Innovation, Growth, and the Optimal Enforcement of the Rule of Law

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Abstract: We study the link between the rule of law, innovation investments, and economic growth in an endogenous growth framework with an expanding set of product varieties. Under a weak rule of law producing firms pay a "mafia tax". We find that a minimum strength of the rule of law is a necessary condition for sustained growth. However, a weak rule of law may be Pareto-improving. A government investing resources in the improvement of the rule of law can shift the economy from a no-growth trap onto a path with positive growth. In terms of welfare no growth might be preferable.

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1 Introduction

The legal framework of an economy is often thought of as defining the rules of the game that economic agents play. However, what really matters for the incentives of these agents is the strength of the "rule of law". While precise definitions of this term are hard to come by, they usually involve the notion of an economy's degree of property rights protection, the enforceability of contracts, the likelihood of crime and violence, and the effectiveness of an economy's judiciary (see, e. g., Kaufmann, Kraay, and Mastruzzi (2007) or Weil (2009), p. 346). The focus of the present paper is on the link between a weak rule of law, the incentives to engage in innovation investments, and economic growth. On the positive side, we want to know whether a minimum of rule of law enforcement is a prerequisite for economic growth. On the normative side, we ask whether a stronger rule of law is desirable and discuss the circumstances under which the government should intervene and direct resources to strengthening the enforcement of the rule of law.

We address these questions in an endogenous growth framework where growth is the result of an expanding set of product varieties in the sense of Grossman and Helpman (1991). The strength of the rule of law is captured by the fraction of profits taken away by, say, mafia activity. This "mafia tax" (Maddison and Pollicino (2003)) deters innovation investments and reduces economic growth. Thus, our framework is consistent with the empirical literature that establishes a positive relationship between the strength of the rule of law and economic growth (see, e. g., Kaufmann and Kraay (2002), Hall and Jones (1999), Clague, Keefer, Knack, and Olson (1999), Knack and Keefer (1995), or Scully (1988)).

In a first step, we take the strength of the rule of law as exogenous and abstract from government interventions. In this scenario, we establish that a weak rule of law is a major reason why an economy may be caught in a no-growth trap. Hence, a minimum strength of the rule of law can be thought of as a necessary condition for sustained growth. Though, on the normative front, we show that a weak rule of law may be Pareto-improving. This is the case when the equilibrium growth rate exceeds the Pareto-efficient one.[1] Then, it is indeed preferable to weaken innovation incentives. A means to accomplish this is a weaker rule of law which essentially acts on innovation incentives as a "mafia tax".

In a second step, we endogenize the strength of the rule of law. More precisely, we allow the government to use tax resources to improve its strength. We find that such government activity can shift the economy from a no-growth path to a path with strictly positive growth. However, such activity may not be optimal from a welfare point of view. Indeed, we characterize environments where no growth is better than some growth even though

[1] Bénassy (1998) establishes this possibility in the variety expansion growth model when the gains from specialization captured by the CES production function are small.

the government would be willing to intervene. This is the case if the instantaneous welfare loss due to reduced current consumption more than outweighs the increasing future consumption possibilities that arise along a path with strictly positive growth. Moreover, we find that a government intervention that strengthens the rule of law is more likely to be desirable if the economic environment is more prone to growth.

The remainder of this paper is organized as follows. Section 2 shows how our work relates and contributes to the existing literature. Section 3 presents a basic analytical setup. Section 4 establishes the dynamic general equilibrium and compares it to the Pareto-optimum for an exogenous strength of the rule of law. In Section 5 we endogenize the strength of the rule of law and determine the welfare-maximizing policy of the government. Section 6 concludes. Proofs are relegated to the Appendix.

2 Related Literature

Our paper relates and contributes to two different strands of the literature on property rights and growth.

First, our study relates to attempts to combine the theory of predation with the theory of economic growth (see, e. g., Grossman and Kim (1996) and Tornell (1997)). Predation in the form of expropriation by the mafia is a key characteristic of our setup. There are only a few theoretical studies that incorporate government activity into such a framework. In general, these studies differ from ours in that they explicitly model individuals' decision how to allocate their time or resources between productive and expropriative activities. Individuals have access to a specific expropriation technology whose exact design determines how many and which type of equilibria exist.

For instance, Economides, Park, and Philippopoulos (2007), incorporate weak property rights and government expenditure on law enforcement into Barro (1990)'s growth model.[2] They find two self-fulfilling perfect-foresight equilibria with different welfare and policy implications when strategic complementarities generate increasing returns to scale in the expropriation technology at the economy level. Then, a government that chooses its policy instruments to maximize the economy's growth rate can solve the expectation-coordination problem of the decentralized equilibrium, but cannot implement fully-protected property rights.

Zak (2002) and Dincer and Ellis (2005) integrate expropriative and government activity into models of overlapping generations. While in Zak (2002) predation occurs intergenerationally, i. e., the young can "steal" some of the capital from the old, Dincer and Ellis (2005) assume that working households expropriate a fraction of output from firms of the

[2] In this framework long-term growth in guaranteed by public expenditure on infrastructure.

same period. In both models the fraction that is expropriated depends on the resources the government spends on the protection of property rights. The government finances this expenditure by taxes, which have to be borne by the victims of expropriation. Then, depending on the effectiveness of the expropriation/protection technology multiple equilibria with different degrees of property rights protection may exist. Assuming that the government aims at maximizing the consumption of the old, Zak (2002) establishes that even if the government allocates an optimal amount of revenues to property rights protection, the economy might be trapped in an equilibrium with low per-capita income and weak property rights. Similarily, in Dincer and Ellis (2005) the government, which is assumed to maximize firms' profits, might not be able to prevent that an economy with a low initial level of output converges to an equilibrium with zero output and zero protection.
In contrast to these papers we do not explicitly model individuals' decision to engage in expropriative activity. The main advantage of our approach is that it allows for a unique equilibrium and hence clear-cut predictions. Moreover, to the best of our knowledge, the present paper is the first that embeds the analysis of the link between a weak rule of law and the incentives to invest in innovations into the framework of an R&D-based growth model. This link is absent in the existing studies that are based on Barro (1990). Moreover, most of the above mentioned studies have a household sector with overlapping generations. Therefore, they restrict their analysis to government policies that maximize the benefits of a subgroup of agents, e. g., consumption of the old or profits of firms. In contrast, we focus on government policies that maximize the welfare of a representative household.

Second, we contribute to the literature that studies intellectual property rights (IPR), growth, and welfare in the framework of the variety expansion growth model. For instance, Kwan and Lai (2003) and Furukawa (2007) analyze the social benefits and costs of IPR protection in closed economy versions of the variety expansion growth model under the assumption that the government can choose the degree of IPR protection. However, these models are solely concerned with intellectual property rights, i. e., the danger of imitation and the erosion of monopoly power. Imperfect property rights in this setting imply that with a certain probability a patent cannot be enforced, the good is imitated and producers loose their monopoly position such that they do not make profits anymore. The shift from monopoly to competition (for some of the varieties) involves a trade-off. On the one hand, it implies a static gain as the provision of existing varieties increases; on the other hand, it involves a dynamic loss as the incentives to innovate and the growth rate of new varieties declines. By contrast, we are interested in the role of property rights over profits. In our model the strength of the rule of law determines the share of monopoly profits that is protected from expropriation. Intermediate-good producers always remain monopolists and weak property rights affect the incentives to innovate by reducing the value of a new variety.

