

Individual vs. Relative Performance Pay with Envious Workers and Non-verifiable Performance*

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Abstract

In a moral-hazard environment, I compare the profitabilities of a rank-order tournament and independent bonus contracts when a firm employs two envious workers whose individual performances are not verifiable. Whereas the bonus scheme must then be self-enforcing, the tournament is contractible. Yet the former incentive regime outperforms the latter as long as credibility problems are not too severe. This is due the fact that the tournament requires unequal pay across peers with certainty, thereby imposing large inequity premium costs on the firm. For a simple example, I show that the more envious the agents are, the larger is the range of interest rates for which the bonus scheme dominates the tournament.

Keywords: principal-agent, relational contract, inequity aversion, bonus, tournament, prize, team, envy

JEL classification: D63, D82, M52, M54

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1 Introduction

Rank-order tournaments are highly competitive incentive schemes based upon relative performance.¹ They are suitable for mitigating moral hazard problems and for the selection of agents under uncertainty about the agents' talents. In the present paper, I focus on the first issue. Compared to other incentive schemes, an important advantage of tournaments is their contractibility in situations where an agent's performance is only known to the principal.² This is due to the fact that the particular outcome of the tournament has no impact on total wage costs because the principal credibly commits to a fixed prize structure *ex ante*.³ However, pitting workers against each other confronts contestants with the certainty of unequal payoffs between peers. Workers though care for relative payoffs as suggested by empirical evidence.⁴ In particular, they frequently exhibit a distaste for inequitable payoff distributions. The prospect of unequal pay then implies additional agency costs for the firm, the so-called inequity premium. In a tournament, these costs cannot be avoided.⁵

By contrast, under individualistic incentive schemes, inequity premium costs are smaller as payoff inequity does not always occur but only with some positive probability. If the individual signals about the workers' performance are, however, not contractible, a double-sided moral hazard prob-

¹Tournaments have been extensively discussed in the literature since the seminal article by Lazear and Rosen (1981). See e.g. Nalebuff and Stiglitz (1983), Malcomson (1984, 1986), O'Keefe, Viscusi, and Zeckhauser (1984), or Bhattacharya and Guasch (1988).

²Third parties, as e.g. a court, are often not able to verify each piece of information that is available to the principal. Moreover, it will often be too costly or even impossible to credibly communicate the agent's contribution to firm value to an outside party. See e.g. Milgrom and Roberts (1992) and Holmström and Milgrom (1994).

³See e.g. Malcomson (1984, 1986). Other advantages of tournaments include the low measurement costs since relative comparisons are often easier to make than absolute judgements. Moreover, random factors that affect all agents equally are automatically filtered such that the risk premium can be lowered without affecting incentives. These issues are, however, not considered in the present paper.

⁴See e.g. Goranson and Berkowitz (1966), Berg, Dickhaut, and McCabe (1995), and Fehr, Kirchsteiger, and Riedl (1998). For an overview of the experimental literature on other-regarding preferences see Camerer (2003) or Fehr and Schmidt (2006).

⁵Tournaments may also induce sabotage activities or collusion. Moreover, once intermediate results are known effort incentives are strongly reduced. These problems are, however, not the subject of the present paper.

lem arises. Specifically, the principal can save wage costs by understating a worker's performance ex post. Workers anticipate the principal's opportunistic behavior and are not willing to work hard. However, given that the contracting parties observe the agent's performance, incentive contracts may yet be sustained in long-term relationships as reputational equilibria.⁶ Such agreements are called relational (or implicit) contracts. Since they are not court-enforceable, the incentive contracts must be self-enforcing.

The purpose of this paper is to compare the aforementioned prominent incentive schemes given that performance measures are non-verifiable and workers are concerned with relative payoffs. Specifically, I analyze the trade-off between the agency costs due to the self-enforcement requirement under a bonus scheme and those due to inequity aversion under a tournament contract. Moreover, I analyze the impact of inequity aversion on the relative profitability of the incentive regimes.

Formally, I analyze an infinitely repeated game between a long-lived firm and a sequence of two homogeneous short-lived workers. The latter are consigned to work on a similar task which is valuable for the firm.⁷ Following Fehr and Schmidt (1999), workers exhibit 'self-centered inequity aversion'. Inequity is specified as inequality, which is suitable provided that agents face symmetrical decision environments. Moreover, I abstract from empathy, which does not affect my qualitative results however. An agent's performance is difficult to measure in the sense that neither is his contribution to firm value observable nor exists a contractible signal on it. But the contracting parties observe an imperfect non-verifiable continuous signal of each worker's effort. To mitigate the moral hazard problem, the firm offers the workers either a rank-order tournament or an individual bonus contract. In the tournament, the agent with the best performance is awarded a winner prize whereas the other receives the smaller loser prize. Under the bonus scheme, an agent obtains a bonus if his performance measure meets or exceeds an ex ante specified standard. In order to guarantee self-enforcement

⁶Reputational equilibria may exist if one party cares about her reputation in future relationships. In particular, the parties may prefer to stick to the implicit agreement if there is a credible future punishment threat in case they renege on the agreement. See e.g. Holmström (1981), Bull (1987), or Baker, Gibbons, and Murphy (1994, 2002).

⁷Typically, workers in such a situation tend to compare their payoffs with those of their colleagues. For the importance of reference groups, see e.g. Loewenstein, Thompson, and Bazerman (1989).

of the bonus contracts, reputation concerns have to restrain the firm from deviating. Specifically, credibility requires the firm's gains from reneging on the bonus to fall short of the discounted profits from continuing the contract (see e.g. Baker et al. (1994)).

Given the two incentive regimes, I first determine the principal's cost of inducing arbitrary levels of effort. Then I deduce the relative profitability of the contracts. I find that the bonus scheme outperforms the tournament for a range of sufficiently small interest rates. This is due to the fact that the latter incentive contract imposes large inequity premium costs on the firm by virtue of a high degree of income inequality. In contrast, the bonus contract entails less expected payoff inequity rendering it superior as long as credibility problems are not too severe. For sufficiently large interest rates, however, credibility requirements restrict the set of implementable effort levels thereby reducing profits. Thus, the firm switches to the tournament contract once the interest rate is such that profits under both schemes coincide.

Moreover, I investigate the impact of a variation in the agents' inequity aversion on the result. For a simple example, I show the range of interest rates for which the bonus scheme is superior to the tournament to be increasing in the agents' propensity for envy. Intuitively, envy affects both incentive regimes differently. Profits in the tournament clearly decrease as agents become more envious. By contrast, envy has an ambiguous impact on the credibility constraint and, thus, on the resulting profits under the bonus scheme. On the one hand, credibility is favored since envy has an incentive-strengthening effect that allows for lowering the bonus and thus reduces the firm's incentive to cheat. On the other hand, the inequity premium is increasing in the agents' propensity for envy which lowers continuation profits and, consequently, makes credibility more difficult. Altogether, I find that envy benefits the dominance of the bonus contract.

Overall, my findings underline that empirically observed cultural differences in social preferences have non-negligible implications for the optimal design of incentive contracts. In particular, the impact of other-regarding preferences proves to be sensitive to the verifiability of the underlying performance measures. When agents have fairness concerns, individualistic pay

schemes clearly outperform tournaments given that performance is verifiable. When performance signals are not verifiable, the result is reversed for purely selfish agents. For envious agents, however, individual performance pay becomes again superior for a considerable range of interest rates even if performance is not verifiable. This result is strengthened the more envious the agents become.

The present paper brings together important aspects of the literature on tournaments, relational contracts, and that on inequity aversion. In their seminal papers, Lazear and Rosen (1981) and Green and Stokey (1983) also compare relative and independent incentive contracts but consider a static environment with purely self-interested agents. The latter authors propose an output function involving a multiplicative common shock. Similarly, I use a multiplicative individual shock in modeling the performance signal. Related to my approach, other papers as e.g. Malcomson (1984, 1986) emphasize the enforceability advantage of tournaments. The present study offers a complementary, preference-dependent explanation as to why either individual pay schemes or tournaments may be superior in repeated employment settings.

The enforceability of incentive schemes under non-verifiable performance is the subject of the literature on relational contracts. Earlier contributions have focused on environments with symmetric information (e.g. Bull (1987), MacLeod and Malcomson (1989), and Levin (2002)). More recent papers analyze self-enforcing contracts under moral hazard in effort (e.g. Baker et al. (1994, 2002), Levin (2003), and Schöttner (2008)). Similar to my work, some papers compare the efficiency of different incentive regimes for multiple agents (Che and Yoo (2001), Kvaløy and Olsen (2006, 2007)). I contribute to that strand of literature by additionally introducing fairness concerns among agents.

During the last decade, there is an evolving literature linking standard incentive theory and social preferences.⁸ Much of the work is associated with the impact of inequity aversion on individual incentive contracts under verifiable performance. Moreover, as I do, the majority of papers fo-

⁸Alternative approaches regarding the formalization of other-regarding preferences have been proposed, e.g. by Rabin (1993), Fehr and Schmidt (1999), Bolton and Ockenfels (2000), and Falk and Fischbacher (2006).

cuses on mutually inequity averse agents (e.g. Demougin, Fluet, and Helm (2006), Bartling and von Siemens (2007), and Neilson and Stowe (2008)).⁹ The effects of such preferences on tournaments are analyzed by Demougin and Fluet (2003), Grund and Sliwka (2005), and Schöttner (2005).¹⁰ More closely related to my analysis are those papers that compare the efficiency of various performance-pay schemes for other-regarding workers (e.g. Bartling (2008), Rey-Biel (2008), Goel and Thakor (2006), and Itoh (2004)). I complement this literature by extending the analysis of different incentive regimes for mutually inequity averse agents to non-verifiable performance measures.

