The Term Structure of Equity Premia in an Affine Arbitrage-Free Model of Bond and Stock Market Dynamics*

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Abstract

We estimate time-varying expected excess returns on the US stock market from 1983 to 2008 using a novel model that jointly captures the arbitrage-free dynamics of stock returns and nominal bond yields. The model nests the popular class of affine term structure models. Expected stock returns and bond yields as well as bond and equity risk premia result as affine functions of the state variables: the dividend yield, two factors driving the one-period real interest rate and the rate of inflation. The estimated series of expected excess stock returns is compared to the evolution of the equity risk premium implied by the prominent three-stage dividend-discount model, which is widely used in practice. This approach employs the same rate of return to discount all expected dividends, whereas our model uses at each point in time the arguably more adequate sequence of stochastic discount factors. While the equity risk premium implied by the arbitrage-free model turns out to exhibit similar dynamics as that implied by the simple dividend discount model, our model comes with the advantage of offering the whole term structure of equity premia at each point in time. It also allows to trace the impact of changes in dividend yields and yield-curve shifts on equity risk premia for different horizons.

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1 Introduction

Following the seminal introduction of the affine class of term structure models by Duffie and Kan (1996), a large and still growing literature on affine term structure models for analyzing the yield curve emerged. The research in this area, e.g. Dai and Singleton (2000), was rather successful in showing that time-varying bond market risk premia can explain observed shortcomings of the expectations hypothesis of the yield curve. As equity valuation essentially requires discounting future dividend cash flows, the integration of stock and bond pricing in a single affine framework could be seen as the natural step forward.

One of the first papers in this respect is Bekaert and Grenadier (2001), who use the the same modeling approach as for affine term structure models, but include the dividend growth associated with a representative portfolio of equities into the state vector. Bond prices are still exponentially affine functions of the state variables but dividend-scaled stock prices are infinite sums of such functions. Hence, while bond yields have an affine representation as usual, stock returns have not. The same holds for the model by Lettau and Wachter (2007).

As an alternative, Mamaysky (2002) proposed a continuous-time affine model which includes the dividend yield (dividend over ex-dividend stock price) as state variable instead of the dividend growth. This comes with the advantage that stock prices have an exponentially affine closed-form solution. Equity returns are consequently affine functions of the state variables in this framework. Besides the fact that exact affine pricing equations for bond yields and equity returns greatly facilitate the estimation of the model, the approach chosen by Mamaysky has the advantage that it does not rely on the forecastability of dividend growth. In fact, as argued forcefully by Cochrane (2008) recently, the empirical evidence does not support the forecastability of this variable for US data. At the same time, dividend yields, albeit rather persistent, show strong evidence of being mean-reverting.

In this paper, we propose a discrete-time arbitrage-free model that jointly captures stock and bond price dynamics. The four-factor model comprises two (latent) state variables driving the real rate of interest, the rate of inflation and the dividend yield. Arbitrage-free bond yields and stock returns as well as bond yield and equity risk premia all result as affine functions of the state variables. The model is estimated on monthly US data from 1983 to 2008.

The paper contributes to the literature on affine asset pricing models and to the empirical finance literature on the US stock market.

First, from a technical point of view, the paper is – as far as we are aware of – the first one developing a general discrete-time affine framework with closed-form solutions for stock returns. It thereby encompasses the class of multi-factor affine term structure models. The paper by d'Addona and Kind (2006) also develops a discrete time model with closed-form affine solutions for stock returns but assumes a more restricted market price of risk. In our paper, the market price of risk specification is very general using the essentially affine approach proposed by Duffee (2002). This allows for various risk factors to affect the equity risk premium. Furthermore, the stock price equation in d'Addona and Kind (2006) is derived as the limit of equities expiring after n periods whereas we derive directly a solution for infinitely living equity, similar to Mamaysky (2002) and Piazzesi (2002) in a continuous-time setting.

Second, our model allows to derive at each point in time the whole 'term structure of equity risk premia'. That is, for an arbitrary investment horizon, it provides the excess expected return of equity over the model-implied real interest rate with the corresponding time to maturity. In addition, the joint affine framework allows to assess the impact of changing dividend yields as well as the impact of yield curve movements on term structure of equity premia.

Third, regarding the empirical analysis of bond and stock markets using an affine framework, our paper is novel in focussing on the time series of the equity risk premium. The aforementioned analysis by d'Addona and Kind (2006), in contrast, gears to understanding the correlation of bond and stock returns, while Lettau and Wachter (2007) focus on explaining the cross-sectional behavior of stock prices.

Our model gives a good fit to US government bond yields, and it implies bond yield risk premia, which are comparable in size and dynamics to those obtained in the affine term structure literature. Hence, enhancing an otherwise standard affine term structure to price also common stock does not adversely affect its capability of capturing salient bond market features.

Concerning stocks, the model-implied equity premia are compared to the equity premium implied by the three-stage dividend discount model, which is a well-established benchmark, especially among practitioners. It turns out that the model-implied premia, exhibit a marked comovement with the equity premium obtained from the dividend-discount model, which supports the empirical validity of our model. At the same time, the equity premia implied by the model can be interpreted as measures of required risk compensation for holding equity over well-defined investment horizons, whereas the equity premium from the dividend-discount model rather represents an average (over a set of horizons) excess return.

Within the sample period, our equity premia show a trend increase towards the early 1990s, followed by a decrease towards very low levels of expected excess returns by the year 2000 when the 'dot-com' euphoria had reached its climax. As a closer exploration of the term structure of equity premia reveals, required risk compensation during that time was extremely low, especially at the shorter end of the spectrum of investment horizons. The subsequent normalization primarily led to an increase of shorter-run premia, while the long end of the term structure of equity premia was hardly affected. The onset of the current financial turmoil, in contrast, has led to a marked upward shift of the equity premium for all investment horizons, while again the short end has shown the most distinct increase.

The following section develops the joint stock-bond-pricing model, where lengthier derivations are delegated to the appendix. Section 3 explains how the model is cast into the statistical state space form, documents the parameter restrictions used for estimation and presents the data. Section 4 contains the empirical results: first, parameter estimates and the empirical fit are reported. This is followed by a discussion of the estimated series of bond and equity risk premia. It also includes an interpretation of the 'term structure of equity risk premia' and how the latter depends on the dividend yield and movements in the term structure of real bond yields. Section 5 concludes and provides perspectives for future research.

2 An affine arbitrage-free model of bond and stock market dynamics

We specify a model for the joint arbitrage-free dynamics of bond yields and stock returns. Time is discrete and the unit time interval can be understood as one month. We will derive the pricing equations for nominal bonds of arbitrary maturity and one dividend-paying stock. This is motivated by the fact that we will use several nominal zero-coupon yields and one broad-based stock index for estimating the model. Hence, while the generalization to a family of dividend-paying securities would be straightforward, we will focus in the following on one stock that will be interchangeably be referred to as 'the stock' or 'the stock index'.

The core component of the model is a pricing kernel that prices assets that pay off in real terms. By also specifying the dynamics of inflation, we obtain the pricing kernel for nominal assets, which is required to compute the arbitrage-free dynamics of the term structure of nominal bond yields. Besides inflation, there are three other risk factors in the model, two of them driving the one-month risk-free real rate and a factor representing the payout yield of the stock index. The term 'payout yield refers to the payout of the stock index divided by its price. Viewed narrowly, this is tantamount to the dividend yield. However, as listed companies also have other measures at their disposal to let stock holders participate in profits (e.g. stock buy backs), the 'payout yield' subsumes all payments to investors, of which dividends may only be a part. Nevertheless, for simplicity, we will interchangeably use the term 'dividend yield' for the same variable, and likewise use the word 'dividends' for what rather refers to the total payout to equity holders.

The solution of the model is a system comprising the linear dynamics of the factor process as well as a a set of affine equations relating bond yields and stock returns to the factors. Hence, the resulting system encompasses the popular class of affine term structure models.

In the following, we first describe the dynamics of the factor process, which is followed by a specification of the pricing kernels. We then turn to the derivation of the arbitrage-free term structure dynamics of nominal bonds and the pricing problem for equity. After that, bond and equity risk premia are derived. The section closes with a synopsis of the model proposed here and the (three-stage) dividend-discount model.

2.1 Factor process

Let $X_t := (\pi_t, \gamma_t, L_{1t}, L_{2t})'$ denote a vector that contains inflation π_t, a log payout yield factor γ_t as well as two additional factors L_{1t} and L_{2t} that constitute the one-period real interest rate r_t. More precisely, π_t is the logarithmic month-on-month change of the level Π_t of a consumer price index, i.e. $\pi_t := \ln \Pi_t - \ln \Pi_{t-1}$. The payout-yield factor is defined as $\Gamma_t := (1 + \frac{D_t}{V_t})$, where D_t is the dividend of the stock in one-period terms and V_t is the ex-dividend stock price. The factor vector contains the log of that, $\gamma_t := \ln \Gamma_t$. Thus, γ_t approximates the dividend yield $\frac{D_t}{V_t}$. Finally, the real interest rate is an affine function of L_{1t} and L_{2t},

$$r_t = \delta_0 + \delta_1' X_t, \quad \delta_1 = (0, 0, \delta_{1,3}, \delta_{1,4})'. \tag{2.1}$$

The factor dynamics is specified as a stationary VAR(1),

$$X_{t+1} = a + \mathcal{K} X_t + \Sigma\, \eta_{t+1}, \quad \eta_t, \underset{i.i.d.}{\sim} N(0, I) \tag{2.2}$$

where a, $\mathcal{K}$ and Σ are a vector and matrices, respectively, of appropriate dimensions. The empirical counterparts of the elements of the factor vector will be discussed in section 3 below.