3 The Basic Setup

The economy has four sectors, a mafia, and a government. *Households* work, consume and save. The *final-good sector* produces a consumption good out of a variety of existing intermediate goods. The *intermediate-good sector* consists of monopolistically competitive firms that manufacture one intermediate good each using labor as the only input. The blueprint for the production of each intermediate good is invented in a *research sector*. The rule of law is imperfect in the sense that property rights in the intermediate-good sector are not fully secured. To fix ideas one may think that intermediate-good firms are subject to some expropriation by an organization such as the *mafia*. We interpret the strength of the rule of law as the fraction of profits in the intermediate-good sector that is protected from expropriation and denote it by $q \in [0,1]$. We consider a closed economy, i. e., mafia income is "laundered" and increases consumption and savings of the household sector. In Section 4 we apply this basic setup to the case where the strength of the rule of law is exogenous. In Section 5 we add a *government* that undertakes investments to improve the rule of law and collects taxes. Hence, the strength of the rule of law, q, becomes endogenous.

3.1 The Household Sector

There is a continuum of identical households of mass 1. We study their behavior through the lens of a single representative household that supplies the time-invariant aggregate labor endowment L inelastically to the intermediate-good and the research sectors. Her consumption-savings decision maximizes intertemporal utility

$$U = \int_0^\infty \ln c(t)\, e^{-\rho t} dt, \tag{3.1}$$

where $c(t)$ is consumption at date t and ρ the subjective discount rate. Henceforth, we suppress time arguments whenever this does not cause confusion. Household income comprises at each t labor income, returns on assets Ω and laundered mafia income M such that the household's flow budget constraint is given by

$$\dot{\Omega} = wL + r\Omega + M - p_c c \quad \text{with} \quad \Omega(0) > 0, \tag{3.2}$$

where w denotes the wage rate at t, r the rate of return on assets, and p_c the price of the consumption good. Hence, mafia income increases household income, i. e., whatever the mafia earns is laundered, re-introduced into the economy, and used for consumption or saving of the household sector.

The household's maximization of (3.1) is subject to (3.2) and a No-Ponzi condition. This gives the following first-order necessary and sufficient conditions

$$\frac{1}{c} = \lambda p_c, \tag{3.3}$$

$$\dot{\lambda} = \lambda(\rho - r), \tag{3.4}$$

and the transversality condition

$$\lim_{t\to\infty} e^{-\rho t}\lambda\Omega = 0. \tag{3.5}$$

Following Grossman and Helpman (1991), we choose consumption expenditure as the numéraire, i. e., $p_c c = 1$ at all t. Then, the Euler condition implies $r = \rho$.

3.2 The Final-Good Sector

The final-good firms produce a homogeneous output y and sell it to the household sector for consumption. Production of y is under constant returns to scale. Therefore, we consider a single representative firm. This firm uses intermediate goods as inputs. At t, there is a continuum of these intermediates $[0, A(t)]$. Each intermediate enters the production function of the final output in a symmetric way, i. e.,

$$y = \left[A^{(\sigma-1)(1-\alpha)} \int_0^A x_j^\alpha \, dj\right]^{1/\alpha}, \tag{3.6}$$

where $A \in \mathbb{R}_+$ is the "number" of available intermediate goods and $x_j \in [0, A]$ denotes the quantity of intermediate-good input j used at t. The parameter $\alpha \in (0, 1)$ determines the elasticity of substitution between any pair of intermediates, $\epsilon \equiv 1/(1-\alpha)$, i. e., intermediates are differentiated goods. Following Ethier (1982) and Bénassy (1998), the term in front of the integral introduces $\sigma \in (0, \infty)$ as a measure of the gains from specialization. As $\sigma \to 0$, these gains vanish. As σ increases, the gains from specialization become more pronounced.

Producers of y are competitive and maximize

$$\max_{\{x(j)\}_{j=0}^A} p_c y - \int_0^A p_j x_j \, dj,$$

where p_j is the price of input j. As the final good corresponds to the consumption good, we have $y = c$. Due to constant returns to scale final-good producers earn zero profits. For future reference, we note that for any symmetric configuration our choice of the numéraire implies

$$Apx = 1. \tag{3.7}$$

3.3 The Intermediate-Good Sector

The intermediate-good sector is monopolistically competitive. Each intermediate-good firm produces a single intermediate good. The downward-sloping demand function for producer j is

$$x_j = \frac{y p_j^{-\epsilon}}{P} \quad \text{for all } j \in [0, A], \tag{3.8}$$

where $P \equiv \left[A^{(\sigma-1)(1-\alpha)} \int_0^A p_{j'}^{1-\epsilon} dj'\right]^{\epsilon/(\epsilon-1)}$. The production function of all varieties j is

$$x_j = l_j, \tag{3.9}$$

where l_j is the amount of labor hired by firm j. The price p_j charged by intermediate-good firm j maximizes his profits subject to (3.8)

$$\pi_j = q\left[p_j - w\right] x_j.$$

Here, $q \in [0, 1]$ denotes the fraction of profits that remains in the hands of intermediate-good producer j after the mafia has made its claims. The weaker the rule of law, i. e., the lower q, the lower are the net profits of intermediate-good producers. Intermediate-good producers treat q, y and P in their maximization problem as exogenous.

Since the cost and demand functions of all intermediate-good producers are identical they choose the same monopoly price

$$p_j = p = \frac{w}{\alpha} \tag{3.10}$$

and supply the same quantity, which follows from (3.8) with (3.10) as

$$x = y A^{\frac{\sigma}{1-\epsilon}-1}. \tag{3.11}$$

Taking into account (3.7), the instantaneous net profits for all monopolists are

$$\pi = \frac{q(1-\alpha)}{A}. \tag{3.12}$$

3.4 The Research Sector

Previous to the marketing of an intermediate good it must be invented through research. We suppose that there is a given number of identical research firms and normalize it to unity. Research firms are indexed by i. A research firm i that devotes $L_{A,i}$ units of labor to R&D produces $\dot{A}_i = L_{A,i}A/a$ new blueprints. This costs the research firm $wL_{A,i}$. The production of $\dot{A}_i$ depends on the productivity of researchers, $1/a$. The existing stock of knowledge A is a public good within the research sector. The sector's production function for blueprints is

$$\dot{A} = \frac{L_A}{a} A, \tag{3.13}$$

where L_A is the aggregate amount of labor used for research. Once a new variety is invented, it is sold by auction to the highest bidder who also receives a perpetual patent. Denote the value of such a patent by v. Then, the highest amount an intermediate-good producer is willing to pay for the patent is equal to the present value of all future net profits generated by the respective innovation, i. e.,

$$v(t) = \int_t^{\infty} \pi(s) e^{-\rho(s-t)} ds. \tag{3.14}$$

The profit-maximization problem of the representative research firm is then to choose L_A that maximizes $vAL_A/a - wL_A$. For a finite equilibrium labor demand of this sector we need

$$v \leq \frac{wa}{A} \quad \text{with} \quad \text{“}=\text{”}, \quad \text{if} \quad \dot{A} > 0. \tag{3.15}$$

4 Exogenous Strength of the Rule of Law

The purpose of this section is to examine the effect of an exogenously given strength of the rule of law on innovation activity and growth. In particular, we show that and how a weak enforcement of the rule of law causes the economy to be trapped in an equilibrium without growth.

