Relative Performance Pay, Bonuses, and Job-Promotion Tournaments *

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Abstract

Empirical studies have challenged tournament theory by pointing out that (1) there is considerable pay variation within hierarchy levels, (2) promotion premiums only in part explain hierarchical wage differences, and (3) external recruitment is even observable on higher hierarchy levels. We explain these puzzles by combining tournaments with bonuses in a two-tier hierarchy. Under certain conditions the firm implements first-best effort on tier 2 although workers earn positive rents. Intuitively, the firm can use second-tier rents for creating incentives on tier 1. Furthermore, with unobserved worker heterogeneity, the firm improves the selection quality of a job-promotion tournament by including bonuses.

Key Words: bonuses; job promotion; limited liability; tournaments

JEL Classification: D82; D86; J33.

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1 Introduction

The empirical literature on internal labor markets has documented stylized facts that are not in line with traditional tournament models. In particular, Baker, Gibbs, and Holmström (1994a, 1994b)[1] have highlighted three empirical puzzles that question the theory of job-promotion tournaments: (1) There is considerable variation in pay on each hierarchy level, which contradicts the important prerequisite of tournaments that wages must be attached to jobs in order to generate incentives. (2) Promotion premiums that are paid to workers when moving to higher levels in the hierarchy can explain only part of the hierarchical wage differences in firms. Baker, Gibbs, and Holmström show that often hierarchical wage differences are even five times higher (or more) than the corresponding promotion premiums. (3) We can observe external recruiting on higher hierarchy levels in many firms from different countries, which would erase incentives from internal job-promotion tournaments.

Our paper follows the advice of Waldman (forthcoming) to develop a more sophisticated tournament model that can explain these empirical findings. We introduce a new approach that combines job-promotion tournaments with additional incentive schemes.[2] Using this model, we can explain the three puzzles mentioned before. Moreover, our approach is also in line with organizational practice, where we frequently observe that job-promotion tournaments and bonus schemes coexist in the same firm.

We analyze two representative periods in the lifespan of a two-tier firm. In the first period, the firm needs to hire two workers for the lower hierarchy level 1. In the second period, the firm has to fill one position on the higher level 2. Workers produce only ordinal performance information on the first

[1] For further empirical evidence, see Lazear (1992), Ariga, Ohkusa and Brunello (1999), Seltzer and Merrett (2000), Treble et al. (2001), Dohmen, Kriechel and Pfann (2004), Gibbs and Hendricks (2004), and Grund (2005).

[2] Hence, our approach builds on the general insight of Baker, Gibbs and Holmström (1994b, p. 921): "None of the major theories of wage determination can alone explain the evidence." As an alternative to our approach, Gibbons and Waldman (1999) have combined job assignment, human capital investments and learning in order to explain the empirical findings.

tier, but are individually visible after promotion to the second tier. Here, they become responsible for certain managerial tasks that lead to individual and verifiable performance signals. Workers are protected by limited liability and, thus, earn rents on each hierarchy level.

Three different instruments are available for stimulating effort incentives. First, the firm can employ relative performance pay (rank-order tournament) on the first hierarchy level. Second, it can install a bonus scheme on hierarchy level 2 contingent on individual performance. Finally, it can combine both tiers by employing a job-promotion scheme that assigns the better performing worker of level 1 to the next hierarchy level.

As a result, the firm has to choose between two possible contractual forms. *First*, the firm can forego the job-promotion scheme. In this case, the firm designs optimal separate contracts for either tier and does not commit itself to promote workers according to past performance. Such a contractual solution gives the firm maximum freedom for job assignment on hierarchy level 2. It can choose the level-1 worker who is better suited for the task on the next tier or it can hire a new worker from the external labor market. *Second*, the firm can integrate a job-promotion scheme into the contract, thereby interlinking the incentive schemes from the two tiers (combined contract). Such a combined contract limits the firm's discretion to fill the vacant position on level 2: It must promote the worker with the best performance on level 1, who, however, does not need to be the best available candidate for level 2. Nevertheless, this self-commitment may also have beneficial effects for the firm since expected rents from hierarchy level 2 induce extra incentives for the workers competing for job promotion on level 1.

Our results point out that either type of contract may be optimal. In particular, we show that, from a pure incentive perspective, a combined contract always dominates separate contracts. However, the firm will prefer separate contracts if efficient job assignment is the primary aim of its personnel policy. Both contract types employ a rank-order tournament on level 1. Under separate contracts, however, the tournament is only used for creating incentives without assigning workers to jobs. By contrast, under a combined contract, the tournament induces incentives and results into job assignment. In both scenarios, the optimal contract includes bonuses contingent on performance on level 2.

Since, ex-post, workers earn high or low bonuses depending on success or failure, we have a natural variation in pay on the second hierarchy level, which resolves puzzle (1). As under either contractual solution a promoted worker earns both relative performance pay and bonuses, hierarchical wage increases are only in part determined by job promotion, hence illuminating puzzle (2). In this context, one of the empirical findings by Dohmen, Kriechel, and Pfann (2004) is interesting. Contrary to other firm studies, they are able to determine the exact point in time when a worker realizes a pay increase, and they find that promotion and wage increase are often not simultaneous. This observation fits quite well to our model. Finally, if the firm chooses separate contracts, it may ex-post prefer external recruitment on level 2 for better job assignment, which explains puzzle (3) on ports of entry at higher hierarchy levels.

The aim of this paper is twofold. On the one hand, it addresses empirical puzzles that cannot be explained by traditional tournament models. On the other hand, we want to add to the theory of rank-order tournaments[3] by combining tournaments with further incentive schemes. In our model, workers are protected by limited liability and, therefore, earn strictly positive rents. By combining bonus pay on hierarchy level 2 with job promotion, the rent earned by a promoted worker provides incentives for level-1 workers: Each worker wants to win the tournament and, hence, the rent on the next level. Interestingly, the use of level-2 rents for creating incentives on level 1 always enhances workers' performances on level 2, but not necessarily on level 1. If the associated rent is not too large, the firm will even implement first-best effort on the second hierarchy level. Recently, contract theorists as Schmitz (2005) have pointed out that optimal bonus payments that lead to positive rents can be reinterpreted as efficiency wages. Since, in general, rents are strictly increasing in effort in single-agent hidden action models with contin-

[3]See the seminal papers by Lazear and Rosen (1981) and Nalebuff and Stiglitz (1983) that discuss tournaments in a contract-theoretic context with application to labor economics.

uous effort, the implemented effort level is inefficiently small. By contrast, in our model the firm implements first-best effort on level 2 even though this effort is associated with a strictly positive rent, which also monotonically increases in effort. This result implies that combining tournaments with bonuses allows for efficiency wages in a more literal sense.

As an extension, we introduce unobserved heterogeneity of workers by assuming symmetric uncertainty about a worker's marginal productivity. We show that, under such heterogeneity, a combined contract has further advantages over separate contracts. When using a combined contract, the firm implements strictly larger efforts than under separate contracts on both hierarchy levels. This is due to the fact that, under the combined contract, higher efforts are desirable for two distinct reasons: First, the higher workers' efforts on level 1 the higher will be the probability that the more productive worker is promoted to the next level. Thus, the selection quality of the job-promotion tournament improves. Second, under a combined contract, all players update their beliefs about the unknown productivity of the promoted worker. Due to the selection properties of the tournament, the posterior expected productivity of the promoted worker is higher than the workers' expected productivity prior to the promotion tournament. As a consequence, the posterior efficient effort on hierarchy level 2 is also higher than the ex-ante efficient one.

Our paper is related to those two tournament models that also combine a rank-order tournament with additional incentive schemes. Tsoulouhas, Knoeber, and Agrawal (2007) analyze optimal handicapping of internal and external candidates in a contest to become CEO. To do so, they consider a promotion tournament where the prize is given by the incentive contract on the next hierarchy level. However, apart from addressing a quite different question, their model also differs from ours in several respects. First, they do not allow for relative performance pay on the first tier of the hierarchy. Second, they assume that the firm cannot commit to a second-period contract at the beginning of the game.[4] Furthermore, even though promoted agents are of limited liability, they do not earn rents due to their high reservation

[4] However, the authors also discuss an extension where commitment is possible.

utility. Schöttner and Thiele (2008) also investigate incentive contracting within a two-tier hierarchy, but consider a production environment with an individual and contractible performance signal on the first tier. They examine the optimal combination of piece rates for level-1 workers and a promotion tournament to the next tier.

Ohlendorf and Schmitz (2008) do not analyze tournaments, but combine two principal-agent contracts in successive periods. As in our model, the agent is wealth-constrained and earns a non-negative rent that can be used for incentive purposes. Compared to our paper, Ohlendorf and Schmitz consider a completely different scenario with a single agent. In their model, the principal is integrated in the production process and can invest in each of the two periods. Hence, the natural application of their model is a supplier-buyer relationship where the principal can terminate the joint project after the first period. In the Ohlendorf-Schmitz paper, optimal second-period incentives serve as a carrot or a stick since they depend on first-period success.

The remainder of the paper is organized as follows. In the next section, we introduce our basic model. Section 3 offers a solution to this model, comparing a combined contract with two separate contracts. In Section 4, we extend the basic model by assuming unobserved worker heterogeneity. Section 5 concludes.

2 The Basic Model

We consider two representative periods in the lifespan of a firm that consists of two hierarchy levels. In the first period, the firm needs to hire two workers for hierarchy level 1. In the second period, the firm has to fill one position on hierarchy level 2. The tasks to be performed on the two hierarchy levels differ in their nature. On level 1, workers fulfill production tasks that do not lead to individually attributable outputs. By contrast, on level 2, we have a managerial task accompanied by personal responsibility, generating an individual performance measure. For example, the position on level 2 may be head of a department or a division. Initially, we assume that all workers share the same abilities in the production task, but differ in their managerial talents. In Section 4, we also discuss worker heterogeneity that persists across hierarchy levels.

We assume that all players are risk neutral. Workers are protected by limited liability, i.e., the firm cannot exact payments from workers. On both tiers of the hierarchy, workers have zero reservation values. For simplicity, we neglect discounting.

On the *first hierarchy level*, each of the two workers i ($i = A, B$) exerts effort $\hat{e}_i \geq 0$. The effort has a non-verifiable monetary value $\hat{v}(\hat{e}_i)$ to the firm with $\hat{v}'(\cdot) > 0$ and $\hat{v}''(\cdot) \leq 0$. The firm neither observes $\hat{e}_i$ nor $\hat{v}(\hat{e}_i)$, but receives a verifiable ordinal signal $\hat{s} \in \{\hat{s}_A, \hat{s}_B\}$ about the relative performance of the two workers. The signal $\hat{s} = \hat{s}_A$ indicates that worker A has performed best, whereas $\hat{s} = \hat{s}_B$ means that worker B has performed better relative to his co-worker. The probability of the event $\hat{s} = \hat{s}_A$ is given by $\hat{p}(\hat{e}_A, \hat{e}_B)$ and that of $\hat{s} = \hat{s}_B$ by $1 - \hat{p}(\hat{e}_A, \hat{e}_B)$.

We assume that the probability function $\hat{p}(\hat{e}_A, \hat{e}_B)$ exhibits the properties of the well-known contest-success function introduced by Dixit (1987):[5]

(i) $\hat{p}(\cdot, \cdot)$ is symmetric, i.e. $\hat{p}(\hat{e}_i, \hat{e}_j) = 1 - \hat{p}(\hat{e}_j, \hat{e}_i)$,

(ii) $\hat{p}_1 > 0$, $\hat{p}_{11} < 0$, $\hat{p}_2 < 0$, $\hat{p}_{22} > 0$,

(iii) $\hat{p}_{12} > 0 \Leftrightarrow \hat{p} > 0.5$.

According to (ii), exerting effort has positive but decreasing marginal returns. Property (iii) implies that if, initially, player A has chosen higher effort than B, a marginal increase in $\hat{e}_B$ will make it more attractive to A to increase $\hat{e}_A$ as well, due to the more intense competition the increase of $\hat{e}_B$ has caused.