2.2 Real pricing kernel

We define the real pricing kernel, or stochastic discount factor (SDF), M_t as

$$M_{t+1} = \exp(-r_t) \frac{\xi_{t+1}}{\xi_t} \tag{2.3}$$

where r_t is the real one-month interest rate, and the risk-adjustment term satisfies

$$\xi_{t+1} = \xi_t \exp\left(-\frac{1}{2}\lambda_t' \lambda_t - \lambda_t' \eta_{t+1}\right), \tag{2.4}$$

with

$$\lambda_t = \lambda_0 + \Lambda_1 X_t \tag{2.5}$$

being the vector of market prices of risk. We take the same approach as in most of the affine term structure literature and let risk prices be spanned by the factors.[1]

Hence, for the log real stochastic discount factor, $m_t := \ln M_t$, we have

$$m_{t+1} = -\frac{1}{2}\lambda_t'\lambda_t - \delta_0 - \delta_1' X_t - \lambda_t'\eta_{t+1}. \tag{2.6}$$

Under the condition of no arbitrage, the price at time t of an asset i with real payoff Z_{t+1}^i in period $t+1$ satisfies

$$P_t^i = E_t(M_{t+1} Z_{t+1}^i). \tag{2.7}$$

2.3 Nominal pricing kernel

Similarly, assets that pay off in nominal terms (i.e. in units of currency) are priced by a nominal SDF $\tilde{M}_t$, so their prices are given by

$$\tilde{P}_t^i = E_t(\tilde{M}_{t+1} \tilde{Z}_{t+1}^i). \tag{2.8}$$

In the following, if not stated otherwise, a tilde on top of a variable (price, return, stochastic discount factor) will denote that it relates to nominal as opposed to real assets. The log nominal and the log real SDF are related by[2]

$$\tilde{m}_{t+1} = m_{t+1} - \pi_{t+1}. \tag{2.9}$$

Let δ_π denote a selection vector that picks inflation from the factor vector, i.e. $\pi_t = \delta_\pi' X_t$. Using (2.9) one obtains for the nominal log SDF $\tilde{m}_{t+1}$:

$$\begin{aligned} \tilde{m}_{t+1} &= m_{t+1} - \delta_\pi' X_{t+1} \\ &= m_{t+1} - \delta_\pi'(a + \mathcal{K} X_t + \Sigma \eta_{t+1}) \\ &= -\frac{1}{2}\lambda_t'\lambda_t - (\delta_0 + \delta_\pi' a) - (\delta_1' + \delta_\pi' \mathcal{K}) X_t - \lambda_t'\eta_{t+1} - \delta_\pi' \Sigma \eta_{t+1} \end{aligned} \tag{2.10}$$

To see that the nominal and the real SDF have a perfectly analogous functional form, we define $\tilde{\lambda}_t := \lambda_t + \Sigma'\delta_\pi$. It satisfies

$$\begin{aligned} \tilde{\lambda}_t'\tilde{\lambda}_t &= (\lambda_t + \Sigma'\delta_\pi)'(\lambda_t + \Sigma'\delta_\pi) \\ &= \lambda_t'\lambda_t + 2\delta_\pi'\Sigma\lambda_t + \delta_\pi'\Sigma\Sigma'\delta_\pi \\ &= \lambda_t'\lambda_t + 2\delta_\pi'\Sigma\lambda_0 + 2\delta_\pi'\Sigma\Lambda_1 X_t + \delta_\pi'\Sigma\Sigma'\delta_\pi. \end{aligned}$$

Replacing λ_t in (2.10) we obtain an expression for the log nominal SDF $\tilde{m}_{t+1}$ which is analogous in structure to (2.6),

$$\tilde{m}_{t+1} = -\frac{1}{2}\tilde{\lambda}_t'\tilde{\lambda}_t - \tilde{\delta}_0 - \tilde{\delta}_1' X_t - \tilde{\lambda}_t'\eta_{t+1}. \tag{2.11}$$

[1] An alternative route is chosen by Lettau and Wachter (2007) who specify a separate process for the market price of dividend(-growth) risk. The dynamics of this variable is a simple AR(1), but its innovations are allowed to be correlated with the innovations of dividend growth and inflation.

[2] See, e.g., Campbell, Lo, and MacKinlay (1997).

The mapping from the 'real' parameters to the 'nominal' parameters (with tilde) is given by:

$$\tilde{\lambda}_t := \lambda_t + \Sigma'\delta_\pi, \quad \text{thus} \quad \tilde{\lambda}_0 \equiv \lambda_0 + \Sigma'\delta_\pi, \quad \tilde{\Lambda}_1 \equiv \Lambda_1 \tag{2.12}$$

$$\tilde{\delta}_0 := \delta_0 + \delta_\pi'(a - \Sigma\lambda_0) - \frac{1}{2}\delta_\pi'\Sigma\Sigma'\delta_\pi, \tag{2.13}$$

$$\tilde{\delta}_1' := \delta_1' + \delta_\pi'(\mathcal{K} - \Sigma\Lambda_1). \tag{2.14}$$

In analogy to (2.6), where $r_t := \delta_0 + \delta_1' X_t$ represents the real interest rate, $i_t := \tilde{\delta}_0 + \tilde{\delta}_1' X_t$ in (2.11) represents the one-period nominal interest rate. One observes that the two are related as

$$r_t = i_t - \delta_\pi'(a + \mathcal{K}X_t) + \delta_\pi'\Sigma\lambda_t + \frac{1}{2}\delta_\pi'\Sigma\Sigma'\delta_\pi,$$

hence, the real short rate equals its nominal counterpart minus expected inflation, plus a risk-premium (which is zero if λ_0 and Λ_1 are both zero) and a convexity term.

2.4 Pricing nominal zero-coupon bonds

Given the factor process and the real as well as the nominal pricing kernel, we can price real and nominal assets. For nominal zero-coupon bonds, relation (2.8) becomes

$$\tilde{P}_t^n = E_t(\tilde{M}_{t+1}\tilde{P}_{t+1}^{n-1}), \tag{2.15}$$

where $\tilde{P}_t^n$ is the price at time t of a zero-coupon bond maturing at time $t+n$, when it pays one unit of currency, i.e $\tilde{P}_{t+n}^0 = 1$. As is well known, the chosen specifications of the factor process, the SDF and the market price of risk imply that bond prices are exponentially-affine functions of the factors[3],

$$\tilde{P}_t^n = \exp\left[\bar{\tilde{A}}_n + \bar{\tilde{B}}_n' X_t\right], \tag{2.16}$$

where the coefficients $\bar{\tilde{A}}_n$ and $\bar{\tilde{B}}_n$ satisfy the following system of difference equations in n,

$$\bar{\tilde{A}}_n = \bar{\tilde{A}}_{n-1} + \bar{\tilde{B}}_{n-1}'(a - \Sigma\tilde{\lambda}_0) + \frac{1}{2}\bar{\tilde{B}}_{n-1}'\Sigma\Sigma'\bar{\tilde{B}}_{n-1} - \tilde{\delta}_0 \tag{2.17}$$

$$\bar{\tilde{B}}_n' = \bar{\tilde{B}}_{n-1}'(\mathcal{K} - \Sigma\tilde{\Lambda}_1) - \tilde{\delta}_1' \tag{2.18}$$

with initial conditions $\bar{\tilde{A}}_0 = 0$ and $\bar{\tilde{B}}_0 = 0_N$.[4] Hence, continuously-compounded bond yields, defined as $\tilde{y}_t^n := -\frac{\ln \tilde{P}_t^n}{n}$, will be affine functions of the factors

$$\tilde{y}_t^n = \tilde{A}_n + \tilde{B}_n' X_t, \tag{2.19}$$

where $\tilde{A}_n = -\frac{\bar{\tilde{A}}_n}{n}$ and $\tilde{B}_n = -\frac{\bar{\tilde{B}}_n}{n}$.

The pricing equation for real bond yields is completely analogous, thus

$$y_t^n = A_n + B_n' X_t, \tag{2.20}$$

where A_n and B_n satisfy (2.17) and (2.18) with all symbols carrying a tilde being replaced by the respective symbol without one.

[3]See, e.g., Ang and Piazzesi (2003).

[4]N denotes the dimension of the factor vector, i.e. here $N = 4$.

2.5 Pricing dividend-paying stocks

Denote by D_t the real dividend of a stock paid at time t and by V_t the stock's real (ex-dividend) price at time t. Buying one unit of the stock at time t at a price of V_t entitles the stock holder to next period's dividend D_{t+1}, and the stock can then be sold for the next period's price V_{t+1}. Hence, the total payoff is given by $D_{t+1} + V_{t+1}$. Therefore, using (2.7), the stock price satisfies

$$V_t = E_t \left\{ M_{t+1}(D_{t+1} + V_{t+1}) \right\}. \tag{2.21}$$

Using the definition of the payout-yield factor $\Gamma_t := (1 + \frac{D_t}{V_t})$, this can be rewritten as

$$V_t = E_t \left\{ M_{t+1} \Gamma_{t+1} V_{t+1} \right\}. \tag{2.22}$$

As derived in appendix A.1, this expectational difference equation has the solution

$$V_t^* = \exp[c \cdot (t - t_0) + D' X_t], \tag{2.23}$$

where

$$D' = \left[\delta_\gamma' (\mathcal{K} - \Sigma\lambda_1) - \delta_1' \right] \cdot \left[I_N - (\mathcal{K} - \Sigma\lambda_1) \right]^{-1}, \tag{2.24}$$

$$c = \delta_0 - (\delta_\gamma + D)' a - \frac{1}{2}(\delta_\gamma + D)' \Sigma\Sigma' (\delta_\gamma + D) + (\delta_\gamma + D)' \Sigma\lambda_0, \tag{2.25}$$

and t_0 is a free parameter. The vector δ_γ has the second element equal to one and zeros elsewhere, i.e. it picks the dividend yield from the state vector, i.e. $\gamma_t = \delta_\gamma' X_t$, where δ_γ is a selection vector with one element equal to unity and zeros elsewhere.