Most closely related to the present paper is the study by Kragl and Schmid (2008), who find that inequity aversion may enhance the profitability of individual relational incentive contracts. In that paper, we also briefly discuss rank-order tournaments and give the intuition for a comparison with the individual payment scheme. The basic model of that paper, however, solely encompasses binary performance measures, which does not allow to satisfactorily embed the results into the standard literature on tournaments. Thus, the present paper complements the former by introducing continuous performance signals and presenting a rigorous analysis of the two incentive schemes in such an environment.

The paper proceeds as follows. The next section describes the basic economic framework. Section 3 introduces the rank-order tournament, and Section 4 derives the optimal individual bonus scheme. In Section 5, I compare the profitabilities of the two incentive regimes and investigate the impact of a variation in the agents' propensity for envy on the results. Section 6 offers some concluding remarks.

2 The Model

I consider an infinitely repeated game between a long-lived firm, hereafter the principal, and a sequence of two homogeneous short-lived workers, hereafter the agents $i = 1, 2$.¹¹ In each period, each of the two agents undertakes

⁹Englmaier and Wambach (2005) and Dur and Glazer (2008) examine incentive contracts when agents care about inequality relative to the principal.

¹⁰More generally, Kräkel (2008) analyzes the role of emotions in tournaments.

¹¹Workers in the sequence are also homogeneous over time.

costly unobservable effort $e_i \geq 0$ that generates some value $v(e_i)$ for the principal. The value function is increasing and concave. An agent's private cost of effort is a strictly increasing and strictly convex function $c(e_i)$ with $c(0) = 0$. Moreover, $c(e_i)$ is twice differentiable for all $e_i > 0$ and $c'(0) = 0$.

An agent's performance is difficult to measure in the sense that neither his contribution to firm value $v(e_i)$ can be observed nor exists a verifiable signal on it. The contracting parties observe, however, a noisy non-verifiable performance measure x_i for each agent:

$$x_i = e_i \varepsilon_i, \quad i = 1, 2, \quad (1)$$

where ε_i is an individual random component. The random components of both agents are independent and identically standard uniformly distributed; $\varepsilon_i \stackrel{iid}{\sim} U(0, 1)$. In other words, effort is measured in terms of the largest possible realization of the performance measure given the amount of work undertaken by the agent.

The agents observe each other's gross wage π_i and exhibit inequity aversion concerning the wage payments.¹² For convenience, I consider a simplified version of the preferences introduced by Fehr and Schmidt (1999). Specifically, I assume that in each period an agent dislikes outcomes where he is worse off than his colleague. Accordingly, in each period agent i 's utility of payoff π_i when his co-worker earns π_j is given by

$$U_i(\pi_i, \pi_j, e_i) = \pi_i - c(e_i) - \alpha \max\{\pi_j - \pi_i; 0\}, \quad i \neq j, \quad (2)$$

where $\alpha \geq 0$ denotes his propensity for envy. Thus, the third term captures his disutility derived from disadvantageous inequity.¹³

¹²Note that dropping the assumption of observable wages would not necessarily resolve the problem of inequity aversion. Agents usually have a belief of a close colleague's income and can moreover infer on wages from observable signals on wealth.

¹³Abstracting from costs, Fehr and Schmidt (1999) propose the following utility function: $U_i = \pi_i - \alpha \max\{\pi_j - \pi_i, 0\} - \beta \max\{\pi_i - \pi_j, 0\}$, $\alpha > \beta > 0$. It is worth pointing out that incorporating empathy via the parameter $\beta > 0$ would not affect my qualitative results. Allowing for status preferences or pride as reflected by $\beta < 0$ would even strengthen the results. In contrast to my setup and that of Fehr and Schmidt (1999), Demougin and Fluet (2006) take effort costs into account when investigating inequity aversion; workers compare net payoffs. As homogeneous workers exert the same effort in equilibrium, an inclusion of effort cost does not affect my results, however.

The sequence of events in each period is as follows. At the beginning of the period, the principal offers both agents one of two compensation contracts; either a rank-order tournament or an individual bonus contract. Second, each agent individually decides whether to accept the contract or reject it in favor of an alternative employment opportunity that provides utility $\bar{u} \geq 0$. Third, if the agents accept the contract, they simultaneously choose their respective effort levels. Fourth, contributions to firm value are realized and the individual performance measures are observed by all contracting parties. Finally, wage payments are made.

3 The Tournament Contract

In the rank-order tournament, in each period the principal ex ante commits to paying out a fixed sum of wages $w + l$. The two agents compete for the winner prize $w > l$. The agent with the higher performance signal wins, and the loser obtains l . Given the continuous distribution of the individual error terms, for positive effort $e_i > 0$, the case of identical signal realizations occurs with zero probability and is, thus, henceforth neglected. Assuming that the loser cannot bribe the principal, the latter cannot manipulate total wage costs ex post by understating performance though signals are not verifiable. Denoting the prize spread by $\Delta := w - l$, agent i 's gross payoff is given by:

$$\pi_i^T = \begin{cases} l & \text{if } x_i < x_j \\ l + \Delta & \text{if } x_i > x_j \end{cases}, \quad i \neq j \quad (3)$$

Accordingly, the utility of agent i upon winning is

$$U_i^w(e_i) = l + \Delta - c(e_i), \quad i = 1, 2, \quad (4)$$

whereas the corresponding utility if he loses is

$$U_i^l(e_i) = l - c(e_i) - \alpha\Delta, \quad i = 1, 2. \quad (5)$$

Hence, the loser not only receives a lower wage but also suffers from being outperformed. Since the probability of equal signal realizations is zero, inequitable payoff occurs with certainty, and the tournament automatically leads to an unequal treatment of the agents ex post, even though agents are

identical ex ante.

3.1 The Winning Probability

For notational convenience, designate the respective effort levels of agent i, j by e, a , and the signal realizations by x, y , respectively. Owing to the structure of the individual performance measures and given the agents' respective effort levels, the signals are independent random variables with support $S(e, a) := \{(x, y) \mid (0, 0) \leq (x, y) \leq (e, a)\}$. The joint signal density obtains:

$$g(x, y|e, a) := \begin{cases} 0 & \text{if } (x, y) \notin S(e, a) \\ \frac{1}{ae} & \text{if } (x, y) \in S(e, a) \end{cases} \quad (6)$$

I denote $p(e|a)$ agent i 's probability of winning the tournament, given that his co-worker exerts effort a . Thus,

$$p(e|a) = \Pr[x > y|e, a] = \Pr[e\varepsilon_i > a\varepsilon_j]. \quad (7)$$

Given the distribution of the error terms, that probability becomes:

$$p(e|a) = \begin{cases} \frac{1}{2} \frac{e}{a} & \text{if } e \leq a \\ 1 - \frac{1}{2} \frac{a}{e} & \text{if } e > a \end{cases} \quad (8)$$

To see how the probabilities are derived from the density function consider Figure 1. The left graph of the figure represents the case $e \leq a$. Due to the tournament structure, player i wins only if signal realizations x, y to the right of the 45°-line occur. Given that the joint probability density function is a constant, the probability of winning multiplies $1/ae$ with the surface of the region where the agent wins; $e^2/2$. Altogether, we thus obtain:

$$\frac{1}{ae} \cdot \left(\frac{e^2}{2}\right) = \frac{1}{2} \frac{e}{a}. \quad (9)$$

The alternative case $e \geq a$ is illustrated by the right graph of the figure. The surface area to the right of the 45°-line is composed of $a^2/2$ and $(e - a)a$.

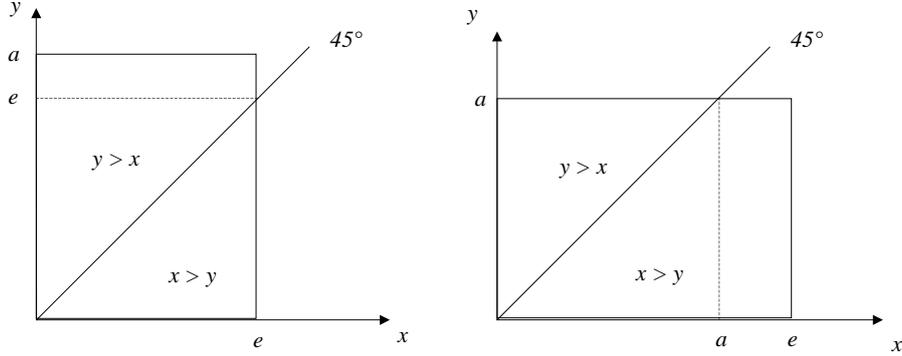


Figure 1: Possible realizations of the signals x, y for $e \leq a$ (left figure) and $e \geq a$ (right figure).

Multiplying the surface again with the density yields

$$\frac{1}{ae} \cdot \left(\frac{a^2}{2} + (e - a)a \right) = 1 - \frac{1}{2} \frac{a}{e}. \quad (10)$$

Altogether, $p(e|a)$ is increasing, concave and continuous in e . Moreover, $p(e|a)$ is continuously differentiable:

$$p'(e|a) := \frac{\partial p(e|a)}{\partial e} = \begin{cases} \frac{1}{2} \frac{1}{a} & \text{if } e \leq a \\ \frac{1}{2} \frac{a}{e^2} & \text{if } e > a \end{cases}, \quad (11)$$

with $p'(e|a = e) = 1/(2a)$. Figure 2 below depicts both functions.