4.1 Dynamic General Equilibrium (DGE)

Given q, the DGE consists of an allocation $\{c(t), y(t), \Omega(t), M(t), x(j,t), l(j,t), L_x(t), L_A(t), A(t)\}_{t=0}^{t=\infty}$ and a price system $\{r(t), p_c(t), w(t), p(j,t), v(j,t)\}_{t=0}^{t=\infty}$ such that:

(E1) Households maximize utility given Ω_0, i. e., conditions (3.3), (3.4) and (3.5) hold;

(E2) Given $A(t)$, final-good firms produce according to (3.6) and maximize profits, i. e., (3.8) holds;

(E3) Intermediate-good producers produce according to (3.9) and maximize profits, i. e., (3.10) holds;

(E4) Given $A_0 > 0$, research firms invent according to (3.13) and maximize profits, i. e., (3.15) holds;

(E5) There is full employment for all t, i. e., $L_x(t) + L_A(t) = L$;

(E6) The capital market values firms according to fundamentals, i. e., (3.14) holds, and for all t the value of households' assets is $\Omega(t) = A(t)v(t)$. Bonds are in zero net supply. Without uncertainty stocks and bonds are perfect substitutes and yield the same rate of return.

(E7) Total mafia income is $M(t) = (1-q)A\left(\frac{\pi(t)}{q}\right)$, i. e., it is equal to the fraction $1-q$ of total gross profits in the intermediate-good sector.

4.1.1 Labor Market Equilibrium

From (E5) and the fact that all intermediate-good producers supply the same quantity x, aggregate labor demand of this sector satisfies

$$L_x = Ax. \tag{4.1}$$

Taking into account (3.7) and (3.10) we obtain $L_x = w/\alpha$. Labor demand for research can be derived from the research technology (3.13). With
$g_A \equiv \dot{A}/A$ we obtain

$$L_A = ag_A. \tag{4.2}$$

Hence, the labor market equilibrium can be expressed as $g_A = L/a - \alpha/wa$. When employment in research is positive, we need $v = wa/A$ (see equation (3.15)). Hence, a necessary condition for positive steady-state growth of A is $v \geq \alpha a/AL$. Defining $V \equiv \Omega^{-1} = 1/Av$ as the inverse of the economy's equity value, we obtain

$$g_A = \max\left\{0, \frac{L}{a} - \alpha V\right\}. \tag{4.3}$$

4.1.2 Capital Market Equilibrium

According to (E6) the return that a shareholder can expect must be equal to the return of a riskless loan. As the former is the sum of dividends and capital gains and the latter is equal to ρ, we obtain as a no-arbitrage condition

$$\rho = \frac{\pi + \dot{v}}{v},$$

where π is given by (3.12).

Then, observing that $\dot{V}/V = -g_A - \dot{v}/v$ we obtain

$$\frac{\dot{V}}{V} = -g_A - \rho + q(1-\alpha)V. \tag{4.4}$$

Equations (4.3) and (4.4) jointly describe the equilibrium paths of V and g_A.

4.1.3 Characterization of the Equilibrium

The following proposition establishes the existence of a steady-state equilibrium with and without growth.

Proposition 1 *The steady-state growth rate of intermediate-good varieties is*

$$g_A^* = max\left\{0, \frac{q(1-\alpha)L - a\alpha\rho}{a\left[(1-\alpha)q + \alpha\right]}\right\} \equiv g_A^*(q). \tag{4.5}$$

The economy immediately jumps to the steady state for any admissible set of initial conditions.

Moreover, the steady-state growth rate of consumption is given by

$$g_c^* \equiv \frac{\dot{c}}{c} = \frac{\sigma}{\epsilon - 1}\, g_A^*. \tag{4.6}$$

Proposition 1 reveals that the steady-state growth rates of the economy depend on the exogenously given strength of the rule of law. If $q = 1$, then the enforcement of the rule of law is perfect. This case is discussed by Grossman and Helpman. Here, it serves as a useful reference point where the economy's growth rate is $g_A^*(1) = max\{0, (1-\alpha)L/a - \alpha\rho\}$. Whether this rate is strictly positive hinges on the environment, in which the economy operates, parameterized by a, ρ, α and L. Naturally, the effect of an imperfectly enforced rule of law on the steady-state allocation depends on whether this set of parameters allows for strictly positive growth or not.

Corollary 1

1. *If $g_A^*(1) > 0$, then there is $q_{min} \in (0,1)$ such that $g_A^* > 0$ and $g_c^* > 0$ if and only if $q > q_{min}$. If $q > q_{min}$, then $dg_A/dq > 0$.*

   Moreover,

$$\frac{\partial q_{min}}{\partial \alpha} > 0, \quad \frac{\partial q_{min}}{\partial \rho} > 0, \quad \frac{\partial q_{min}}{\partial a} > 0, \quad \frac{\partial q_{min}}{\partial L} < 0. \tag{4.7}$$

2. *If $g_A^*(1) = 0$, then the strength of the rule of law has no growth effects.*

Statement 1 of Corollary 1 refers to all situations in which the environment of the economy is such that under a perfect rule of law the economy grows at a strictly positive rate. Then, a less than perfect rule of law can be responsible for an economy to be trapped in a no-growth equilibrium. The presence of the mafia and the prospect of expropriation reduce the value of an innovation, i. e., of a patent v (see equation (3.14)). Hence, the incentives to engage in research declines. For the economy to grow at a positive rate, the strength of the rule of law has to surpass a threshold level q_{min}. Thus, we can assert that a certain strength of the rule of law is an underlying prerequisite for sustained growth. If $q > q_{min}$, then raising q speeds up economic growth.

The threshold level q_{min} also depends on the environment of the economy. The comparative statics of (4.7) reveal that q_{min} increases in α and ρ and declines in $1/a$ and L. Intuitively, the lower the degree of substitutability of intermediate goods, α, the lower are the monopoly profits in the intermediate-good sector. The greater the discount rate, ρ, the lower is the incentive to save and to acquire equity shares issued by research firms. The lower the productivity of researchers, $1/a$, the lower is the research output. Finally, the smaller the aggregate labor endowment, L, the less labor is available for research. All these factors have negative effects on the incentives to engage in research. The greater these countervailing forces on the invention of new products, the stronger the rule of law has to be to ensure a strictly positive growth rate. For instance, larger economies with very productive researchers can more easily afford to have a weak rule of law. Clearly, if the economy does not admit a strictly positive growth rate under a perfect enforcement of the rule of law, improvements in q have no growth effects. However, even in this case, q affects the equilibrium income distribution.

Corollary 2 *For all $q \in [0, 1]$, mafia income accounts for the fraction $\mu = (1 - q)(1 - \alpha)$ of aggregate final-good output while legal income constitutes the fraction $1 - \mu$. For any $g_A^* > 0$, the functional distribution of income is independent of q. If $g_A^* = 0$, labor income is independent of q and a rise in q increases the share of capital income in legal household income.*

4.2 Welfare Analysis

In this section we derive the Pareto-efficient growth rate of the economy and compare it to the dynamic general equilibrium. We establish cases where the presence of a mafia is Pareto-improving. As a benchmark we consider a social planner who allocates the factors of production and outputs to households and firms disregarding the rule of law.