The arbitrage-free stock price consists of a deterministic exponential trend and a stochastic fluctuation around this trend. Note that the absolute magnitude of the stock price is not pinned down by the model. Hence, it is useful to think of the solution $V_t = V_t^*$ as describing the dynamics of an index in arbitrary units of measurements that can be altered via t_0.

The comment is in order that (2.23) is not the only solution to the pricing relation (2.22). In fact, there is a wider family of solutions, for which the stock price does not only depend on the four factors but also on an unrelated random walk process. Such a solution would be characterized as a 'rational bubble'. While it would in principle be interesting to allow for the presence of bubbles for explaining stock returns, we decided to exclude them in this paper and to take a 'purely fundamental' view on stock pricing. How this affects the estimation approach will be outlined in section (3) below.

Gross one-period stock returns are given by

$$R_{t+1}^{(1)} = \frac{V_{t+1} + D_{t+1}}{V_t} = \frac{V_{t+1}(1 + \frac{D_{t+1}}{V_{t+1}})}{V_t}.$$

Accordingly, one-period log-returns equal the capital gain (change of ex-dividend log stock price), $\Delta v_{t+1} = c + D'\Delta X_{t+1}$, plus the next period's dividend yield:

$$r_{t+1}^{(1)} = \Delta v_{t+1} + \gamma_{t+1} = c + D'\Delta X_{t+1} + \delta_\gamma' X_{t+1}, \tag{2.26}$$

Thus, conditionally expected returns are affine functions of the state vector

$$\begin{aligned} E_t\, r_{t+1}^{(1)} &= c + (D' + \delta_\gamma')a + (D'(\mathcal{K} - I) + \delta_\gamma'\mathcal{K})X_t \\ &=: f_1 + F_1' X_t \end{aligned} \tag{2.27}$$

with obvious definitions of f_1 and F_1.[5] From (2.26), it follows immediately that the unconditionally expected stock return equals

$$E\, r_t^{(1)} = c + \mu_\gamma \tag{2.28}$$

where $\mu_\gamma := E\gamma_t$.

With $\lambda_0 = 0$ and $\Lambda_1 = 0$ (risk neutrality), we have[6]

$$r_{t+1}^{(1)} = r_t + (D' + \delta_\gamma')\Sigma\eta_{t+1} - J \tag{2.29}$$

where the variance (Jensen) term is $J = \frac{1}{2}(\delta_\gamma + D)'\Sigma\Sigma'(\delta_\gamma + D)$. Taking conditional expectations yields

$$E_t r_{t+1}^{(1)} = r_t - J. \tag{2.30}$$

That is, under risk neutrality the expected real stock return equals the real interest rate (plus a Jensen adjustment).

For defining multi-period returns, one has to make an assumption on how investors treat dividends that they receive during the considered investment horizon. One possibility is to assume that dividends are always reinvested in the stock that they are associated with.[7] That is, for an n-period horizon, the investor would buy the stock (say 100 units) at some time t for the ex-dividend price of V_t per share. He would then receive dividends in $t+1$, which he would use for buying new pieces of the stock for the then prevailing ex-dividend price V_{t+1} and so forth. His total payoff in the last period $(t+n)$ consists of the number of stocks carried over from period $t+n-1$ multiplied by $(D_{t+n} + V_{t+n})$, i.e. the dividend per share plus the ex-dividend stock price prevailing in the last period. This investment strategy is formally analyzed in appendix A.3. One obtains for n-period returns (scaled, i.e. in per-one-period terms)

$$r_{t+n}^{(n)} = \frac{1}{n}\left(v_{t+n} - v_t + \sum_{i=1}^{n}\gamma_{t+i}\right). \tag{2.31}$$

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[5] For the derivation, it has been used that $E_t\, X_{t+1} = a + \mathcal{K}X_t$ and $E_t\, \Delta X_{t+1} = a + (\mathcal{K} - I)X_t$.

[6] As shown in Appendix A.2.

[7] Taking again the perspective that the stock considered here can be conveniently considered as an index, the assumption implies that all receipts are reinvested into the index.

Thus, in analogy to the one-period case, they equal the n-period capital gain plus the average of dividend yields over the horizon considered. Since conditional expectations of v_t, v_{t+n} and the γ_{t+i} are all affine in X_t, conditional expectations of n-period returns have likewise an affine representation of the form:[8]

$$E_t r_{t+n}^{(n)} = f_n + F_n' X_t. \tag{2.32}$$

Note that this implies at each time t a 'term structure of expected stock returns'. Finally, *unconditionally* expected n-period stock returns are independent of n and equal to (2.28), which directly follows from taking unconditional expectations of (2.31). That is, the term structure of unconditional expectations of stock returns is flat.

2.6 Risk premia

The model implies the dynamics of equity and bond yield risk premia. We define the one-period equity risk premium *ERP* at time t as the expected excess one-period log return - as defined in (2.26) - over the one-period real bond yield,

$$ERP_t^{(1)} := E_t r_{t+1}^{(1)} - r_t. \tag{2.33}$$

As both the one-period real interest rate and the expected stock return are affine functions of the state vector, the equity risk premium inherits this convenient property:

$$\begin{aligned} ERP_t^{(1)} &= f_1 + F_1' X_t - \delta_0 - \delta_1' X_t \\ &=: g_1 + G_1' X_t. \end{aligned}$$

The n-period *ERP* can be defined as the difference of expected n-period stock returns, as defined above, and the n-period real bond yield:

$$\begin{aligned} ERP_t^{(n)} &= E_t r_{t+1}^{(n)} - y_t^n \\ &= f_n + F_n' X_t - A_n - B_n' X_t \\ &=: g_n + G_n' X_t, \end{aligned} \tag{2.34}$$

again an affine function of the state vector.

For a given point in time t, (2.34) defines a 'term structure of equity risk premia'. From the fact that unconditional expectations of stock returns are independent of n, the shape of the unconditional expectation of the term structure of *ERP*s depends solely on the shape of the term structure of unconditional expectations of real bond yields. If this is upward-sloping, the term structure of unconditional means of ERPs is downward sloping, since the term structure of unconditional expectations of stock returns is flat.

[8] Explicit expressions for f_n and F_n are straightforwardly obtained.

As stated above, our model nests the popular class of affine term structure models. One of their uses is the provision of nominal term premia, i.e. the differences between nominal long-term bond yields and their hypothetical counterparts that would prevail under the expectations hypothesis. Thus, it is a useful validation exercise for our encompassing model to compare the resulting term premia to those stemming from more specialized 'term-structure-only' models. The n-period nominal term premium (or yield risk premium YRP) is defined as:

$$YRP_t^n = \tilde{y}_t^n - \frac{1}{n} E_t \sum_{i=0}^{n-1} i_{t+1}. \tag{2.35}$$

Again, since our model implies that arbitrage-free bond yields as well as current and expected nominal short rates are affine functions of the state vector, yield risk premia (and likewise forward premia and excess expected holding-period returns) are also affine functions of X_t.[9]

2.7 Comparing the arbitrage-free model and the dividend discount model

Since the equity premium from the widely-used dividend discount model will be employed as a yardstick of comparison for our estimated equity risk premium in the empirical application, it may be useful at this point to compare the two approaches with respect to their embodied stock pricing formulae. In fact, they both share the same core property, namely that stock prices are represented as discounted sums of future dividends. However, the distinctive features are the way in which expectations enter the models and – closely related – the respective notion of a 'discount factor'.

For the dividend-discount model, the stock price is the sum of *discounted expected* future dividends

$$V_t = \sum_{i=1}^{\infty} \left(\frac{1}{1 + \overline{r\!\!œ}_t} \right)^i E_t D_{t+i} \tag{2.36}$$

where $\overline{r\!\!œ}_t$ is the one-period required rate of return, which is taken as constant for all future periods.

In the arbitrage-free model, in contrast, the discount factor is inside the expectations operator, implying that the stock price is the sum of *expected discounted* future dividends

$$V_t = \sum_{i=1}^{\infty} E_t \left\{ \bar{M}_{t+i} D_{t+i} \right\} \tag{2.37}$$

where $\bar{M}_{t+i} = M_{t+1} \cdot \ldots \cdot M_{t+i}$.[10] Before turning to the comparison of the two approaches, we will first expound in more detail how the equity risk premium is extracted in the three-stage dividend discount model.

[9] See, e.g., Hördahl, Tristani, and Vestin (2006) for the various definitions of bond-related risk premia

[10] Note that equation (2.37) results as the forward solution of (2.21), and utilizing the transversality condition $\lim_{n\to\infty} E_t \left[\left(\prod_{i=1}^{n} M_{t+i} \right) V_{t+n} \right] = 0$.