Spending effort $\hat{e}_i$ leads to costs $\hat{c}(\hat{e}_i)$ for worker i ($i = A, B$) with $\hat{c}(0) = \hat{c}'(0) = 0$ and $\hat{c}'(\hat{e}_i) > 0$, $\hat{c}''(\hat{e}_i) > 0$ for all $\hat{e}_i > 0$. Furthermore, to guarantee some regularity conditions, we make the following technical assumptions. To ensure concavity of the firm's objective function, we assume that $\hat{c}'''(\hat{e}_i) > 0$ and $\frac{\partial^2}{\partial \hat{e}^2}\hat{p}_1(\hat{e}, \hat{e}) \leq 0$. Finally, to obtain an interior solution, we assume that $\hat{c}''(0) = 0$.

On the *second hierarchy level*, a worker's effort generates an individual performance signal. Following the binary-signal model by Demougin and

[5] Subscripts of $p(\cdot, \cdot)$ denote partial derivatives.

Garvie (1991) and Demougin and Fluet (2001), we assume that the worker on level 2 chooses effort $e \geq 0$ leading to a contractible signal $s \in \{s^L, s^H\}$ on the worker's performance with $s^H > s^L$. The observation $s = s^H$ is favorable information about the worker's effort choice in the sense of Milgrom (1981). Let the probability of this favorable outcome be $p(e)$ with $p'(e) > 0$ (strict monotone likelihood ratio property) and $p''(e) < 0$ (convexity of the distribution function condition).

If the firm assigns worker i to the management task, i's effort choice e yields the firm a non-verifiable monetary value $v(e) + \mu_i$, with $v'(\cdot) > 0$ and $v''(\cdot) \leq 0$. Here, μ_A and μ_B are independent draws from a probability distribution of a random variable μ, which reflects workers' different talents for the management task. We assume that μ has a differentiable c.d.f. At the beginning of the first period, nobody knows μ_i. However, during the course of this period, the firm gets to know the workers and, finally, can tell who is better suited for the managerial task. Hence, the firm observes μ_i at the end of the first period. No other party is able to assess the workers' suitability for level 2 and, thus, μ_i is non-verifiable. If the firm fills the management position with an external candidate of unknown talent, the expected monetary value of his effort is $v(e) + E[\mu]$. Again, neither e nor its monetary value is observable by the firm.[6] Exerting effort e entails costs $c(e)$ to the worker on level 2 with $c(0) = c'(0) = 0$ and $c'(e) > 0$, $c''(e) > 0$ for all $e > 0$. Furthermore, analogous to the technical assumptions for the first hierarchy level, $c'''(e) > 0$, $p'''(e) \leq 0$, and $c''(0) = 0$.

In the given setting, the firm can use three different instruments to provide incentives: First, it can employ *relative performance pay* (i.e., a rank-order tournament) on hierarchy level 1. Under relative performance pay, the better performing worker receives a high wage w_H whereas the other worker obtains a low wage w_L. Hence, worker i earns w_H if $\hat{s} = s_i$. Otherwise, he obtains w_L. Second, the firm can install a *bonus scheme* on hierarchy level 2. In case of a favorable signal ($s = s^H$) the worker gets a high bonus b_H, whereas he receives a low bonus b_L if the signal is bad news ($s = s^L$). Due to limited liability,

[6] Note that $v(\cdot) + \mu_i$ measures the worker's contribution to total firm profits and is not identical with department or division profits.

payments must always be non-negative ($w_L, w_H, b_L, b_H \geq 0$). Finally, the firm can interlink the two hierarchy levels by committing to a *job-promotion scheme* where the better performing worker from level 1 will be promoted to level 2 at the end of the first period.

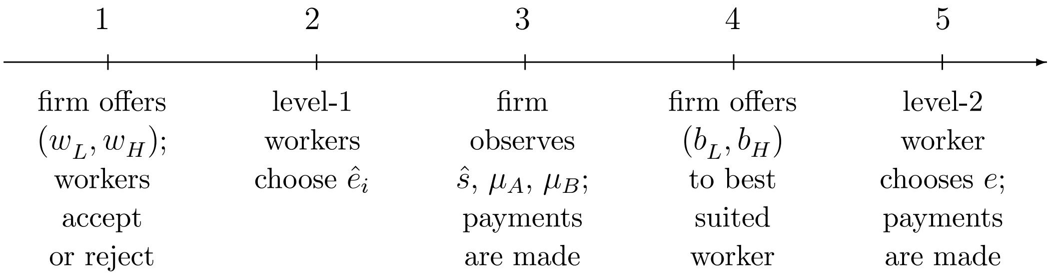
According to these incentive devices, the firm can offer one of the following two types of contracts. Under the first type, the firm designs *separate contracts* for each tier of the hierarchy, thereby foregoing a job-promotion scheme. By contrast, the second type of contract combines both hierarchy levels via a job-promotion scheme *(combined contract)*. The contract details are specified in Sections 3.1 and 3.2, respectively, where we analyze incentives and worker behavior under each contractual form.

3 Worker Behavior and the Optimal Contract

3.1 Separate Contracts

We start our analysis with the case where the firm designs separate contracts for each tier of the hierarchy. Then, the time schedule of the game is as follows. First, the firm offers two workers a one-period contract specifying relative performance pay (w_L, w_H) for employment on hierarchy level 1. Provided that the workers accepted the contract, they exert efforts $\hat{e}_A$ and $\hat{e}_B$. Afterwards, $\hat{s}$ is observed. Furthermore, the firm learns workers' abilities μ_A and μ_B. The workers then get w_L or w_H, respectively, whereas the firm receives $\hat{v}(\hat{e}_A) + \hat{v}(\hat{e}_B)$.

Next, the firm has to hire an individual for the management job at hierarchy level 2. The firm can either choose one of the internal candidates or an external worker. In the latter case, the worker's ability is unknown to the firm. The firm only knows that an external candidate has expected ability $E[\mu] = \bar{\mu}$. If $\max\{\mu_A, \mu_B\} > \bar{\mu}$ the firm will select the better internal candidate for the job on level 2; otherwise an external candidate is chosen. In any case, the firm offers to the preferred candidate a one-period contract (b_L, b_H) associated with hierarchy level 2. After acceptance of the contract, the level-2 worker chooses effort e yielding either a low or a high bonus payment. The firm earns $v(e) + \max\{\mu_A, \mu_B, \bar{\mu}\}$. The timing is summarized in the following figure.



We solve the game by backwards induction and thus first analyze hierarchy level 2. For this tier, the firm's optimization problem is

$$\max_{b_L, b_H, e} \{v(e) + \max\{\mu_A, \mu_B, \bar{\mu}\} - b_L - p(e) \cdot (b_H - b_L)\}$$

$$\text{s.t.} \quad e = \arg\max_z \{b_L + p(z) \cdot (b_H - b_L) - c(z)\} \tag{1}$$

$$b_L + p(e) \cdot (b_H - b_L) - c(e) \geq 0 \tag{2}$$

$$b_L, b_H \geq 0. \tag{3}$$

The firm maximizes its profit net of wage payments taking into account the incentive compatibility constraint (1), the participation constraint (2), and the limited-liability constraints (3). Due to the monotone likelihood ratio property and the convexity of the distribution function condition, the incentive constraint (1) is equivalent to its first-order condition

$$b_H - b_L = \frac{c'(e)}{p'(e)}. \tag{4}$$

Using this relationship, the firm's problem can be transformed to

$$\max_{b_L,e}\left\{v\left(e\right)+\max\{\mu_A,\mu_B,\bar{\mu}\}-b_L-p\left(e\right)\cdot\frac{c'\left(e\right)}{p'\left(e\right)}\right\}$$
$$\text{s.t. } b_L+p\left(e\right)\cdot\frac{c'\left(e\right)}{p'\left(e\right)}-c\left(e\right)\geq 0 \qquad (5)$$
$$b_L\geq 0.$$

Regarding the participation constraint, we can make the following observation, which is important for our further analysis.

Lemma 1 *The term*

$$r(e):=p\left(e\right)\frac{c'\left(e\right)}{p'\left(e\right)}-c\left(e\right) \qquad (6)$$

is strictly positive and monotonically increasing for all $e>0$.

Proof. $r(e)>0$ can be rewritten as $c\left(e\right)-c'\left(e\right)\frac{p(e)}{p'(e)}<0$. Note that $\frac{p(e)}{p'(e)}>e\Leftrightarrow p\left(e\right)-ep'\left(e\right)>0$ is true since $p\left(\cdot\right)$ is strictly concave. But then we also have $c\left(e\right)-c'\left(e\right)\frac{p(e)}{p'(e)}<c\left(e\right)-ec'\left(e\right)<0$ from the strict convexity of $c\left(\cdot\right)$. The derivative $r'(e)=p(e)\left[\frac{c''(e)p'(e)-p''(e)c'(e)}{[p'(e)]^2}\right]$ is positive for all $e>0$ by strict concavity of $p(e)$ and strict convexity of $c(e)$. ■

Hence, given e, the transformed participation constraint (5) is satisfied for all $b_L\geq 0$. Therefore, the firm optimally sets $b_L^s=0$, where superscript s denotes optimal contract parameters under separate contracts. After substituting b_L^s into the firm's objective function, we obtain that the firm induces the effort level $e^s>0$ given by[7]

$$e^s=\arg\max_e\left\{v\left(e\right)+\max\{\mu_A,\mu_B,\bar{\mu}\}-r(e)-c(e)\right\}.$$

Intuitively, since the worker is protected by limited liability, the firm has to leave him a rent. As a result, the firm's costs for inducing effort e are composed of the worker's effort costs, $c(e)$, and his rent $r(e)$.

Now we turn to hierarchy level 1. Here, two workers compete in a tour-

[7] Due to our technical assumptions, the objective function is strictly concave. Furthermore, the assumption $c''(0)=0$ ensures an interior solution. For $r''\left(e\right)>0$ see the additional pages for the referees.

nament for relative performance pay w_H and w_L. Furthermore, each worker anticipates that, in the following period, he will be assigned to the management position and earn the expected rent $r(e^s)$ if he is assessed to be the best future manager. Since workers' abilities are unknown ex ante, this case occurs with probability $\tilde{p} :=\text{prob}\{\mu_i > \max\{\mu_j, E[\mu]\}\}$ ($i, j = A, B$; $i \neq j$). Otherwise, the worker realizes his reservation value of zero.

