant on $\Theta'$ except possibly at $\hat{\theta} = \inf \Theta'$, and this implies (i).

(ii) Let $(q^*, x^*)$ have the form described under (i). Let the exit option contract be given as follows: $M = \{m_\ell, m_h\}$, $Z_B = \{T, N\}$ and $Z_S = \emptyset$, and define $x(m, N) = 0$ and $x(m, T) = 1$ for all $m$. Moreover, let $p(m_\ell, N) = p(m_\ell, T) = 0$ and $p(m_h, N) = 0$ and $p(m_\ell, T) = v(\hat{q}, \hat{\theta})$. It is easy to verify that $(M, Z, x, p)$ implements $(q^*, x^*)$. Q.E.D.

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