The role of mobility in tax and subsidy competition*

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Abstract

In this paper, we analyse the role of mobility in tax and subsidy competition. Our primary result is that increasing ‘relocation’ mobility of firms leads to increasing ‘net’ tax revenues under fairly weak conditions. While enhanced relocation mobility intensifies tax competition, it weakens subsidy competition. The resulting fall in the governments’ subsidy payments overcompensates the decline in tax revenues, leading to a rise in net tax revenues. We derive this conclusion in a model in which two governments are first engaged in subsidy competition and thereafter in tax competition, and firms locate and potentially relocate in response to the two political choices.

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1 Motivation

In this paper, we analyse the role of mobility in international tax and subsidy competition for firms. More specifically, we distinguish between two different concepts of mobility - ‘location’ and ‘relocation’ mobility. The first concept, location mobility, refers to the additional costs that accrue to investors when they set up a new firm or plant in a foreign country rather than in their home country. The second concept, relocation mobility, refers to the costs that arise when an already established firm or plant moves to another jurisdiction. These two types of mobility jointly shape the countries’ subsidy and tax competition. They thus affect each country’s ‘net’ tax revenues, defined as the difference between a government’s tax revenues and its subsidy payments.

Our primary result is that increasing relocation mobility leads to increasing net tax revenues under fairly weak conditions. We derive this conclusion in a four-stage model in which two symmetric jurisdictions compete for firms with subsidies and taxes, each aiming at maximising its net revenues. In the first stage, the non-cooperative governments simultaneously set subsidies for attracting investors. In the second stage, the investors decide where they will set up their firms and receive subsidies. After subsidies have been phased out, in the third stage governments simultaneously choose corporate taxes. In the fourth stage, firms decide whether to stay or to relocate, and pay taxes accordingly.

A key feature of the model is that investors face location costs in the second stage, reflecting imperfect location mobility, and relocation costs in the fourth stage, reflecting imperfect relocation mobility. The location costs, i.e., the cost disadvantage from investing abroad, imply that investors are, on average, home biased. This is an empirically well established result (e.g., French and Poterba, 1991; Lewis, 1999; Pinkowitz et al., 2001). The relocation costs imply that firms are, in general, ‘locked in’ once they are operating in a country because, for instance, they develop ties with the regional economy and acquire location-specific knowledge. Reversing the initial location choice is possible but costly. The resulting lock-in effect allows governments to levy higher taxes on firms than is otherwise possible, and it provides incentives to pay subsidies to attract new firms in the first place.

Surprisingly, a decline in relocation costs leads to a rise in net revenues in the two countries under ‘reasonable’ assumptions although it weakens the lock-in effect and intensifies tax competition. This outcome occurs because the induced fall in taxes weakens the preceding subsidy competition and is more than offset by the resulting decline in subsidy payments. By contrast, a decline in location costs negatively affects each country’s net revenues, since it intensifies subsidy competition without weakening tax competition. It thus increases government payments without
enhancing revenues.

Distinguishing between location and relocation costs allows us to disentangle the different channels through which the different types of mobility affect net tax revenues. This is particularly important, because we cannot expect the two types of mobility costs to decrease in line with one another, since the decline in location costs is at least partly driven by forces other than those which determine the decline in relocation costs. We now briefly illustrate this point.

Let us first look at the initial location choice. Investors are, on average, home biased. For a variety of reasons, they prefer to set up new firms or plants in their home region. There are, for instance, international information asymmetries which mean that even large investors are simply better informed about the economic and legal conditions at home than abroad, and this leads to higher transaction costs and greater uncertainties for foreign direct investments (FDIs). This feature is captured by our location costs.

These costs, however, have been decreasing in recent years. International legal and economic harmonisation, the progress of communication and information technologies, and the liberalisation of the world capital markets are the main reasons for this decline. All these measures make the international movement of financial capital less costly and less risky, thereby facilitating foreign investments.

Next, let us consider briefly the relocation choice. Relocation is an option, but it causes substantial opportunity costs. A firm often forges strong links with local business networks and suppliers and acquires location-specific knowledge once it has become established in a region. Local links and knowledge are both worthless in the case of relocation. Also, relocation requires not only the transfer of financial capital, but also the movement of real capital goods and human capital, which is particularly costly.

Nevertheless, we argue that the relocation costs have also been declining over time. Consider the case of a smaller high-tech or services firm initially located in, say, the Netherlands.¹ The main assets of such smaller firms in the high-tech and services sectors are often their highly skilled employees with a very product-specific know-how, who cannot easily be replaced. In this case, the introduction of the common European labour market substantially reduced the costs of relocating such a firm, including its key employees, to adjacent Belgium. Additionally, the development of modern communication and transportation technologies and the internationalisation

¹This firm might be an academic or corporate spin-off, or a ‘regular’ start-up. In the late 1990s, almost 1.8 million start-ups were established in eight European OECD countries in one year, compared to approximately 1.1 million closures. About 230,000 of the start-ups were corporate spin-offs. See Moncada-Paternò-Castello et al. (2000) and, for further discussion on spin-offs from public sector research institutions, Callan (2001).
of the former national economies have been diminishing the role of the established local networks.

Alternatively, consider the case of large firms in the semiconductor industry. Here the pace of the technological progress has, in some sense, substantially reduced relocation costs. In this industry, the development has been so dynamic that product life cycles are nowadays extremely short. They are, in fact, now measured in months (cf. Henisz and Macher, 2004). Consequently, new production lines are set up very frequently, for example, in order to produce a new generation of microprocessors. Once production facilities have to anyway be replaced, it is only a small step to relocate, or rather replace, the entire factory. In this sense, the relocation costs have been declining as a result of the accelerating speed of technological innovations. These costs are, in general, still positive, given the partial loss of a skilled workforce and the other downsides of relocation. But the crucial point here is the general downward trend.

The semiconductor industry also provides a striking example for the relevance of considering subsidy competition along with tax competition. Subsidy payments to this industry are common (Henisz and Macher, 2004). For instance, the *AMD Fab 36* project in Dresden in 2003 was officially subsidised by almost €550 million (cf. Grundig et al., 2008). Now, only a few years later, the future of AMD in Dresden is very uncertain as a result of low relocation costs. Politicians and the public, having noticed that the lock-in effect is much weaker than initially thought, are more and more critical of such subsidies. And this is in line with our model.2

Our paper is related to the ‘tax holiday’ literature. In this strand of literature, governments initially grant tax holidays, or upfront subsidies, to attract foreign direct investments and to compensate firms for high time-consistent taxes in the future (e.g., Bond and Samuelson, 1986; Doyle and van Wijnbergen, 1994; Janeba, 2002; Marjit et al., 1999; Thomas and Worrall, 1994). The resulting policy outcome in these papers, i.e., subsidies or low taxes initially followed by high taxes, is similar to our subsidy and tax structure. But, unlike these papers, we analyse the impact of changes in location mobility and relocation mobility on net tax revenues. We also examine how the mobility of firms affects the strategic interactions between the governments in the subsidy and tax stages. By contrast, the articles referred to cannot explore this issue, as they either consider the unilateral policies of a single

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2The German-based semiconductor memory producer Qimonda in its 2006 IPO prospectus explicitly mentions that “[r]eductions in the amount of government subsidies we receive or demands for repayment could increase our reported expenses. (…) The availability of government subsidies is largely outside our control. (…) As a general rule, we believe that government subsidies are becoming less available in each of the countries in which we have received funding in the past” (Qimonda, 2006, pp. 26-27).
host country or assume a large number of potential host countries, thus excluding strategic interactions from the outset.\textsuperscript{3}

Like our paper, the literature on tax competition in models of the ‘new economic geography’ raises some doubts about whether increasing economic integration necessarily erodes government revenues (for instance, Baldwin and Krugman, 2004; Borck and Pflüger, 2006; Kind et al., 2000). In this strand of literature, the arguments hinge on the presence of significant agglomeration economies, which are totally absent in our framework. By contrast, our conclusion that rising relocation mobility does not harm the governments’ budgets follows from the interaction between tax and subsidy competition, which is not considered in the ‘new economic geography’ literature.\textsuperscript{4}

Konrad and Kovenock (2008) is related to both the tax holiday and the new economic geography literature. They analyse tax competition for ‘overlapping FDIs’ in a dynamic model with agglomeration advantages. The vintage property of the FDI prevents a ruinous race to the bottom as long as governments only have non-discriminatory taxes at their disposal. But if governments can also offer subsidies to new FDI, international competition will again be “cut-throat in nature.” Konrad and Kovenock (2008), however, are not interested in the implications of increasing mobility. By contrast, we analyse how rising location and relocation mobility re-shapes tax and subsidy competition, and how it ultimately affects net tax revenues.