Due to decreasing marginal productivity of the intermediate goods in the production of the final good, it is obvious that the social planner will also choose a symmetric solution $c = A^{\sigma/(\epsilon-1)}L_x$. Then, the planner's task is to choose the allocation of labor between the production of intermediate goods and research to maximize U of (3.1). Invoking full employment and in view of (4.2), the planner maximizes U subject to $\dot{A} = ag_A$ and $c = A^{\sigma/(\epsilon-1)}(L - ag_A)$. This problem has previously been solved by Bénassy (1998). In our notation his result appears in the following proposition.

Proposition 2 *(Bénassy (1998)) The Pareto-efficient growth rate of intermediate goods is*

$$g_A^P = \max\left\{0, \frac{L}{a} - \frac{(\epsilon-1)\rho}{\sigma}\right\}. \tag{4.8}$$

The economy immediately settles at this steady-state growth rate. For all $g_A^P > 0$ holds that $\partial g_A^P/\partial\sigma > 0$.

Corollary 3 compares the Pareto-efficient allocation to the equilibrium allocation for all $\sigma > 0$ and $q \in [0,1]$.

Corollary 3 *Let $g_A^*(1) > 0$. Then, there are threshold values $\underline{\sigma}$ and $\bar{\sigma}$ with $0 < \underline{\sigma} < \bar{\sigma} < 1$ such that*

- *if $\sigma > \bar{\sigma}$, then $g_A^P > g_A^*$.*
- *if $\sigma \in (\underline{\sigma}, \bar{\sigma}]$, then there is a unique $q^P \equiv q^P(\sigma) \in (q_{min}, 1]$ such that $g_A^*(q^P) = g_A^P$. Moreover, $q^P(\sigma)$ is a function satisfying*

$$\frac{dq^P}{d\sigma} > 0, \quad \lim_{\sigma\to\underline{\sigma}} q^P(\sigma) = q_{min}, \quad \text{and} \quad q^P(\bar{\sigma}) = 1.$$

- *if $\sigma \leq \underline{\sigma}$, we have $g_A^P = 0$. It follows $g_A^*(q) = g_A^P$ for all $q < q_{min}$.*

According to Corollary 3 the Pareto-efficient growth rate may be smaller than the equilibrium growth rate if the gains from specialization are sufficiently small, i. e., $\sigma \leq \bar{\sigma}$. The intuition for this is the following.

For a perfect rule of law, it is well known that the equilibrium outcome may differ from the Pareto-efficient one because of three opposing effects, namely, the consumer-surplus effect[3], the profit-destruction effect, and the intertemporal-spillover effect. In the standard Grossman-Helpman model, with $\sigma = 1$ and $q = 1$, the consumer-surplus effect and the profit-destruction effect exactly cancel out and due to the intertemporal-spillover effect the efficient growth rate exceeds the equilibrium growth rate. For $\sigma > 1$, this gap widens. Intuitively, the surplus that accrues to households when a new product is added in the production of the consumption good increases with σ. By contrast, if $\sigma < 1$, the profit-destruction effect surpasses the consumer-surplus effect. Nevertheless, the equilibrium

[3]The term consumer-surplus effect stems from an alternative interpretation of the model where per-period utility is directly defined over the set of differentiated consumption goods $x(j)$, i. e.,

$$\ln c(t) = \ln\left[A^{(\sigma-1)(1-\alpha)} \int_0^{A(t)} x(j,t)^\alpha \, dj\right]^{1/\alpha}.$$

growth rate might still fall short of the efficient one due to the intertemporal-spillover effect. However, for $\sigma < \bar{\sigma}$, the consumer-surplus effect becomes so small that the absolute value of the profit-destruction effect is greater than the sum of the values of the consumer-surplus and intertemporal-spillover effects such that the equilibrium growth rate exceeds the Pareto-efficient one.

In this case, the Pareto-optimum can be implemented by having an imperfect rule of law, i. e., $q < 1$, which introduces a fourth externality into the model. Mafia activity under a weak rule of law lowers the value of an innovation and potentially causes the equilibrium to underinvest in research.
In other words, if $\sigma < \bar{\sigma}$, it is socially optimal to allow for a positive degree of mafia activity, $q^P(\sigma) < 1$. Intuitively, in this scenario mafia activity acts as a tax that the government levies on intermediate-good firms and which helps to internalize the externality described by the profit-destruction effect. In this context, mafia income, M, corresponds to tax revenues that are redistributed to the household in a lump-sum fashion and $\mu(q^P)$ represents the ratio of government expenditure to GDP.

5 Endogenous Strength of the Rule of Law

In this section, the government plays an active role. It uses tax resources to pay police officers, judges, etc., that help to enforce the rule of law. More precisely, we stipulate that the strength of the rule of law, q, depends on the strength of the government, which is measured at each t in terms of government expenditure on the rule of law, G, as a share of total final-good output, y,

$$q = F\left(\frac{G}{y}\right) \quad \text{with} \quad F : [0, 1] \to [0, 1]. \tag{5.1}$$

Moreover, F is $\mathcal{C}^2$ with $F(0) = q_0 \in (0, q_{min})$, $F(1) = 1$, $F' > 0 > F''$, and $\lim_{G/y \to 0} F' = \infty$.

This reduced form relationship captures the idea that the government via increased spending relative to the size of the economy can improve the rule of law, though at a declining rate. Naturally, government expenditure is bounded by aggregate output. Without government spending on the rule of law firms keep a fraction q_0 of their profits. The fraction q_0 has an interpretation as the minimal strength of the rule of law guaranteed by the norms of the society that does not require any enforcement by the government. However, in this case the economy won't grow as $q_0 < q_{min}$. If the government spent total aggregate output on the enforcement of the rule of law, property rights in the intermediate-good sector would be fully secured. Moreover, the function F fulfills an Inada-type condition. A function that complies with all these properties is $F = (G/y)^\gamma$ with $\gamma \in (0, 1)$. Note also that q is a flow variable, i. e., the enforcement level of the rule of law has to be maintained constantly.

Finally, we assume that the government finances its expenditure by levying a tax $\tau \in [0,1]$ on final-good output y, i. e.,[4]

$$G = \tau y \tag{5.2}$$

such that $G/y = \tau$.

5.1 Dynamic General Equilibrium

The tax on final-good output acts as a tax on consumption. The final good y is manufactured according to (3.6), but final-good producers have to pay the tax τ on their total production. In other words, final-good producers are aware that they will only be able to sell $c = (1-\tau)y$ as private consumption goods to the households. The remaining fraction of output, $G = \tau y$, is claimed by the government and used for the enhancement of the rule of law. In equilibrium, final-good producers pass on the tax to consumers via a higher price p_c.

Then, given τ, the equilibrium consists of an allocation $\{y(t), c(t), \Omega(t), M(t), G(t), x(j,t), l(j,t), L_x(t), L_A(t), A(t)\}_{t=0}^{t=\infty}$ and a price system $\{r(t), p_c(t), w(t), p(j,t), v(j,t)\}_{t=0}^{t=\infty}$ such that (5.2) and conditions (E1) to (E7) of the DGE of Section 4.1 with q specified by (5.1) hold.