For extracting a risk premium measure for stocks, the dividend-discount model takes the observed stock price as given, uses some assumptions concerning future dividend growth and solves for the discount rate $\overline{roe}$. In the last step, one then subtracts from $\overline{roe}$ a risk-free rate (usually a long-term government bond yield, say y_t^m) and treats the difference as the equity risk premium:

$$\overline{ERP}_t^{DDM} = \overline{roe}_t - y_t^m \tag{2.38}$$

Hence, given the observed stock price (index) V_t, the only ingredient needed to back out $\overline{roe}_t$ from (2.36) is the sequence of expected future dividends. Equivalently, the equation can be written in terms of the current dividend D_t and future dividend *growth* rates,

$$V_t = D_t \sum_{i=1}^{\infty} \left(\frac{1}{1+\overline{roe}_t} \right)^i E_t \left\{ \prod_{j=1}^{i} (1+g_{t+j}) \right\}, \quad 1 + g_{t+j} = \frac{D_{t+j}}{D_{t+j-1}}, \tag{2.39}$$

hence inferring the equity premium requires an assumption on expected future dividend growth rates. The simplest approach is to assume these growth rates to be constant for all future horizons from t onwards, $g_{t+j} = \bar{g}_t$, which is the famous Gordon growth model endowed with some quantification of $\bar{g}_t$.

A popular refinement used in practice is the so-called three-stage dividend discount model.[11] The version employed by many practitioners and central banks uses IBES (Institutional Brokers Estimate System) forecasts of *earnings* growth rates as a central input. These survey figures are understood as 'long-run' forecasts relating to a time horizon of 'three to five years', which for the purpose of estimating the equity premium is usually taken to correspond to a four-year horizon. Furthermore, it is assumed that the ratio of dividends to earnings is roughly constant, so that earnings growth forecasts proxy well for dividend growth forecasts. Denote this growth rate to be plugged into (2.39) for g_{t+j} over the first four years as g_t^{IBES}. For the very long run, say from twelve years henceforth, a constant dividend growth rate g^{LR} is used.[12] This is either quantified to equal $\overline{roe}_t$ multiplied by one minus a constant dividend-earnings ratio, or it is equated with some other (ad hoc) long-run dividend growth assumption. For the time period of eight years ('second stage') between the four years , for which the IBES forecast is used ('first stage'), and the time starting after twelve years ('third stage'), the dividend growth rate is linearly interpolated between g_t^{IBES} and g^{LR}. Under these assumptions, the stock-valuation formula (2.39) becomes

$$V_t = \frac{D_t}{\overline{roe}_t - g^{LR}} \left((1 + g^{LR}) + 8 \cdot (g_t^{IBES} - g^{LR}) \right), \tag{2.40}$$

[11] See Fuller and Hsia (1984) and the exposition of the simplified version by Panigirtzoglou and Scammell (2002).

[12] The subscript t denotes that the survey forecasts as well as the long-run assumptions may change depending on the month, in which the computation is conducted. However, the crucial point to note is that they are treated fixed for the coming four years or twelve years and beyond, respectively.

which can be inverted to obtain the desired $\overline{roe}_t$. Note that (2.40) becomes the well-known formula for the Gordon model if $g_t^{IBES} = g^{LR}$, i.e. when the expected dividend growth rate is assumed constant in all three stages.

Proceeding in this fashion, the return on equity $\overline{roe}_t$ is arguably similar in nature to a 'yield to maturity' that one would derive from the price of a coupon-bearing bond and information about its future coupon and principal payments. While this approach makes broad intuitive sense, it neglects the term structure and the intertemporal risk structure of discount rates. The resulting equity risk premium represents an average over the whole set of future horizons, at which dividend payments are expected.

The approach proposed in this paper, in contrast, acknowledges the risk and term structure of discount factors; given the estimated dynamics of the random process $\{M_t\}$ (implied by the factor process and the dependence of M_t on factors and factor innovations), well-defined one- or multi-period measures of excess returns serve as equity risk premia, as shown in the previous section. The second crucial difference is the fact that our model does not employ any survey-based information on future dividend developments. Rather, all conditional expectations on future dividend yields, stock prices and interest rates follow from the arbitrage-free model dynamics; accordingly, they are fully determined by current state variables.

3 Estimation approach and data

3.1 The empirical model in state space form

Regarding estimation, the advantage of choosing dividend yield as opposed to dividend growth as part of the state vector becomes evident. Unlike with the approach of, e.g., Bekaert and Grenadier (2001), stock returns in our model are affine function of the state vector. Hence, as common for affine term structure models, the combined stock-bond model can be estimated in a state space framework.[13]

The measurement equation reads

$$\begin{pmatrix} \pi_t \\ \gamma_t \\ \tilde{y}_t^{n_1} \\ \vdots \\ \tilde{y}_t^{n_K} \\ \Delta v_t \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \tilde{A}_{n_1} \\ \vdots \\ \tilde{A}_{n_K} \\ c \end{pmatrix} + \begin{pmatrix} \delta_\pi' & 0 \\ \delta_\gamma' & 0 \\ \tilde{B}_{n_1}' & 0 \\ \vdots & \vdots \\ \tilde{B}_{n_K}' & 0 \\ D' & -D' \end{pmatrix} \begin{pmatrix} X_t \\ X_{t-1} \end{pmatrix} + \begin{pmatrix} 0 \\ w_t \\ \tilde{\epsilon}_{1t} \\ \vdots \\ \tilde{\epsilon}_{Kt} \\ 0 \end{pmatrix}. \tag{3.1}$$

[13]See, e.g., Lemke (2006) for an overview. The model by Bekaert and Grenadier (2001) is estimated using an iterated GMM method. d'Addona and Kind (2006) estimate the factor process parameters by maximum likelihood (they do not have latent factors) and calibrate the remaining parameters.

The measurement vector on the left-hand side comprises inflation, the dividend yield of a broad-based stock index, nominal bond yields of K different maturities and the real ex-dividend return of the stock index. The data used are discussed in more detail below. The right-hand side contains the model-implied counterparts, which are functions of the states, and adds – except for inflation – measurement errors. Note that the state vector contains both the factor vector and its first lag. The latter is needed for explaining capital gains on the stock index, which are a function of ΔX_t, whereas inflation, dividend yield and bond yields depend on contemporaneous X_t only.

Examining the system of measurement equations in detail, the first equation identifies the first element of the state vector as observed inflation. The second equation links the second element of the state vector to the observed dividend yield, but the two can differ in each period by a measurement error w_t.[14] This measurement error is introduced for two reasons.

First, it shall capture the possible wedge between the theoretical concept of the 'payout yield' of the stock and the observed dividend yields. The two can differ since dividends are not the only means by which stock investors can be made participating in profits. Most prominently, stock buy-backs are an important alternative to providing cash flow to equity holders. In fact, as shown by Boudoukh, Michaely, Richardson, and Roberts (2007), the fraction of the payout to equity holders that is due to stock buy-backs on the part of the issuing company has been increasing over time.

The second role of the measurement error in the dividend-yield equation is related to the fact that we do not allow for a measurement error in the last equation of the system, which relates stock returns to the state vector. Hence, we assume that realized stock returns are fully explained by the dynamics of the state variables, i.e. the two latent real-interest-rate factors and the payout yield – as it follows from the model solution. It may be reasonably argued that these factors will never account for all observed movements of monthly stock returns, especially so because they cannot capture periods of 'irrational' investment behavior, and also because we rule out rational bubbles, as noted above. In fact, the approach chosen here takes a completely 'fundamental' and rational view of pricing equity. Thus, as stock returns are always perfectly matched, the conditional moments of the joint evolution of current and future dividend yields and discount factors have to align in such a way that stock returns are perfectly priced given the dynamics of the state process and given the arbitrage-free pricing relation for equity. Since factor dynamics are Markovian, the future distribution of real rates and dividend yields is completely determined by the current realization of the state vector. Moreover, since risk prices are affine functions of the factor vector, the expectation of these risk prices is also determined

[14] We do not distinguish in notation between the observed dividend yield, say γ_t^{obs} and the model counterpart γ_t but rather use the latter notation for both of them as it is always clear from the context, which one is referred to. The same holds for bond yields and stock returns.

by current state variables. Summing up, given the model structure, a set of parameters, and realized inflation, the two latent real-rate factors and the state variable representing payout yield adjust in each period in such a way that observed stock returns are aligned with their model-implied counterparts. In this respect, the size of the measurement error w_t in the second equation indicates by how far the payout yield has to deviate from the observed dividend yield in order to 'support' the observed stock return.

Taking this approach, the natural question arises whether the latent factors determining the real interest rate and the state variable representing payout yield have to 'bend too much' in order to equate realized empirical stock returns with their model-implied counterparts. This question is largely an empirical issue. One part of the answer is given by the standard deviation of the measurement error, which is itself an (estimated) parameter. If it is close to zero, measurement errors are small on average and the dynamics of observed dividend yields are sufficient to explain the variation in observed stock returns. If it is very large, empirical dividend yields are not a very useful representative of the stochastic process representing what is considered as 'payout yield' from the viewpoint of the model.

The latent real-rate factors also play a part for explaining stock returns by shaping the sequence of risk-free real interest rates. Their main empirical task, though, is to fit – in conjunction with inflation – the evolution of nominal bond yields. Hence, in order to check whether their additional 'obligation' of fitting observed stock returns has pushed real-rate factors in a strange direction, one can assess whether the implied evolution of the real rate itself makes sense, but also whether the two factors have generated a good fit of the nominal term structure. Anticipating the empirical results discussed below, it turns out that bond yields are fitted well and the deviation of the dividend yield from the estimated model-implied payout yield can be considered as moderate.