3.2 The Agent's Problem

Both agents simultaneously decide on their effort choice. I determine the equilibrium effort levels using the Nash-equilibrium concept. In the remainder, a denotes the amount of effort agent j exerts at the Nash-equilibrium. Agent i 's optimization problem is thus given by

$$\max_e EU_i(e, a; \alpha) = l + p(e|a) \Delta - c(e) - \alpha (1 - p(e|a)) \Delta. \quad (\text{I})$$

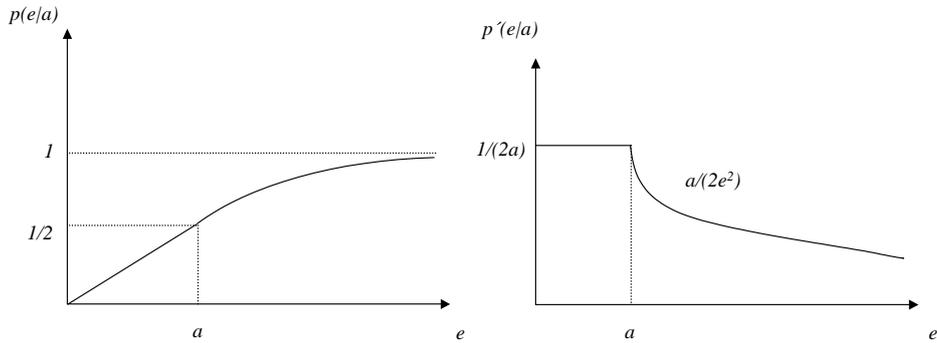


Figure 2: Winning-probability $p(e|a)$ and marginal winning probability $p'(e|a)$.

The first-order condition yields

$$p'(e|a)(1 + \alpha)\Delta = c'(e). \quad (12)$$

The Nash equilibrium of the agents' effort choices is symmetric and unique.¹⁴ Thus, in order to elicit effort a , the principal offers the prize spread

$$\Delta(a; \alpha) = \frac{2ac'(a)}{(1 + \alpha)}. \quad (\text{ICT})$$

It follows that $\partial\Delta/\partial\alpha < 0$. Alternatively, for a given prize spread, the agents' effort incentives increase in the agents' propensity for envy. This observation is known as the *incentive effect of envy* (see e.g. Demougin and Fluet (2003) and Grund and Sliwka (2005)).¹⁵

3.3 The Principal's Wage Cost

In each period, the principal wishes to minimize her cost for implementing a given level of effort. Denote by $C^T(a)$ her average cost of implementing effort a . Solving the game by backward induction, the minimization problem is subject to the agents' incentive-compatibility and participation

¹⁴For a verification see the appendix. Moreover, it is worth pointing out that I use the difference in gross payoffs as a measure for inequity which is, however, only meaningful at the symmetric equilibrium where workers face identical cost of effort.

¹⁵The authors derive the effect for agents that are also compassionate. As in their setups envy dominates the latter emotion, altogether inequity aversion has a positive effort-strengthening effect.

constraints. Her per-period objective is thus given by

$$\begin{aligned}
& \min_{l, \Delta} && 2C^T(a, l, \Delta) = 2l + \Delta \\
& \text{s.t.} && \\
& \text{(ICT)} && \Delta = \frac{2ac'(a)}{1 + \alpha} \\
& \text{(PCT)} && l + \frac{\Delta}{2} - c(a) - \alpha \frac{\Delta}{2} \geq \bar{u},
\end{aligned} \tag{II}$$

where (PCT) ensures the agents' participation in the contract. Note that, in expectation, each agent wins the tournament with probability 0.5. Since the loser prize l positively enters the principal's cost function, the participation constraint is binding in the optimal tournament contract, leading to zero rent for the agents. Using (ICT) and (PCT) in order to substitute l and Δ in the principal's objective function, we obtain the following result:

Lemma 1 *In a rank-order tournament, the principal's cost for implementing effort a is given by*

$$C^T(a; \alpha, \bar{u}) = c(a) + \frac{\alpha}{1 + \alpha} ac'(a) + \bar{u}. \tag{13}$$

For a given effort level a , these wage costs are increasing in the parameter capturing envy. In the literature, these agency costs of inequity aversion are known as *inequity premium* (see e.g. Grund and Sliwka (2005)). They are represented by the second term of the principal's cost function (13). The preceding observations lead to the following conclusion.

Proposition 1 *In a rank-order tournament, the principal implements first-best effort a^* when agents are not envious. Once agents are envious, she implements second-best effort $a_T^{**} < a^*$. Per-period profits as well as implemented effort levels decrease in the agents' propensity for envy.*

Proof. The principal's profit maximization problem is given by

$$\max_a \Pi^T(a; \alpha, \bar{u}) = v(a) - c(a) - \frac{\alpha}{1 + \alpha} ac'(a) - \bar{u}. \tag{PI}$$

The first-order condition of the above problem yields:

$$v'(a_T^{**}) - \frac{\alpha}{1 + \alpha} (c'(a_T^{**}) + a_T^{**} c''(a_T^{**})) = c'(a_T^{**}) \tag{14}$$

For $\alpha = 0$, the equation reduces to $v'(a^*) = c'(a^*)$ implying first-best effort levels. For $\alpha > 0$, by the implicit-function theorem, effort a_T^{**} is strictly decreasing in α . Using the envelope theorem, profits also decrease in α . ■

In comparing the incentive regimes in Section 5, I focus on stationary contracts. That is, I embed the above derived one-period problem into an infinitely repeated game.

4 The Bonus Contract

In the individual bonus contract, in each period the principal pays a fixed base wage A with certainty and promises to pay a bonus B whenever an agent's individual performance measure in the respective period meets or exceeds some ex ante fixed standard z . Keeping the foregoing notation, agent i 's per-period gross monetary payoff is thus given by:

$$\pi_i^B = \begin{cases} A & \text{if } x < z \\ A + B & \text{if } x \geq z \end{cases} \quad (15)$$

Unlike in the tournament contract, agent i suffers from uneven payoffs only in the case that he does not obtain the bonus whereas his co-worker does. In particular, the additional loss due to inequity aversion amounts to αB .

4.1 The Benchmark Case: Verifiable Performance

In this section I initially analyze the benchmark case of verifiable performance signals. As credibility issues do not arise in this case, I only consider the single-period game.

4.1.1 The Agent's Problem

Given the contract and the underlying distribution function, the probability that agent i gets a bonus, $p(e|z) = \Pr[x \geq z|e]$, is given by

$$p(e|z) = \max\{0, 1 - \frac{z}{e}\}. \quad (16)$$

To see how equation (16) is obtained, consider Figure 3.

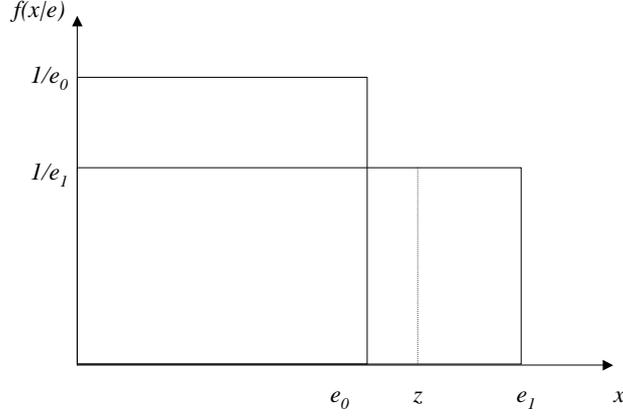


Figure 3: Signal densities $f(x|e)$ for two effort levels $e_0 < z < e_1$.

For effort $e < z$ as depicted by e_0 , the agent never obtains the bonus. In contrast, for effort $e > z$, e.g. e_1 in the figure, the agents receives the bonus with probability

$$(e - z) \frac{1}{e} = 1 - \frac{z}{e}. \quad (17)$$

For any $e \geq z$, the function $p(e|z)$ is increasing and strictly concave in effort.

Following the same conventions as in the foregoing section, agent i 's expected utility is

$$EU_i(e, a, z; \alpha) = A + p(e|z)B - c(e) - \alpha(1 - p(e|z))p(a|z)B. \quad (18)$$

where a denotes the other agent's effort at the Nash equilibrium. The expected disutility from being outperformed is captured by the last term in the above equation. Rewriting the agent's utility as

$$EU_i(e, a, z; \alpha) = A + p(e|z)\{1 + \alpha p(a|z)\}B - c(e) - \alpha p(a|z)B \quad (19)$$

we see that the agent will undertake a positive effort $e > 0$ only if

$$p(e|z)\{1 + \alpha p(a|z)\}B \geq c(e). \quad (\text{IntC})$$

Otherwise, the worker is better off by choosing $e = 0$ (see Figure 4). In the remaining, the above requirement will be referred to as the interior-solution constraint.

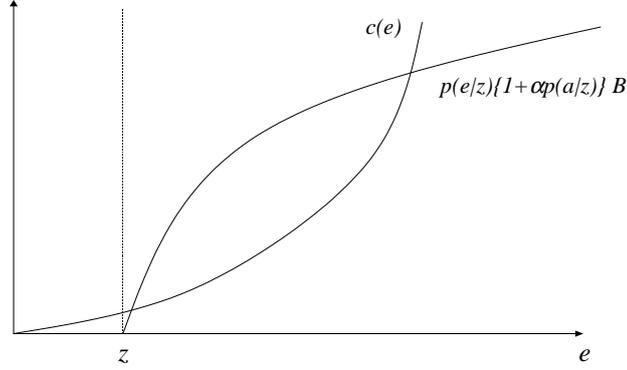


Figure 4: Interior-solution constraint of an agent's maximization problem.