We first characterize the workers' effort choices. Given the wages w_H and w_L, worker A chooses his effort level to solve

$$\max_{\hat{e}_A} w_L + \hat{p}\left(\hat{e}_A, \hat{e}_B\right) \cdot [w_H - w_L] - \hat{c}\left(\hat{e}_A\right) + \tilde{p} \cdot r(e^s) \tag{7}$$

whereas worker B solves

$$\max_{\hat{e}_B} w_L + [1 - \hat{p}\left(\hat{e}_A, \hat{e}_B\right)] \cdot [w_H - w_L] - \hat{c}\left(\hat{e}_B\right) + \tilde{p} \cdot r(e^s). \tag{8}$$

The equilibrium effort levels must satisfy the first-order conditions

$$(w_H - w_L)\, \hat{p}_1\left(\hat{e}_A, \hat{e}_B\right) = \hat{c}'\left(\hat{e}_A\right) \quad \text{and} \quad -\left(w_H - w_L\right) \hat{p}_2\left(\hat{e}_A, \hat{e}_B\right) = \hat{c}'\left(\hat{e}_B\right).$$

Recall that, due to the symmetry property (i) of the probability function $\hat{p}(\cdot, \cdot)$ we have $\hat{p}\left(\hat{e}_B, \hat{e}_A\right) = 1 - \hat{p}\left(\hat{e}_A, \hat{e}_B\right)$. Differentiating both sides with respect to $\hat{e}_B$ yields $\hat{p}_1\left(\hat{e}_B, \hat{e}_A\right) = -\hat{p}_2\left(\hat{e}_A, \hat{e}_B\right)$ so that the first-order conditions can be rewritten as

$$w_H - w_L = \frac{\hat{c}'\left(\hat{e}_A\right)}{\hat{p}_1\left(\hat{e}_A, \hat{e}_B\right)} = \frac{\hat{c}'\left(\hat{e}_B\right)}{\hat{p}_1\left(\hat{e}_B, \hat{e}_A\right)}.$$

Thus, we have a unique symmetric equilibrium $\left(\hat{e}_A, \hat{e}_B\right) = \left(\hat{e}, \hat{e}\right)$ given by[8]

$$w_H - w_L = \frac{\hat{c}'\left(\hat{e}\right)}{\hat{p}_1\left(\hat{e}, \hat{e}\right)}. \tag{9}$$

Our assumptions do not rule out the existence of additional asymmetric equilibria. However, we restrict attention to the symmetric equilibrium, which

[8] By strict concavity of $\hat{p}$ and convexity of $\hat{c}$, this condition is necessary and sufficient for $(\hat{e}, \hat{e})$ to be an equilibrium.

seems plausible in the given setting with ex-ante homogeneous contestants.[9] Condition (9) shows that equilibrium efforts increase in the tournament prize spread $w_H - w_L$.[10] To simplify notation, we denote by $\Delta w(\hat{e})$ the prize spread that induces effort $\hat{e}$, i.e.,

$$\Delta w(\hat{e}) := \frac{\hat{c}'(\hat{e})}{\hat{p}_1(\hat{e}, \hat{e})}. \tag{10}$$

The firm maximizes $2\hat{v}(\hat{e}) - w_L - w_H$ subject to the incentive constraint (9), the participation constraint[11]

$$w_L + \frac{1}{2}(w_H - w_L) - c(\hat{e}) + \tilde{p} \cdot r(e^s) \geq 0, \tag{11}$$

and the limited-liability constraints

$$w_L, w_H \geq 0. \tag{12}$$

Note that, when choosing the equilibrium effort $\hat{e}$, a worker must obtain at least the same expected payment as if he exerted zero effort, i.e.,

$$w_L + \frac{1}{2}\Delta w(\hat{e}) - c(\hat{e}) + \tilde{p} \cdot r(e^s) \geq w_L + \hat{p}(0, \hat{e})\Delta w(\hat{e}) - c(0) + \tilde{p} \cdot r(e^s). \tag{13}$$

Hence, $\frac{1}{2}\Delta w(\hat{e}) - c(\hat{e}) \geq 0$, implying that the firm chooses $w_L^s = 0$. Together with (9), it follows that $w_H^s = \Delta w(\hat{e})$ is optimal. Thus, the firm implements the effort level $\hat{e}^s > 0$ given by[12]

$$\hat{e}^s = \arg\max_{\hat{e}} 2\hat{v}(\hat{e}) - \Delta w(\hat{e}).$$

The results of this section are summarized in the following proposition.

[9] For example, asymmetric equilibria do not exist if the probability function is described by the well-known Tullock or logit-form contest-success function, $\hat{p}(\hat{e}_A, \hat{e}_B) = \frac{\hat{e}_A}{\hat{e}_A + \hat{e}_B}$.

[10] We have $\frac{\partial}{\partial \hat{e}}\hat{p}_1(\hat{e}, \hat{e}) = \hat{p}_{11}(\hat{e}, \hat{e}) + \hat{p}_{12}(\hat{e}, \hat{e}) < 0$ due to properties (ii) and (iii) of the probability function.

[11] In the symmetric equilibrium, each worker's winning probability is 1/2.

[12] The second-order condition $2\hat{v}''(\hat{e}) - \Delta w''(\hat{e}) < 0$ is satisfied due to our technical assumptions $\hat{c}'''(\hat{e}) > 0$ and $\frac{\partial^2}{\partial \hat{e}^2}\hat{p}_1(\hat{e}, \hat{e}) \leq 0$. An interior solution is guaranteed by the assumption $\hat{c}''(0) = 0$. In the additional pages for the referees, we verify that $\Delta w''(\hat{e}) > 0$.

Proposition 1 *Under separate contracts, the firm implements the effort levels*

$$\hat{e}^s = \arg\max_{\hat{e}} \{2\hat{v}(\hat{e}) - \Delta w(\hat{e})\}, \quad (14)$$

$$e^s = \arg\max_{e} \{v(e) + \max\{\mu_A, \mu_B, \bar{\mu}\} - r(e) - c(e)\}. \quad (15)$$

The optimal contract elements are

$$w_L^s = 0,\ w_H^s = \Delta w(\hat{e}^s),\ b_L^s = 0,\ b_H^s = \frac{c'(e^s)}{p'(e^s)}, \quad (16)$$

where $\Delta w(\hat{e})$ and $r(e)$ are given by (10) and (6), respectively.

From Lemma 1, it follows that the worker on level 2 earns a strictly positive rent $r(e^s)$. This suggests that the firm may benefit from a job-promotion scheme where the better performing level-1 worker is promoted to the next hierarchy level. Then, the level-2 rent provides additional effort incentives for the first hierarchy level. However, improved first-level incentives come at the cost of a possible suboptimal task-assignment on level 2 since the best internal production worker does not need to be the best manager. An approach that uses a strict internal promotion rule according to past performance corresponds to our combined contract, which we analyze in the following section.

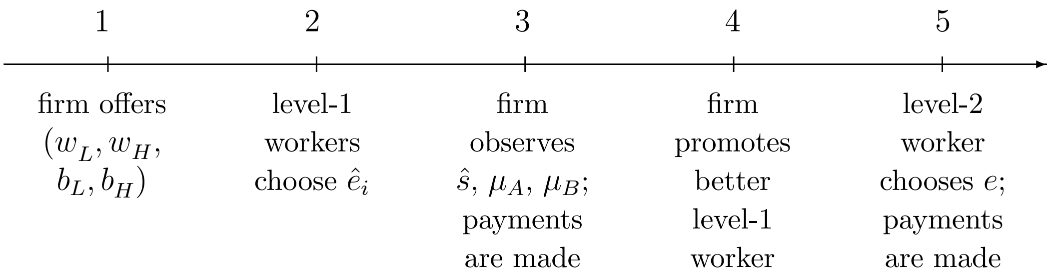
3.2 Combined Contract

This section considers combined contracts. Under a combined contract, the firm offers two workers a contract (w_L, w_H, b_L, b_H) at the start of the first period. The contract includes the commitment to promote the better performing level-1 worker to level 2 in the second period, i.e., worker i will be promoted if and only if $\hat{s} = \hat{s}_i$. Then, in the second period, the promoted worker will be paid according to the pre-specified bonus scheme.[13] For simplicity, we assume that the worker who did not achieve promotion

[13] Since μ_i is non-verifiable, the promotion rule as well as second-period payments cannot be contingent on μ_i.

is dismissed. Furthermore, the worker selected for promotion can quit and realize his zero reservation value in the second period.

The following figure summarizes the timing under a combined contract.



The time schedule differs from the one under separate contracts only with respect to stages 1 and 4. Under a combined contract, at stage 1, the firm offers two workers a contract (w_L, w_H, b_L, b_H) that covers the following two periods. At stage 4, the firm promotes the better level-1 worker to the next tier. In period 2, the firm's payoff is $v(e) + \mu_i$ if worker i was promoted.

Again, we solve the game by backwards induction. In the second period, given the bonus payments b_L and b_H, the promoted worker faces the same kind of decision problem as under separate contracts. Provided that his participation constraint (2) is satisfied, he chooses the effort level characterized by (1). In the first period, however, workers' optimization problems fundamentally differ from the case of two separate contracts. Now, increasing effort also raises the chance of being promoted and, consequently, earning a rent under the bonus contract. Hence, worker A's and B's optimization problems, respectively, are:

$$\max_{\hat{e}_A} \; w_L + \hat{p}(\hat{e}_A, \hat{e}_B) \cdot [w_H - w_L + b_L + p(e)(b_H - b_L) - c(e)] - \hat{c}(\hat{e}_A) \quad (17)$$

$$\max_{\hat{e}_B} \; w_L + [1 - \hat{p}(\hat{e}_A, \hat{e}_B)] \cdot [w_H - w_L + b_L + p(e)(b_H - b_L) - c(e)] - \hat{c}(\hat{e}_B). \quad (18)$$

Comparing the workers' objective functions with those under separate contracts, (7) and (8), we can see that, under combined contracts, the "prize" for performing better at level 1 increases by the expected payment to the promoted worker, $b_L + p(e)(b_H - b_L) - c(e)$. Analogously to the case of separate contracts, one can show that there is a unique symmetric equilibrium given by

$$\hat{p}_1(\hat{e}, \hat{e})\left[w_H - w_L + b_L + p(e)(b_H - b_L) - c(e)\right] = \hat{c}'(\hat{e})\,. \tag{19}$$

The first-period participation constraint thus is

$$w_L + \frac{1}{2}[w_H - w_L + b_L + p(e)(b_H - b_L) - c(e)] - \hat{c}(\hat{e}) \geq 0. \tag{20}$$

Now we can state the firm's optimization problem. Since suitability for the management task is not correlated with performance as a production worker,[14] the promotion rule induces efficient assignment of internal workers to level 2 with probability 1/2. Consequently, the firm's expected monetary payoff associated with level 2 is

$$v(e) + E\left[\frac{1}{2}(\mu_{(1)} + \mu_{(2)})\right],$$

where $\mu_{(1)}$ and $\mu_{(2)}$ denote the highest and second-highest order statistic, respectively, of the two random draws μ_A and μ_B. Since $E\left[\frac{1}{2}(\mu_{(1)} + \mu_{(2)})\right] = \bar{\mu}$, the firm's problem is[15]

$$\max_{e,\hat{e},w_L,w_H,b_H,b_L} [2\hat{v}(\hat{e}) - w_L - w_H] + [v(e) + \bar{\mu} - b_L - p(e)(b_H - b_L)] \tag{21}$$

$$\text{subject to (1), (2), (19), (20),} \tag{22}$$

$$w_L, w_H, b_L, b_H \geq 0. \tag{23}$$

[14] A case with correlation is discussed in Section 4.

[15] The limited-liability constraints $b_L, b_H \geq 0$ imply that a promoted worker cannot be held liable to the extent of his tournament prize w_H. This assumption is justified when workers can use their tournament prizes for consumption before the second period ends or, alternatively, when workers are protected by a strict liability limit of zero after failure at the bonus stage. However, in the additional pages for the referees we show that replacing (23) by $w_L \geq 0$, $w_H + b_L \geq 0$ and $w_H + b_H \geq 0$ would not alter our results.

By solving this problem, we obtain the following result.

Proposition 2 *Under a combined contract, the firm implements the effort levels*

$$(\hat{e}^c, e^c) \in \arg\max_{\hat{e},e} \{2\hat{v}(\hat{e}) - \Delta w(\hat{e}) + v(e) + \bar{\mu} - c(e)\} \quad (24)$$

$$\textit{subject to } \Delta w(\hat{e}) - r(e) \geq 0. \quad (25)$$

The optimal contract elements are

$$w_L^c = 0,\ w_H^c = \Delta w(\hat{e}^c) - r(e^c),\ b_L^c = 0,\ b_H^c = \frac{c'(e^c)}{p'(e^c)}, \quad (26)$$

where $\Delta w(\hat{e})$ *and* $r(e)$ *are given by (10) and (6), respectively.*

Proof. See Appendix. ■

3.3 Comparison of the Two Contracts

Given Propositions 1 and 2, we are now able to investigate the question which of the two contracts the firm prefers to implement. Our conjecture was that the combined contract may have the advantage of partially substituting direct first-level incentives $w_H - w_L$ for indirect incentives which arise due to the prospect of the expected second-period rent $r(e)$. By comparing the optimal contract elements (16) and (26), it becomes clear that this is indeed the case because we have $w_H^c = \Delta w(\hat{e}^c) - r(e^c)$. When we examine the firm's objective functions under separate contracts (see (14) and (15)) and a combined contract (see (24)), we can see that this substitution of incentives has the consequence that the firm's costs of inducing a given pair of effort levels $(\hat{e}, e)$ are strictly lower under combined contracts: $\Delta w(\hat{e}) + c(e)$ as opposed to $\Delta w(\hat{e}) + r(e) + c(e)$. But, in contrast to the case of separate contracts, the firm's optimization problem under combined contracts exhibits a constraint, (25). At first sight, one might think that this constraint restricts the set of feasible effort pairs $(\hat{e}, e)$ under combined contracts and is, thus, detrimental. However, such a conclusion would be wrong. As the proof of

Proposition 2 shows, constraint (25) arises because, for any given level-2 effort e, the firm always wants to use the entire associated rent to enhance first-level incentives. In other words, given e, the firm wishes to implement at least the first-level effort $\hat{e}$ that workers are willing to spend to win $r(e)$, i.e., $\Delta w(\hat{e}) \geq r(e)$. To induce a level-1 effort with $\Delta w(\hat{e}) < r(e)$, the firm would have to punish good performance on the first tier by setting $w_H < w_L$. This cannot be optimal because the firm would actually pay for reducing effort.