Further measurement errors occur in the relation of model-implied and observed zero-coupon bond yields. Again, the measurement errors account for the fact that the factors may not be able to match all bond yields perfectly. Regarding the average magnitudes of these measurement errors, we make the assumption that the respective standard deviations are equal across maturities. This is not uncommon in the literature on affine term structure models and mainly serves to reduce the number of free parameters. At the same time, however, this approach amounts to imposing the restriction that the model's fit of bond yields is similar across maturities.

Collecting all measurement errors in a vector $u_t := (w_t, \tilde{\epsilon}_{1t}, \ldots, \tilde{\epsilon}_{Kt})'$, we assume that u_t is serially uncorrelated and

$$u_t \sim N(0, H), \quad H = diag(h_1^2,\, h_2^2, \ldots, h_2^2) \tag{3.2}$$

Moreover, u_t is assumed to be independent of the factor innovations η_t at all times and lags.

Finally, the transition equation of the state space model represents the dynamics of the factor vector X_t and its first lag, which is implied by (2.2),

$$\begin{pmatrix} X_t \\ X_{t-1} \end{pmatrix} = \begin{pmatrix} a \\ 0 \end{pmatrix} + \begin{pmatrix} \mathcal{K} & 0 \\ I & 0 \end{pmatrix} \begin{pmatrix} X_{t-1} \\ X_{t-2} \end{pmatrix} + \begin{pmatrix} \Sigma \\ 0 \end{pmatrix} \eta_t. \tag{3.3}$$

3.2 Parameter restrictions

The number of parameters in the model is fairly large and not all parameters are separately identifiable. In order to reduce the number of free parameters, we will impose the following parameter restrictions on the factor dynamics:

$$\begin{pmatrix} \pi_t \\ \gamma_t \\ L_{1t} \\ L_{2t} \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \mathcal{K}_{11} & 0 & 0 & 0 \\ 0 & \mathcal{K}_{22} & \mathcal{K}_{23} & \mathcal{K}_{24} \\ 0 & 0 & \mathcal{K}_{33} & 0 \\ 0 & 0 & \mathcal{K}_{43} & \mathcal{K}_{44} \end{pmatrix} \begin{pmatrix} \pi_{t-1} \\ \gamma_{t-1} \\ L_{1,t-1} \\ L_{2,t-1} \end{pmatrix} + \begin{pmatrix} \Sigma_{11} & 0 & 0 & 0 \\ 0 & \Sigma_{22} & 0 & 0 \\ 0 & 0 & 0.001 & 0 \\ 0 & 0 & 0 & 0.001 \end{pmatrix} \begin{pmatrix} \epsilon_{1t} \\ \epsilon_{2t} \\ \epsilon_{3t} \\ \epsilon_{4t} \end{pmatrix} \tag{3.4}$$

The real-rate factors are assumed to depend solely on their own past but not on lags of inflation or the dividend yield. The respective autoregressive matrix is taken as lower-triangular, which is an innocuous assumption: as these two factors are unobservable, for each law of motion with full autoregressive matrix and full variance-covariance matrix of innovations, there is an observationally equivalent representation with lower-triangular autoregressive matrix and diagonal factor innovations. The standard deviation of factor innovations is normalized to 1E-3: any re-scaling of the latent factors could be accommodated by re-sizing the respective loadings in δ_1 in (2.1). Finally, since the two latent factors are mean-zero processes, the unconditional expectation of the risk-free one-period rate is given by δ_0 in (2.1).

Inflation is assumed to evolve independently of the factors driving the real interest rates and the dividend yield, hence the inflation process is modelled as a simple AR(1). Together with contemporaneously independent factor innovations, this implies that we have a strict separation between the real and the nominal sphere in the model and the Fisher hypothesis will hold.[15]

For the payout yield, we allow it to be driven by its own past as well as by lagged levels of the real-rate factors. Hence, we make it an empirical issue whether real rates have forecasting power for future dividend yields.

[15]This assumption is also made in Bekaert and Grenadier (2001), while d'Addona and Kind (2006) and Lettau and Wachter (2007) allow for correlated real and nominal factors or innovations.

Concerning the parameterization of the dynamics of the market prices of risk, the elements of λ_0 and Λ_1 in (2.5) are generally difficult to estimate. Moreover, there is no universally applicable set of conditions under which a certain parameter structure, i.e. a set of (zero) restrictions, guarantees identifiability. Some authors of the affine term structure literature try to estimate the respective parameters using a heuristic iterative approach, starting with a full parametrization and subsequently setting parameters with low t-values to zero. However, there has not yet been established a 'best practice' of estimating market-price-of-risk parameters in the literature. For our model, we decided to allow only the diagonal elements of Λ_1 to be different from zero:

$$\begin{pmatrix} \lambda_{\pi,t} \\ \lambda_{\gamma,t} \\ \lambda_{L_1,t} \\ \lambda_{L_2,t} \end{pmatrix} = \begin{pmatrix} \lambda_{0,1} \\ 0 \\ \lambda_{0,3} \\ \lambda_{0,4} \end{pmatrix} + \begin{pmatrix} \Lambda_{1,11} & 0 & 0 & 0 \\ 0 & \Lambda_{1,22} & 0 & 0 \\ 0 & 0 & \Lambda_{1,33} & 0 \\ 0 & 0 & 0 & \Lambda_{1,44} \end{pmatrix} \begin{pmatrix} \pi_t \\ \gamma_t \\ L_{1t} \\ L_{2t} \end{pmatrix}.$$

We allow the time-invariant parts of the market prices of risk to be non-zero, with the exception of the market price of dividend-yield risk. The single role of $\lambda_{0,2}$ is to shift the mean of stock returns, i.e. changing $\lambda_{0,2}$ only affects c in (2.25). Hence, given all other parameters affecting the average capital gain c – one could use $\lambda_{0,2}$ to shift the average capital gain to any desired value without affecting D or the pricing equations for bonds. However, using our Maximum Likelihood estimation below, it turned out that $\lambda_{0,2}$ is difficult to estimate. Accordingly, we decided to let only the other model parameters determine c.

3.3 Estimation of parameters

For the model in state space form, the likelihood of $(Y_1, \ldots, Y_T)$, where $Y_t = (\pi_t, \gamma_t, \tilde{y}_t^{n_1}, \ldots, \tilde{y}_t^{n_K}, \Delta v_t)$, can be constructed using the Kalman filter and the free parameters can be estimated by Maximum Likelihood. However, even after imposing the restrictions expounded in the previous sub-section, the number of parameters is still large, so we use the following two-step procedure.

In the first step, we calibrate the mean of the real interest rate δ_0 to equal the difference between the average realized nominal one-month rate and the average of realized inflation. In addition, we estimate the AR(1) process of inflation, which in the model is independent of the dynamics of the other state variables. Proceeding in this way, we obtain consistent estimates of the inflation-process parameters, but may forfeit some efficiency.[16] However, facing the trade-off between efficiency and numerical stability we opted for delegating the estimation of inflation parameters to the first step.

[16] This is because the parameters steering inflation dynamics (a_1, $\mathcal{K}_{11}$ and Σ_{11}) also appear in the pricing relation for bonds. Hence, in a full-system estimation, the variation in bond yields is informative on the inflation parameters.

The second step consists of maximizing the likelihood with respect to the remaining parameters. In order to prevent getting stuck at local maxima, the corresponding optimization routine has been run from different starting vectors. Given the whole set of estimated parameters, the paths of the unobservable factors are backed out using the Kalman filter and based on these, the time series of bond and equity risk premia are obtained.

3.4 The data

Estimation is based on monthly data for the United States, spanning 24 years from January 1983 to December 2006. The following months are excluded for estimation as the results turned out to be somewhat sensitive to the inclusion of the recent financial crisis starting in mid-2007. However, given parameters obtained from the indicated sample ending in 2006, most of the results (premia, fitted series etc.) are provided for the 18 additional months until June 2008.

For inflation, we use the year-on-year log difference of the consumer price index (all urban consumers, all items). Strictly speaking, the model rather requires to take the month-on-month changes. However, as this produces a very erratic series that will not prove very useful for the bond pricing exercise, we take the same route as others in the term structure literature and employ the smoother year-on-year series.[17] Stock variables are for the US S&P 500 index. We use the corresponding monthly dividend yield, and as real ex-dividend stock returns we compute month-on-month differences of the price index, from which the inflation rate is subtracted. Nominal zero-coupon bond yields for maturities of one-, three-, seven- and ten-years are taken from Gurkaynak, Sack, and Wright (2007).[18] The data are shown in figure 1. For estimation, all variables are expressed in monthly terms, i.e. a bond yield of 3.6% would enter as 3.6/1200=0.003.

[Figure 1 about here]

4 Empirical results

4.1 Parameter estimates and fit

Parameter estimates and associated t-statistics are provided in table 1. As the estimates of the diagonal elements of $\mathcal{K}$ show, all four factors are highly persistent. In particular, the autoregressive parameter of the dividend yield $\mathcal{K}_{22}$ is very close to unity. Economically, it is

[17] As an alternative approach, one may use the more jagged month-on-month series and allow for a measurement error, which would be tantamount to using some smoothed or 'core-inflation'-type measure for pricing bonds. However, this would require some additional parameters to be estimated and the results are not expected to be hugely different.