In the appendix, I verify that in case condition (IntC) is satisfied at the Nash-equilibrium, the equilibrium is unique and symmetric:¹⁶

$$a = \arg \max_e A + p(e|z) \{1 + \alpha p(a|z)\} B - c(e) - \alpha p(a|z) B \quad (20)$$

In the unique symmetric interior equilibrium, the first-order condition yields

$$p'(a|z) \{1 + \alpha p(a|z)\} B - c'(a) = 0. \quad (\text{ICB})$$

The condition again reveals the *incentive-strengthening effect of envy*. Intuitively, an increase in α has the same effect as raising the bonus.¹⁷

4.1.2 The Principal's Wage Cost

In this subsection, I analyze the cost minimization problem of the principal if she wants to implement effort a . From the foregoing, the principal solves:

¹⁶Eventhough there exist contracts that do not lead to an interior solution, these are not interesting since the principal will want the agents to undertake positive effort. Therefore I ignore these contracts in the following analysis of the agents' behavior.

¹⁷Again, the result is in line with the literature. Neilson and Stowe (2008) find a similar effect for piece-rate contracts. For the incentive effect under bonus contracts with binary signals see Demougin and Fluet (2006) and Kragl and Schmid (2008).

$$\begin{aligned}
& \min_{A,B,z} && C^B(a, A, B, z) = A + p(a|z) B \\
& \text{s.t.} && \\
& \text{(IntC)} && p(a|z) \{1 + \alpha p(a|z)\} B \geq c(a), \\
& \text{(ICB)} && p'(a|z) \{1 + \alpha p(a|z)\} B = c'(a), \\
& \text{(PCB)} && A + p(a|z) B - c(a) - \alpha (1 - p(a|z)) p(a|z) B \geq \bar{u}
\end{aligned} \tag{III}$$

where (IntC) guarantees that the agents are better off undertaking the desired effort level rather than no effort at all. Condition (ICB) is the standard incentive-compatibility constraint, equalizing marginal benefit and marginal cost of effort, and (PCB) ensures the agents' participation. Just as before (PCB) will be binding at the optimum, which allows to substitute A into the principal's objective function. Rewriting the problem yields:

$$\begin{aligned}
& \min_{B,z} && C^B(a, B, z; \alpha, \bar{u}) = c(a) + \alpha (1 - p(a|z)) p(a|z) B + \bar{u} \\
& \text{s.t.} && \\
& \text{(IntC)} && p(a|z) \{1 + \alpha p(a|z)\} B \geq c(a), \\
& \text{(ICB)} && p'(a|z) \{1 + \alpha p(a|z)\} B = c'(a)
\end{aligned} \tag{IV}$$

Lemma 2 *Assume that performance measures are verifiable and the principal wishes to implement effort a . Then solving problem (IV) for the optimal bonus contract B^*, z^* requires that the interior-solution constraint (IntC) is binding.*

Proof. Consider the principal's problem as given in (IV) and assume that condition (IntC) is not binding. Substituting B from condition (ICB) yields:

$$\min_z C^B(a, B, z; \alpha, \bar{u}) = c(a) + \frac{\alpha (1 - p(a|z)) p(a|z) c'(a)}{p'(a|z) \{1 + \alpha p(a|z)\}} + \bar{u} \tag{PII}$$

The principal's objective becomes minimizing the inequity premium by the choice of z :

$$\min_z \frac{(1 - p(a|z)) p(a|z)}{p'(a|z) \{1 + \alpha p(a|z)\}} \tag{PIII}$$

Plugging in the bonus probability as given in equation (16) and simplifying yields:

$$\min_z \frac{a - z}{1 + \alpha \left(1 - \frac{z}{a}\right)} \tag{PIV}$$

The first-order condition of the above problem is given by

$$0 = - \left(1 + \alpha \left(1 - \frac{z^+}{a} \right) \right) + \left(-\frac{\alpha}{a} \right) (a - z^+), \quad (21)$$

implying

$$z^+ = a. \quad (22)$$

With the second-order condition of problem (PIV) $2\alpha/a > 0$, we thus have a minimum. With $z^+ = a$, however $p(a|z^+) = 0$ while $c(a) > 0$ for any $a > 0$. This contradicts condition (IntC) as $0 \geq c(a)$ cannot be satisfied for any positive value of a . As a result, condition (IntC) must be binding. ■

To illustrate the intuition of the proof, consider Figure 5. It depicts the constraints of the principal's minimization problem as given in (IV) for a given level of effort.

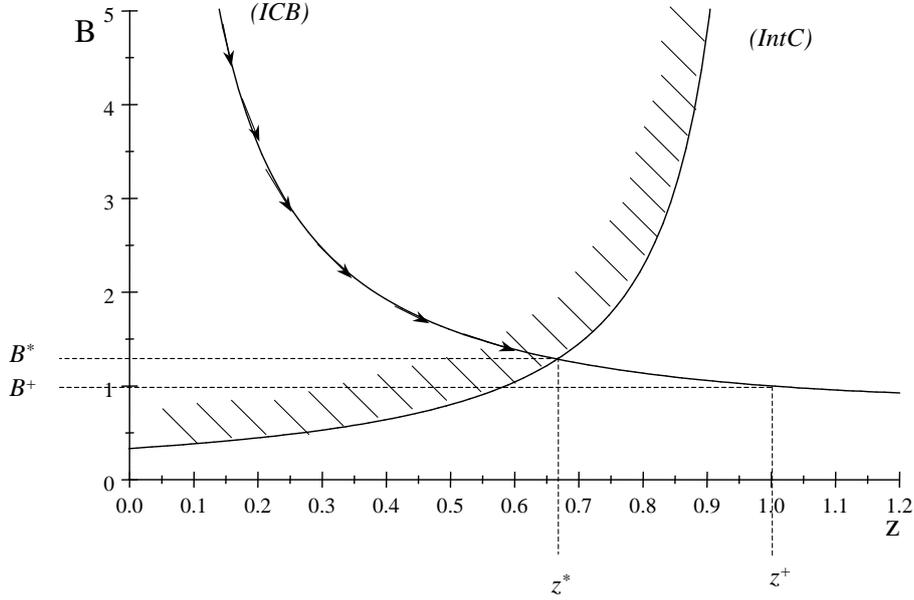


Figure 5: Conditions (IC) and (IntC) with $c(a) = \frac{a^2}{2}$, $a = 1$, and $\alpha = 0.5$.

In the figure, observe that condition (ICB) implies that reducing the bonus B requires raising the performance standard z .¹⁸ However, B, z must also satisfy the interior-solution constraint. The shaded area depicts combinations B, z for which inequality (IntC) is satisfied.

Intuitively, if constraint (IntC) were not binding, the principal would want to choose z such that the inequity premium, i.e. the second term of her objective function in problem (IV), becomes zero. This implies $z = a$ as then $p(a|z) = 0$. However, zero bonus probability violates condition (IntC) as, with $a > 0$, the agents incur positive costs of effort. As can be seen in the figure, the solution of the relaxed problem denoted by B^+, z^+ is thus located outside the shaded area. As a result, condition (IntC) must be binding.

From the foregoing follows that z^*, B^* are implicitly defined by the two constraints (ICB) and (IntC):

$$\begin{aligned} \frac{z^*}{a^2} \left\{ 1 + \alpha \left(1 - \frac{z^*}{a} \right) \right\} B^* &= c'(a) \\ \left(\frac{a - z^*}{a} \right) \left\{ 1 + \alpha \left(1 - \frac{z^*}{a} \right) \right\} B^* &= c(a) \end{aligned} \tag{23}$$

Figure 6 illustrates the solution to the above equation system. In particular, condition (ICB) requires the slope of the two curves to coincide while the (IntC)-constraint stipulates their intersection. As a result, the curves must be tangent. Solving for z^*, B^* , calculating $p(a|z^*)$ and $p'(a|z^*)$ and substituting the solutions in the principal's cost function yields the following result. For an explicit derivation see the appendix.

Lemma 3 *Assume that performance measures are verifiable and the principal wishes to implement effort a . Then the associated cost-minimizing bonus contract is given by*

$$B^*(a; \alpha) = \frac{(c(a) + c'(a)a)^2}{(1 + \alpha)c(a) + c'(a)a}, \tag{24}$$

$$z^*(a) = \frac{a^2 c'(a)}{c(a) + c'(a)a}. \tag{25}$$

¹⁸Note that a necessary condition for constraint (IntC) to be satisfied is $a > z$. The figure thus illustrates the constraints for these values of z .

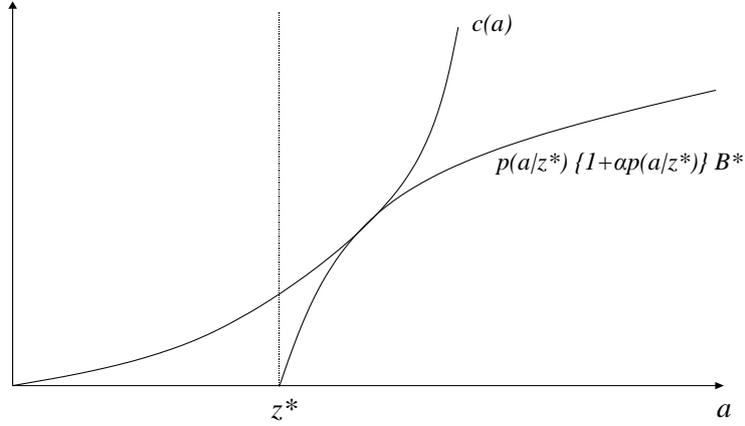


Figure 6: Solution of the 2×2 -system for z^* , B^* .