To compare the two contractual forms, we now denote the expected profit under the optimal separate contracts by π^s, i.e.,

$$\pi^s := 2\hat{v}(\hat{e}^s) - \Delta w(\hat{e}^s) + v(e^s) + E[\max\{\mu_{(1)}, \bar{\mu}\}] - r(e^s) - c(e^s), \qquad (27)$$

and the expected profit under the optimal combined contract by π^c, i.e.,

$$\pi^c := 2\hat{v}(\hat{e}^c) - \Delta w(\hat{e}^c) + v(e^c) + \bar{\mu} - c(e^c). \qquad (28)$$

We obtain the following result.

Proposition 3 *There exists a cut-off value $\tau > 0$ such that the firm will prefer a combined contract to separate contracts if and only if*

$$E[\max\{\mu_{(1)}, \bar{\mu}\}] - \bar{\mu} \leq \tau.$$

Proof. Assume, for a moment, that all internal and external workers are homogeneous with respect to the managerial task, i.e., μ is deterministic and hence $E[\max\{\mu_{(1)}, \bar{\mu}\}] = \bar{\mu}$. Under a combined contract, the firm can induce the same level-2 effort as under the optimal separate contracts by offering the bonuses b_L^s, b_H^s. If the corresponding rent $r(e^s) > 0$ does not exceed $\Delta w(\hat{e}^s)$, setting $w_H = \Delta w(\hat{e}^s) - r(e^s)$ implements $\hat{e}^s$ on level 1. Thus, if $r(e^s) \leq w_H^s$, there is a combined contract that replicates the effort choices under the optimal separate contracts at strictly lower costs. It follows that $\pi^c > \pi^s$. If $r(e^s) > \Delta w(\hat{e}^s)$, the combined contract that induces e^s on level 2 entails $\hat{e} > \hat{e}^s$ on level 1. Thus, profit under the combined contract $(0, 0, b_L^s, b_H^s)$ is strictly larger than π^s. It follows that, for an arbitrary distribution of μ, we have $\pi^c \geq \pi^s$ as long as selection is sufficiently unimportant, i.e., if and only if $E[\max\{\mu_{(1)}, \bar{\mu}] - \bar{\mu} \leq \tau$, where

$$\begin{aligned} \tau := & [2\hat{v}(\hat{e}^c) - \Delta w(\hat{e}^c) + v(e^c) - c(e^c)] \\ & - [2\hat{v}(\hat{e}^s) - \Delta w(\hat{e}^s) + v(e^s) - r(e^s) - c(e^s)] > 0. \end{aligned} \tag{29}$$

■

Proposition 4 shows that the firm may prefer either contract type. If selection plays no role because all production workers will be equally good managers, the combined contract yields a higher profit than separate contracts due to the extra incentives via rent $r(e)$. Hence, the incentive effect of a combined contract, which can be characterized by τ, is always positive. Consequently, from a pure incentive perspective, combined contracts are always superior. However, separate contracts become optimal if it is likely that production workers differ strongly in their suitability for the management task, i.e., the variance of μ is high. Then, the selection effect of separate contracts, $E[\max\{\mu_{(1)}, \bar{\mu}\}] - \bar{\mu}$, is large and dominates the incentive effect τ. Therefore, depending on whether incentives or selection issues prevail in a certain situation, we should observe either type of contract in practice. To see under which circumstances the incentive effect τ is large, we need to know how optimal effort levels compare under the two contracts. Therefore, in the next step, we analyze worker behavior under the two types of contract.

To characterize effort under the optimal combined contract, it is necessary to distinguish whether restriction (25) is binding or not at the optimum. First, assume the constraint is not binding. Then, effort on the second hierarchy level corresponds to the first-best effort level, i.e.,

$$e^c = e^{FB} = \arg\max_e \{v(e) - c(e)\},$$

as can be seen from (24). Hence, under the combined contract, level-2 effort is larger than under separate contracts. Concerning the first hierarchy level, however, a comparison of (14) and (24) points out that $\hat{e}^c = \hat{e}^s$. Thus, interestingly, the use of second-level rents for incentive purposes on hierarchy level 1 does *not* lead to higher effort on that hierarchy level. Instead, only second-tier effort increases. This result is due to the fact that raising incentives on the second tier increases efforts on both levels, but level-1 efforts are then decreased again by reducing $w_H - w_L$. Hence, direct first-level incentives stemming from relative performance pay are simply replaced by indirect ones.

This observation can be related to the concept of *efficiency wages,* which has been reconsidered by contract theorists in the last decade. According to Tirole (1999, p. 745), Laffont and Martimort (2002, p. 174), and Schmitz (2005), efficiency wages occur if workers are protected by limited liability and earn positive rents under the optimal contract. In their models, the implemented effort level is inefficiently small. By contrast, in our setting the firm implements the efficient effort level e^{FB} although this entails a strictly positive rent that is monotonically increasing in effort. Hence, combining both hierarchy levels for creating optimal incentives allows for efficiency wages in a more literal sense. As a crucial condition, the associated rent $r(e^{FB})$ must not be too large, i.e., $\Delta w(\hat{e}^s) > r(e^{FB})$. Otherwise, restriction (25) is binding at the optimal solution. In this case, $r(e^{FB})$ exceeds the costs for inducing $\hat{e}^s$ under separate contracts. Implementing $e = e^{FB}$ would then yield $\hat{e} > \hat{e}^s$. But level-1 effort is not so valuable that the firm is willing to spend $r(e^{FB})$. Therefore, the firm induces $e^c < e^{FB}$, which still leads to higher first-level effort than the optimal separate contracts. Altogether, we have the following results:

Proposition 4 *(i) If restriction (25) is non-binding, i.e.,* $\Delta w(\hat{e}^s) > r(e^{FB})$*, effort levels under the two contracts compare as follows:* $\hat{e}^c = \hat{e}^s$ *and* $e^c = e^{FB} > e^s$*. (ii) If restriction (25) is binding, the firm implements* $\hat{e}^c > \hat{e}^s$ *and* $e^{FB} > e^c > e^s$*.*

Proof. See Appendix. ■

Proposition 4 helps us to further characterize the incentive effect of a combined contract, τ. Together with the definition of τ, equation (29), we obtain that, in case $\Delta w(\hat{e}^s) > r(e^{FB})$,

$$\tau = [v(e^{FB}) - c(e^{FB})] - [v(\hat{e}^s) - r(\hat{e}^s) - c(\hat{e}^s)]. \tag{30}$$

Hence, τ is equal to the difference between first-best level-2 profit and the level-2 profit under optimal separate contracts. As a result, the incentive effect is large and, therefore, combined contracts are more likely to be optimal if the rent extraction problem under separate contracts is severe. In general, this tends to be the case if the quality of the performance signal s is poor.[16] If $r(e^{FB})$ is large so that the firm implements different level-1 efforts under the two contracts, the size of τ depends, among other factors, on the relative importance of level-1 and level-2 effort and the relative quality of the performance signals. Thus, since we assume fairly general functional forms, we cannot make any clear-cut predictions.

We are now able to fully characterize the contracting environments that may arise in the firm. First, consider the case where the incentive effect of the combined contract dominates the selection effect of separate contracts, i.e., $E[\max\{\mu_{(1)}, \bar{\mu}\}] - \bar{\mu} \leq \tau$. Then, a combined contract is optimal and, hence, the firm employs a job-promotion scheme. In addition, the firm will implement both a bonus scheme and relative performance pay if the rent for implementing first-best effort on hierarchy level 2 is not too large (case (i) of Proposition 4). In such a situation, the firm makes use of moderate relative performance pay on the first tier by choosing a tournament winner prize $w_H^c = \Delta w(\hat{e}^c) - r(e^{FB}) > 0$. Since $\hat{e}^s = \hat{e}^c$, the winner prize w_H^c is smaller than the winner prize under separate contracts, $w_H^s = \Delta w(\hat{e}^s) = \Delta w(\hat{e}^c)$. Moreover, the firm installs high-powered incentives via a bonus system on level 2 of the hierarchy. The optimal bonus is zero in case of an unfavorable performance signal ($b_L^c = 0$). In case of a favorable signal, the worker receives the bonus $b_H^c = \frac{c'\left(e^{FB}\right)}{p'(e^{FB})}$, which is larger than the high bonus under separate contracts ($b_H^s = \frac{c'(e^s)}{p'(e^s)}$). However, if the rent for implementing e^{FB} is rather

[16] For example, for $v(e) = e$, $c(e) = e^3$, $p(e) = e^\theta$ $(0 < \theta < 1)$, we obtain that the level-2 profit under separate contracts is increasing in θ, while first-best surplus is independent of θ. Hence, τ is large for $\theta = \frac{dp(e)}{de}\frac{e}{p(e)}$ close to zero, implying that $p(e)$ is not very responsive to changes in effort.

large (case (ii) of Proposition 4), the firm foregoes relative performance pay on the first tier[17] and, thus, solely relies on indirect level-1 incentives through the second-period rent.

If, however, selection issues play the dominant role (i.e., $E[\max\{\mu_{(1)}, \bar{\mu}\}] - \bar{\mu} > \tau$), separate contracts are optimal. The firm then makes use of both relative performance pay and a bonus system but renounces a job-promotion scheme.

These results are nicely in line with the three empirical observations from the introduction that contradict standard models on job-promotion tournaments. Baker, Gibbs, and Holmström (1994a, 1994b) analyze the internal structure and the wage policy of a US corporation.[18] As a first puzzling result, they find considerable variation in pay on each hierarchy level (see Figure VI in Baker, Gibbs and, Holmström 1994a, p. 906). This finding contradicts the important prerequisite of standard job-promotion tournaments that wages must be attached to jobs and, therefore, to hierarchy levels in order to generate incentives. In our model, under the combined contract we have a job-promotion tournament with pay variation because the promoted worker may or may not receive a bonus on hierarchy level 2. Under separate contracts, we may also observe internal promotion, but then the job assignment decision does not follow a promotion rule that strictly honors past performance. Nevertheless, a promoted worker receives a wage increase due to relative performance pay on the first level and, in addition, a variable bonus payment on the next level. These results are quite in line with empirical studies that observe substantial variation in annual compensations of workers at the same hierarchy level.

Furthermore, according to standard job-promotion tournaments, hierarchical wage differences should be completely explained by promotion premi-

[17] Note that, since $\Delta w(\hat{e}^c) = r(e^c)$, we must have $w_H^c = 0$.