Then, following the same steps as in Section 4.1 with $q = F(\tau)$ we obtain the steady-state growth rate of intermediates and the growth rate of consumption. The tax on final-good output affects the steady-state growth rates of A and c only through its effect on q. In fact, for a given tax rate $\tau \in [0,1]$, we find

$$g_A^* \quad = \quad \max\left\{0, \frac{(1-\alpha)F(\tau)L - a\alpha\rho}{a\left[F(\tau)(1-\alpha)+\alpha\right]}\right\} \equiv g_A^*(F(\tau)) \tag{5.3}$$

and

$$g_c^* \quad = \quad \frac{\sigma}{\epsilon - 1}\, g_A^*. \tag{5.4}$$

Henceforth, we assume that the environment of our economy is such that there would be positive growth if the rule of law were perfect, i. e., $g_A^*(F(1)) > 0$. Then, analogously to Corollary 1, there exists a $\tau_{min} \in (0,1)$ such that for all $\tau > \tau_{min}$ the steady-state growth rates are positive. Thus, a strong government that spends sufficient resources on the enforcement of the rule of law can increase the incentives to engage in research such that the economy moves to an equilibrium with a positive growth rate.

[4]For a balanced growth path to exist government spending has to be proportionate to the size of the economy.

5.2 Welfare Analysis

From (5.3) and (5.4) it is clear that $\partial g_c^*/\partial\tau > 0$ for any $g_c^* > 0$ and $\tau \in (0,1)$. However, there is little reason why the tax rate should be arbitrarily large since a higher tax and faster consumption growth have a cost in terms of foregone current consumption. In this section, we analyze which tax rate the government should choose in order to maximize the welfare of the representative household in equilibrium. By integrating over time the utility function (3.1) with $c_t = c_0\, e^{g_c^* t}$ individual welfare in equilibrium obtains as

$$U = \frac{1}{\rho}\left(\ln c_0 + \frac{g_c^*}{\rho}\right),$$

where c_0 denotes the initial level of consumption at $t = 0$. By combining $c = (1-\tau)y$ with (3.11) and (4.1) we obtain c_0, for a given initial quantity of intermediates A_0, as

$$c_0 = (1-\tau)A_0^{\frac{\sigma}{\epsilon-1}} L_x. \tag{5.5}$$

When maximizing the welfare function we have to take into account that U is continuous, yet piecewise-defined on $[0,\tau_{min}]$ and $[\tau_{min},1]$

$$U = \begin{cases} \frac{1}{\rho}\ln\left[(1-\tau)A_0^{\frac{\sigma}{\epsilon-1}} L\right] & \text{if } \tau \in [0,\tau_{min}] \\ \\ \frac{1}{\rho}\ln\left[\frac{(1-\tau)\alpha(L+a\rho)A_0^{\frac{\sigma}{\epsilon-1}}}{[F(\tau)(1-\alpha)+\alpha]}\right] + \frac{g_c^*}{\rho^2} & \text{if } \tau \in [\tau_{min},1]. \end{cases} \tag{5.6}$$

The reason for this is that the steady-state consumption growth rate is only positive for $\tau > \tau_{min}$. More specifically, for values of τ smaller than τ_{min}, the rule of law is so weak that no research takes place. Hence, the whole labor supply is devoted to intermediate-good production, i. e., $L_x = L$, and $g_A^* = g_c^* = 0$. Thus, on the interval $[0,\tau_{min}]$, a rise in τ just reduces consumption and welfare declines monotonically in τ (see Figure 1).

For values of τ greater than τ_{min}, the rule of law is sufficiently strong to provide incentives to profitably conduct in research. This has two effects on welfare. On the one hand, the growth rates of intermediates and consumption become positive. On the other hand, less labor is allocated to manufacturing, $L_x = \alpha/w$, which translates into lower levels of initial consumption.[5]

[5]One obtains the left term of U in (5.6) for $\tau \geq \tau_{min}$ upon substitution of $L_x = \alpha/w$ in (5.5). The steady-state wage rate is determined by condition (3.15) which has to hold with equality in a steady state with positive R&D activity, i. e., $w = vA/a$. The aggregate value of equities, $\Omega = vA$, is constant in the steady state. From $A(s) = A_0\, e^{g_A s}$ and (3.14) one finds that $vA = F(\tau)(1-\alpha)/(g_A^* + \rho)$ and thus $w = [F(\tau)(1-\alpha)+\alpha]/(L + a\rho)$. Note, that the former also implies that the initial value $A_0 > 0$ determines $v(0)$ such that $\Omega_0 = F(\tau)(1-\alpha)/(g_A^* + \rho)$.

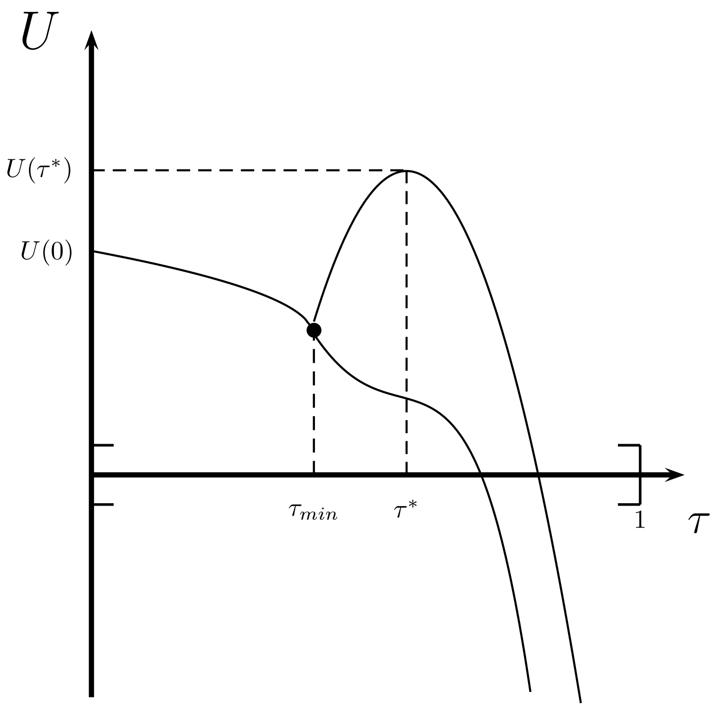

Figure 1: Welfare U as a function of $\tau \in [0,1]$.

If the positive growth effect of a higher tax rate outweighs the negative effects on the level of initial consumption near τ_{min}, then the welfare function is inversely U-shaped on $[\tau_{min}, 1]$. Otherwise, U continues to decline in τ (see Figure 1).

The following proposition establishes the welfare-maximizing tax rate.

Proposition 3 *Let* $g_A^*(F(1)) > 0$.