Our paper proceeds as follows. In Section 2, the basic set up of the model is presented. Section 3 investigates the outcome of the subsidy and tax competition stages. We analyse the effects of increasing location and relocation mobility on net revenues in Section 4. Section 5 concludes with a brief discussion of some policy implications.

2 Governments and Firms

We start by presenting our two-period, four-stage, model of tax and subsidy competition for imperfectly mobile firms. In the first period (consisting of the first and

\textsuperscript{3}Haufler and Wooton (2006) analyse regional tax and subsidy coordination within an economic union when the two members of this union compete with a third country. In their model, however, each government has only one policy instrument at its disposal, which can be either a subsidy or a corporate tax. Their paper thus differs considerably from the tax holiday literature and from our contribution.

\textsuperscript{4}Wilson (2005) provides another argument that explains why tax competition can be welfare-enhancing. In his model, the presence of tax competition implies that selfish government officials intensify their efforts in expenditure competition in order to attract mobile capital, and this second type of competition makes residents better off by reducing government “waste”.

4
second stages; see below), the governments of two jurisdictions grant subsidies to attract investors non-cooperatively. Given these subsidies, investors then decide which country they will set up their firms in. In the second period (consisting of the third and fourth stages), the two governments levy corporate taxes. Since the firms are now established in a country, they are locked-in, but only imperfectly, as we will explain in more detail below. Firms can still relocate in response to the tax policies of the jurisdictions. So there is competition for mobile firms in both periods, albeit to a different degree.

Our framework draws on Haupt and Peters (2005). They, however, deal with tax competition only. But since their model is very tractable, we can enrich the tax competition stages and, more importantly, incorporate the new subsidy competition stages. Let us now look at the model in more detail.

**Firms** Consider two symmetric countries, $A$ and $B$. In each of these jurisdictions, there is a continuum of home investors, normalised to 1. Here, the term ‘home’ refers to the fact that there are already some links between investors and a country. For instance, the investors might simply reside in this country.

Each of the investors sets up a single firm. Despite these existing links, firms can initially be located either in the investors’ home country or abroad. A firm’s set up costs that occur in the first period are $c$ if it stays in its home country, and $c + m_1$ if it moves abroad. While all firms face identical cost components $c$, they differ with respect to their $m_1$. (For notational convenience, firm indices are not used.) We label the location costs $m_1$ and interpret them as the mobility costs or the cost disadvantage of investing abroad in the first period. This characteristic is distributed according to the distribution function $F_1(m_1)$, whose properties are described below.

In the second period, each firm realises the (gross) return $\pi$ if it continues to stay in the country where it was established in the first period. Its return is $\pi - m_2$ if it relocates in the second period. Again, $\pi$ is the same for all firms, while the component $m_2$ differs across firms. We label the relocation costs $m_2$ and interpret them as the mobility costs or the cost disadvantage of relocating in the second period.\footnote{Relocation costs $m_2$ can contain a cost component $c_2$ that is identical for all relocation firms. For simplicity, we ignore such a cost component. Analogously, we could set $c$ equal to zero.} Denote the ‘number’ or, more correctly, mass of firms which locate in jurisdiction $i$ in period 1 by $N_i$. Then, the characteristic $m_2$ is distributed across these $N_i$ firms according to the new distribution function $F_2(m_2)$.

The distribution functions $F_1(m_1)$ and $F_2(m_2)$ are twice continuously differentiable and strictly increasing functions over the intervals $[m_1, \bar{m}_1]$ and $[m_2, \bar{m}_2]$,
respectively. They fulfil

Assumption 1:

(i) \( F_k(m_k) = 0 \) and \( F_k(\bar{m}_k) = 1 \), \( k = 1, 2 \), (ii) \( m_k < 0 < \bar{m}_k \), (iii) \( F_k(0) < 0.5 \), (iv) \( m_1 < m_2 \) and \( \bar{m}_1 < \bar{m}_2 \), (v) \( F_1(m) > F_2(m) \) for all \( m \in (m_1, \bar{m}_2) \), (vi) \( F'_k(m_k) \in \left( -2 \left( F'_k(m_k) \right)^2 / \left[ 1 - F_k(m_k) \right] , 2 \left( F'_k(m_k) \right)^2 / F_k(m_k) \right) \).

Properties (i) and (ii) restrict the relevant domains of the distribution functions, allowing for both positive and negative values of \( m_1 \) and \( m_2 \). In most cases, set up costs are lower in an investor’s home region, since investors are more familiar with their domestic business environment than with the foreign one. This situation corresponds with a positive \( m_1 \). But for some firms, set up costs are lower abroad. They might be able to take advantage of a particularly specialised foreign labour force. Or entrepreneurs might be able to make profitable use of their business ideas only in very specific places. For instance, a fashion label might be successful only in cities such as New York or Paris. These cases are captured by a negative \( m_1 \). Property (iii), however, implies that the set up costs of the majority of firms indeed favour their home country. Similarly, relocation costs \( m_2 \) are positive for the majority of firms. For instance, relocation after the start up phase causes the loss of immobile input factors and regional networks built up in the first period. This relocation costs, however, need not be prohibitive. Firms are thus only imperfectly locked in. Moreover, some firms might even benefit from relocating and thus increase their returns. They might, for instance, be closer to clients or suppliers.

Properties (iv) and (v) are most important for our analysis. They capture the feature that second period mobility costs \( m_2 \) exceed first period mobility costs \( m_1 \), meaning that distribution function \( F_2 \) lies to the right of \( F_1 \), as illustrated in figure 1. In other words, firms become decreasingly mobile over their life span. This ‘natural’ assumption reflects the imperfect lock-in effect once a firm is located in a country. It drives our results. By contrast, the properties \( m_1 < 0 \) and \( m_2 < 0 \) are not important for our economic mechanisms. In fact, our results would go through with \( m_1 = m_2 = 0 \). \(^6\)

Finally, property (vi) is a ‘purely technical’ restriction on the density functions’ slopes that guarantees well-behaved objective functions. A uniform distribution, among others, fulfils this condition.

The functions \( F_1 \) and \( F_2 \) are common knowledge. Each firm learns about the realisation of its specific location costs \( m_1 \) and relocation costs \( m_2 \) before it makes its location decision in the first period and its relocation decision in the second period, \(^6\)The ‘technical’ advantage of allowing negative mobility costs is that the distribution functions, and thus the governments’ objective functions below, are ‘smooth’ for a wider range of tax and subsidy differentials. This simplifies our proofs.
respectively. For simplicity, we assume that a firm’s first period and second period mobility costs are not correlated. This assumption enables us to put forward our arguments as simply as possible.\footnote{In fact, it is far from clear whether location and relocation costs are correlated. Take the example of a large, internationally experienced, investor. The location costs of this investor can be minor. But if it sets up a steel factory, the relocation costs will be substantial - if not prohibitive. Low location costs do not imply low relocation costs, and vice versa.}

**Governments** When competing for mobile firms, the non-cooperative governments have subsidies and corporate taxes at their disposal. Subsidies are used in period 1, while taxes are levied in period 2. Governments can implement preferential subsidy and tax regimes. That is, in each country subsidies would then be different for firms of home investors that receive subsidy $s^i_n$, and ‘incoming’ firms of foreign investors that receive subsidy $s^m_i$, where $i = A, B$. Similarly, governments might set differentiated taxes. Firms that have already had their subsidised start up phase in country $i$ then pay tax $t^i_n$, while those firms that relocate ‘newly’ to country $i$ in the second period pay tax $t^m_i$.\footnote{A firm is ‘domestic’ in the country where it is set up, and it is taxed accordingly in the second period. At this stage, a government discriminates between domestic and foreign firms, i.e., according to the firms’ initial location, but it treats all domestically set up firms equally. Importantly, in our setting, there are no incentives for governments to discriminate between domestic firms - as defined above - according to the home base of their investors.}
Figure 2: The timing of decisions

**Objectives and Timing** Each country maximises its ‘net’ revenues $NR_i$, i.e., the difference between tax revenues $R_i$ and subsidy payments $P_i$, given the decisions of its opponent. As usual, investors maximise the net profits of their firms, taking into account (gross) return $\pi$, set up costs $c$, firm specific mobility costs $m_1$ and $m_2$, subsidies $s^n_i$ and $s^m_i$, and taxes $t^n_i$ and $t^m_i$.