Altogether, the principal's cost for implementing effort a is

$$C^B(a; \alpha, \bar{u}) = c(a) + \bar{u} + \frac{\alpha}{1+\alpha} ac'(a) \cdot \frac{c(a)}{c(a) + \frac{ac'(a)}{1+\alpha}}. \quad (26)$$

Note the incentive effect; $B^*(a; \alpha)$ is decreasing in α . However, for a given effort level a , overall wage costs are increasing in the parameter capturing envy.¹⁹ With $\alpha > 0$, the principal incurs *inequity premium* costs because the agents must be compensated for the expected disutility from inequity.²⁰ The following proposition gives the main results of the foregoing analysis.

Proposition 2 *With verifiable performance measures and restricting the analysis to the individual bonus scheme, the principal implements first-best effort a^* when agents are not envious. Once agents are envious, she implements second-best effort $a_B^{**} < a^*$. Per-period profits decrease in the agents' propensity for envy.*

Proof. See the appendix. ■

¹⁹For a proof see the appendix.

²⁰The result is in line with the agency literature. See Bartling and von Siemens (2007), Kragl and Schmid (2008), and Neilson and Stowe (2008) for similar results in different setups.

4.1.3 Comparison with the Tournament

In the one-shot game with verifiable performance, the principal's wage cost for implementing a given effort level, differ in both types of contract only in the amount of the inequity premium. Naturally, this results from the characteristics of the two incentive regimes. Comparing the costs of implementing a given effort level as given by equations (13) and (26) directly yields the following result:

Proposition 3 *Assume that performance measures are verifiable and agents are envious. Then the wage cost for implementing an arbitrary effort level is strictly larger under the tournament contract than the wage cost under the individual bonus scheme.*

When $\alpha = 0$, wage costs are $C^B(a) = C^T(a) = c(a) + \bar{u}$, and the firm implements the first-best solution under either incentive regime. When $\alpha > 0$, however, the firm must compensate the agents for the expected disutilities implied by the respective pay structures. Intuitively, under the rank-order tournament, inequity occurs with certainty whereas in the bonus contract it arises only with some positive probability. As a result, the latter scheme dominates the former when performance measures are verifiable.

4.2 Non-verifiable Performance

The cost-minimizing tournament contract derived in Section 3 is not affected by the non-verifiability of the performance measures as the fixed sum of prizes is contractible. By contrast, in the individual bonus scheme, the principal may have an incentive to renege on the bonus ex post by understating the agent's performance. Thus, individual bonus contracts are feasible only if the principal is credible to keep her promise regarding the agreed terms of payments. In other words, the contracts must be self-enforcing. Mathematically, this requires introducing a credibility constraint on the side of the principal.

In order to do so, I embed the one-shot model analyzed above into an infinitely repeated game between the firm and an infinite sequence of workers.²¹ Modeling trigger-strategy equilibria, I assume that, if the firm reneges on

²¹In particular, I focus on stationary contracts.

the bonus once, no agent believes the principal to adhere to the contract in any subsequent period of the game.²² In particular, for simplicity, I assume that after a single contract breach the firm is not able to conclude another employment contract. Altogether, the principal's per-period objective thus becomes:

$$\begin{aligned}
& \max_{a,B,z} && \Pi^B(a, B, z; \alpha, \bar{u}) = v(a) - C^B(a, B, z; \alpha, \bar{u}) \\
& \text{s.t.} && \\
(\text{IntC}) & && B \geq \frac{c(a)}{p(a|z) \{1 + \alpha p(a|z)\}}, \\
(\text{ICB}) & && B = \frac{c'(a)}{p'(a|z) \{1 + \alpha p(a|z)\}}, \\
(\text{CC}) & && B \leq \frac{\Pi^B(a, B, z; \alpha, \bar{u})}{r},
\end{aligned} \tag{V}$$

where $C^B(a, z, B; \alpha, \bar{u})$ is the firm's wage cost as defined in problem (IV). With r designating the firm's interest rate, condition (CC) guarantees credibility. The constraint requires it to be worthwhile to stick to the agreement; i.e. the gains from renegeing must fall short of the discounted gains from continuing the contract.

In order to highlight the impact of the credibility constraint on the optimization problem, consider the size of the firm's interest rate. Given that r is sufficiently small, (CC) is not binding, and the principal implements the same contract as under verifiability, i.e. with $\alpha > 0$ she implements effort a_B^{**} and the associated bonus payment and performance standard; $B^*(a_B^{**}; \alpha), z^*(a_B^{**})$. By contrast, for sufficiently large r , the foregoing contract is no more credible. In order to reestablish credibility, the principal must thus reduce the bonus payment. The following lemma implies that this requires lowering the implemented effort level.

Lemma 4 *Suppose that performance measures are non-verifiable. Assuming that the optimal bonus contract solving problem (V) implements credible*

²²In modeling reputation, I follow Baker et al. (1994). Implicitly, I assume the information on a principal's deviation from the contract to be rapidly transmitted to the labor market. Alternatively, as Baker et al. (1994) note, each period's agent learns the history of play before the period begins.

effort a^c , the principal uses the bonus $B^*(a^c; \alpha)$ and the standard $z^*(a^c)$, where B^*, z^* are defined by equations (24) and (25).

Proof. To verify the claim, all we need to show is that condition (IntC) is binding in problem (V) for an arbitrary effort level. To prove this, I again use Figure 5 from Section 4.1, which depicts the constraints (ICB) and (IntC) as given in problem (V) for a fixed effort level. First, consider the case that r is such that condition (CC) is not binding for the bonus that implements the desired effort level. Problem (V) then resembles the problem under verifiability, and by Lemma 2 condition (IntC) is binding. Secondly, consider the case that condition (CC) is binding. Denote by B_{\max} the bonus payment that makes (CC) binding for the desired effort level and a given interest rate r . Initially, suppose that $B_{\max} \geq B^*$ in Figure 5. Then the desired effort level can always be implemented by choosing B^*, z^* (or any combination B, z on the (ICB)-curve for which $B^* \leq B \leq B_{\max}$). By contrast, if $B_{\max} < B^*$, the desired effort level is not implementable by any choice of B, z . Consequently, the principal must assure that $B_{\max} = B^*$ by adapting the induced effort level. In the appendix, I verify that the system of the two binding constraints (ICB) and (IntC) defining B^*, z^* implies that $\partial B^*/\partial a, \partial z^*/\partial a > 0$. To reestablish implementability, the principal must thus reduce effort. For the reduced effort level, Figure 5 then looks alike, and the logic from above applies. Consequently, without loss of generality, the optimal credibility-constrained bonus contract B^*, z^* is given by the equation system (23), as depicted in Figure 5 by the intersection of the conditions (ICB) and (IntC). ■

Given the above result, the credibility constraint can now be written as

$$rB^*(a; \alpha) \leq \Pi^B(a; \alpha, \bar{u}). \quad (\text{CC}^*)$$

As discussed above, for sufficiently small r , the condition (CC*) is not binding, and the firm implements effort a_B^{**} . As r increases, at a particular point, $B^*(a_B^{**}; \alpha)$ is no longer credible, and the firm needs to lower the induced effort level in order to reduce the bonus payment. In particular, the largest credible effort level is decreasing in r . By concavity of the profit function, profits must thus also decrease. Altogether, we obtain the following result.

Proposition 4 *Assume that performance measures are non-verifiable. Then,*

under the individual bonus scheme, there is an interest rate \hat{r} such that

$$\hat{r}B^*(a_B^{**}; \alpha) = \Pi^B(a_B^{**}; \alpha). \quad (27)$$

(i) For any interest rate $r \leq \hat{r}$, the principal implements effort a_B^{**} and realizes profits $\Pi^B(a_B^{**}; \alpha)$ as under verifiability.

(ii) For any interest rate $r > \hat{r}$, she implements an effort level $a^c(r) < a_B^{**}$ that just satisfies condition (CC*) for the given interest rate. Profits are strictly smaller than under verifiability; $\Pi^B(a^c(r); \alpha) < \Pi^B(a_B^{**}; \alpha)$.

5 Comparison of the Incentive Schemes

In section 4.1 I verified that the principal is better off with an individual bonus scheme when performance measures are verifiable and agents are envious. In section 4.2 we saw that the advantage of the bonus scheme is, however, weakened when performance measures are non-verifiable and the firm runs into credibility problems. This is due to the fact that the choice of effort levels is then restricted by the credibility constraint.

In the present section, I first compare the two incentive schemes for non-verifiable performance and a given positive degree of envy. Moreover, I investigate the impact of a variation in the agents' propensity for envy on the relative profitability of the two regimes. In order to keep the analysis tractable, in the remaining, I consider a simple example with $v(a) = a$, $c(a) = 0.5a^2$, and $\bar{u} = 0$.²³ The results are generalizable, but using the example, however, greatly simplifies the analysis.