[18] They are downloaded from the website http://www.bilkent.edu.tr/ refet/research.html.

plausible that the dividend yield is stationary. However, for the most part of our sample, the observed dividend yield has a clearly falling trend, hence explaining the near-unity estimate of $\mathcal{K}_{22}$.[19]

[Table 1 about here]

In analogy to econometric analyses of affine term structure models, where factors often show close to I(1) dynamics, we nevertheless treat all factors as stationary. Concerning other drivers of dividend yields, we allowed $\mathcal{K}_{23}$ and $\mathcal{K}_{24}$ to differ from zero. In fact, the lagged real-rate factors turn out to load significantly on the dividend yield. The latent factors themselves are also persistent with estimates of $\mathcal{K}_{33}$ and $\mathcal{K}_{44}$ being of a dimension well in line with those obtained in the literature for affine term structure models.

The standard deviation of the measurement error of dividend yields in annualized percentage terms, i.e. $1200 \cdot h_1$, amounts to 0.17 percentage points. That is, the difference between observed and model-implied dividend yields is relatively small on average. In fact, the model-implied dividend yield, which supports observed stock returns is fairly close to its counterpart in the data: figure 2 shows the observed dividend yield together with the Kalman-filtered payout-yield factor. Although there are protracted phases of deviation, the two series show a strong overall comovement.

[Figure 2 about here]

Concerning the fit of bond yields, the respective standard deviation of the measurement error is very small, as $1200 \cdot h_2$ amounts to less than eight basis points. The good fit of bond yields is confirmed in figure 3, that plots observed and estimated (Kalman-filtered) bond yields, as well as in figure 4, which compares the mean of observed yields (not only the four used for estimation) to those implied by the filtered model-implied yields.

[Figure 3 about here]

[Figure 4 about here]

4.2 Estimated term premia

As our joint stock-bond model nests an essentially affine term structure model, a plausibility check of the results is given by comparing the model-implied term premia (yield risk premia) with comparable ones obtained from a well-known affine term structure model: figure 5 plots the model-implied ten-year nominal term premia as defined in (2.35) together with those obtained by Kim and Wright (2005). From 1992 to 2006, the end of our

[19] The high persistence of dividend yields is also found in other studies. For instance, Lewellen (2004) obtains an autocorrelation coefficient of 0.991 for dividend yield over the sample 1973-2000, and 0.999 for log dividend yield.

estimation sample, the two series of estimated premia share similar dynamics (correlation coefficient of 0.8), a similar level (mean of our model 1.67% p.a. vs. 1.36%), and similar variability (standard deviations 0.61% vs. 0.71%). However, while the two series show a distinct comovement, they tend to diverge occasionally, in particular for the recent past: after 2005, the Kim-Wright premia showed a marked downturn to very low levels, while the premia implied by our model ranged considerably higher.

[Figure 5 about here]

4.3 The time series of estimated equity risk premia

Turning to the equity premium, figure 6 shows the estimated time series of three-month and ten-year premia i.e $\hat{ERP}_t^3$ and $\hat{ERP}_t^{120}$ defined by (2.34). For comparison, the figure also shows the equity risk premium implied by the three-stage dividend discount model.

[Figure 6 about here]

The two time series of estimated short- and long-run equity risk premia allow to broadly distinguish five phases. After being on an upward trend since the mid-eighties (beginning of estimation period), the equity risk premium – especially at the short horizon – displayed a distinct downward movement, reaching a long-term low towards the peak of the 'dot-com boom'.[20] With the onset of the sharp correction of this booming period, the estimated series of premia implied strong reversals with the short-term measure of required equity risk compensation reaching a peak in 2003. The following reversal came to a sudden halt when first signs of the current financial turmoil, which arose from strains in the U.S market for (subprime) mortgage-backed securities, emerged. In fact, since mid-2007, equity risk premia – especially for the short horizon – showed its most remarkable increase over the sample period, reflecting how strongly the most current events impacted on the required risk compensation of stock market investors.

Throughout the period, for which the equity risk premium obtained from the dividend-discount model was likewise available (since 1990), the latter estimate and its counterparts implied by our model showed a strong comovement.[21] In fact, for all horizons of our estimated equity risk premium, ranging from one month towards 50 years, the correlation ranges between 0.82 and 0.90.

As argued above, the equity premium derived from the dividend-discount model may be broadly interpreted as an average across various investment horizons. Broadly in line with this interpretation is the result that the mean of of model-implied premia is closest

[20] Note that we abstain from using the term 'bubble', as from the perspective of our model, all movements in stock returns are fully rationalized implicitly by respective dividend expectations.

[21] The availability of the equity risk premium obtained from the dividend discount model is limited by our access to the IBES survey data for earnings.

to that of the dividend-discount model (over the shared sample period) for a horizon of about five years. Similarly, the correlation between the two series reaches a maximum at a horizon of about two and a half years.

However, the standard deviation of model-implied premia is generally higher than that of the dividend-discount model. Figure 7 shows that it is a monotonically decreasing function of the horizon. Only for a horizon of about 20 years the model-implied volatility of the equity risk premium has reached a similarly low level.

[Figure 7 about here]

The comovement of the two measures of the equity risk premium (dividend-discount model vs. our model) is remarkable, given the two different approaches of estimation. The dividend-discount model makes explicit use of a forward-looking measure of future earnings, which is based on the IBES survey measure, to construct the equity risk premium. In contrast, in the affine arbitrage-free model, the equity premium for any horizon is a function of current observed state variables only. Expected stock returns result as mathematical expectations, given the estimated law of motion of the state variables and the no-arbitrage pricing relations. Hence, one interpretation of the marked comovement of the two series is that the forward-looking content of the IBES survey variable regarding the expected stock return can also be exploited from the combination of observed state variables. In the affine model, this information is extracted from the observed dividend yields and the observed term structure of interest rates. More precisely, since we actually use the *filtered* dividend yield (second state variable) and *filtered* latent real rate factors (third and fourth state variables), our equity risk premia are functions of current and past observed stock returns, dividend yields and bond yields.

4.4 The term structure of equity risk premia

As stressed above, the model allows to trace out at each point t in time the family of equity risk premia ERP_t^n for different horizons n. Figure 8 illustrates such 'term structures of equity risk premia'. In January 1999, for instance, at the climax of the 'dot-com' euphoria, the equity premium for short horizons was extremely low but then increasing to more normal levels at the longer end. In 2002, after the stock market correction, the longer end (beyond eight years) was essentially the same as in 1999, but the short end of the term structure of equity risk premia had caught up by about two percentage points. Regarding the beginning of 2008, the subprime crisis had an impact especially on the short end of the term structure of equity premia. However, also for all longer-term horizons required risk compensation has become strongly elevated.

[Figure 8 about here]

Finally, the model allows to quantify the contemporaneous impact of the state variables (interest rate factors and dividend yield) on the equity risk premium of different horizons.[22] Figure 9 displays the effect of three scenarios.

[Figure 9 about here]

The first is an increase of the dividend yield relative to its unconditional mean by one percentage point, i.e. a shock to $\eta_{2,t}$ in (2.2) of appropriate size.[23] The effect on the equity risk premium in the model is channelled via the time varying market price of dividend-yield risk. Following from our parameterization of the model, a change in dividend yield has an effect on the equity premium only via the expected stock return and not via the risk-free long-term real rate, since the latent real rate factors are unaffected by the dividend yield factor. If the market-price-of-risk parameter Λ_{22} were zero, the effect on the premium would be zero. For the one-period premium, this follows from the fact that the the impact on the conditional expectation of the future dividend yield would be positive, but the effect would be exactly offset by the resulting expected negative capital gain. This makes the conditional expectation of (2.26) zero. A similar mechanism applies to multi-period expected stock returns implied by (2.31).

If Λ_{22} is sufficiently negative – as resulting from our estimation – the effect on the equity premium can be positive, and even exceed the size of the increase in the dividend yield, see figure 9.[24] The one-percentage point increase of the dividend yield, roughly increases the equity premium by about 1.5 percentage points.

The second and third scenario are changes in the term structure of real bond yields.[25] Any change of the real yield curve has two effects on the term structure of equity risk premia. First, since the real yields represent the risk-free rates for the respective investment horizons, an increase of them leads – *ceteris paribus* – to a same-sized decrease of the corresponding equity risk premia. Second, at the same time, our parametrization of the factor process (3.5) implies that changes in the interest rate factors, $X_{3,t}$ and $X_{4,t}$, have

[22] We do not report impulse responses, i.e. dynamic responses of the premia. Thus, 'horizon' always refers to the investment horizon associated with the equity premium, whereas the impact is always meant contemporaneously.

[23] Hence, in the model written in terms of non-annualized decimal returns, $\eta_{2,t} = (1200 \cdot \Sigma_{22})^{-1}$.

[24] Due to the mean reversion of the dividend yield, an increase at time t implies an increase of the expected future dividend yield $X_{2,t+1} := \gamma_{t+1}$. At the same time, again due to mean reversion, $E_t \Delta X_{2,t+1}$ is negative. For sufficiently negative Λ_{22}, the coefficient D for the capital gain component in (2.26) is negative – see (2.24) – thus, the expected capital gain is positive, and hence the disproportionately strong overall increase in expected stock returns.