The precise timing of the subsidy and tax competition game between the two governments is as follows. In the first stage, the non-cooperative governments simultaneously set subsidies $s^n_A$, $s^m_A$, $s^n_B$ and $s^m_B$. Given these subsidies, in the second stage investors decide whether their firms locate and receive subsidies in either country $A$ or country $B$. In the third stage, the governments simultaneously set their taxes $t^n_A$, $t^m_A$, $t^n_B$ and $t^m_B$, again non-cooperatively. In the fourth stage, firms decide whether they stay or relocate, and pay their taxes accordingly.

This decision structure is illustrated in figure 2. In terms of time periods, the first two stages can be interpreted as constituting period 1, the third and fourth stages as constituting period 2. As mentioned above, the specific location costs for each firm are revealed prior to the location decision at the beginning of the second stage. Similarly, the relocation costs are revealed to each firm prior to the relocation decision at the beginning of the fourth stage. The distribution of these costs is common knowledge.

## 3 Subsidy and tax competition

As usual, we solve our model by backward induction, starting with the tax competition stages and then going on the subsidy competition stages.

### 3.1 Tax competition

The firms’ decisions in the fourth stage are straightforward. A firm that was set up in region $i$ in the first period can stay in this region and receive net return $\pi - t^n_i$.
(first period costs and subsidies are sunk at this stage). Alternatively it can move
to region \( j \) and gain the net return \( \pi - m_2 - t_j^m \). A profit maximising firm thus stays
in region \( i \) (relocates to region \( j \)) if, and only if,
\[
m_2 \geq t_i^n - t_j^m \quad (m_2 < t_i^n - t_j^m),
\]
i.e., if, and only if, the tax differential between the countries is smaller (strictly
larger) than the firm specific relocation costs. Consequently, the share of firms
relocating from region \( i \) to \( j \) is \( F_2(t_i^n - t_j^m) \).

Then the tax revenues of government \( i \) are
\[
R_i(t_i^n, t_j^m) = t_i^n \left[ 1 - F_2(t_i^n - t_j^m) \right] N_i + t_i^m F_2(t_j^m - t_i^n) N_j,
\]
where \( N_i \) and \( N_j \) result from the firms’ decisions in the second stage. The first term
on the right-hand side captures the tax revenues from all firms that were already
located in country \( i \) in the first period (indicated by \( N_i \)) and stay there in the second
period.\(^9\) By contrast, the second term refers to the revenues from those firms that
were initially located in country \( j \) (indicated by \( N_j \)) and only enter country \( i \) in the
second period.

In the third stage, government \( i \) chooses taxes \( t_i^n \) and \( t_i^m \) that maximise revenues
\( R_i \), given the choices of its competitor (previous subsidy payments \( P_i \) are sunk at
this stage). The optimal taxes are characterised by the first-order conditions
\[
\frac{\partial R_i}{\partial t_i^n} = 0 \iff \varepsilon_i^n := \frac{F_2'(t_i^n - t_j^m) t_i^n}{1 - F_2(t_i^n - t_j^m)} = 1, \quad (3)
\]
\[
\frac{\partial R_i}{\partial t_i^m} = 0 \iff \varepsilon_i^m := \frac{F_2'(t_j^m - t_i^n) t_i^m}{F_2(t_j^m - t_i^n)} = 1, \quad (4)
\]
where \( \varepsilon_i^n \) and \( \varepsilon_i^m \) denote the elasticities of the respective tax bases. These elasticity
rules reflect the traditional trade-off: a higher tax rate increases the revenues from
the firms ultimately located in country \( i \), but reduces the number of those firms.

The first-order conditions (3) and (4) give the governments’ reaction functions
implicitly. The resultant equilibrium taxes are symmetric, i.e., \( t_A^n = t_B^n =: t^n \) and
\( t_A^m = t_B^m =: t^m \), and given by
\[
t^n = \frac{1 - F_2(t^n - t^m)}{F_2'(t^n - t^m)} \quad \text{and} \quad t^m = \frac{F_2(t^n - t^m)}{F_2'(t^n - t^m)}, \quad (5)
\]
yielding a positive tax differential\(^10\)
\[
t^n - t^m = \frac{1 - 2F_2(t^n - t^m)}{F_2'(t^n - t^m)} =: \Delta t > 0. \quad (6)
\]
\(^9\)Recall that function \( F_2 \) characterises the distribution of relocation costs of all firms whose start
up phase was in the same country, independent of their original home region.
\(^10\)We can exclude \( t^n - t^m < 0 \), since this implies \( F_2(t^n - t^m) < 0.5 \) and thus
\([1 - 2F_2(t^n - t^m)]/F_2'(t^n - t^m) > 0 \), which is obviously a contradiction. Therefore, \( t^n - t^m > 0 \)
results (see Haupt and Peters, 2005).
These solutions contain two important conclusions. First, government $i$’s tax on firms already established in country $i$ in the first period exceeds the tax on firms that move to region $i$ only in the second period, i.e., $t^n > t^m$. This tax differential arises because firms are locked in, at least imperfectly, once they have settled in a country. Since firms respond less elastically to an increase in the ‘domestic’ tax $t^n$ than to one in the ‘foreign’ tax $t^m$, they end up with higher tax payments if they stick to their initial location choice.

Second, taxes are independent of the number of firms $N_i$ and thus independent of subsidies. By contrast, the optimal subsidies in the first stage are shaped by the future taxes, as will soon become evident. In this sense, there is a one-way link between tax and subsidy competition.

The equilibrium values (5) and (6) are analogous to the results in Haupt and Peters (2005). We derive these results in a more general setting than Haupt and Peters (2005) with respect to mobility. More importantly, they only consider tax competition and completely ignore subsidy competition while we are interested precisely in the relationship between tax and subsidy competition, and we analyse the resulting net revenues. Let us therefore turn next to the subsidy competition between the governments.

### 3.2 Subsidy competition

Since the tax $t^n_A (t^n_B)$ is equal to $t^n_B (t^n_B)$, and since the distributions of migration costs $m_2$ are the same in the two countries, a firm’s expected performance in the second period is independent of its location in the first period. The location choice in the second stage, however, affects a firm’s overall net profit through its location costs and received subsidy. A home investor of country $i$ has net costs of $c - s^n_i (c + m_1 - s^m_j)$ in the first period if its firm is set up in country $i$ (country $j$). This firm is thus located in country $i$ (country $j$) in the second stage if, and only if,

$$m_1 \geq s^m_j - s^n_i \quad (m_1 < s^m_j - s^n_i), \quad (7)$$

i.e., if, and only if, the subsidy differential between the countries is smaller (strictly larger) than the firm specific location costs. The resultant share of $i$’s investors who locate their firms in country $j$ is $F_1(s^m_j - s^n_i)$. Consequently, the number of firms established in country $i$ is

$$N_i = [1 - F_1(s^m_j - s^n_i)] + F_1(s^m_i - s^n_j), \quad \text{where} \quad H_i = 1 - H_j \quad \text{and} \quad (8)$$

where $H_i$ is the number of $i$’s investors setting up their firms in country $i$ and $(1 - H_j)$ is the number of $j$’s investors locating their firms in country $i$.  


In the first stage, each government chooses its subsidies $s_i^n$ and $s_i^m$, given the subsidies of its opponent. Government $i$ maximises its net revenues

$$NR_i = \Omega^n[H_i + (1 - H_j)] + \Omega^m[(1 - H_i) + H_j] - s_i^nH_i - s_i^m(1 - H_j),$$

where $\Omega^n := t^n[1 - F_2(t^n - t^m)]$ and $\Omega^m := t^mF_2(t^n - t^m)$. The first two terms on the right-hand side capture future tax revenues while the third and the fourth term give the subsidy payments to home and foreign investors.

The optimal subsidies are given by the first-order conditions

$$\frac{dNR_i}{ds_i^n} = -[1 - F_1(s_j^n - s_i^n)] + [(\Omega^n - \Omega^m) - s_i^n]F'_1(s_j^n - s_i^n) = 0,$$

$$\frac{dNR_i}{ds_i^m} = -F_1(s_i^m - s_j^n) + [(\Omega^n - \Omega^m) - s_i^n]F'_1(s_i^m - s_j^n) = 0.$$  

A marginal rise in the subsidies $s_i^n$ and $s_i^m$ increases government spending by the number of recipients $H_i$ and $1 - H_j$, respectively. This negative effect of today’s subsidies on net revenues is captured by the first term of each of the two derivatives.