5.1 Profits

In each period, the principal wishes to maximize expected per-agent profits. From the foregoing, for the given example, her objective under the rank-order tournament is given by

$$\max_a \quad \Pi^T(a; \alpha) = a - \left(\frac{1}{2} + \frac{\alpha}{1+\alpha} \right) a^2, \quad (VI)$$

²³For traceability, I give the solutions of the model variables derived in the preceding sections for the example in the appendix.

whereas under the bonus scheme her problem becomes

$$\begin{aligned}
& \max_{a, B^*} \quad \Pi^B(a; \alpha) = a - \left(\frac{1}{2} + \frac{\alpha}{3 + \alpha} \right) a^2 \\
& \text{s.t.} \\
& \text{(CC}^*) \quad rB^*(a; \alpha) \leq \Pi^B(a; \alpha), \\
& \text{(ICB)} \quad B^*(a; \alpha) = \frac{9}{2} \frac{a^2}{3 + \alpha}.
\end{aligned} \tag{VII}$$

To shed light on the interest rate's impact on the profitability of the individual bonus contract and allow for a comparison with the tournament, I illustrate the credibility constraint as given in (CC*) in Figure 7. In the figure, I plot the profit functions under both incentive regimes for a given value of envy. Moreover, the convex curves depict $rB^*(a)$ for different interest rates, $r^S > r > \hat{r}$.

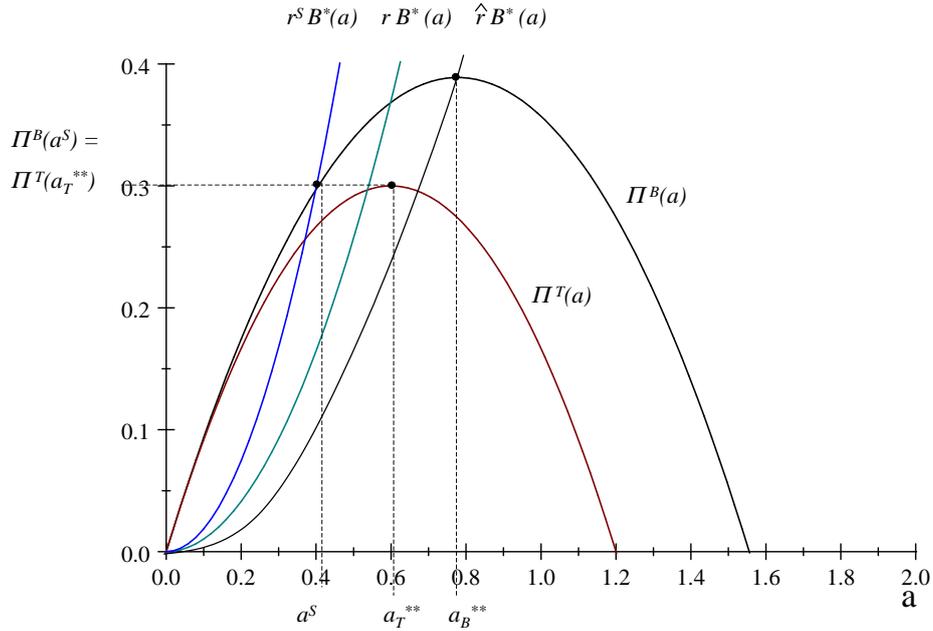


Figure 7: Profit functions in the bonus contract and the tournament, and the credibility constraint with $\alpha = 0.5$.

In the tournament contract, the principal needs not account for a credibility constraint such that she implements a_T^{**} and realizes profits $\Pi^T(a_T^{**})$

for any interest rate r . Under the bonus contract, as long as $r \leq \hat{r}$, profits are also not affected by the interest rate; the firm implements a_B^{**} and realizes profit $\Pi^B(a_B^{**}) > \Pi^T(a_T^{**})$. However, once $r > \hat{r}$, the size of r has a negative impact on the firm's profit under the bonus contract. In the figure, for a given value of r , the realized profit and the corresponding credible effort level $a^c(r)$ are determined by the intersection of the two curves $\Pi^B(a)$ and $rB^*(a)$. Observe that with increasing r , the firm lowers effort below a_B^{**} , thereby realizing reduced profit $\Pi^B(a^c(r)) < \Pi^B(a_B^{**})$.

Importantly, the figure shows that there is a critical interest rate r^S for which effort $a^c(r^S) =: a^S$ is implemented, and profit $\Pi^B(a^S)$ under the bonus scheme corresponds to profit $\Pi^T(a_T^{**})$ under the tournament.²⁴ Note that it is optimal for the principal to switch to the tournament contract for any interest rate $r > r^S$. The preceding observations directly yield the following result.

Proposition 5 *Assume that performance measures are non-verifiable and agents are envious. Then there is an interest rate r^S such that*

$$\Pi^B(a^c(r^S); \alpha) = \Pi^T(a_T^{**}; \alpha). \quad (28)$$

- (i) *For any interest rate $r < r^S$, the firm is better off under an individual bonus contract.*
- (ii) *For any interest rate $r > r^S$, the firm is better off under the rank-order tournament.*

Intuitively, profit in the tournament suffers from large inequity premium costs as inequitable payoff distributions cannot be avoided. The individual bonus scheme outperforms the tournament in that respect since expected payoff distributions are more even. As a result, the latter incentive regime is more profitable as long as credibility problems are not too severe. For sufficiently large interest rates, however, credibly implementable effort levels

²⁴In the present analysis, I assume $\bar{u} = 0$. Note that with $\bar{u} > 0$, the switching point r^S, a^S may become the point where $rB^*(a)$ is tangent to $\Pi^B(a)$. Then for any $r > r^S$, individual bonus contracts are no longer feasible. Note that with $\bar{u} > 0$, it may be the case that $\Pi^B(a^S) > \Pi^T(a_T^{**})$. In that case, the interest rate r^S would depend on \bar{u} . For analysis of the impact of envy on the interest rate for which bonus contracts become infeasible see Kragl and Schmid (2008).

in the bonus contract lead to a profit smaller than that under the tournament such that the latter contract becomes superior. Interestingly, at the switching point, implemented effort increases from a^S to a_T^{**} . Thus, under the tournament, agents must work harder albeit the firm receives the same profit as under the bonus contract. Intuitively, the firm must pay the agents a larger wage in order to compensate them for the increased expected pay-off inequity under the tournament. The principal is compensated for these higher wage payments by an increased output.

5.2 The Impact of Envy on the Relative Profitability

In the foregoing subsection, I analyzed the relative profitability of the two incentive schemes for a given degree of envy. By equation (28), the parameter capturing envy, however, endogenously determines the interest rate r^S for which it is optimal for the firm to switch from the individual bonus contract to the tournament. In order to explicitly investigate the issue, we solve the principal's optimization programs as given in problems (VI) and (VII). This yields the respective optimal effort levels under the two incentive schemes for given values of α and r . Moreover, solving for the switching point (r^S, a^S) as defined by equation (28) then implicitly yields $r^S(\alpha)$. This allows to directly analyze the impact of envy on that critical interest rate. As a first step, the following lemma gives the solutions to the respective optimization problems.

Lemma 5 *Assume that performance is not verifiable and agents are envious. Moreover, suppose $v(a) = a$, $c(a) = 0.5a^2$, and $\bar{u} = 0$.*

(i) In the rank-order tournament, the principal implements an effort level

$$a_T^{**}(\alpha) = \left(1 + \frac{2\alpha}{1 + \alpha}\right)^{-1}. \quad (29)$$

(ii) In the individual bonus scheme, for any $r \leq \hat{r} = (\frac{1}{3}\alpha + \frac{1}{3})$, the credibility constraint is not binding, and the principal implements an effort level

$$a_B^{**}(\alpha) = \left(1 + \frac{2\alpha}{3 + \alpha}\right)^{-1}. \quad (30)$$

(iii) In the individual bonus scheme, for any $r > \hat{r} = (\frac{1}{3}\alpha + \frac{1}{3})$, the credibil-

ity constraint is binding, and the principal implements an effort level

$$a^c(\alpha, r) = \left(1 + \frac{0.5\alpha + 4.5r - 1.5}{3 + \alpha}\right)^{-1}. \quad (31)$$

Proof. See the appendix. ■

Next, plugging in the effort levels $a_T^{**}(\alpha)$ and $a^c(\alpha, r)$ in equation (28), implicitly yields $r^S(\alpha)$. In the appendix, I derive that implicit function and, moreover, verify that $\partial r^S / \partial \alpha > 0$. Thus, envy has a positive impact on the critical interest rate for which the firm switches from the bonus contract to the tournament. The following proposition summarizes this result.

Proposition 6 *Assume performance is not verifiable and agents are envious. Moreover, suppose $v(a) = a$, $c(a) = 0.5a^2$, and $\bar{u} = 0$. Then the more envious the agents are, the larger is the critical interest rate $r^S(\alpha)$ and, consequently, also the range of interest rates for which the individual bonus scheme dominates the rank-order tournament.*

Hence, the degree of envy impacts the relative profitability of the two considered incentive contracts in favor of the bonus scheme. Intuitively, an increasing propensity for envy affects both incentive regimes to a different extent. Profits in the tournament clearly decrease. In the individual bonus scheme, however, envy has an ambiguous impact on the credibility constraint and, thus, on profits in the optimum. In section 4.1, we derived two particular implications of envy. Specifically, the incentive effect of envy allows for lowering the bonus for a given effort level. As a result, the left-hand side of the credibility constraint is decreasing in the degree of envy which favors credibility. By contrast, due to the inequity premium effect the right-hand side of the constraint is also decreasing in envy, thereby making credibility more difficult. Thus, from the outset, it is not clear, the relative profitability of which contract is favored by an increasing propensity for envy. However, my analysis shows that envy clearly benefits the relative performance of the individual bonus contract.