[25] The second and third scenarios are implemented as follows. For the increase of the short end of the yield curve, the shocks $\eta_{3,t}$ and $\eta_{4,t}$ are chosen such that the real three-month yield increases by one percentage point, while the real ten-year yield remains unaffected. For the third scenario, the two innovations are set such that both yields increase by one percentage point.

an impact on future expected dividend yields and therefore also on the market price of dividend-yield risk, which in turn triggers the effects discussed in the previous paragraphs.

An increase of the real three-month rate, that leaves the long end of the real yield curve unaffected, triggers a decrease of the equity risk premium for the shortest horizons of about one half percentage points (dashed red line in figure 9). Hence, the effect working via the risk-free rate channel is counteracted to some extent by the channel working via expected stock returns. With increasing horizon, the effect peters out, reaching essentially zero at the eight-year horizon. A parallel shift of the real yield curve (dashed-dotted green line in figure 9) leads to an almost symmetric downward shift of the term structure of equity risk premia.

5 Conclusion

We presented a novel approach for estimating the time series of equity risk premia. For this, we proposed a discrete-time arbitrage-free model that jointly captures stock and bond price dynamics. There is one pricing kernel that prices bonds of all maturities as well as stocks. The model is driven by four factors: the dividend yield, the rate of inflation, and two latent factors that make up the one-period real interest rate. The way the model is set up implies that it has an affine structure: nominal and real bond yields, yield risk premia, realized and expected stock returns, as well as equity risk premia are all affine functions of the factors. The model nests the popular class of essentially affine term structure models.

With this set-up, it is possible to infer the evolution of bond premia (yield risk premia, forward premia etc) and the equity premia in a coherent framework. In addition, at each point in time, the system dynamics imply a whole 'term structure of equity risk premia'. That is, for any investment horizon n, the model provides the excess expected return of stocks over the model-implied n-period real interest rate. Moreover, the integrated stock-bond modeling approach enables one to assess the impact of changing dividend yields as well as the impact of yield curve movements on the term structure of equity premia.

Estimation is based on monthly US data from 1983 to 2008. The results make economic sense, as they comply with the intuition for prominent stock market phases such as the 'dot-com' boom phase (decreasing equity risk premia, especially over short investment horizons), the following correction (normalization of equity risk premia), as well as the onset of the current financial turmoil (sharp increase of equity premia of all horizons). Furthermore, the time series of estimated equity risk premia, are strongly correlated with those based on the prominent dividend-discount model, which gives further trust to the results. Also, the model-implied bond yield risk premia are reasonable with respect to size and dynamics as they are broadly comparable to those obtained by Kim and Wright (2005).

The comovement of the two measures of the equity risk premium (dividend-discount model vs. our model) is especially remarkable against the background that the first approach employs a forward-looking measure of future earnings (which is based on the IBES survey measure) while the affine arbitrage-free model does not. In our model, the equity premium for any horizon is a function of estimated state variables only, which are in turn filtered from current and past observations of interest rates and dividend yields. Expected stock returns result as mathematical expectations, given the estimated law of motion of the state variables and the no-arbitrage pricing relations. Hence, the results suggest that any information on expected stock returns coming from survey information can also be obtained from observed asset prices – channelled through the no-arbitrage equations of the model.

The results point to various avenues of future research. First, since the model jointly captures the bond yield curve and stock returns as well as inflation (required to match the nominal observed yield curve), it may be enhanced to shed light on the nexus between real stock returns and inflation, which is he subject of a broad literature.[26] Second, instead of working with one representative stock index only, it is conceivable to apply the model to different stock portfolios in order to analyze the cross-section of equity premia in the joint stock-bond framework. Finally, as there is an active literature on so-called macro-finance models of the term structure – mostly in an affine framework – this approach seems promising to be applied to our framework as well. This would allow to trace the joint effect of macroeconomic shocks (output gap, monetary policy, natural real interest rate, etc.) on bond as well as stock market risk premia.

[26] See the survey by Sellin (2001). Note that our specification excludes any nexus between real stock returns and inflation by assumption, i.e. through the parameter restrictions on factor dynamics.

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A Details of derivations

A.1 Arbitrage-free stock prices

We derive the solution (2.23) for arbitrage-free stock prices with coefficients defined in (2.24) and (2.25).

The derivation starts with the guess that stock prices satisfy

$$V_t = \exp[c(t-t_0) + D'X_t), \quad \text{or} \quad v_t := \ln V_t = c(t-t_0) + D'X_t \tag{A.1}$$

and then chooses D and c that makes (2.22) hold as an almost-sure identity.

Plugging the guess (A.1) into the right-hand-side of (2.22) yields

$$\begin{aligned}
& E_t\left\{M_{t+1}\Gamma_{t+1}V_{t+1}\right\} \\
= \;& E_t\left\{\exp[m_{t+1} + \gamma_{t+1} + v_{t+1}]\right\} \\
= \;& E_t\left\{\exp[-\frac{1}{2}\lambda_t'\lambda_t - \delta_0 - \delta_1'X_t - \lambda_t'\epsilon_{t+1} + \delta_\gamma'X_{t+1} + c(t+1-t_0) + D'X_{t+1}]\right\} \\
= \;& E_t\left\{\exp[-\frac{1}{2}\lambda_t'\lambda_t - \delta_0 + (\delta_\gamma + D)'a + c(t+1-t_0) + (\delta_\gamma + D)'\mathcal{K}X_t - \delta_1'X_t]\right. \\
& \left. + \left((\delta_\gamma + D)'\Sigma - \lambda_t'\right)\epsilon_{t+1}]\right\}
\end{aligned}$$

This expression is of the form $E_t \exp[W_{t+1}]$, where W_{t+1} is conditionally normal. For the conditional expectation and the conditional variance we obtain

$$E_t W_{t+1} = -\frac{1}{2}\lambda_t'\lambda_t - \delta_0 + (\delta_\gamma + D)'a + c(t+1-t_0) + (\delta_\gamma + D)'\mathcal{K}X_t - \delta_1'X_t$$

and

$$Var_t W_{t+1} = \left((\delta_\gamma + D)'\Sigma - \lambda_t'\right)\left(\Sigma'(\delta_\gamma + D) - \lambda_t\right) = (\delta_\gamma + D)'\Sigma\Sigma'(\delta_\gamma + D) + \lambda_t'\lambda_t - 2(\delta_\gamma + D)'\Sigma\lambda_t$$

respectively. Using that $E_t \exp[W_{t+1}] = \exp\left[E_t W_{t+1} + \frac{1}{2}Var_t W_{t+1}\right]$, we finally get

$$\begin{aligned}
& E_t\left\{M_{t+1}\Gamma_{t+1}V_{t+1}\right\} \\
= \;& \exp\Big[-\delta_0 + (\delta_\gamma + D)'a + \frac{1}{2}(\delta_\gamma + D)'\Sigma\Sigma'(\delta_\gamma + D) - (\delta_\gamma + D)'\Sigma\lambda_0 \\
& + c\,(t+1-t_0) \\
& + \Big((\delta_\gamma + D)'\mathcal{K} - \delta_1' - (\delta_\gamma + D)'\Sigma\Lambda_1\Big)X_t\Big]
\end{aligned} \tag{A.2}$$

which completes our computation of the right-hand-side of (2.22). Using the guess (A.1), the left-hand side of (2.22) reads

$$\exp\Big[c\,(t-t_0) + D'X_t\Big].$$

In order for (2.22) to hold as an identity, the coefficients D and c have to satisfy

$$(\delta_\gamma + D)'\mathcal{K} - \delta_1' - (\delta_\gamma + D)'\Sigma\Lambda_1 = D' \tag{A.3}$$

and

$$c\,(t+1-t_0) - \delta_0 + (\delta_\gamma + D)'a + \frac{1}{2}(\delta_\gamma + D)'\Sigma\Sigma'(\delta_\gamma + D) - (\delta_\gamma + D)'\Sigma\lambda_0 = c\,(t-t_0) \quad \text{(A.4)}$$

respectively, for all t. Solving (A.3) for D yields (2.24), and given this solution, the expression (2.25) for c is then obtained from (A.4).

A.2 Stock returns when $\lambda_0 = 0$ and $\Lambda_1 = 0$

We derive (2.29). For $\Lambda_1 = 0$ and $\lambda_0 = 0$,

$$\begin{aligned} D' &= (\delta_\gamma'\mathcal{K} - \delta_1')(I-\mathcal{K})^{-1} \\ c &= \delta_0 - (\delta_\gamma + D)'a - \frac{1}{2}(\delta_\gamma + D')\Sigma\Sigma'(\delta_\gamma + D). \end{aligned}$$

Noting that in this case $\delta_\gamma' + D' = (\delta_\gamma' - \delta_1')(I-\mathcal{K})^{-1}$ and recalling that $EX_t = (I-\mathcal{K})^{-1}a =: \mu_X$ is the unconditional expectation of the stationary factor process (2.2), we have

$$c = \mu_r - \mu_\gamma - J$$

where $\mu_r := Er_t$, $\mu_\gamma = E\gamma_t$, and $J = \frac{1}{2}(\delta_\gamma + D)'\Sigma\Sigma'(\delta_\gamma + D)$. Hence, for the one-period stock return

$$\begin{aligned} & \Delta v_{t+1} + \gamma_{+1} \\ = \;& c + D'\Delta X_{t+1} + \delta_\gamma' X_{t+1} \\ = \;& c - D'X_t + (\delta_\gamma + D)'a + (\delta_\gamma + D)'\mathcal{K}X_t + (\delta_\gamma + D)'\Sigma\eta_{t+1} + \delta_\gamma' X_t - \delta_\gamma' X_t \\ = \;& c - (\delta_\gamma + D)'(I-\mathcal{K})X_t + (\delta_\gamma + D)'a + \delta_\gamma' X_t + (\delta_\gamma + D)'\Sigma\eta_{t+1} \\ = \;& \mu_r - \mu_\gamma - J - (\delta_\gamma' - \delta_1')X_t + (\delta_\gamma' - \delta_1')(I-\mathcal{K})^{-1}a + \delta_\gamma' X_t + (\delta_\gamma + D)'\Sigma\eta_{t+1} \\ = \;& \mu_r - \mu_\gamma - J - \gamma_t + r_t + \mu_\gamma - \mu_r + \gamma_t + (\delta_\gamma + D)'\Sigma\eta_{t+1} \\ = \;& r_t - J + (\delta_\gamma + D)'\Sigma\eta_{t+1} \end{aligned}$$

The second equality plugs in the law of motion for X_t, the third regroups, the fourth plugs in the expressions derived above for c and $\delta_\gamma' + D'$, the fifth uses the definition of the real short rate and the dividend yield as well as their unconditional expectations.