By contrast, the second terms show the positive impact of today’s subsidies on future revenues. Note that government $i$’s expected future tax revenue from a firm is $\Omega^n$ if this firm is set up in country $i$, but only $\Omega^m$ if the firm is set up in country $j$. Using (5) and (6), the expected revenue differential is

$$\Omega^n - \Omega^m = t^n - t^m > 0.$$  

That is, country $i$’s revenue increase caused by attracting an additional investor in the first period is exactly equal to the positive tax differential. Taking into account the subsidy payments, the net benefit of attracting an additional home and foreign investor is $(t^n - t^m) - s_i^n$ and $(t^n - t^m) - s_i^m$, respectively. Finally, the derivatives $F'_i(s_j^n - s_i^n)$ and $F'_i(s_i^m - s_j^n)$ tell us how the number of firms established in country $i$ changes in response to a marginal rise in subsidies $s_i^n$ and $s_i^m$.

There is also an alternative interpretation of the optimality conditions. Defining hypothetical taxes $\tau_i^n := (t^n - t^m) - s_i^n$ and $\tau_i^m := (t^n - t^m) - s_i^m$, we can reformulate the first-order conditions (10) and (11):

$$\eta_i^n := \frac{F'_i(\tau_i^n - \tau_j^n)\tau_i^n}{1 - F_1(\tau_i^n - \tau_j^n)} = 1 \quad \text{and} \quad \eta_i^m := \frac{F'_i(\tau_i^m - \tau_j^m)\tau_i^m}{F_1(\tau_i^m - \tau_j^m)} = 1.$$  

The similarity between the elasticity rules (3) and (4) on the one hand and (13) on the other hand is striking and proves to be convenient later on.

From the first-order conditions, the equilibrium subsidies and hypothetical taxes follow immediately. Not surprisingly, the solution is symmetric, i.e., $s_A^n = s_B^n =: s^n$,
These equilibrium values have a straightforward interpretation. If there were no tax differential $\Delta t$, firms would have had to pay the hypothetical taxes $\tau^n$ and $\tau^m$ in the first period (cf. equilibrium taxes (5)). This tax is ‘cut’ by the expected revenue differential (12). In this sense, governments give up current revenues for the benefit of having future ones. But only if the future gain $t^n - t^m$ strictly exceeds the hypothetical tax $\tau^n$ or $\tau^m$, will the subsidy indeed be positive. This outcome, in turn, requires a sufficiently strong lock-in effect.

In any case, the equilibrium levels (14) and (15) directly imply a positive subsidy and hypothetical tax differential

$$s^m - s^n = \tau^n - \tau^m = \frac{1 - 2F_1(\tau^n - \tau^m)}{F'_1(\tau^n - \tau^m)} =: \Delta \tau > 0. \quad (16)$$

Each government grants a higher subsidy to foreign investor than to domestic ones. This preferential treatment reflects the initial home bias and corresponds to our previous result (cf. tax differential (6)). Since investors respond less elastically to subsidy changes at home than to those abroad, they receive less public support for setting up their firms in their home country than for doing the same thing in the other country.\(^{11}\)

We have so far side-stepped the more technical topics of existence and uniqueness of the equilibrium. These issues are taken up in

**Lemma 1** Tax and subsidy competition.

There exists a unique subgame perfect equilibrium. This equilibrium is given by (5), (6), (14), (15), and (16). Moreover, $N_i = N_j = 1$ results.

**Proof:** See Appendix. \(\square\)

4 Net tax revenues and mobility

We now turn to our key issue, the relationship between mobility and net revenues. To analyse the emerging links, we first consider in more detail the net revenues in equilibrium.

\(^{11}\)Alternatively, the differential (16) can be explained in terms of hypothetical taxes.
4.1 Net tax revenues

Using the equilibrium values (5), (12), (14) and (15), each country’s net revenues can be expressed as

\[ NR = \left( \frac{\Delta t}{\text{rev diff}} + 2^m F_2(\Delta t) \right) - \left[ \frac{\Delta t}{\text{hyp sub}} - \left( \tau^n (1 - F_1(\Delta \tau)) + \tau^m F_1(\Delta \tau) \right) \right]. \]  

(17)

The revenues can be decomposed into two elements. First, the basic revenues give the tax revenues that would occur in a country if no firm had been located there in the first period. In this case, all firms would be set up in the other country, but the share \( F_2(\Delta t) \) would relocate in the second period, generating revenue \( t^m 2 F_2(\Delta t) \).

Second, the revenue differential (\( \text{rev diff} \)) captures the additional revenues that arise because some firms are initially set up in the respective country and thus pay higher taxes due to the lock-in effect.

The subsidy payments can also be split up into two components: First, the hypothetical tax payments reflect the tax revenues that would result in the first period in the absence of any lock-in effects. In the case of \( \Delta t = 0 \), countries would tax firms similarly in the two periods, as the optimality conditions (3) and (4) on the one hand and (13) on the other hand show. The similarity becomes even more evident if we express the hypothetical tax payments as \( \Delta \tau + 2^m F_1(\Delta \tau) \) and compare these formulation with revenues \( R \).\(^{12}\)

Second, there are hypothetical subsidy payments (\( \text{hyp sub} \)) that reduce these hypothetical tax payments in order to attract firms. This second element – which eventually gives rise to positive real subsidies – constitutes each government’s opportunity costs of attracting firms and generating the revenue differential. These opportunity costs are, in equilibrium, equal to the revenue differential. That is, the costs and benefits of attracting firms exactly cancel out. We refer to this outcome as the What-You-Give-Is-What-You-Get (WYGIWYG) principle. Taking WYGIWYG into account, net revenues are

\[ NR = 2^m F_2(\Delta t) + \tau^n (1 - F_1(\Delta \tau)) + \tau^m F_1(\Delta \tau). \]  

(18)

With this simple expression, investigating the impact of mobility on net revenues is straightforward. We distinguish between increasing location mobility and increasing relocation mobility. This distinction proves to be crucial.

\(^{12}\)Using eqs. (14), (15) and (16), we can rearrange the hypothetical tax payments: \( \tau^n (1 - F_1) + \tau^m F_1 = (1 - F_1)^2/F_1' - F_1^2/F_1' + 2^m F_1 = \Delta \tau + 2^m F_1. \)
4.2 Net tax revenues and relocation mobility

In this section, we look at the implications of increasing relocation mobility for net revenues. As already argued above, even firms that are well established in a country are for various reasons becoming more and more mobile. In our model, the increase of mobility comes as a reduction in the firms’ relocation costs. More specifically, we capture the rise in mobility as a change in the value of the distribution function $F_2(\Delta t; z_2)$ in equilibrium $(t^n, t^m)$ which is formally caused by a marginal increase in a parameter $z_2$. In particular, we start by considering

Scenario 1: $dF_2(\Delta t; z_2)/dz_2 > 0$ and $dF_2'(\Delta t; z_2)/dz_2 = 0$

at the ‘old’ equilibrium level $\Delta t$. We stick, for convenience, to our notation $F' = \partial F/\partial \Delta t$, $F'' = \partial^2 F/\partial \Delta t^2$, etc. All derivatives with respect to the parameter $z_2$ are explicitly expressed as $dF/dz_2$, etc.

Scenario 1 means that we consider an upward shift of the distribution curve that leaves its slope, i.e., the density $F_2'$, at the ‘old’ equilibrium level $\Delta t$ unaltered, as illustrated in figure 3. The corresponding rise in mobility weakens the lock-in effect. Since established firms are more inclined to relocate and to respond more elastically to international tax differentials, the old tax differential $\Delta t$ cannot be maintained. In this sense, tax competition is intensified and erodes the revenue differential in equation (17).

Nevertheless, this revenue differential is always identical in magnitude to the hypothetical subsidy, as the WYGIWYG principle stresses. That is, any decline in the revenue differential does not matter, since it is matched by an equal fall in sub-
sidy payments. Attracting firms in the first period is simply less beneficial if these firms are more mobile and pay fewer taxes in the second period. Consequently, subsidy competition is reduced. All that ultimately matters is the impact of relocation mobility on basic revenues, as reflected in the derivative\textsuperscript{13}

\[
\frac{dNR}{dz_2} = 2t^m \frac{dF_2(\Delta t; z_2)}{dz_2} + 2t^m F_2(\Delta t; z_2) \frac{dt^n}{dz_2}. \tag{19}
\]

The first term on the right-hand side captures the direct effect of increasing mobility in the second period. For given taxes \(t^n\) and \(t^m\), the number of relocating firms \(F_2(\Delta t; z_2)\) rises, since the lock-in effect is weakened. This positive effect on country \(i\)’s ‘basic’ tax base drives net revenues up.