[18] The empirical puzzles documented by Baker, Gibbs, and Holmström (1994a, 1994b) are also found by Treble et al. (2001), who analyzed a British firm. Considerable wage variation within job levels is also documented by the empirical studies of Seltzer and Merrett (2000), Dohmen, Kriechel and Pfann (2004), Gibbs and Hendricks (2004) and Grund (2005). Moreover, Dohmen, Kriechel and Pfann (2004) show that promotion and wage increase are often not simultaneous, which gives further evidence that salaries are also determined by bonuses and not solely by promotion premiums.

ums paid to workers when moving to higher levels in the hierarchy. However, Baker, Gibbs, and Holmström (1994a) find that "promotion premiums explain only part of the differences in pay between levels" (p. 909). In fact, often hierarchical wage differences are even five times higher (or more) than the corresponding promotion premiums. This second puzzling observation is also in line with our modified tournament model. Under either contractual solution, a promoted worker does not only earn the promotion premium $w_H - w_L$ but may also receive a bonus. In particular, under a combined contract, the higher the expected rent on hierarchy level 2, the smaller will be the optimal promotion premium. The reason is that indirect incentives replace direct ones. Presuming that effort on higher hierarchy levels is more valuable to firms than effort choices on lower levels,[19] we will have considerable rents on higher tiers, thus reducing corresponding promotion premiums.

Finally, Baker, Gibbs and Holmström document that there is significant external recruiting on higher hierarchy levels. This observation contradicts traditional models on job-promotion tournaments as external hiring destroys internal career incentives. However, in our model the observation on ports of entry on higher levels can be explained by separate contracts dominating a combined contract in specific situations. If for certain positions in the hierarchy efficient job assignment is the dominating feature of the firm's personnel policy and there is sufficient heterogeneity among workers, the firm prefers separate contracts and may hire an external candidate for level 2. In that case, the firm still applies a tournament as relative performance pay but strictly separates incentive provision on hierarchy level 1 from assignment of workers to level 2.

4 Unobserved Worker Heterogeneity

So far, we have assumed that workers differ only in their capacities for the managerial task. Moreover, the respective characteristics of a worker's ability are revealed to the firm after one period of interaction. Such characteristics

[19] That is, the firm's value function for effort increases more steeply on higher hierarchy levels.

could be soft skills such as social competence, the capability to lead and motivate people, or to oversee complex production processes. However, workers usually also differ along unobservable dimensions. To take this into account, we now introduce unobserved worker heterogeneity with respect to marginal productivity. We assume that this aspect of ability persists over time and hierarchy levels, i.e., it is not task specific. Because it is not possible to observe a worker's individual output on the first hierarchy level, the firm cannot deduce marginal productivity at the end of the first period.

In the previous section, we have seen that separate contracts are superior when selecting the worker with the higher observable ability for the managerial task is sufficiently important. By contrast, in this section we will show that, with unobserved heterogeneity, combined contracts exhibit a new comparative advantage: By tying promotion to first-level performance, the firm increases the chances of assigning the worker with the higher unobservable talent to the management job.

4.1 Modifications of the Basic Model

We assume that each worker has either high unobservable talent t_1 or low unobservable talent t_0 with $t_1 > t_0 > 0$. Neither the workers nor the firm observe the workers' individual talents during the whole game. In other words, we introduce symmetric uncertainty about the quality of the workers.[20] All players (i.e., the workers and the firm) have the same prior distribution about worker talent. For simplicity, let each talent be equally likely so that unobservable talent can be described by a random variable t that takes values t_0 and t_1 with probability $\frac{1}{2}$, respectively, and has mean $E[t] = (t_0 + t_1)/2$.

On each hierarchy level, a worker's talent influences both the value of effort for the firm and the probability of generating a favorable signal. Let the value of worker i $(i = A, B)$ to the firm when exerting effort $\hat{e}_i$ on level 1 be $t \cdot \hat{v}(\hat{e}_i)$, and that on level 2 when choosing effort e_i be $t \cdot v(e_i)$. In analogy, the probability of a favorable signal on level 2 is now given by $t \cdot p(e)$, with

[20] The assumption of symmetric talent uncertainty is widespread in labor economics. See, among many others, Harris and Holmström (1982), Murphy (1986), Holmström (1999) and Gibbons and Waldman (1999).

$t_1 \cdot p(e) \leq 1$ for all e. For a relative performance signal on level 1 we have to differentiate four possible situations. If both workers have equal talents, A's probability of winning the tournament will again be described by the function $\hat{p}(\hat{e}_A, \hat{e}_B)$. In addition, now we also have two possible asymmetric pairings. If worker A has high talent t_1 and worker B low talent t_0, A's probability of getting the better evaluation will be described by $\hat{p}(\hat{e}_A, \hat{e}_B; t_1)$ whereas B's one is given by $1 - \hat{p}(\hat{e}_A, \hat{e}_B; t_1)$. In the opposite asymmetric case with B being more talented than A, worker A wins the tournament with probability $\hat{p}(\hat{e}_A, \hat{e}_B; t_0)$ and B with probability $1 - \hat{p}(\hat{e}_A, \hat{e}_B; t_0)$.

We assume that the new probability functions have analogous properties (i)–(iii) as the function $\hat{p}(\cdot, \cdot)$ (see Section 2). For example, in the basic model we have $\hat{p}_1(\hat{e}_j, \hat{e}_i) = -\hat{p}_2(\hat{e}_i, \hat{e}_j)$, which follows from the symmetry assumption (i). In analogy, we assume that also in heterogeneous pairings the specific identity of a certain worker does not have any influence on his (marginal) winning probability, that is whether a worker acts on the first or on the second position in $\hat{p}(\cdot, \cdot; t)$ does not influence the (marginal) returns of his effort choice for a given asymmetric pairing. Technically, this means that $\hat{p}(\hat{e}_i, \hat{e}_j; t_1) = 1 - \hat{p}(\hat{e}_j, \hat{e}_i; t_0)$, implying

$$\hat{p}_1(\hat{e}_i, \hat{e}_j; t_1) = -\hat{p}_2(\hat{e}_j, \hat{e}_i; t_0) \quad \text{and} \quad \hat{p}_2(\hat{e}_i, \hat{e}_j; t_1) = -\hat{p}_1(\hat{e}_j, \hat{e}_i; t_0) \tag{31}$$

for $i, j = A, B$; $i \neq j$. Of course, talent should have an impact on a worker's absolute winning probability and his marginal one. In particular, we assume that, for given effort levels, the more talented worker has a higher winning probability than the less talented one, i.e.,

$$\hat{p}(\hat{e}_i, \hat{e}_j; t_1) > \hat{p}(\hat{e}_i, \hat{e}_j; t_0). \tag{32}$$

Furthermore, let effort and talent be complements in the sense of

$$\hat{p}_1(\hat{e}_i, \hat{e}_j; t_1) > \hat{p}_1(\hat{e}_i, \hat{e}_j; t_0) \quad \text{and} \quad -\hat{p}_2(\hat{e}_i, \hat{e}_j; t_0) > -\hat{p}_2(\hat{e}_i, \hat{e}_j; t_1), \tag{33}$$

that is marginally increasing effort is more effective under high talent than under low one. Properties (ii) and (iii) from the basic model also hold analogously for heterogeneous workers. Note that property (iii) together with symmetry here implies that $\hat{p}_{12}(\hat{e}, \hat{e}; t_1) = -\hat{p}_{12}(\hat{e}, \hat{e}; t_0)$: If workers choose identical efforts the more able one has a higher winning probability; if now the other worker increases his effort, competition becomes more intense so that the more able worker raises his effort, too. Again, this effect is assumed to be independent of whether a worker acts on the first or on the second position in $\hat{p}(\cdot, \cdot; t)$. Finally, we assume analogous regularity conditions to hold as in the basic model of Section 2.

In the following, we will investigate how the comparison between separate contracts and a combined contract will change when workers are characterized by unobserved heterogeneity.

4.2 Separate Contracts

We first consider the case of separate contracts. The equilibrium on hierarchy level 1 is now described by the first-order conditions

$$(w_H - w_L)\frac{1}{4}\left(\hat{p}_1(\hat{e}_A, \hat{e}_B; t_1) + \hat{p}_1(\hat{e}_A, \hat{e}_B; t_0) + 2\hat{p}_1(\hat{e}_A, \hat{e}_B)\right) = \hat{c}'(\hat{e}_A),$$
$$(w_H - w_L)\frac{1}{4}\left(-\hat{p}_2(\hat{e}_A, \hat{e}_B; t_1) - \hat{p}_2(\hat{e}_A, \hat{e}_B; t_0) - 2\hat{p}_2(\hat{e}_A, \hat{e}_B)\right) = \hat{c}'(\hat{e}_B).$$

Using $\hat{p}_1(\hat{e}_B, \hat{e}_A) = -\hat{p}_2(\hat{e}_A, \hat{e}_B)$ and (31) shows that there exists a symmetric equilibrium in which each worker chooses $\hat{e}$ characterized by

$$w_H - w_L = \Delta\tilde{w}(\hat{e}) \tag{34}$$
$$\text{with} \quad \Delta\tilde{w}(\hat{e}) := \frac{4\hat{c}'(\hat{e})}{\hat{p}_1(\hat{e}, \hat{e}; t_1) + \hat{p}_1(\hat{e}, \hat{e}; t_0) + 2\hat{p}_1(\hat{e}, \hat{e})} \tag{35}$$

and $\Delta\tilde{w}'(\hat{e}) > 0$.[21] The firm maximizes $2E[t]\,\hat{v}(\hat{e}) - w_L - w_H$ subject to the participation constraint (11),[22] the limited-liability constraints (12) and the incentive constraint (34). The optimal tournament prizes are, therefore,

[21] Note that $\frac{\partial}{\partial \hat{e}}\left(\hat{p}_1(\hat{e}, \hat{e}; t_1) + \hat{p}_1(\hat{e}, \hat{e}; t_0) + 2\hat{p}_1(\hat{e}, \hat{e})\right) = \hat{p}_{11}(\hat{e}, \hat{e}; t_1) + \hat{p}_{12}(\hat{e}, \hat{e}; t_1) + \hat{p}_{11}(\hat{e}, \hat{e}; t_0) + \hat{p}_{12}(\hat{e}, \hat{e}; t_0) + 2\hat{p}_{11}(\hat{e}, \hat{e}) + 2\hat{p}_{12}(\hat{e}, \hat{e}) < 0$.

[22] Note that, due to the symmetric equilibrium, the participation constraint will be the same as in the basic model.

given by $w_L^s = 0$ and $w_H^s = \Delta\tilde{w}(\hat{e})$, and the firm implements the effort level[23] $\hat{e}_h^s$ that solves

$$\max_{\hat{e}} 2E[t]\,\hat{v}(\hat{e}) - \Delta\tilde{w}(\hat{e}). \tag{36}$$

On hierarchy level 2, the firm's optimization problem now reads as

$$\begin{aligned} &\max_{b_L, b_H, e} \{E[t]\,v(e) + \max\{\mu_A, \mu_B, \bar{\mu}\} - b_L - E[t]\,p(e)(b_H - b_L)\} \\ &\text{subject to } \; e = \arg\max_z \{b_L + E[t]\,p(z)(b_H - b_L) - c(z)\} \\ &\qquad b_L + E[t]\,p(e)(b_H - b_L) - c(e) \geq 0 \\ &\qquad b_L, b_H \geq 0. \end{aligned}$$

In analogy to the basic model, the incentive constraint can be replaced with the first-order condition $b_H - b_L = \frac{c'(e)}{E[t]p'(e)}$. It is straightforward to show that, under the optimal bonus contract, we have $b_L^s = 0$. Furthermore, the participation constraint is identical to (5) and the firm thus implements effort e_h^s with

$$e_h^s = \arg\max_e \{E[t]\,v(e) + \max\{\mu_A, \mu_B, \bar{\mu}\} - r(e) - c(e)\} \tag{37}$$

and $r(e)$ being defined in (6). Altogether, the comparison of (36) and (37) with (14) and (15) from the basic model shows that introducing unobserved heterogeneity leads to changes in the expected values of the workers' effort choices and in the optimal winner prize w_H^*, but leaves the implementation costs on level 2 unchanged for a given effort level e.