1. *In the interval* $[0, \tau_{min}]$, *U is maximized at* $\tau = 0$.
2. *If* $dU/d\tau|_{\tau=\tau_{min}} > 0$, *then* $\underset{\tau \in [\tau_{min},1]}{\arg\max} U = \tau^* \in (\tau_{min}, 1)$. *Otherwise* $\tau_{min} = \underset{\tau \in [\tau_{min},1]}{\arg\max} U$.
3. *If* $\tau^* \in (\tau_{min}, 1)$ *and* $U(\tau^*) > U(0)$, *then* τ^* *maximizes U on* $[0,1]$. *Otherwise* $\tau = 0$ *maximizes U on* $[0,1]$.[6]

If a positive globally welfare-maximizing tax rate exists, it is strictly smaller than one. Thus, Proposition 3 implies that the optimal tax rate will never fully enforce the rule of law which in turn means that there is a positive optimal level of mafia activity. At this point we can draw a comparison to the literature on optimal law enforcement initiated by the seminal paper of Becker (1968). Most of the models in this literature (see, e. g.,

[6] There is a non-generic case where $U(0) = U(\tau^*)$. Then, the solution of $\max_{\tau \in [0,1]} U$ is not unique.

Garoupa (1997) for a survey) find that the optimal amount of crime deterrence does not eliminate crime altogether. The principal reason for this is that eradicating crime is costly and has a declining social benefit. In our model, the enforcement of the rule of law has a social cost in form of tax payments. Moreover, Statement 3 of Proposition 3 reveals that in terms of welfare no growth can be better than some growth[7]. Observe that $U(\tau^*)$ may not be a global maximum because $U(0) > U(\tau^*)$. Then, it is preferable to set $\tau = 0$. This is the case if the negative static welfare effect of the consumption tax is so large that it is optimal for the government not to levy any taxes, leave the rule of law unchanged and remain in an equilibrium without growth.

Corollary 4 *The lower τ_{min} and the greater σ, the more likely it is that τ^* is the global maximizer of U on [0,1]. Moreover, it holds that*

$$\frac{\partial \tau^*}{\partial \alpha} < 0, \quad \frac{\partial \tau^*}{\partial a} < 0, \quad \frac{\partial \tau^*}{\partial \rho} < 0, \quad \frac{\partial \tau^*}{\partial L} > 0, \quad \frac{\partial \tau^*}{\partial \sigma} > 0.$$

Intuitively, the lower the threshold investment necessary to move an economy to a positive growth path, the more likely it is that a welfare-maximizing government should indeed intervene and use tax revenues to improve the rule of law. This can be interpreted as a self-reinforcing mechanism. Similarly, for large gains from specialization, σ, the utility gain from additional varieties is sufficient to outweigh the utility loss due to lower levels of initial consumption. Finally, the comparative statics of Corollary 4 reveal that a lower degree of substitutability of intermediates, higher productivity of researcher, a lower discount rate, a greater labor supply, and greater gains from specialization increase the welfare-maximizing tax rate.

6 Concluding Remarks

We have studied the interdependence between innovation, economic growth, and the rule of law in an economy where growth results from an expanding set of product varieties. The strength of the rule of law determines the profit that firms expect from an innovation investment. Our results may be summarized as follows.

1. We find that a weak rule of law can be responsible for an economy to be caught in a "no-growth trap". In other words, in our setting a minimum strength of the rule of law is a prerequisite for sustained growth.

[7] Arnold and Bauer (2008) draw a similar conclusion in Grossman-Helpman type variety expansion growth model with erosion of monopoly power due to exogenous imitation and a non-innovative traditional sector.

2. We establish that a weak rule of law may be Pareto-improving. This is the case when the equilibrium growth rate exceeds the Pareto-efficient one. Then, it is desirable to weaken the incentive to innovate.

3. When the rule of law is endogenous, we show that a strong government willing to spend an appropriate amount of resources on the improvement of the rule of law can shift the economy from a no-growth into a welfare-improving equilibrium with strictly positive growth rates.

4. Depending on the economic environment described by preference, endowment, and technology parameters, no growth might be better than some growth even though the government is willing to intervene.

7 Appendix

7.1 Proof of Proposition 1

Setting $\dot{V} = 0$ in (4.4) and substituting $V = \frac{g_A+\rho}{q(1-\alpha)}$ in (4.3) delivers equation (4.5).

As to the transitional dynamics consider the phase-diagram in the (g_A, V)-plane depicted in Figure 2. The kinked curve LL depicts the labor market equilibrium as expressed by equation (4.3) and has to be satisfied at every moment in time. The lines VV_1 and VV_q reflect the combinations of V and g_A that imply $\dot{V} = 0$. While the VV_1-locus represents the case without mafia activity, the VV_q-locus corresponds to a situation with an imperfect rule of law. The intersection of the LL-locus with the VV_1 or the VV_q-locus, respectively, determines the equilibrium growth rate of intermediate goods. Figure 2 has been drawn to depict the case in which the rule of law is so weak that there is no growth in equilibrium. Moreover, from the phase diagram one sees that starting the economy outside of the steady state leads either to $V \to \infty$, $g_A = 0$ or $V \to 0$, $g_A \to L/a > 0$. Both cases violate rational expectations. We abstract from a more detailed discussion of the transitional dynamics because it is a mirror of the dynamics of the Grossman-Helpman model that have been discussed extensively in Arnold (1997).

To obtain the consumption growth rate, consider that from (3.11), (4.1) and $y = c$ follows $c = A^{\sigma/(\epsilon-1)}L_x$. Then, $\dot{c}/c = \sigma/(\epsilon-1)g_A + \dot{L}_x/L_x$. By (E5) and in view of (4.2), L_A and L_x are constant in the steady state such that $\dot{c}/c = \sigma/(\epsilon-1)g_A^*$. ■

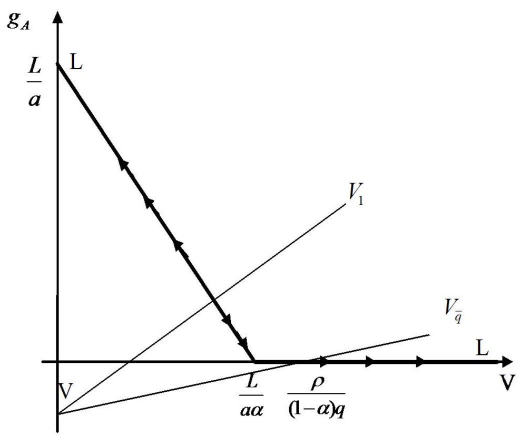

Figure 2: Equilibrium growth rate with exogenous institutions

7.2 Proof of Corollary 1

Setting $q = 1$ in (4.5) delivers $g_A^*(1) = \max\{0, (1-\alpha)L/a - \alpha\rho\}$.

Thus, with respect to the first statement, $g_A^*(1) > 0$ if and only if $a\rho\alpha/(1-\alpha)L < 1$. Moreover, from (4.5) follows $g_A^*(q) > 0$ if and only if $q > a\rho\alpha/(1-\alpha)L$. Denote $q_{min} \equiv a\rho\alpha/(1-\alpha)L$. Then, $g_A^* > 0$ if and only if $q > q_{min}$, where it is obvious that $q_{min} \in (0,1)$. Moreover, $\forall g_A^* > 0$ it follows from (4.5) that

$$\frac{\partial g_A^*(q)}{\partial q} = \frac{(1-\alpha)\alpha(L + a\rho)}{[q(1-\alpha)+\alpha]^2} > 0.$$

The comparative statics obtain from differentiation of q_{min} with respect to α, ρ, a and L.