The second term shows the indirect effect of increasing relocation mobility through the tax change in equilibrium. If the tax \(t^n\) decreases (increases) with mobility parameter \(z_2\), revenues decline (further increase), thus counteracting (reinforcing) the direct effect. In general, the outcome is undetermined. The slope of the density function – or, equivalently, the curvature of the distribution function – turns out to be decisive, as Proposition 1 states more precisely.

**Proposition 1** Net tax revenues and relocation mobility.

*In scenario 1, the net revenues \(NR\) increase (decrease) with the firms’ mobility parameter \(z_2\) if the density function’s slope \(F_2''(\Delta t; z_2)\) is greater (smaller) than the threshold level \(\gamma_2\):

\[
\frac{dNR}{dz_2} \gtrless 0 \iff F_2''(\Delta t; z_2) \gtrless -\left[\frac{F_2'(\Delta t; z_2)}{1 - F_2(\Delta t; z_2)}\right]^2 := \gamma_2 < 0. \tag{20}
\]

**Proof:** See appendix. \(\Box\)

To understand the role of the density function’s slope, recall that the tax differential drops in response to greater mobility. Consequently, the density \(F_2'\) increases (decreases) at the new equilibrium tax differential if \(F_2'' < 0\) (\(F_2'' > 0\)) holds. That is, the tax base becomes more (less) elastic. This leads to lower (higher) taxes, as a glance at the optimality conditions (3) and (4) and the equilibrium values (5) shows. If the density function’s slope \(F_2''\) is below the critical value \(\gamma_2\), tax \(t^n\) falls so drastically that net revenues decline. By contrast, for moderately negative or positive values of \(F_2''\), net revenues rise.\textsuperscript{14}

\textsuperscript{13}Here, we made use of the envelope theorem, i.e., \(\partial R_i/\partial t_i^m = 0\). Note that, considering \(NR_i\), to be exact we have to read \(dt^n\) as \(dt^n_j\) in (19).

\textsuperscript{14}Alternatively, the condition (20) can be expressed in terms of the elasticity of the elasticity \(\varepsilon^n\):

\[
\frac{dNR}{dz_2} \gtrless 0 \iff \frac{dt^n}{dt^n} \frac{\varepsilon^n}{\Delta t} \gtrless 1. \] The intuitive explanation for this relationship is as follows. The rise in
In other words, net revenues \( NR \) increase with the mobility parameter \( z_2 \) if the distribution function is not too concave at the equilibrium levels \( t^n \) and \( t^m \). This condition is fulfilled, for instance, in the case of a uniform distribution, as our following example illustrates.

**Example:** Mobility costs are assumed to be uniformly distributed, i.e., \( F_k(m_k) = \frac{m_k - m_e}{m_k - m_c} \) and \( F'_k(m_k) = \frac{1}{m_k - m_c} \), where \( k = 1, 2 \), \( m_2 > m_1 > 0 > m_2 > m_1 \), and \( m_k > |m_k| \). These relationships capture the fact that relocation costs exceed location costs, and that investors are home biased and firms are locked in (since \( F_k(0) < 0 \Leftrightarrow m_k > |m_k| \)). They are consistent with assumption 1 and figure 1. In the current example, of course, the two distribution curves are straight lines, with support \([m_1, m_1]\) and \([m_2, m_2]\), respectively.

Following our previous line of reasoning, the equilibrium tax and subsidy rates are

\[
\begin{align*}
t^n &= \frac{2m_2 - m_2}{3} > \frac{m_2 - 2m_2}{3} = t^m, \\
s^n &= \frac{m_2 + m_2}{3} - \frac{2m_1 - m_1}{3} < \frac{m_2 + m_2}{3} - \frac{m_1 - 2m_1}{3} = s^m.
\end{align*}
\]

The home bias of investors and the lock-in effect that established firms experience (implied by \( m_1 > |m_1| \) and \( m_2 > |m_2| \), respectively) lead to preferential subsidy and tax regimes in favour of foreign investors and firms.\(^{15}\) Using equilibrium taxes and subsidies and the equilibrium outcome \( N_i = N_j = 1 \), the resulting net revenues can be determined:

\[
NR_i = \frac{2(m_2 - 2m_2)^2}{9(m_2 - m_2)} + \frac{(2m_1 - m_1)^2}{9(m_1 - m_1)} + \frac{(m_1 - 2m_1)^2}{9(m_1 - m_1)}. \tag{23}
\]

Let us define \( m_k = \omega_k - z_k \) and \( \bar{m}_k = \bar{\omega}_k - z_k \). Then, we can formally capture an increase in relocation mobility, i.e., an decline in relocation costs, by an increase in mobility \( dF_2(\Delta t; z_2)/dz_2 > 0 \) increases the elasticity \( \varepsilon^n \) for given taxes and thus distorts the initial equilibrium, as the first-order condition (3) reveals. To restore the equilibrium, the tax \( t^n \) has to adjust the more, the less elastic the elasticity \( \varepsilon^n \) responds to changes in \( t^n \). If the elasticity of the elasticity is sufficiently small (i.e., below one), the tax \( t^n \) declines so drastically that the indirect effect dominates, and the net revenues fall.

\(^{15}\)Both subsidies, \( s^n \) and \( s^m \), are indeed positive if \( m_2 > 2m_1 - m_1 - m_2 \) holds. In turn, if this condition does not hold (but assumption 1 is still valid) we have the case of a tax holiday in a narrow sense, i.e., tax rates are positive in both periods but relatively lower during period 1 when the firm is established and higher later on in period 2.
the parameter $z_2$, shifting the distribution $F_2(m_2)$ to the left. Differentiating (23) then yields

$$
\frac{dN_{R_i}}{dz_2} = \frac{4(m_2 - 2m_2)}{9(m_2 - m_2)} > 0.
$$

(24)

Hence, a decrease in relocation costs, resulting in a higher relocation mobility, unambiguously increases net revenues. □

Returning to our general discussion, we now take into account the fact that changes in relocation mobility might also affect the slope of the distribution function. The additional effects that arise if $dF_2'(\Delta t; z_2)/dz_2 \neq 0$ holds at the ‘old’ equilibrium level $\Delta t$ are stated in

**Proposition 2** Net tax revenues and relocation mobility (continued).

The revenue increasing effect of a marginal change in relocation mobility is reinforced (counteracted) if $dF_2'(\Delta t; z_2)/dz_2 < 0$ ($dF_2'(\Delta t; z_2)/dz_2 > 0$) holds.

Proof: See appendix. □

The economic explanation for this conclusion is straightforward. If the density $F'_2$ decreases (increases) with the mobility parameter, the firms’ response to tax increases becomes less (more) elastic, causing a rise (decline) in tax $t^n$. Such a tax change, however, increases (erodes) the basic revenues.

### 4.3 Net tax revenues and location mobility

Next, we investigate the implications of rising location mobility. That is, we analyse the case in which investors are more mobile and less home biased when they decide where their firms are set up in the first period.

Analogously to scenario 1, we now consider

**Scenario 2:** $dF_1'(\Delta \tau; z_1)/dz_1 > 0$ and $dF_1'(\Delta \tau; z_1)/dz_1 = 0$.

We formally express this scenario in terms of hypothetical taxes instead of subsidies. The two interpretations are equivalent, since a rise in the hypothetical taxes $\tau^n$ and $\tau^m$ corresponds with a decline in subsidies $s^n$ and $s^m$ of the same magnitude. Referring to taxes, however, proves to be more convenient and allows us to compare the differences between rising location and relocation mobilities more explicitly.