4.3 Combined Contract

Now we turn to the analysis of the combined contract. Solving the game by backwards induction, we first consider the actions on hierarchy level 2. Here, all players update their beliefs about the unknown talent of the promoted worker. Let $E[t|\hat{s}]$ denote the expected talent of the promoted worker, that is each player calculates a new expectation depending on the realization of

[23] Here and in the following, the subscript "h" for optimal efforts indicates heterogeneity of workers.

the relative performance signal $\hat{s}$. Note that at any prior point in time the workers as well as the firm already know that they have to update their beliefs in light of the promotion decision and that they will not receive further information. Hence, when designing the optimal combined contract, the firm has to include the incentive constraint

$$b_H - b_L = \frac{c'(e)}{E[t|\hat{s}]\, p'(e)} \tag{38}$$

and the participation constraint

$$b_L + E[t|\hat{s}]\, p(e)(b_H - b_L) - c(e) \geq 0 \Leftrightarrow b_L + r(e) \geq 0, \tag{39}$$

where the last inequality follows from (6) and (38).

At level 1, worker A and worker B maximize

$$\begin{aligned} & w_L + (w_H - w_L + b_L + E[t|\hat{s}]\, p(e)(b_H - b_L) - c(e)) \\ & \times \frac{1}{4}\left(\hat{p}(\hat{e}_A, \hat{e}_B; t_1) + \hat{p}(\hat{e}_A, \hat{e}_B; t_0) + 2\hat{p}(\hat{e}_A, \hat{e}_B)\right) - \hat{c}(\hat{e}_A) \qquad \text{and} \\ & w_L + (w_H - w_L + b_L + E[t|\hat{s}]\, p(e)(b_H - b_L) - c(e)) \\ & \times \frac{1}{4}\left((1 - \hat{p}(\hat{e}_A, \hat{e}_B; t_1)) + (1 - \hat{p}(\hat{e}_A, \hat{e}_B; t_0)) + 2(1 - \hat{p}(\hat{e}_A, \hat{e}_B))\right) - \hat{c}(\hat{e}_B), \end{aligned}$$

respectively. Equations (38) and (6) together with the first-order conditions, $\hat{p}_1(\hat{e}_B, \hat{e}_A) = -\hat{p}_2(\hat{e}_A, \hat{e}_B)$ and (31) yield

$$\begin{aligned} (w_H - w_L + b_L + r(e)) \frac{\hat{p}_1(\hat{e}_A, \hat{e}_B; t_1) + \hat{p}_1(\hat{e}_A, \hat{e}_B; t_0) + 2\hat{p}_1(\hat{e}_A, \hat{e}_B)}{4} &= \hat{c}'(\hat{e}_A) \\ (w_H - w_L + b_L + r(e)) \frac{\hat{p}_1(\hat{e}_B, \hat{e}_A; t_0) + \hat{p}_1(\hat{e}_B, \hat{e}_A; t_1) + 2\hat{p}(\hat{e}_B, \hat{e}_A)}{4} &= \hat{c}'(\hat{e}_B). \end{aligned}$$

Thus, in the symmetric equilibrium each worker exerts $\hat{e}$ described by

$$w_H - w_L + b_L + r(e) = \Delta\tilde{w}(\hat{e}) \tag{40}$$

with $\Delta\tilde{w}(\hat{e})$ being defined in (35).

Now we can state the firm's problem. It maximizes[24]

$$
\begin{aligned}
&2E[t]\,\hat{v}(\hat{e}) - 2w_L - (w_H - w_L) + E[t|\hat{s}]\,v(e) - b_L - E[t|\hat{s}]\,p(e)\,(b_H - b_L) \\
&\overset{(6),(38),(40)}{=} 2E[t]\,\hat{v}(\hat{e}) - \Delta\tilde{w}(\hat{e}) + E[t|\hat{s}]\,v(e) - 2w_L - c(e)
\end{aligned}
$$

subject to the limited-liability constraints (23), the incentive compatibility constraints (38) and (40), the participation constraint for the second hierarchy level (39) and the participation constraint for the first level,

$$
\begin{aligned}
&w_L + \frac{1}{2}\left(w_H - w_L + b_L + E[t|\hat{s}]\,p(e)\,(b_H - b_L) - c(e)\right) - \hat{c}(\hat{e}) \geq 0 \\
&\overset{(6),(38),(40)}{\Leftrightarrow} w_L + \frac{1}{2}\Delta\tilde{w}(\hat{e}) - \hat{c}(\hat{e}) \geq 0.
\end{aligned}
$$

Moreover, the firm has to take into account that $E[t|\hat{s}]$ depends on the workers' equilibrium efforts chosen on hierarchy level 1:

$$
\begin{aligned}
E[t|\hat{s}] &= \frac{1}{4}t_1 + \frac{1}{4}t_0 + \frac{1}{4}\left(\hat{p}(\hat{e},\hat{e};t_1)\,t_1 + (1 - \hat{p}(\hat{e},\hat{e};t_1))\,t_0\right) \\
&\quad + \frac{1}{4}\left(\hat{p}(\hat{e},\hat{e};t_0)\,t_0 + (1 - \hat{p}(\hat{e},\hat{e};t_0))\,t_1\right) \\
&= E[t] + \frac{\Delta t\left(\hat{p}(\hat{e},\hat{e};t_1) - \hat{p}(\hat{e},\hat{e};t_0)\right)}{4} \overset{(32)}{>} E[t] \qquad (41)
\end{aligned}
$$

with $\Delta t := t_1 - t_0$. Thus, the posterior expectation is larger than the prior one because the more talented worker is promoted with higher probability in case of an asymmetric pairing in the tournament. Furthermore, the posterior mean strictly increases in level-1 equilibrium efforts as talent and effort are complements:

$$
\begin{aligned}
\frac{\partial E[t|\hat{s}]}{\partial \hat{e}} &= \frac{\Delta t}{4}\left(\hat{p}_1(\hat{e},\hat{e};t_1) + \hat{p}_2(\hat{e},\hat{e};t_1) - \hat{p}_1(\hat{e},\hat{e};t_0) - \hat{p}_2(\hat{e},\hat{e};t_0)\right) \\
&\overset{(31)}{=} \frac{\Delta t}{2}\left(\hat{p}_1(\hat{e},\hat{e};t_1) - \hat{p}_1(\hat{e},\hat{e};t_0)\right) \overset{(33)}{>} 0. \qquad (42)
\end{aligned}
$$

Applying the same two-step procedure as in the basic model yields that the

[24] For simplicity, we drop the constant $\bar{\mu}$ in the firm's objective function.

firm implements the effort pair $(\hat{e}_h^c, e_h^c)$ with[25]

$$(\hat{e}_h^c, e_h^c) \in \arg\max_{\hat{e},e} \{2E[t]\,\hat{v}(\hat{e}) + E[t|\hat{s}]\,v(e) - \Delta\tilde{w}(\hat{e}) - c(e)\} \quad (43)$$

$$\text{subject to } \Delta\tilde{w}(\hat{e}) - r(e) \geq 0. \quad (44)$$

When comparing optimal efforts under the combined contract with those under two separate contracts, we have to distinguish whether the restriction (44) is binding or not at the optimum. In case of a *non-binding restriction*, optimal efforts $(\hat{e}_h^c, e_h^c)$ are described by the first-order conditions

$$2E[t]\,\hat{v}'(\hat{e}) + \frac{\partial E[t|\hat{s}]}{\partial \hat{e}} v(e) = \Delta\tilde{w}'(\hat{e}) \quad \text{and} \quad E[t|\hat{s}]\,v'(e) = c'(e). \quad (45)$$

Comparing the first equation with (36) clearly shows that $\hat{e}_h^c > \hat{e}_h^s$ as $\partial E[t|\hat{s}] / \partial \hat{e} > 0$. The comparison of the second equation with (37) points out that $e_h^c > e_h^s$, due to Lemma 1 and the fact that $E[t|\hat{s}] > E[t]$. Now, we have to consider the case of a *binding restriction* (44). Using this restriction, we can express level-2 effort as a function of level-1 effort, $e(\hat{e})$, with $\frac{\partial e}{\partial \hat{e}} = \frac{\Delta\tilde{w}'(\hat{e})}{r'(e)} > 0$. The firm's objective function under a combined contract can be rewritten as

$$2E[t]\,\hat{v}(\hat{e}) + E[t|\hat{s}]\,v(e(\hat{e})) - \Delta\tilde{w}(\hat{e}) - c(e(\hat{e})).$$

The first-order condition yields

$$2E[t]\,\hat{v}'(\hat{e}) + \frac{\partial E[t|\hat{s}]}{\partial \hat{e}} v(e(\hat{e})) - \Delta\tilde{w}'(\hat{e}) + [E[t|\hat{s}]\,v'(e(\hat{e})) - c'(e(\hat{e}))]\frac{\partial e}{\partial \hat{e}} = 0.$$

Inserting for $\partial e/\partial \hat{e}$ leads to

$$2E[t]\,\hat{v}'(\hat{e}) + \frac{\partial E[t|\hat{s}]}{\partial \hat{e}} v(e(\hat{e})) + \frac{E[t|\hat{s}]\,v'(e(\hat{e})) - c'(e(\hat{e})) - r'(e(\hat{e}))}{r'(e(\hat{e}))}\Delta\tilde{w}'(\hat{e}) = 0.$$

Since the first two expressions as well as $r'(e(\hat{e}))$ and $\Delta\tilde{w}'(\hat{e})$ are positive, the numerator of the last expression is negative. As this numerator is a strictly concave function of $e(\hat{e})$ and since $E[t|\hat{s}] > E[t]$, we obtain from the

[25] See the Appendix.

comparison with (37) that $e_h^c > e_h^s$.

Finally, we have to consider optimal effort implementation on hierarchy level 1. Since (44) is binding, the effort $\hat{e}$ that would maximize level 1-profit corresponds to a level-2 effort that is below the effort e that maximizes level-2 profit $E[t|\hat{s}]\,v(e) - c(e)$. Hence, the firm may be interested in further raising $\hat{e}$. As both profit functions are strictly concave, we can apply the same argument as in the proof of Proposition 4: The firm would, thus, never implement a smaller $\hat{e}$ than the optimal effort under a non-binding restriction. Since that effort was larger than the optimal level-1 effort under separate contracts, we have proved that $\hat{e}_h^c > \hat{e}_h^s$ also holds under a binding restriction.

Proposition 5 *Irrespective of whether restriction (44) is binding or not at the optimum, we have* $\hat{e}_h^c > \hat{e}_h^s$ *and* $e_h^c > e_h^s$.

Proposition 5 points out that, under a combined contract, the firm implements strictly larger efforts on hierarchy level 1 than under separate contracts. This result sharply contrasts with our findings in Proposition 4, where workers differ only with respect to observable characteristics for the level-2 task. The intuition comes from the fact that, in case of unobservable talent, the firm has an additional motive for implementing large efforts on hierarchy level 1: The larger $\hat{e}$ the higher will be the probability that the worker of higher unobserved talent is promoted to level 2 in case of a heterogeneous pairing, i.e., $\hat{p}_1(\hat{e}, \hat{e}; t_1) > 0$. This, in turn, increases the posterior expected talent of the promoted worker: $\partial E[t|\hat{s}] / \partial \hat{e} > 0$ according to (42) since $E[t|\hat{s}]$ monotonically increases in $\hat{p}(\hat{e}, \hat{e}; t_1)$. In other words, if workers have unobservable characteristics that persist across hierarchy levels, higher incentives on level 1 improve worker selection for level 2. The reason is that incentives and selection are strictly interlinked.

Again, from a pure incentive perspective, the firm is strictly better off by choosing a combined contract. Analogously to the basic model, the combined contract will lead to first-best effort on hierarchy level 2, i.e. $e_h^c = \arg\max_e \{E[t|\hat{s}]\,v(e) - c(e)\}$, if restriction (44) is not binding. However, there is a crucial difference in comparison to the basic model. With unobserved talents, we have the additional effect that combining both hierarchy levels via a job-promotion scheme even improves on first-best implementation under uncertainty as $E[t|\hat{s}] > E[t]$. By inducing large efforts $\hat{e}$ on level 1, the firm raises the posterior expected talent of the promoted worker (i.e. $\partial E[t|\hat{s}]/\partial\hat{e} > 0$) which, in turn, increases the *efficient* effort level e_h^c on level 2 that maximizes $E[t|\hat{s}]\,v(e) - c(e)$.