6 Concluding Remarks

In a moral-hazard environment, I compare the profitabilities of relative and individual performance pay when a firm employs two envious workers whose

respective performances are not verifiable. My findings underline that social preferences play a non-negligible role for the design of incentive schemes.²⁵ In particular, when agents do not care about relative payoffs, a rank-order tournament clearly outperforms individual bonus contracts as the former solves the non-verifiability problem altogether. The present analysis shows that this result is reversed for a considerable range of interest rates once agents are envious.

The paper highlights an interesting trade-off. With envious agents, the tournament becomes more costly than the bonus contract in terms of inequity premium costs. Thus, for a range of sufficiently small interest rates, the latter incentive contract dominates the former. For sufficiently large interest rates, however, credibility requirements restrict the set of implementable effort levels under the bonus scheme thereby reducing profits. Hence, the firm switches to the tournament contract at some level of interest rate. Moreover, my analysis suggests that the more envious the agents are the more likely is an individual bonus scheme to be superior. For a simple example, I show that the range of interest rates for which the bonus contract dominates the tournament is increasing in the agents' propensity for envy. Thus, fairness concerns render the individual pay scheme relatively more profitable even though it must be self-enforcing.

It is worth briefly discussing some assumptions of my model. First, regarding the shape of the agents' inequity aversion I have solely focused on envy. However, the trade-off concerning the relative performance of the two incentive regimes presented in my paper still carries over to the case that agents are also compassionate as proposed by Fehr and Schmidt (1999). Specifically, inequity premium costs increase under both contracts even further since the agents must not only be compensated for the expected inequity from being outperformed but also for that from being ahead. This makes the tournament even less profitable and impedes the firm's credibility under the bonus contract. In addition, empathy counteracts the incentive effect (see e.g. Grund and Sliwka (2005)). However, as has been found by e.g. Loewenstein et al. (1989), agents dislike being outperformed to a larger extent than

²⁵Not surprisingly, empirical evidence shows that social preferences differ between cultures. For instance, Alesina, Di Tella, and MacCulloch (2004) and Corneo (2001) find Europeans to exhibit a higher propensity for inequity aversion in comparison to U.S.-Americans.

they resent being ahead. Formalizing the notion of compassion by the parameter β , Fehr and Schmidt (1999) therefore assume $\alpha > \beta$. As a result, inequity aversion still has an, albeit smaller, overall incentive-strengthening effect. Altogether, credibility in the bonus scheme thus becomes more difficult to achieve when empathy is additionally introduced which makes that contract relatively less profitable. However, the firm still prefers the bonus scheme for small interest rates but switches to the tournament for sufficiently large ones.

Secondly, it is worth pointing out that in modeling reputation I have made a restrictive assumption. Specifically, I have assumed that the firm cannot enter another employment contract after once reneging on the individual bonus contract. It is, however, plausible to assume that the firm can still contract with the agents using a rank-order tournament. Such an assumption indeed affects the firm's credibility constraint under the bonus contract. Particularly, her loss from reneging on the agreement becomes smaller. However, my results reestablish for this case. Specifically, as long as the credibility constraint is not binding, the individual bonus scheme still dominates the tournament as it entails smaller inequity premium costs. The interest rate for which the constraint becomes binding will, however, be smaller as a positive fallback profit decreases the right-hand side of the credibility constraint. Consequently, profits under the bonus contract will start to decrease for smaller interest rates compared to the case analyzed in the present paper. Yet the firm will switch to the tournament once the interest rate is such that profits under the bonus contract undercut those in the tournament. Indeed, that critical interest rate must then be smaller as well.

Appendix

Proofs for Section 3

Proof of symmetry and uniqueness of the Nash-equilibrium. The agents' respective first-order conditions are given by

$$p'(e|a)(1 + \alpha)\Delta = c'(e), \quad (32)$$

$$p'(a|e)(1 + \alpha)\Delta = c'(a). \quad (33)$$

Combining both equations implies

$$\frac{p'(e|a)}{p'(a|e)} = \frac{c'(e)}{c'(a)}. \quad (34)$$

Consider the case $e \leq a$. By equation (11), the marginal probabilities are then given by

$$p'(e|a) = \frac{1}{2} \frac{1}{a}, \quad (35)$$

$$p'(a|e) = \frac{1}{2} \frac{e}{a^2}. \quad (36)$$

Equation (34) thus becomes

$$\frac{a}{e} = \frac{c'(e)}{c'(a)}. \quad (37)$$

Reformulation yields

$$c'(a)a = c'(e)e. \quad (38)$$

Note that $c'(e)e$ is a monotonically increasing function of effort:

$$\frac{\partial (c'(e)e)}{\partial e} = c''(e)e + c'(e) > 0$$

Thus, equation (38) is satisfied if and only if $e = a$. Hence, the Nash-equilibrium is symmetric. It is also unique as

$$a = \arg \max_e EU_i(e, a). \quad (39)$$

The proof for the case $e \geq a$, is conducted equivalently by simply reversing the effort variables e, a . ■

Proofs for Section 4.1

Proof of symmetry and uniqueness of the Nash-equilibrium. Assume that condition (IntC) is satisfied. Both agents maximize their expected utility:

$$EU_i(e, a, z; \alpha) = A + p(e|z) \{1 + \alpha p(a|z)\} B - c(e) - \alpha p(a|z) B$$

The respective first-order conditions are given by

$$p'(e|z) \{1 + \alpha p(a|z)\} B - c'(e) = 0, \quad (40)$$

$$p'(a|z) \{1 + \alpha p(e|z)\} B - c'(a) = 0. \quad (41)$$

Combining both equations implies

$$\frac{c'(e)}{p'(e|z) (1 + \alpha p(a|z))} = \frac{c'(a)}{p'(a|z) (1 + \alpha p(e|z))} \quad (42)$$

$$\Leftrightarrow \frac{c'(e) (1 + \alpha p(e|z))}{p'(e|z)} = \frac{c'(a) (1 + \alpha p(a|z))}{p'(a|z)}. \quad (43)$$

Both sides of equation (43) represent a function of an agent's effort level:

$$\frac{c'(\cdot) (1 + \alpha p(\cdot))}{p'(\cdot)} \quad (44)$$

The above function is monotonically increasing in effort. To see this, consider the derivative of (44) with respect to effort:

$$\frac{(1 + \alpha p(\cdot)) p'(\cdot) c''(\cdot) + \alpha p'(\cdot)^2 c'(\cdot) - p''(\cdot) c'(\cdot) (1 + \alpha p(\cdot))}{p'(\cdot)^2}. \quad (45)$$

Note that for an interior solution to exist it must hold that $a > z$. As then $\alpha, p(a|z), p'(a|z), c''(a), c'(a) > 0$, and $p''(a|z) < 0$, expression (45) is strictly positive. Thus, equation (43) is satisfied if and only if $e = a$. Hence, the equilibrium is symmetric. Moreover, as

$$a = \arg \max_e EU_i(e, a, z; \alpha) \quad (46)$$

the equilibrium is also unique. ■

Proof of Lemma 3. The equation system (23) implies

$$\frac{c(a) a}{(a - z^*)} = \frac{c'(a) a^2}{z^*}. \quad (47)$$

Solving for z^* yields

$$z^*(a) = \frac{a^2 c'(a)}{c(a) + c'(a) a}. \quad (48)$$

By equation (16), the probability of receiving a bonus is then positive:

$$p(a|z^*) = 1 - \frac{z^*(a)}{a} \quad (49)$$

$$\Rightarrow p(a) = \frac{c(a)}{c(a) + c'(a)a} \quad (50)$$

The marginal probability of receiving a bonus becomes:

$$p'(a|z^*) = \frac{z^*(a)}{a^2} \quad (51)$$

$$\Rightarrow p'(a) = \frac{c'(a)}{c(a) + c'(a)a} \quad (52)$$

Substituting the above results into condition (ICB) yields the incentive-compatible bonus as given in (24):

$$B(a, z^*; \alpha) = \frac{c'(a)}{p'(a|z^*)(1 + \alpha p(a|z^*))} \quad (53)$$

$$\Rightarrow B^*(a) = \frac{(c(a) + c'(a)a)^2}{(1 + \alpha)c(a) + c'(a)a} \quad (54)$$

From the foregoing, the principal's per-worker cost function is

$$C^B(a, B^*, z^*; \alpha, \bar{u}) = c(a) + \alpha(1 - p(a|z^*))p(a|z^*)B^* + \bar{u}. \quad (55)$$

Substituting $B(a, z^*; \alpha)$ yields

$$C^B(a, z^*; \alpha, \bar{u}) = c(a) + \alpha c'(a) \cdot \frac{(1 - p(a|z^*))p(a|z^*)}{p'(a|z^*)\{1 + \alpha p(a|z^*)\}} + \bar{u}. \quad (56)$$

Plugging in $p(a|z^*)$ and $p'(a|z^*)$, the per-worker costs for implementing effort a become

$$C^B(a; \alpha, \bar{u}) = c(a) + \bar{u} + \alpha \frac{ac(a)c'(a)}{(1 + \alpha)c(a) + ac'(a)}. \quad (57)$$

Rearranging terms yields the expression given in equation (26). ■

Proof that wage costs are increasing in α . Differentiating equation

(57) wrt α yields a positive expression:

$$\frac{\partial C^B(a; \alpha, \bar{u})}{\partial \alpha} = ac(a)c'(a) \cdot \frac{c(a) + ac'(a)}{((1 + \alpha)c(a) + ac'(a))^2} \quad (58)$$