Increasing location mobility does not affect future real taxes, but only current
hypothetical tax revenues or, equivalently, real subsidy payments:

\[
\frac{dNR}{dz_1} = - (\tau^n - \tau^m) \frac{dF_1(\Delta \tau; z_1)}{dz_1} + \tau^n F_1'(\Delta \tau; z_1) \frac{d\tau^n}{dz_1} + \tau^m F_1'(\Delta \tau; z_1) \frac{d\tau^m}{dz_1}. \tag{25}
\]

The first term on the right-hand side again reflects the direct impact of mobility on the tax bases. In contrast to its counterpart in derivative (19), this effect is now negative. For given hypothetical taxes, and thus subsidies, increasing mobility reduces the number of home firms located in each country \(1 - F_1(\Delta \tau; z_1)\), but it increases the number of foreign firms \(F_1(\Delta \tau; z_1)\) by the same amount. The impact of these changes on net revenues is negative, since the former firms pay more hypothetical taxes than the later ones. To put it differently, increasing mobility implies that highly subsidised foreign investors who take advantage of the subsidy differential replace less subsidised home investors who set up their firms abroad, thereby increasing each country’s overall subsidy payments.

The second and third term capture the indirect effects of location mobility via its influence on equilibrium taxes \(\tau^n\) and \(\tau^m\). These indirect effects are positive (negative) if the hypothetical taxes \(\tau^n\) and \(\tau^m\) increase (decrease) and thus real subsidies decline (rise).\(^{17}\) Only if the increase in taxes, and thus the decline in subsidies, is sufficiently large, will these indirect effects overcompensate the negative direct effect and cause an increase in net revenues. Otherwise, net revenues fall.

Therefore, the overall impact on net revenues is not clear cut. The slope of the density function, i.e., the curvature of the distribution function, again proves to be crucial. This is not surprising, given the similarity between real and hypothetical taxes.

**Proposition 3** Net tax revenues and location mobility.

In scenario 2, the net revenues \(NR\) increase (decrease) with the investors’ mobility parameter \(z_1\) if the density function’s slope \(F''_1(\Delta \tau; z_1)\) is greater (smaller) than the threshold level \(\gamma_1\):

\[
\frac{dNR}{dz_1} \geq 0 \Leftrightarrow F''_1 \geq \frac{[1 - 2F'_1(\Delta \tau; z_1)] [F'_1(\Delta \tau; z_1)]^2}{2 [1 - F_1(\Delta \tau; z_1)] F_1(\Delta \tau; z_1)} =: \gamma_1 > 0. \tag{26}
\]

Proof: See appendix. □

\(^{16}\) We take advantage of the fact that \(\partial R_i/\partial t^n_i = 0\) and \(\partial R_i/\partial t^m_i = 0\) hold in equilibrium. Again, considering \(NR_i\), to be exact we have to read \(d\tau^n\) and \(d\tau^m\) as \(d\tau^n_j\) and \(d\tau^m_j\) in (25).

\(^{17}\) Analogously to scenario 1, we know that the hypothetical tax (or subsidy) differential (16) decreases with mobility. The previous discrimination against home investors is simply no longer viable once they become less attached to their home country. Depending on the curvature of the distribution function, however, both taxes \(\tau^n\) and \(\tau^m\) might rise or fall, or \(\tau^n\) falls and \(\tau^m\) rises in response to a larger location mobility.
This conclusion mirrors our previous one. Now, net revenues $NR$ decrease with the mobility parameter $z_1$ if the distribution function is not ‘too’ convex. That is, the negative direct impact dominates as long as the rise in hypothetical taxes is not ‘too’ drastic. In proposition 1, we stated that net revenues $NR$ increase with the mobility parameter $z_2$ if the distribution function is not ‘too’ concave. That is, the positive direct impact prevails as long as the decline in real taxes is not ‘too’ drastic.\[18\]

The sign of the direct effect constitutes the major difference between the impact of increasing location mobility and relocation mobility. This direct effect is now negative because, for given hypothetical taxes, hypothetical revenues from home firms decline with location mobility. This negative effect has no counterpart in the case of changes in relocation mobility. Then, the revenue differential arising from domestic firms (i.e., from firms that were already set up in the country considered) anyway does not count because it is offset by subsidy payments, as already stated by the WYGIWYG principle. The ‘remaining’ direct effect is thus positive in the second period.

It is straightforward to extend our analysis to the case $dF_1'((\Delta t; z_1))/dz \neq 0$. Analogously to proposition 2, the revenue increasing effect of a marginal change in mobility is strengthened (weakened) if $dF_2'((\Delta t; z_2))/dz_2 < 0 \ (dF_2'((\Delta t; z_2))/dz_2 > 0)$ holds. Again, this conclusion reflects the fact that the tax base becomes less (more) elastic, thereby pushing up (pushing down) hypothetical taxes.

**Example (continued):** Let us return briefly to our example. In line with our argumentation above, we now consider the impact of an increase in the location mobility, i.e., an decline in location costs, captured by a marginal shift of the distribution $F_1(m_1)$ to the left. Formally, we analyse a marginal change in $z_1$. Differentiating the net-revenue function (23) yields

$$dNR_i/dz_1 = -\frac{2(m_1 + m_1)}{9(m_1 - m_1)} < 0.$$  

Hence, a decline in location costs, resulting in a higher location mobility, unambiguously lowers net tax revenues. \(\square\)

### 5 Concluding Remarks

Governments compete for mobile firms with both subsidies and taxes. We have analysed the resulting interplay between tax competition and subsidy competition,
leading to the WYGIWYG principle. That is, the additional revenues generated by attracting firms through subsidies are exactly offset by the opportunity costs of these subsidies. This result has helped us to shed some light on the impact of rising mobility on net tax revenues, thereby distinguishing between location mobility and relocation mobility. Our key conclusion is that a rise in relocation mobility increases net tax revenues under fairly weak conditions. A higher relocation mobility reinforces tax competition, but weakens subsidy competition. Overall, the fall in subsidy payments overcompensates for the decline in tax revenues, yielding higher net tax revenues.

This conclusion is in sharp contrast to the common belief that increasing mobility erodes national revenues – a belief that is backed by ‘pure’ tax competition models. Notably, our contrasting conclusions are derived in a ‘conventional’ tax competition framework, but in one that is supplemented by subsidy competition stages. In this setting, we also argue that rising location mobility indeed reduces net tax revenues, somewhat in line with the ‘conventional’ tax competition literature and common beliefs.

Our findings have important policy implications. They directly imply that fiercer tax competition (here, due to rising relocation mobility) might be advantageous to the governments because of its feedback effect on subsidy competition. In the public debate, however, the focus is on weakening tax competition, or preventing harmful tax competition, through various measures (cf. OECD, 1998). In our model, weakening tax competition actually implies intensifying subsidy competition, with potentially adverse effects on net tax revenues. So an exclusive concentration on tax harmonisation might be misleading and thus detrimental to future revenues. In this sense, our paper cautions politicians against narrow minded tax harmonisation on grounds different from those previously discussed in the literature.19 Our paper also indicates that more attention should be paid to subsidy competition and its interaction with tax competition. Reducing subsidy competition might indeed be a more successful avenue for larger tax revenues than restrictions on tax competition.

Exploring the implication of various forms of harmonisation and cooperation in our framework in detail can be a promising extension of our analysis. Such an extension would also include the discussion of limitations on preferential tax and subsidy regimes – as far as such limitations are enforceable, given that subsidies are frequently granted in form of somewhat hidden and indirect transfers, and even preferential tax treatments are often hidden.20 As a further extension, the impact

19See, for instance, Zodrow (2003) for a survey on tax competition in the European Union and the standard arguments against tax harmonisation.

of correlated location and relocation costs could be checked. Firms might then sort themselves according to their mobility characteristics, and multiple equilibria might arise. Nevertheless, the underlying mechanisms explored in our simplified version should remain the same, and our conclusions should therefore still be valid, perhaps with some modifications.

References


tax regimes. See also Krieger and Lange (2008) for a discussion of the implications of preferential and non-preferential regimes in the context of student and graduate mobility.


Krieger, T. and T. Lange (2008), Education policy and tax competition with imperfect student and labor mobility, Discussion paper no. 08/01, DFG Research Group “Heterogeneous labor: Positive and normative aspects of the skill structure of labor”, University of Konstanz.


OECD (1998), Harmful tax competition: An emerging global issue, Paris, OECD.


Appendix

Proof of Lemma 1 We start by analysing the second period equilibrium (third and fourth stage). As argued above, this equilibrium is independent of the governments’ subsidies (first stage) and the investors’ initial location choice (second stage). In step 1, we prove the existence and the characteristics of this equilibrium. Uniqueness is proved in step 2. In step 3, we show that our lines of reasoning can easily be repeated to prove existence, characteristics and uniqueness of the subsidy competition equilibrium, and thus of the subgame perfect equilibrium.