Finally, we want to compare the selection properties of the different contractual forms with respect to unobserved worker heterogeneity. To do so, we assume that there are no task-specific differences in ability, i.e., μ is deterministic. Only if workers are promoted internally, selection can be improved by appropriate contract design. Therefore, we focus on internal recruitment for level 2. Comparing the combined contract with separate contracts for a heterogeneous match of workers, we see that the probability of promoting the more talented worker is strictly larger under the former. The reason is that, due to random selection, under separate contracts the probability of promoting the better worker is 1/2. By contrast, under the combined contract, we have $\hat{p}(\hat{e}_h^c, \hat{e}_h^c; t_1) > 1/2$ due to $\hat{p}(\hat{e}_i, \hat{e}_j; t_1) = 1 - \hat{p}(\hat{e}_j, \hat{e}_i; t_0)$ and (32).

Furthermore, it is interesting to contrast our combined contract with a standard job-promotion tournament where wages are attached to jobs, i.e., tournament prizes are fixed rather than determined by the incentive scheme for the next level. Note that, in our model, the separate contract for hierarchy level 1 corresponds to a standard promotion scheme: The relative performance pay w_H can also be interpreted as a fixed wage attached to the next hierarchy level. Since level-1 effort is higher under the combined contract, $\hat{e}_h^c > \hat{e}_h^s$, we obtain the following result.

Corollary 1 *Combining job-promotion with incentive pay on the next hierarchy level always improves the selection quality of a job-promotion tournament.*

Proof. $\hat{p}(\hat{e}_h^c, \hat{e}_h^c; t_1) > \hat{p}(\hat{e}_h^s, \hat{e}_h^s; t_1)$ since $\frac{\partial}{\partial \hat{e}}\hat{p}(\hat{e}, \hat{e}; t_1) = \hat{p}_1(\hat{e}, \hat{e}; t_1) + \hat{p}_2(\hat{e}, \hat{e}; t_1) \overset{(31)}{=} \hat{p}_1(\hat{e}, \hat{e}; t_1) - \hat{p}_1(\hat{e}, \hat{e}; t_0) \overset{(33)}{>} 0$. ■

In the introduction and at the end of Section 3, we mentioned empirical puzzles that contradict standard tournament theory but can be explained in our model. One of these puzzles was that wages are not attached to jobs and, therefore, to hierarchy levels. As has been shown in this section, the selection quality of standard job-promotion tournaments can be significantly improved by replacing wages that are attached to jobs with incentive pay such as a bonus scheme. Hence, missing wages-attached-to-jobs in the empirical literature on firms' wage policies can be nicely explained by the existence of unobserved worker heterogeneity.

5 Conclusion

We analyzed a two-tier hierarchy where workers compete in a rank-order tournament on level 1. On the second tier, a worker is hired from outside or promoted from the first tier to carry out a managerial task that leads to an individual performance signal. Workers are protected by limited liability on either hierarchy level. We have shown that combining a job-promotion tournament on level 1 with bonus payments on level 2 has two advantages: First, rents from level 2 can be used to create incentives for level 1. As a consequence, the firm may even implement first-best effort on the second hierarchy level although the worker earns a strictly positive rent on this level. Second, in case of unobserved heterogeneity, a complementary bonus scheme has the additional advantage of improving the tournament's selection quality in promoting the most talented internal worker.

Combining a tournament with a bonus scheme might lead to further advantages if there is the possibility of sabotage among heterogeneous workers. For example, Münster (2007) shows that more able workers may be deterred from participating in a tournament if contestants can sabotage each other. Then, the advantage of higher talent is completely erased since more able workers are sabotaged more heavily than less able ones, thereby equalizing the winning probabilities of the heterogeneous workers. However, if the winner prize of the tournament is a bonus contract that entails higher rents for more able workers, the problem of adverse participation may be mitigated.

In a different setting, the combination of a tournament with a bonus scheme may be useful to make the competition between heterogeneous contestants more even. As is known from the tournament literature, the more uneven competition the less effort will be chosen in equilibrium. Suppose that unobserved talent and effort are substitutes on each hierarchy level and not complements as in our paper. Then, workers' rents from a bonus contract on the second hierarchy level may be decreasing in ability. As a result, introducing a bonus scheme would mitigate the problem of uneven competition on level 1. The reason is that more able workers have lower expected rents from winning the tournament than less able ones. If the firm cannot use handicaps (e.g., due to only ordinal information) to counterbalance ability differences, such decreasing rents would be an appropriate instrument for regulating competition.

6 Appendix

6.1 Proof of Proposition 2

We can solve problem (21)-(23) in two steps: *First*, we derive the firm's minimum cost for inducing a given pair of effort levels $(\hat{e}, e)$. *Then*, we use the optimal cost function to solve the profit maximization problem and determine the optimal effort pair $(\hat{e}^c, e^c)$. The cost minimization problem for a given effort pair $(\hat{e}, e)$ reads as

$$\min_{w_L, w_H, b_L, b_H} \quad 2w_L + (w_H - w_L) + b_L + p(e)(b_H - b_L)$$
$$\text{subject to (1), (2), (19), (20), } w_L, w_H, b_L, b_H \geq 0.$$

By the incentive compatibility constraint (1), $b_H - b_L = \frac{c'(e)}{p'(e)}$. Thus, in combination with the incentive compatibility constraint (19), we obtain

$$w_H - w_L = \frac{\hat{c}'(\hat{e})}{\hat{p}_1(\hat{e}, \hat{e})} - b_L - p(e)\frac{c'(e)}{p'(e)} + c(e) = \Delta w(\hat{e}) - b_L - r(e), \quad (46)$$

where $\Delta w(\hat{e})$ is given by (10) and $r(e)$ by (6).[26]

Using (46), the first-level participation constraint (20) boils down to

$$w_L + \frac{1}{2}\Delta w(\hat{e}) - \hat{c}(\hat{e}) \geq 0. \quad (47)$$

Furthermore, the second-level participation constraint (2) becomes

$$b_L + p(e)\frac{c'(e)}{p'(e)} - c(e) = b_L + r(e) \geq 0. \quad (48)$$

Thus, substituting for the tournament prize spread $w_H - w_L$ and the bonus

[26] Recall that $\Delta w(\hat{e})$ is the prize spread necessary to induce $\hat{e}$ under *separate* contracts. However, note that $\Delta w(\hat{e})$ will usually be different from $w_H^c - w_L^c$.

spread $b_H - b_L$, the cost minimization problem can be simplified to[27]

$$\min_{w_L, b_L} 2w_L + \Delta w(\hat{e}) + c(e) \quad \text{subject to (47), (48) and}$$
$$\Delta w(\hat{e}) - b_L - r(e) + w_L,\ w_L,\ b_L \geq 0. \tag{49}$$

By Lemma 1, we obtain $b_L^c = 0$ for the optimal low bonus: This satisfies the participation constraint for the second hierarchy level (48) and is also best for ensuring that $w_H = \Delta w(\hat{e}) - b_L - r(e) + w_L \geq 0$. Hence, we can skip constraint (48) and obtain

$$\min_{w_L} 2w_L + \Delta w(\hat{e}) + c(e) \quad \text{subject to (47) and}$$
$$\Delta w(\hat{e}) - r(e) + w_L,\ w_L \geq 0.$$

The cost-minimizing w_L is given by

$$w_L = \max\left\{0,\ \hat{c}(\hat{e}) - \frac{1}{2}\Delta w(\hat{e}),\ r(e) - \Delta w(\hat{e})\right\}.$$

From (13), we know that $\frac{1}{2}\Delta w(\hat{e}) - \hat{c}(\hat{e}) \geq 0$. Therefore,

$$w_L = \max\{0,\ r(e) - \Delta w(\hat{e})\}.$$

We now have to distinguish two cases. The first case is

$$w_H - w_L = \Delta w(\hat{e}) - r(e) \geq 0.$$

Then, $w_L = 0$ and $w_H = \Delta w(\hat{e}) - r(e)$. In the second case,

$$w_H - w_L = \Delta w(\hat{e}) - r(e) < 0.$$

Hence, $w_L = r(e) - \Delta w(\hat{e})$ and $w_H = 0$. In the first case, the firm's expected labor costs are

$$2w_L + \Delta w(\hat{e}) + c(e) = \Delta w(\hat{e}) + c(e),$$

[27]Note that the optimal high bonus, $b_H = \frac{c'(e)}{p'(e)} + b_L$, is non-negative due to $b_L \geq 0$.

and in the second scenario the firm's costs amount to

$$2w_L + \Delta w\,(\hat{e}) + c(e) = 2r(e) - \Delta w\,(\hat{e}) + c(e).$$

We can now turn to the second step of the solution procedure, the solution of the firm's profit maximization problem. The optimal effort pair $(\hat{e}^c, e^c)$ solves

$$\max_{e,\hat{e}} \begin{cases} 2\hat{v}(\hat{e}) + v(e) + \bar{\mu} - \Delta w\,(\hat{e}) - c(e) & \text{if} \quad \Delta w\,(\hat{e}) - r(e) \geq 0 \\ 2\hat{v}(\hat{e}) + v\,(e) + \bar{\mu} - [2r(e) - \Delta w\,(\hat{e}) + c(e)] & \text{otherwise.} \end{cases}$$

We can see that in case 2 (i.e., the second line of the maximization problem) the firm's objective function is monotonically increasing in $\hat{e}$. Hence, for each e, the firm chooses the maximum possible $\hat{e}$, which makes the given restriction just binding, i.e., $\Delta w\,(\hat{e}) = r(e)$. This implies that case 2 becomes a special case of case 1. Thus, the firm never wants to induce effort levels $(\hat{e}, e)$ such that $\Delta w\,(\hat{e}) < r(e)$. Doing so would imply that $0 = w_H^c < w_L^c$. Intuitively, this means that, by implementing an adverse relative performance scheme, the firm pays for reducing first-level incentives that stem from the second-level rent $r(e)$. Such a contract cannot be optimal. The firm would be better off by setting $0 = w_H^c = w_L^c$, thereby increasing first-level effort and reducing workers' first-period rents.

Hence, we are always in the first case. Consequently, $w_L^c = 0$ and the results of the proposition follow.

6.2 Proof of Proposition 4

(i) $\hat{e}^c = \hat{e}^s$ immediately follows from examining the objective functions (14) and (24). $e^c > e^s$ follows from $r'(e) > 0$, which we have proven in Lemma 1, and $r''(e) > 0$, which follows from our regularity assumptions and is straightforward to check.[28]

It remains to prove result (ii). Due to the binding restriction, we can

[28]See the additional pages for the referees.

consider e as an implicitly defined function of e, i.e., $e(\hat{e})$ with

$$\frac{\partial e}{\partial \hat{e}} = \frac{\Delta w'(\hat{e})}{r'(e)} > 0.$$

Moreover, the firm's objective function (24) becomes

$$2\hat{v}(\hat{e}) + v(e(\hat{e})) + \bar{\mu} - \Delta w(\hat{e}) - c(e(\hat{e})).$$

The respective first-order condition is

$$2\hat{v}'(\hat{e}) - \Delta w'(\hat{e}) + \left[v'(e(\hat{e})) - c'(e(\hat{e}))\right] \frac{\partial e}{\partial \hat{e}} = 0. \tag{50}$$

Hence, compared to the case where the restriction is non-binding, we either have higher effort at hierarchy level 1 and lower effort at level 2, or vice versa. Inserting $\partial e/\partial \hat{e}$ in (50) yields

$$2\hat{v}'(\hat{e}) + \frac{v'(e(\hat{e})) - c'(e(\hat{e})) - r'(e(\hat{e}))}{r'(e(\hat{e}))} \Delta w'(\hat{e}) = 0.$$

Recall that $\Delta w'(\hat{e}) > 0$ and $r'(e) > 0$. The optimal effort, e^c, must therefore satisfy $v'(e^c) - c'(e^c) - r'(e^c) < 0$. Under separate contracts, we have $v'(e^s) - c'(e^s) - r'(e^s) = 0$. Thus, since $v(e) - c(e) - r(e)$ is strictly concave, it follows that $e^c > e^s$.