As to the second statement, note that the second term in brackets of (4.5) positively depends on q. Thus, if $g_A^* = 0$ for a perfect rule of law, i. e., $q = 1$, then it also has to be zero for all $q < 1$.■

7.3 Proof of Corollary 2

The aggregate value of equities Ω in this economy is constant over time. From $A(s) = A_0 e^{g_A s}$ and (3.14) one finds that

$$v(t) \;=\; \frac{q(1-\alpha)}{A_0(g_A+\rho)} e^{-g_A t} \quad \text{so that} \quad \Omega(t) = A(t)v(t) = \frac{q(1-\alpha)}{g_A+\rho}.$$

The mafia expropriates $(1-q)(1-\alpha)/A$ from each of A intermediate-good firms. Thus, total mafia income is given by $M = (1-q)(1-\alpha)$. As we have chosen consumption expenditure as the numéraire, $(1-q)(1-\alpha) = \mu$ represents a fraction of aggregate output.

As Ω is constant in the steady state also has to hold $\dot{\Omega} = 0$. Total legal income then obtains from the household's budget constraint (3.2) as

$$r\Omega + wL = 1 - M = q(1-\alpha) + \alpha.$$

In a steady state with positive R&D activity ($g_A^* > 0$) condition (3.15) has to hold with equality such that $w = vA/a$. Thus, $wL = L\Omega/a$ and $r\Omega = \rho\Omega$. Consequently, a change in q does not affect the distribution between these two types of income. However, if $g_A = 0$, then $L_x = \alpha/w$ with $L_x = L$ implies $wL = \alpha$. Moreover, $r\Omega = q(1-\alpha)$. Thus, in this case an increase in q raises the share of income from equities in legal household income. ■

7.4 Proof of Proposition 2

The program of the social planner is to

$$\max_{g_A, A} \int_0^\infty \ln c\, e^{-\rho t} dt, \quad \text{where} \quad c = A^{\sigma/(\epsilon-1)} \left(L - a g_A\right)$$
$$\text{s.t.} \qquad \dot{A} = A g_A.$$

Hence, the following current-value Hamiltonian obtains

$$\mathcal{H} \equiv \frac{\sigma}{\epsilon - 1} \ln A + \ln\left(L - a g_A\right) + \lambda A g_A.$$

The necessary and sufficient conditions for an optimum are

$$\frac{\partial \mathcal{H}}{\partial g_A} = \frac{-a}{L - a g_A} + \lambda A \leq 0 \text{ with “=”, if } g_A > 0, \tag{7.1}$$

$$\dot{\lambda} = \rho\lambda - \frac{\sigma}{\epsilon - 1}\frac{1}{A} - \lambda g_A, \tag{7.2}$$

$$0 = \lim_{t\to\infty} e^{-\rho t} \lambda A. \tag{7.3}$$

Denote $S \equiv 1/\left(\lambda A\right)$. Then, the first-order condition (7.1)-(7.3) become

$$g_A = \max\left\{0, \frac{L}{a} - S\right\} \tag{7.4}$$

$$\frac{\dot{S}}{S} = \frac{\sigma}{\epsilon - 1} S - \rho \tag{7.5}$$

$$0 = \lim_{t\to\infty} \frac{e^{-\rho t}}{S}. \tag{7.6}$$

In a steady-state $\dot{S} = 0$ has to hold. From (7.5) one has $S = (\epsilon - 1)\rho/\sigma$ so that (7.4) becomes (4.8). There are no transitional dynamics. The proof of this mirrors Arnold (1997), p. 132-134, for $\sigma \neq 1$. ■

7.5 Proof of Corollary 3

While the Pareto-efficient growth rate (4.8) is monotonically increasing in σ, the equilibrium growth rate (4.5) is independent of σ and increasing in q. Then, there must exist a $\bar{\sigma}$ such that $\forall \sigma > \bar{\sigma}$ holds $g_A^P > g_A^*(1) > g_A^*(q < 1)$. From (4.8) and (4.5) with $q = 1$ follows

$$g_A^P > g_A^*(1) \quad \Leftrightarrow \quad \sigma > \frac{a\rho}{(1-\alpha)(L+a\rho)}.$$

Denote $\bar{\sigma} \equiv \frac{a\rho}{(1-\alpha)(L+a\rho)}$. It is straightforward to see that $\bar{\sigma} < 1$ if $g_A^*(1) > 0$ holds (remember that $g_A^*(1) > 0$ implies $q_{min} = a\alpha\rho/(1-\alpha)L < 1$). Thus, the first statement has been proven.

For small σ, g_A^P drops to zero. More specifically, $g_A^P = 0$ for all $\sigma < a\alpha\rho/(1-\alpha)L$. Denote $\underline{\sigma} = \frac{a\alpha\rho}{(1-\alpha)L}$. As $a\alpha\rho/(1-\alpha)L < 1$, one readily verifies that $\underline{\sigma} < \bar{\sigma}$. Thus, in the interval $(\underline{\sigma},\bar{\sigma}]$, $g_A^P > 0$. Moreover, as $dg_A^*/dq > 0$, $g_A^*(q_{min}) = 0$ and $g_A^*(1) = g_A^P(\bar{\sigma})$, for each $\sigma \in (\underline{\sigma},\bar{\sigma}]$ must exist a $q^P \in (q_{min}, 1)$ such that $g_A^*(q^P) = g_A^P$. Equalizing (4.8) and (4.5) and solving for q delivers

$$q^P(\sigma) = \frac{\sigma(L+a\rho)(1-\alpha) - a\alpha\rho}{(1-\alpha)a\rho}. \tag{7.7}$$

From (7.7) one can verify that q^P is increasing in σ, $q^P(\underline{\sigma}) = a\alpha\rho/(1-\alpha)L = q_{min}$ and $q^P(\bar{\sigma}) = 1$. Then, the second statement follows directly.

As to the third statement, remember that $g_A^P = 0$ for all $\sigma < \underline{\sigma}$. Thus, for $g_A^* = g_A^P$ to hold g_A^* has to be zero. This is the case for all $q < q_{min}$ (see Corollary 1). ■

7.6 Proof of Proposition 3

We proof each statement of the Proposition separately, starting with Statement 1.

1. In the interval $[0, \tau_{min}]$, U is a monotonically declining function in τ. Thus, in this interval U has its global maximum at $\tau = 0$. ■

2. In the interval $[\tau_{min}, 1]$, increasing τ has two opposing effects on U. A higher τ negatively impinges on welfare by lowering initial consumption c_0 and positively affects welfare by enabling a higher consumption growth rate g_c^*. For large values of τ, the former effect dominates the latter and $\lim\limits_{\tau\to 1} \to -\infty$. For values of τ close to τ_{min} it is not clear a priori which effect dominates. In the following we demonstrate that the maximization of U in $[\tau_{min}, 1]$ has a corner (unique interior) solution if a marginal increase in τ at $\tau = \tau_{min}$ has a negative (positive) effect on utility, i.e., $dU/d\tau|_{\tau=\tau_{min}} < 0$ (>0).