Step 1 (Existence and Characteristics of Tax Competition Equilibrium) The first-order conditions

\[
\frac{\partial R_i}{\partial t^n_i} = \left\{ \left[ 1 - F_2(t^n_i - t^n_j) \right] - t^n_i F'_2(t^n_i - t^n_j) \right\} N_i = 0, \\
\frac{\partial R_j}{\partial t^m_j} = \left[ F_2(t^n_i - t^m_j) - t^m_j F'_2(t^n_i - t^m_j) \right] N_i = 0,
\]

are fulfilled for all taxes that constitute a solution to (28) and (29) according to assumption 1 (vi) and, second, \( F \) is a twice continuously differentiable function.
Obviously, negative taxes can never be revenue maximising so that we can focus on non-negative solutions, i.e., \( t^m_A, t^m_B, t^m_A, t^m_B \geq 0 \). Moreover, \( \partial NR_i / \partial t^m_i \big|_{t^m_i = 0} = [1 - F_2(-t^m_i)] N_i > 0 \) and \( \partial NR_i / \partial t^m_i \big|_{t^m_i = t^m - m_2} = -t^m_i F'_2(m_2) < 0 \), implying \( 0 < t^m_i = G^m_i(t^m_j) < t^m_j + m_2 \). Similarly, \( \partial NR_j / \partial t^m_j \big|_{t^m_j = 0} = F_2(t^n_i) N_i > 0 \) and \( \partial NR_j / \partial t^m_j \big|_{t^m_j = t^n_m - m_2} = -t^m_j F'_2(m_2) < 0 \), implying \( 0 < t^m_j = G^m_j(t^n_i) < t^n_m - m_2 \). (We implicitly assume that the firms’ gross returns \( \pi \) are sufficiently large so that they do not constrain government taxation.)

In addition, the unique intersection between the reaction curve \( G^m_i \) and the \( t^m_i = t^m_j \)-line and the unique intersection between the reaction curve \( G^m_j \) and the \( t^m_i = t^m_j \)-line are characterised by \( t^m_i |_{t^m_j = t^m} = [1 - F_2(0)] / F'_2(0) \) and \( t^m_j |_{t^m_i = t^m} = F_2(0) / F'_2(0) \), respectively. From these unique intersections, \( G^m_i(t^m_j) > 0 \) for all \( t^m_j \geq 0 \) and \( G^m_j(t^n_i) > 0 \) for all \( t^n_i \geq 0 \) follow that \( G^m_i(t^m_j) \geq t^m_j < [1 - F_2(0)] / F'_2(0) \) and \( G^m_j(t^n_i) \geq t^n_i < F_2(0) / F'_2(0) \). Combining these relationships with \( t^m_i |_{t^m_j = t^m} > t^m_j |_{t^m_i = t^m} \) (which results form \( F_2(0) < 0.5 \), see assumption 1 (i)), and given the upper and lower boundaries of the taxes as explored above, we can conclude: (i) there exists at least one equilibrium, (ii) \( t^m_i > t^m_j \in (0, [1 - F_2(0)] / F'_2(0)) \) and thus \( t^m_i < [1 - F_2(0)] / F'_2(0) + m_2 \) hold in equilibrium, and (iii) taxes are characterised by (5) and thus (6).

**Step 2 (Uniqueness)**  
As taxes are given by (5), the tax differential is characterised by (6), or equivalently by \( \Delta t - [1 - 2F_2(\Delta t)] / F'_2(\Delta t) = 0 \). This tax differential is positive (see step 1 and remarks in footnote 10). To show that it is uniquely determined, we differentiate the term \( [1 - 2F_2(\Delta t)] / F'_2(\Delta t) =: \Phi(\Delta t) \) with respect to \( \Delta t \), leading to

\[
\frac{\partial}{\partial \Delta t} \left[ \frac{[1 - 2F_2(\Delta t)]}{F'_2(\Delta t)} \right] < 0 \Leftrightarrow F'_2(\Delta t) > -2 \frac{[F'_2(\Delta t)]^2}{1 - 2F_2(\Delta t)},
\]

(32)

The latter inequality is fulfilled, since \( F'_2(\Delta t) > -2 [F'_2(\Delta t)]^2 / [1 - 2F_2(\Delta t)] \) holds by assumption 1 (vi) and \( -2 [F'_2(\Delta t)]^2 / [1 - 2F_2(\Delta t)] \geq -2 [F'_2(\Delta t)]^2 / [1 - 2F_2(\Delta t)] \Leftrightarrow F_2(\Delta t) \geq 0 \) results. Thus, the term \( [1 - 2F_2(\Delta t)] / F'_2(\Delta t) \) continuously decreases with \( \Delta t \), with \( \Phi(0) = [1 - 2F_2(0)] / F'_2(0) > 0 \) (see assumption 1 (iii)) and \( \Phi(m_2) = -1/F'_2(m_2) < 0 \) (see assumption 1 (i)). Also, in the interval \([0, m_2] \) the term \( \Delta t \) is obviously continuously increasing from 0 to \( m_2 \). As a consequence of the intermediate value theorem, the tax differential \( \Delta t \) is then uniquely determined by \( \Delta t - [1 - 2F_2(\Delta t)] / F'_2(\Delta t) = 0 \) or (6), and so are then the taxes \( t^n_A = t^n_B = t^n \) and \( t^n_A = t^n_B = t^n \) by (5).

**Step 3 (Subsidy Competition and Subgame-Perfect Equilibrium)**  
The first-order
conditions (10) and (11) are equivalent to
\[
\frac{\partial NR_i}{\partial \tau^n_i} = [1 - F_1(t^n_i - \tau^n_j)] - \tau^n_i F'_1(t^n_i - \tau^n_j) = 0, \tag{33}
\]
\[
\frac{\partial NR_j}{\partial \tau^m_j} = F_1(t^n_i - \tau^m_j) - \tau^m_j F'_1(t^n_i - \tau^m_j) = 0, \tag{34}
\]
where (12), the definitions \( \tau^n_i := (t^n - t^m) - s^n_i \) and \( \tau^m_i := (t^n - t^m) - s^m_i \), and (16) are used. The similarity between (33) and (34) on the one hand and (28) and (29) on the other hand is striking. Not surprisingly, the proof of existence, characteristics and uniqueness of the subsidy competition equilibrium follows the lines of reasoning explored in step 1 and 2, which need not to be repeated here. The hypothetical taxes \( \tau^n_i \) and \( \tau^m_i \) are independent of the second period equilibrium. The only impact of the second period equilibrium on the first period equilibrium is that the taxes \( t^n \) and \( t^m \) raise the resulting subsidies \( s^n \) and \( s^m \) by the tax differential \( \Delta t \). The symmetric nature of the framework and the resulting equilibrium imply \( N_i = N_j = 1 \).

Consequently, we can conclude that (i) there exists a unique subgame perfect equilibrium which is characterised by (5), (6), (14), (15), (16), and \( N_i = N_j = 1 \).

**Proof of Propositions 1 and 2**

**Preliminary Results** Inserting the optimal taxes (5), (14) and (15) into the net tax revenues (18) and rearranging to resulting terms lead to
\[
NR = 2 F_2' (\Delta t; z_2) F_2 (\Delta t; z_2) + 1 - 2 F_1 (\Delta \tau; z_1) \frac{\partial F_2 (\Delta t; z_2)}{\partial z_2} - [F_2 (\Delta t; z_2)]^2 \frac{\partial F_2' (\Delta t; z_2)}{\partial z_2}, \tag{35}
\]
Differentiating net tax revenues (35) with respect to mobility parameter \( z_2 \) yields
\[
\frac{dNR}{dz_2} = \frac{\partial NR}{\partial z_2} + \frac{\partial NR}{\partial \Delta t} \frac{d\Delta t}{dz_2}. \tag{36}
\]
The components of this derivative are given by
\[
\frac{\partial NR}{\partial z_2} = 2 F_2' (\Delta t; z_2) F_2 (\Delta t; z_2) \frac{\partial F_2 (\Delta t; z_2)}{\partial z_2} - [F_2 (\Delta t; z_2)]^2 \frac{\partial F_2' (\Delta t; z_2)}{\partial z_2}, \tag{37}
\]
\[
\frac{\partial NR}{\partial \Delta t} = -2 [F_2 (\Delta t; z_2)]^2 F_2 (\Delta t; z_2) - [F_2 (\Delta t; z_2)]^2 F_2'' (\Delta t; z_2), \tag{38}
\]
\[
\frac{d\Delta t}{dz_2} = \frac{2 F_2' (\Delta t; z_2) \frac{\partial F_2 (\Delta t; z_2)}{\partial z_2} + [1 - 2 F_2 (\Delta t; z_2)] \frac{\partial F_2' (\Delta t; z_2)}{\partial z_2}}{[F_2' (\Delta t; z_2)]^2 [3 + \rho_2]}, \tag{39}
\]
where
\[
\rho_2 = \frac{\Delta t F_2'' (\Delta t; z_2)}{F_2' (\Delta t; z_2)} = \frac{[1 - 2 F_2 (\Delta t; z_2)] F_2'' (\Delta t; z_2)}{[F_2' (\Delta t; z_2)]^2}. \tag{40}
\]
is the elasticity of the density function $F'_t(\Delta t; z_2)$ with respect to changes in the tax differential $\Delta t$. Note that derivative (39) follows from tax differential (6) and the associated comparative statics: $d\Delta t / dz_2 = -(\partial g_2 / \partial z_2) / (\partial g_2 / \partial \Delta t)$, where $g_2(\Delta t; z_2) := \Delta t - [1 - 2F_2(\Delta t; z_2)] / F'_2(\Delta t; z_2)$ and $\partial g_2 / \partial \Delta t = 3 + \rho_2$.