Now consider the effort choice on hierarchy level 1 under a binding restriction (25). Suppose that the firm wants to implement the same effort level as under a non-binding restriction, i.e., $\hat{e}^s = \arg\max_{\hat{e}} \{2\hat{v}(\hat{e}) - \Delta w(\hat{e})\}$. However, since (25) is binding in this situation, the corresponding level-2 effort is below the optimal one, e^{FB}. Of course, the firm can raise e to increase $v(e) - c(e)$, but then it has to increase $\hat{e}$ as well because of $\partial e/\partial \hat{e} > 0$. Whether such an adjustment is beneficial to the firm or not depends on the functional forms. In any case, since both functions $2\hat{v}(\hat{e}) - \Delta w(\hat{e})$ and $v(e) - c(e)$ are strictly concave, the firm will never raise e above e^{FB}. This is because, if $e > e^{FB}$ and $\hat{e} > \hat{e}^s$, the firm can increase profits by decreasing both effort levels, while keeping (25) binding. This proves $e^c < e^{FB}$.

Since $e^c < e^{FB}$ implies $v'(e^c) - c'(e^c) > 0$, from (50) we obtain that the corresponding optimal effort on hierarchy level 1 must satisfy $2\hat{v}'(\hat{e}) - \Delta w'(\hat{e}) < 0$. Thus, this effort must be larger than the optimal level-1 effort under a non-binding restriction (25). Since that effort was identical with the optimal level-1 effort under separate contracts, $\hat{e}^s$, we have $\hat{e}^c > \hat{e}^s$ under the binding restriction.

6.3 Combined Contract with Heterogeneous Workers

Step 1: Minimizing costs
Since $b_H \geq 0$ is ensured by the incentive constraint for hierarchy level 2 in combination with $b_L \geq 0$ the problem of minimizing implementation costs reduces to

$$\begin{gathered}\min_{w_L, w_H, b_L} \Delta\tilde{w}(\hat{e}) + 2w_L + c(e) \\ \text{subject to } \; b_L + r(e) \geq 0 \\ w_L + \frac{1}{2}\Delta\tilde{w}(\hat{e}) - \hat{c}(\hat{e}) \geq 0 \\ w_H - w_L + b_L + r(e) = \Delta\tilde{w}(\hat{e}) \\ w_H, w_L, b_L \geq 0.\end{gathered}$$

Replacing w_H yields:

$$\begin{gathered}\min_{w_L, b_L} \Delta\tilde{w}(\hat{e}) + 2w_L + c(e) \\ \text{subject to } \; b_L + r(e) \geq 0 \\ w_L + \frac{1}{2}\Delta\tilde{w}(\hat{e}) - \hat{c}(\hat{e}) \geq 0 \\ \Delta\tilde{w}(\hat{e}) - b_L - r(e) + w_L, \; w_L, \; b_L \geq 0.\end{gathered}$$

From Lemma 1 we know that $r(e) \geq 0$ so that $b_L^c = 0$ and the minimization problem further reduces to

$$\min_{w_L} \Delta\tilde{w}(\hat{e}) + 2w_L + c(e)$$
$$\text{s.t.} \quad w_L + \frac{1}{2}\Delta\tilde{w}(\hat{e}) - \hat{c}(\hat{e}) \geq 0$$
$$\Delta\tilde{w}(\hat{e}) - r(e) + w_L, \; w_L \geq 0.$$

Hence,

$$w_L = \max\left\{0, \; \hat{c}(\hat{e}) - \frac{1}{2}\Delta\tilde{w}(\hat{e}), \; r(e) - \Delta\tilde{w}(\hat{e})\right\}.$$

We know that $\frac{1}{2}\Delta\tilde{w}(\hat{e}) - \hat{c}(\hat{e}) \geq 0$; otherwise, $\hat{e}$ would not be an equilibrium strategy. Thus,

$$w_L = \max\{0, \; r(e) - \Delta\tilde{w}(\hat{e})\}.$$

We have to distinguish two cases. First, $w_H - w_L = \Delta\tilde{w}(\hat{e}) - r(e) \geq 0$. Then,

$$w_L = 0 \quad \text{and} \quad w_H = \Delta\tilde{w}(\hat{e}) - r(e).$$

Second, $w_H - w_L = \Delta\tilde{w}(\hat{e}) - r(e) < 0$. Then,

$$w_L = r(e) - \Delta\tilde{w}(\hat{e}) \quad \text{and} \quad w_H = 0.$$

In the first case, the firm's expected labor costs are

$$\Delta\tilde{w}(\hat{e}) + 2w_L + c(e) = \Delta\tilde{w}(\hat{e}) + c(e)$$

and in the second they amount to

$$\Delta\tilde{w}(\hat{e}) + 2w_L + c(e) = 2r(e) - \Delta\tilde{w}(\hat{e}) + c(e).$$

Step 2: Maximizing expected profits
Therefore, the optimal effort pair $(\hat{e}^c, e^c)$ solves

$$\max_{\hat{e},e} \begin{cases} 2E[t]\,\hat{v}(\hat{e}) + E[t|\hat{s}]\,v(e) - \Delta\tilde{w}(\hat{e}) - c(e) & \text{if } \Delta\tilde{w}(\hat{e}) - r(e) \geq 0 \\ 2E[t]\,\hat{v}(\hat{e}) + E[t|\hat{s}]\,v(e) - 2r(e) + \Delta\tilde{w}(\hat{e}) - c(e) & \text{otherwise.} \end{cases}$$

In analogy to the basic model, again the firm's objective function in the second line is monotonically increasing in $\hat{e}$ (recall that $\partial E[t|\hat{s}]/\partial\hat{e} > 0$ according to (42)). Hence, for each e the firm chooses the maximum possible $\hat{e}$ that makes the given restriction just bind so that the second line becomes a special case of the problem in line 1. The firm chooses $w_L^c = 0$ and implements the effort pair $(\hat{e}_h^c, e_h^c)$ with

$$(\hat{e}_h^c, e_h^c) \in \arg\max_{\hat{e},e} \{2E[t]\,\hat{v}(\hat{e}) + E[t|\hat{s}]\,v(e) - \Delta\tilde{w}(\hat{e}) - c(e)\}$$
$$\text{subject to } \Delta\tilde{w}(\hat{e}) - r(e) \geq 0.$$

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7 Appendix for Referees

7.1 Separate Contracts in the Basic Model

Second-order condition for the firm's problem on the first hierarchy level, $\max_{\hat{e}}\ \hat{v}(\hat{e}) - \Delta w(\hat{e})$.

$$\begin{aligned}
\Delta w(\hat{e}) &= \frac{\hat{c}'}{\hat{p}_1} \\
\Delta w'(\hat{e}) &= \frac{\hat{c}''\hat{p}_1 - \frac{\partial \hat{p}_1}{\partial \hat{e}}\hat{c}'}{[\hat{p}_1]^2} = \frac{\hat{c}''}{\hat{p}_1} - \frac{\frac{\partial \hat{p}_1}{\partial \hat{e}}\hat{c}'}{[\hat{p}_1]^2} \\
\Delta w''(\hat{e}) &= \frac{\hat{c}'''\hat{p}_1 - \frac{\partial \hat{p}_1}{\partial \hat{e}}\hat{c}''}{[\hat{p}_1]^2} - \frac{\left[\frac{\partial^2 \hat{p}_1}{\partial \hat{e}^2}\hat{c}' + \frac{\partial \hat{p}_1}{\partial \hat{e}}\hat{c}''\right][\hat{p}_1]^2 - 2\hat{p}_1\frac{\partial \hat{p}_1}{\partial \hat{e}}\frac{\partial \hat{p}_1}{\partial \hat{e}}\hat{c}'}{[\hat{p}_1]^4} > 0
\end{aligned}$$

The last inequality follows since $\hat{c}''' > 0$, $\frac{\partial \hat{p}_1}{\partial \hat{e}} < 0$, $\frac{\partial^2 \hat{p}_1}{\partial \hat{e}^2} \leq 0$.

Second-order condition for the firm's problem on the second hierarchy level, $\max_e v(e) - r(e) - c(e)$.

$$\begin{aligned}
r(e) &= p\frac{c'}{p'} - c \\
r'(e) &= \frac{c''p' - p''c'}{[p']^2} = \frac{c''}{p'} - \frac{p''c'}{[p']^2} \\
r''(e) &= \frac{c'''p' - p''c''}{[p']^2} - \frac{[p'''c' + p''c''][p']^2 - 2p'p''p''c'}{[p']^4} > 0.
\end{aligned}$$

The last inequality follows since $c''' > 0$, $p'' < 0$, $p''' \leq 0$.

7.2 Modified Limited-Liability Constraints

In this subsection, we reconsider the problem (21)–(22) of a combined contract in the basic model where the limited-liability constraints (23) are replaced by $w_L \geq 0$, $w_H + b_L \geq 0$ and $w_H + b_H \geq 0$. We will show that these modifications do not change our results. Again, we start with minimizing the firm's cost for inducing a given pair of effort levels $(\hat{e}, e)$:

$$\min_{w_L, w_H, b_L, b_H} 2w_L + (w_H - w_L) + b_L + p(e)(b_H - b_L)$$
$$\text{subject to (1), (2), (19), (20), } w_L, b_L + w_H, b_H + w_H \geq 0.$$

From the incentive constraint (1) we obtain $b_H - b_L = \frac{c'(e)}{p'(e)}$, which, in combination with the incentive constraint (19), yields

$$w_H - w_L = \frac{\hat{c}'(\hat{e})}{\hat{p}_1(\hat{e}, \hat{e})} - b_L - p(e)\frac{c'(e)}{p'(e)} + c(e) = \Delta w(\hat{e}) - b_L - r(e),$$

where $\Delta w(\hat{e})$ is given by (10) and $r(e)$ by (6). Using this expression, (20) can be rewritten as $w_L + \frac{1}{2}\Delta w(\hat{e}) - \hat{c}(\hat{e}) \geq 0$. Furthermore, (2) becomes $b_L + r(e) \geq 0$. In addition, we have

$$b_L + w_H = b_L + \Delta w(\hat{e}) - b_L - r(e) + w_L = \Delta w(\hat{e}) - r(e) + w_L$$
$$b_H + w_H = b_H + \Delta w(\hat{e}) - b_L - r(e) + w_L = \frac{c'(e)}{p'(e)} + \Delta w(\hat{e}) - r(e) + w_L.$$

Substituting for $w_H - w_L$ and $b_H - b_L$ in the objective function, the cost minimization problem can be summarized as follows:

$$\min_{w_L, b_L} 2w_L + \Delta w(\hat{e}) + c(e)$$
$$\text{subject to } w_L + \frac{1}{2}\Delta w(\hat{e}) - \hat{c}(\hat{e}) \geq 0$$
$$b_L + r(e) \geq 0$$
$$w_L, \Delta w(\hat{e}) - r(e) + w_L, \frac{c'(e)}{p'(e)} + \Delta w(\hat{e}) - r(e) + w_L \geq 0.$$

The last non-negativity constraint is less strong than the second one and can thus be skipped. Since b_L does not appear in the objective function but only in the second-level participation constraint, we can set $b_L = 0$ (or any other $b_L \geq -r(e)$). Moreover, since $\frac{1}{2}\Delta w(\hat{e}) - \hat{c}(\hat{e}) \geq 0$, the first constraint is satisfied whenever $w_L \geq 0$ and can, therefore, also be skipped. Altogether, we obtain the same cost minimization problem as in Subsection 7.1 (Proof of Proposition 2), where we assumed $w_L, w_H, b_L, b_H \geq 0$. The intuition is as follows. If $w_L, b_L + w_H, b_H + w_H \geq 0$, a negative bonus b_L can be used to decrease rents on the second tier. However, all these rents serve as indirect incentives for the first tier. Hence, these rents do not constitute costs for the firm so that it cannot benefit from lowering them. Finally, if the rents are so high that they provide too strong incentives for the first tier, $w_H = 0$ anyway.