Let $dU/d\tau|_{\tau=\tau_{min}} < 0$. In this case U is a monotonically declining function in $[\tau_{min}, 1]$ and U is maximized at τ_{min}.

$$\begin{aligned} \left.\frac{dU}{d\tau}\right|_{\tau=\tau_{min}} &< 0 \\ \Leftrightarrow \quad \left|\frac{\partial \ln c_0}{\partial \tau}\right|_{\tau=\tau_{min}} &> \left.\frac{1}{\rho}\frac{\partial g_c}{\partial \tau}\right|_{\tau=\tau_{min}} \\ \Leftrightarrow \quad \frac{a\alpha^2\rho(L+a\rho)}{F'(\tau_{min})(1-\tau_{min})} &> \sigma(1-\alpha)^2L^2 - (1-\alpha)a\rho\alpha L, \end{aligned} \tag{7.8}$$

where we have substituted $F(\tau_{min}) = \frac{a\alpha\rho}{(1-\alpha)L}$. For future reference, note that $\sigma < F(\tau_{min})$ is a sufficient but not necessary condition for (7.8) to hold.

Inversely,

$$\begin{aligned} \left.\frac{dU}{d\tau}\right|_{\tau=\tau_{min}} &> 0 \\ \Leftrightarrow \quad \left.\frac{1}{\rho}\frac{\partial g_c}{\partial \tau}\right|_{\tau=\tau_{min}} &> \left|\frac{\partial \ln c_0}{\partial \tau}\right|_{\tau=\tau_{min}} \\ \Leftrightarrow \quad \frac{a\alpha^2\rho(L+a\rho)}{F'(\tau_{min})(1-\tau_{min})} &< \sigma(1-\alpha)^2L^2 - (1-\alpha)a\rho\alpha L. \end{aligned} \tag{7.9}$$

In what follows we show that a unique interior local extremum in $[\tau_{min}, 1]$ exists if (7.9) holds. In $[\tau_{min}, 1]$,

$$\begin{aligned} \frac{dU}{d\tau} &= 0 \quad \Leftrightarrow \\ \frac{F'(\tau)(1-\tau)}{[F(\tau)(1-\alpha)+\alpha]^2} &= \frac{a\rho}{\sigma(1-\alpha)^2(L+a\rho) - (1-\alpha)a\rho\,[F(\tau)(1-\alpha)+\alpha]} \end{aligned} \tag{7.10}$$

Define the left-hand side of (7.10) as $LHS(\tau)$ and the right-hand side of (7.10) as $RHS(\tau)$. A unique solution to (7.10), $\tau^* \in (\tau_{min}, 1)$, exists if $LHS(\tau)$ and $RHS(\tau)$ intersect exactly once in $[\tau_{min}, 1]$. While $LHS(\tau)$ monotonically decreases in τ, $RHS(\tau)$ is a monotonically increasing function in τ. Moreover,

$$LHS(\tau_{min}) > RHS(\tau_{min})$$

$$\text{as} \quad \frac{F'(\tau_{min})(1-\tau_{min})L^2}{\alpha^2(L+a\rho)^2} > \frac{a\rho L^2}{(L+a\rho)\,[\sigma(1-\alpha)^2L^2 - (1-\alpha)a\alpha\rho L]}. \tag{7.11}$$

Condition (7.11) is equivalent to condition (7.9) if the right-hand side of (7.11) is positive which is true for all $\sigma > F(\tau_{min}) = a\alpha\rho/(1-\alpha)L$. The latter has to hold because $\sigma < F(\tau_{min})$ would imply $dU/d\tau|_{\tau=\tau_{min}} < 0$ which is a contradiction. $\sigma > F(\tau_{min})$ also entails that $RHS(\tau_{min}) > 0$. Thus, $LHS(\tau)$ and $RHS(\tau)$ intersect exactly once in $(\tau_{min}, 1)$. As U is a continuous function in $[\tau_{min}, 1]$ with $\lim_{\tau\to 1} \to -\infty$ and $dU/d\tau|_{\tau=\tau_{min}} > 0$, the unique local extremum at τ^* has to be a global maximum in $[\tau_{min}, 1]$. ■

3. Statements 1 and 2 determine the welfare-maximizing tax rates in the intervals $[0, \tau_{min}]$ and $[\tau_{min}, 1]$, respectively. To find the globally welfare-maximizing tax rate in $[0, 1]$ we have to compare the welfare associated with these two optimal rates. In the interval $[0, \tau_{min}]$, welfare is always maximized at $\tau = 0$. In the interval $[\tau_{min}, 1]$, welfare is either maximized at τ_{min} or at τ^*. Thus, if τ^* exists, it is the globally welfare-maximizing tax rate in the whole interval $[0, 1]$ if and only if $W(\tau^*) > W(0)$. This if fulfilled if

$$\frac{g_c^*(\tau^*)}{\rho} > -ln(1-\tau^*) - ln\left[\frac{\alpha(L+a\rho)}{L\left(F(\tau^*)(1-\alpha)+\alpha\right)}\right]. \tag{7.12}$$

Note that the right-hand side of (7.12) is positive as $\tau^* < 1$ and $a\alpha\rho/(1-\alpha)L < F(\tau^*)$. If τ^* exists and $U(\tau^*) < U(0)$, then $\tau = 0$ is globally welfare-maximizing. If $\tau_{min} = \underset{\tau^* \in [\tau_{min}, 1]}{\arg\max}\, U$, $\tau = 0$ is again globally welfare-maximizing as $U(0) > U(\tau_{min})$.
■

7.7 Proof of Corollary 4

From $F(\tau_{min}) = a\alpha\rho/(1-\alpha)L$ follows

$$\tau_{min} = F^{-1}\left(\frac{a\alpha\rho}{(1-\alpha)L}\right),$$

where F^{-1} is an increasing function. Then,

$$\frac{\partial\tau_{min}}{\partial\alpha} > 0, \quad \frac{\partial\tau_{min}}{\partial a} > 0, \quad \frac{\partial\tau_{min}}{\partial\rho} > 0, \quad \frac{\partial\tau_{min}}{\partial L} < 0.$$

Condition (7.9), which guarantees the existence of an interior welfare maximum in the interval $[\tau_{min}, 1]$, can be rearranged to

$$\frac{a\alpha^2\rho(1+\frac{a\rho}{L})}{F'(\tau_{min})(1-\tau_{min})} \quad < \quad (1-\alpha)\left[\sigma(1-\alpha)L - a\rho\alpha\right]. \tag{7.13}$$

One readily verifies that the left-hand side of (7.13) increases in α, a, ρ and decreases in L whereas the right-hand side of (7.13) decreases in α, a, ρ and increases in L and σ. Thus, the smaller τ_{min}, i. e., the smaller α, a, ρ and the greater L, and the greater σ the more likely it is that condition (7.13) is fulfilled and an interior maximum exists. A similar argument applies for condition (7.12), which assures that $U(\tau^*) > U(0)$. As the right-hand side of (7.12) increases in a, α, ρ, and decreases in L, it is more likely that this condition is satisfied the smaller a, α, ρ, and the greater L.

If $\tau^* \in (\tau_{min}, 1)$ exists, it is determined by condition (7.10) (see proof of Proposition 3). Then, the comparative statics results follow from implicit differentiation of (7.10). ■

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