We can prove propositions 1 and 2 in a more convenient and shorter manner by making use of the derivatives (36)-(39) instead of the more intuitive derivative (19) and the tedious comparative statics that leads to $dt^n / dz_2$.

**Proposition 1** We now consider scenario 1 with $\partial F_2(\Delta t; z_2) / \partial z_2 > 0$ and $\partial F'_2(\Delta t; z_2) / \partial z_2 = 0$ at the equilibrium value of $\Delta t$, which simplifies the derivatives (37) and (39). Inserting (37), (38) and (39) into derivative (36) and rearranging the resulting terms lead to

$$
\frac{dNR}{dz_2} = 4 \frac{F_2 F'_2 \partial F}{(F'_2)^2} \left[ 1 - \frac{2(F'_2)^2 - F_2 F''}{(F'_2)^2 (3 + \rho_2)} \right] \geq 0, \quad (41)
$$

where the functions’ argument $\Delta t$ and parameter $z_2$ are suppressed for notational convenience. The sign of this derivative depends on the terms in the square brackets, since all other terms are positive. Rearranging the terms in the square brackets and using (40) result in relation (20) from proposition 1.

**Proposition 2** To calculate the additional impact of a change in the mobility parameter $z_2$ on the net tax revenues $NR$ that arises if $\partial F'_2(\Delta t; z_2) / \partial z_2 > 0$, we evaluate the derivatives (37) and (39) for $\partial F_2(\Delta t; z_2) / \partial z_2 = 0$ and $\partial F'_2(\Delta t; z_2) / \partial z_2 > 0$ at the equilibrium value of $\Delta t$. Inserting again (37)-(39) into derivative (36) yields, after some rearrangements,

$$
\frac{dNR}{dz_2} = -2 \frac{\partial F}{\partial z_2} \left[ F_2^2 + \frac{2(F'_2)^2 F_2 - F_2^2 F''}{(F'_2)^2} \left( \frac{1 - 2F_2}{3 + \rho_2} \right) \right] \geq 0, \quad (42)
$$

where we again suppress the functions’ argument $\Delta t$ and parameter $z_2$. Recall that both second-order conditions (30) and (31) are fulfilled if, and only if, $F''_2 \in (-2(F'_2)^2 / (1 - F_2), 2(F'_2)^2 / F_2)$ holds, which is assumed to be the case by assumption 1 (vi). Then, $F''_2 < 2(F'_2)^2 / F_2$ implies that $2(F'_2)^2 F_2 - (F_2)^2 F''_2 > 0$ holds. Also, $F''_2 > -2(F'_2)^2 / (1 - F_2)$ implies that the inequality $3 + \rho_2 > 3 - 2[(1 - 2F_2) / (1 - F_2)] > 1$ is fulfilled, where we use (40). Finally, $F_2 < 0.5$ and thus $1 - 2F_2 > 0$ hold in equilibrium (see tax differential (6) and the explanation in footnote 10). Thus, all terms in the square brackets are positive, resulting in

$$
\frac{dNR}{dz_2} \geq 0 \iff \frac{\partial F'_2(\Delta t; z_2)}{\partial z_2} \leq 0, \quad (43)
$$

which proves Proposition 2.
Proof of Proposition 3  This proof follows along the lines of the previous reasoning. Now, equilibrium net tax revenues (35) are affected by a change in the mobility parameter $z_1$ according to

$$\frac{dNR}{dz_1} = \frac{\partial NR}{\partial z_1} + \frac{\partial NR}{\partial \Delta \tau} \frac{d\Delta \tau}{dz_1}. \quad (44)$$

We consider scenario 2; that is, $dF_1'(\Delta \tau; z_1)/dz_1 > 0$ and $dF_1''(\Delta \tau; z_1)/dz_1 = 0$ hold at the equilibrium differential $\Delta \tau$. Then, the three terms in (44) are given by

$$\frac{\partial NR}{\partial z_1} = -2 \frac{F_1'(\Delta \tau; z_1)}{[F_1'(\Delta \tau; z_1)]} [1 - 2F_1'(\Delta \tau; z_1)] \frac{\partial F_1'(\Delta \tau; z_1)}{\partial z_1}, \quad (45)$$

$$\frac{\partial NR}{\partial \Delta \tau} = -2 \frac{[F_1'(\Delta \tau; z_1)]^2 [1 - 2F_1'(\Delta \tau; z_1)]}{[F_1'(\Delta \tau; z_1)]^2} \left[ 1 - 2F_1'(\Delta \tau; z_1) + 2 \frac{F_1''(\Delta \tau; z_1)}{F_1'(\Delta \tau; z_1)} \right] F_1''(\Delta \tau; z_1), \quad (46)$$

$$\frac{d\Delta \tau}{dz_1} = \frac{2F_1'(\Delta \tau; z_1)}{[F_1'(\Delta \tau; z_1)]^2} \frac{\partial F_1'(\Delta \tau; z_1)}{\partial z_1} \frac{[1 - 2F_1'(\Delta \tau; z_1)] + 2 \frac{F_1''(\Delta \tau; z_1)}{F_1'(\Delta \tau; z_1)} \left[ 3 + \rho_1 \right]}{[3 + \rho_1]}, \quad (47)$$

where

$$\rho_1 = \frac{\Delta \tau F_1''(\Delta \tau; z_1)}{F_1'(\Delta \tau; z_1)} = \frac{[1 - 2F_1'(\Delta \tau; z_1)] F_1''(\Delta \tau; z_1)}{[F_1'(\Delta \tau; z_1)]^2}. \quad (48)$$

is the elasticity of the density function $F_1'(\Delta \tau; z_1)$ with respect to changes in the differential $\Delta \tau$. Analogously to (39), derivative (47) follows from differential (16) and the respective comparative statics: $d\Delta \tau/dz_1 = -(\partial g_1/\partial z_1)/(\partial g_1/\partial \Delta \tau)$, where $g_1(\Delta \tau; z_1) := \Delta \tau - [1 - 2F_1'(\Delta \tau; z_1)]/F_1'(\Delta \tau; z_1)$ and $\partial g_1/\partial \Delta \tau = 3 + \rho_1$.

Inserting (45), (46) and (47) into derivative (44) and rearranging the resulting terms yield

$$\frac{dNR}{dz_1} = \frac{-2\frac{\partial F_1}{\partial z_1}}{F_1'(3 + \rho_1)} \left[ 1 - \frac{F_1''(1 - F_1)}{(F_1')^2} \right] \left( 1 - F_1 \right) - \left( 1 + \frac{F_1''(1 - F_1)}{(F_1')^2} \right) F_1 \gg 0, \quad (49)$$

where we make use of elasticity (48) and, for notational convenience, omit the functions’ argument $\Delta \tau$ and parameter $z_1$. Note that $F_1'' > -2(F_1')^2/(1 - F_1)$ holds according to assumption 1 (vi) – otherwise the second-order condition would not be fulfilled. Thus, the inequality $3 + \rho_1 > 3 - 2[(1 - 2F_1)/(1 - F_1)] > 1$ results, where we use (48). Since $F_1'$ and $\partial F_1/\partial z_1$ are also positive, the quotient outside the square brackets is definitely negative. The overall sign of (49) then depends on the terms in the brackets. Rearranging them leads to condition (26) in Proposition 3.