■

Proof of Proposition 2. The principal's profit maximization problem is given by

$$\max_a \Pi^B(a; \alpha, \bar{u}) = v(a) - c(a) - \bar{u} - \frac{\alpha}{1 + \alpha} \cdot \frac{ac'(a)c(a)}{c(a) + \frac{ac'(a)}{1 + \alpha}}. \quad (\text{AI})$$

For notational convenience, denote $\frac{ac'(a)c(a)}{c(a) + \frac{ac'(a)}{1 + \alpha}} = X(a)$. Then the first-order condition of the above problem yields:

$$v'(a_B^{**}) = c'(a_B^{**}) + \frac{\alpha}{1 + \alpha} \cdot X'(a_B^{**}) \quad (59)$$

For $\alpha = 0$, the equation reduces to $v'(a_B^{**}) = c'(a_B^{**})$ implying first-best effort levels $a_B^{**} = a^*$. For $\alpha > 0$, the last term of the above equation is given by

$$X'(a_B^{**}) = \frac{[c'c + ac''c + ac'c'] \left[c + \frac{ac'}{1 + \alpha} \right] - ac'c \left[c' + \frac{c' + ac''}{1 + \alpha} \right]}{\left[c + \frac{ac'}{1 + \alpha} \right]^2}, \quad (60)$$

where $c' = c'(a_B^{**})$ and $c = c(a_B^{**})$. Reformulation verifies that the term is strictly positive:

$$X'(a_B^{**}) = \frac{c'cc + ac''cc + ac'c' \frac{ac'}{1 + \alpha}}{\left[c + \frac{ac'}{1 + \alpha} \right]^2} > 0 \quad (61)$$

Due to the concavity of the value function $v(a)$ and strict convexity of the cost function $c(a)$, equation (59) is satisfied only for values $a_B^{**} < a^*$. Moreover, by inequality (58) wage costs and thus $\frac{\alpha}{1 + \alpha} \cdot X(a_B^{**})$ strictly increase in α . Using the envelope theorem, profits must consequently decrease in that parameter. ■

Proofs for Section 4.2

Proof of Lemma 4 ctd. As derived in the proof of Lemma 3 above, the equation system (23) consisting of the two binding constraints (IntC) and (ICB) implies

$$z^*(a) = \frac{a^2 c'(a)}{c(a) + c'(a)a}. \quad (62)$$

Differentiating this expression with respect to effort yields a positive expression:

$$\frac{\partial z^*(a)}{\partial a} = ac(a) \cdot \frac{ac''(a) + 2c'(a)}{(c(a) + ac'(a))^2} \quad (63)$$

Moreover, given that condition (IntC) is binding, the constraint (ICB) can be written as:

$$\frac{z^*}{a^2} \left\{ 1 + \alpha \left(1 - \frac{z^*}{a} \right) \right\} B^* - c'(a) = 0 \quad (64)$$

Substituting $z^*(a)$ implicitly yields the bonus B^* , implied by system (23):

$$\frac{c'(a)}{c(a) + c'(a)a} \left\{ 1 + \alpha \left(\frac{c(a)}{c(a) + c'(a)a} \right) \right\} B^* - c'(a) = 0 \quad (65)$$

Applying the implicit-function theorem yields the effect of a variation in effort a on the incentive-compatible bonus B^* :

$$\frac{\partial B^*}{\partial a} = - \frac{soc}{\frac{c'(a)}{c(a)+c'(a)a} \left\{ 1 + \alpha \left(\frac{c(a)}{c(a)+c'(a)a} \right) \right\}}, \quad (66)$$

where soc denotes the second-order condition of the agent's maximization problem. Assuming concavity of the utility function, that term must be negative. Given that the denominator is positive, also expression (66) is positive. Altogether, the effect of an increase in the induced effort level a on the bonus $B^*(a; \alpha)$ as well as on the performance standard $z^*(a)$ is thus positive. ■

Solutions to the model for $c(a) = \frac{1}{2}a^2$

Plugging in $c(a) = 0.5a^2$ in the solutions to the model variables given in the text yields the following values:

Solutions to the rank-order tournament.

$$\Delta(a) = \frac{2a^2}{(1+\alpha)} \quad (67)$$

$$C^T(a) = \left(\frac{1}{2} + \frac{\alpha}{1+\alpha}\right) a^2 + \bar{u} \quad (68)$$

Solutions to the individual bonus contract.

$$z^*(a) = \frac{2a}{3} \quad (69)$$

$$p(a) = \frac{1}{3} \quad (70)$$

$$p'(a) = \frac{2}{3a} \quad (71)$$

$$B^*(a) = \frac{9}{2} \frac{a^2}{3+\alpha} \quad (72)$$

$$C^B(a) = \left(\frac{1}{2} + \frac{\alpha}{3+\alpha}\right) a^2 + \bar{u} \quad (73)$$

Proofs for Section 5

Proof of Lemma 5. (i) With $\bar{u} = 0$, in the tournament, the principal's objective is:

$$\max_a \Pi^T(a; \alpha) = a - \left(\frac{1}{2} + \frac{\alpha}{1+\alpha}\right) a^2 \quad (\text{AII})$$

The first-order condition is given by

$$0 = 1 - 2 \left(\frac{1}{2} + \frac{\alpha}{1+\alpha}\right) a_T^{**}. \quad (74)$$

Reformulation directly yields $a_T^{**}(\alpha)$ as given in equation (29).

(ii) In the bonus scheme, given that (CC*) is not binding, the principal's objective is:

$$\max_a \Pi^B(a; \alpha) = a - \left(\frac{1}{2} + \frac{\alpha}{3+\alpha}\right) a^2 \quad (\text{AIII})$$

The first-order condition is given by:

$$0 = 1 - 2 \left(\frac{1}{2} + \frac{\alpha}{3 + \alpha} \right) a_B^{**}. \quad (75)$$

Reformulation directly yields the profit-maximizing effort level $a_B^{**}(\alpha)$ as given in equation (30). By Proposition 4, that effort level can be implemented only for values of $r \leq \hat{r}$. The interest rate \hat{r} is implicitly defined in equation (27):

$$\hat{r} B(a_B^{**}; \alpha) = \Pi^B(a_B^{**}; \alpha) \quad (76)$$

Calculating $B(a_B^{**}; \alpha)$ and $\Pi^B(a_B^{**}; \alpha)$ by plugging in $a = a_B^{**}$ in the functions given in problem (VII), and then solving equation (76) for \hat{r} yields

$$\hat{r} = \frac{1}{3}\alpha + \frac{1}{3}. \quad (77)$$

(iii) In the credibility-constrained bonus scheme, the maximal credibly implementable effort level a^c depends on r and is defined by:

$$r B(a^c; \alpha) = \Pi^B(a^c; \alpha) \quad (78)$$

Plugging in $B(\cdot)$ and $\Pi^B(\cdot)$ as given in problem (VII), the condition becomes:

$$r \frac{9}{2} \frac{(a^c)^2}{3 + \alpha} = a^c - \left(\frac{1}{2} + \frac{\alpha}{3 + \alpha} \right) (a^c)^2 \quad (79)$$

Reformulation yields $a^c(\alpha, r)$ as given in equation (31):

$$a^c(\alpha, r) = \left(\frac{1}{2} + \frac{\alpha + 4.5r}{3 + \alpha} \right)^{-1} = \left(1 + \frac{0.5\alpha + 4.5r - 1.5}{3 + \alpha} \right)^{-1} \quad (80)$$

■

The implicit function $r^S(\alpha)$. Given the calculations above, the switching point is implicitly defined by:

$$\Pi^B(a^c(\alpha, r^S); \alpha) = \Pi^T(a_T^{**}(\alpha); \alpha) \quad (81)$$

From Figure 7, recall that for any $\alpha > 0$, there are two values of r^S for which the above equation is satisfied; one left-hand and one right-hand of

the individual profit curve's maximum. However, only the larger of the two solutions is of interest as the smaller one undercuts \hat{r} and, consequently, does not constitute a credibility restriction of the individual bonus scheme. Plugging in the profit functions from problems (VII) and (VI), equation (81) becomes:

$$a^c(\alpha, r^S) - \left(\frac{1}{2} + \frac{\alpha}{3 + \alpha}\right) (a^c(\alpha, r^S))^2 = a_T^{**} - \left(\frac{1}{2} + \frac{\alpha}{1 + \alpha}\right) (a_T^{**})^2 \quad (82)$$

Plugging in $a^c(\alpha, r^S)$ and a_T^{**} as given in equations (31) and (29), implicitly defines $r^S(\alpha)$. Explicitly solving equation (82) for $r^S(\alpha)$ yields two solutions, the larger (and thus relevant) of which is given by:

$$r^S(\alpha) = \frac{1}{9\alpha + 9} \left(3 + 14\alpha + 3\alpha^2 + 4\sqrt{\alpha(3 + \alpha)(1 + 3\alpha)}\right) \quad (83)$$

Differentiating r^S with respect to α yields a cumbersome but clearly positive expression; i.e. $\frac{\partial r^S}{\partial \alpha} > 0$:

$$\frac{\partial r^S}{\partial \alpha} = \frac{\left(6 + 34\alpha + 18\alpha^2 + 6\alpha^3 + \sqrt{\alpha(3 + \alpha)(1 + 3\alpha)}(11 + 6\alpha + 3\alpha^2)\right)}{9(1 + \alpha)^2 \sqrt{\alpha(3 + \alpha)(1 + 3\alpha)}} \quad (84)$$

■

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