A.3 Multi-period stock returns

We derive $n-$period stock market returns under the assumption that any dividends paid out between period t and $t+n$ are fully reinvested in the stock (index).

In period t, the investor buys $H_t = 1$ unit of the stock at the ex-dividend price V_t. In the next period, he receives total dividends $H_t \cdot D_{t+1}$ which – following our assumption – are used for buying new stock at the new ex-dividend price V_{t+1}. Hence, the number of stocks to be bought is $\Delta H_{t+1} = \frac{H_t \cdot D_{t+1}}{V_{t+1}}$, and the new number of stocks held is $H_{t+1} =$

$H_t + \Delta H_{t+1} = H_t(1 + \frac{D_{t+1}}{V_{t+1}})$. Reinvestment in the next period $t+2$ follows the same pattern, and the number of stocks held when entering period $t+3$ is $H_{t+2} = H_{t+1}(1 + \frac{D_{t+2}}{V_{t+2}})$. It is straightforward to see that the number of stocks held when entering period $t+i$ is recursively obtained as

$$H_{t+i} = H_{t+i-1}\left(1 + \frac{D_{t+i}}{V_{t+i}}\right). \quad \text{(A.5)}$$

The final period of the investment horizon $t+n$ is entered with H_{t+n-1} units of the stock, then the investor obtains total dividends $H_{t+n-1} \cdot D_{t+n}$, finally he sells his stocks and obtains the revenue $H_{t+n-1} \cdot V_{t+n}$.

Thus, the overall (random) log-return of this investment strategy equals

$$\begin{aligned} r_{t+n}^{(n)} &= \ln\left(H_{t+n-1}(V_{t+n} + D_{t+n})\right) - \ln(V_t) \\ &= \ln\left(H_{t+n-1}V_{t+n}\left(1 + \frac{V_{t+n}}{D_{t+n}}\right)\right) - \ln(V_t) \end{aligned}$$

Using the definitions from the main text and $\ln(H_t) =: h_t$,

$$r_{t+n}^{(n)} = h_{t+n-1} + v_{t+n} + \gamma_{t+n} - v_t.$$

From (A.5) with $H_t = 1$, one obtains

$$h_{t+n-1} = \gamma_{t+1} + \gamma_{t+2} + \ldots + \gamma_{t+n-1}$$

Hence,

$$r_{t+n}^{(n)} = v_{t+n} - v_t + \sum_{i=1}^{n} \gamma_{t+i} \quad \text{(A.6)}$$

as had to be shown.

B Tables

Parameter	Estimate	t-value
a_1	1.113E-4	.
a_2	2.097E-06	1.04
$\mathcal{K}_{11}$	0.954	.
$\mathcal{K}_{22}$	0.999	1.95
$\mathcal{K}_{23}$	-8.713E-4	-7.51
$\mathcal{K}_{24}$	-7.793E-4	-14.96
$\mathcal{K}_{33}$	0.992	1.46
$\mathcal{K}_{43}$	-0.030	-13.76
$\mathcal{K}_{44}$	0.970	6.29
Σ_{11}	2.589E-4	.
Σ_{22}	8.335E-5	21.93
δ_0	2.081E-3	.
δ_{L1}	0.112	4.95
δ_{L2}	0.317	21.79
$\lambda_{0,1}$	-0.387	-3.35
$\lambda_{0,3}$	-0.014	-0.18
$\lambda_{0,4}$	0.059	0.63
$\Lambda_{1,11}$	-9.201	-1.05
$\Lambda_{1,22}$	-35.536	-2.61
$\Lambda_{1,33}$	15.792	1.47
$\Lambda_{1,44}$	22.205	2.37
h_1	6.298E-5	39.17
h_2	1.392E-4	17.60

Table 1: Maximum likelihood parameter estimates and estimated asymptotic t-statistics (based on outer-product of gradient estimator). Note that the parameters a_1, $\mathcal{K}_{11}$ and Σ_{11} of the AR(1) for inflation are estimated by OLS in the first step, and δ_0 is equated to the mean real rate. For estimating the parameters $\mathcal{K}_{22}$, $\mathcal{K}_{33}$ and $\mathcal{K}_{44}$, the reparameterization $\mathcal{K}_{ii} = \psi_i^2/(1+\psi_i^2)$ has been used to guarantee that $\mathcal{K}_{ii} \in [0,1)$. The respective t-values correspond to the auxiliary parameters ψ_i.

C Figures

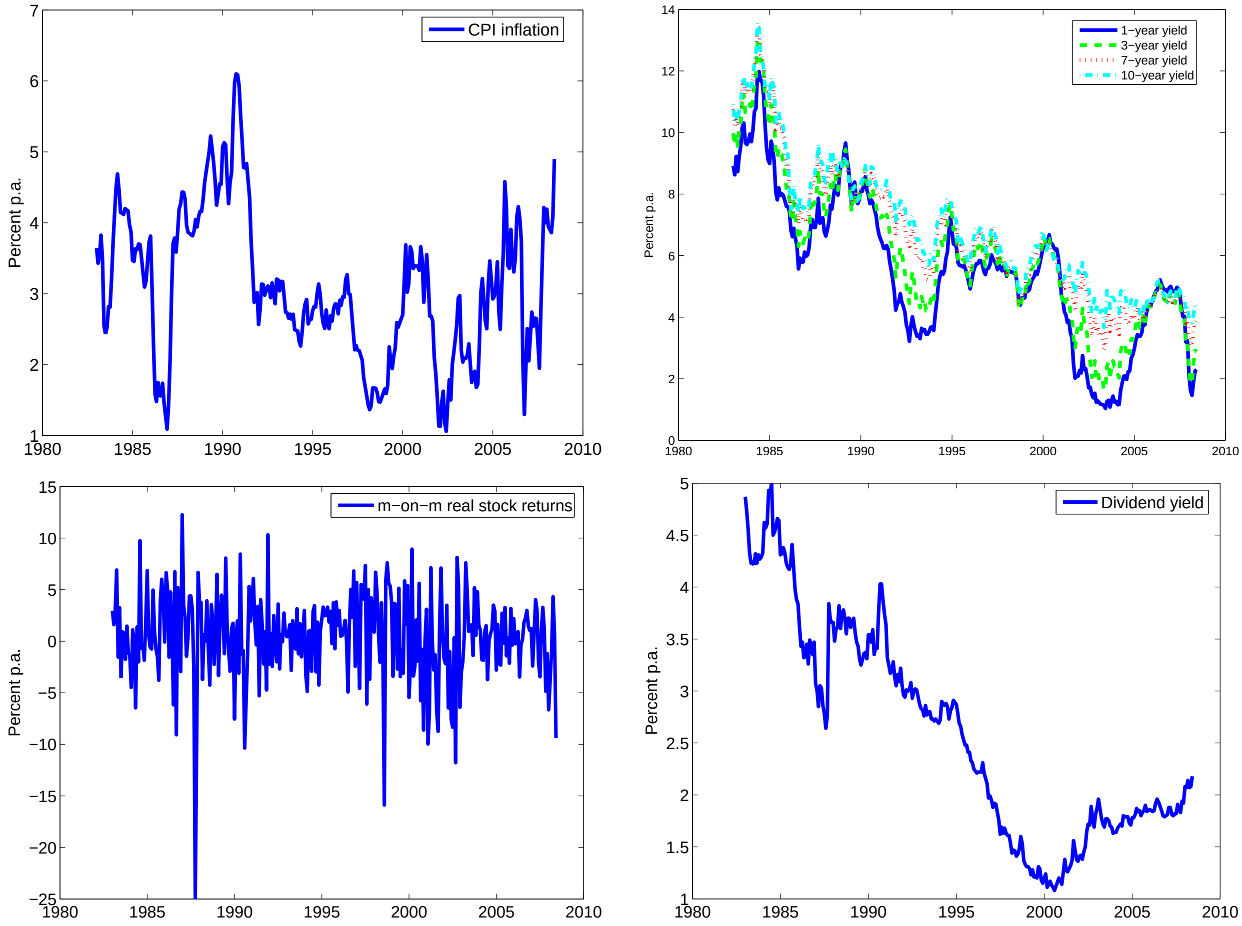

Figure 1: Data used for estimation. CPI inflation (top left), nominal bond yields (top right), real stock returns (bottom left), dividend yield (bottom right).

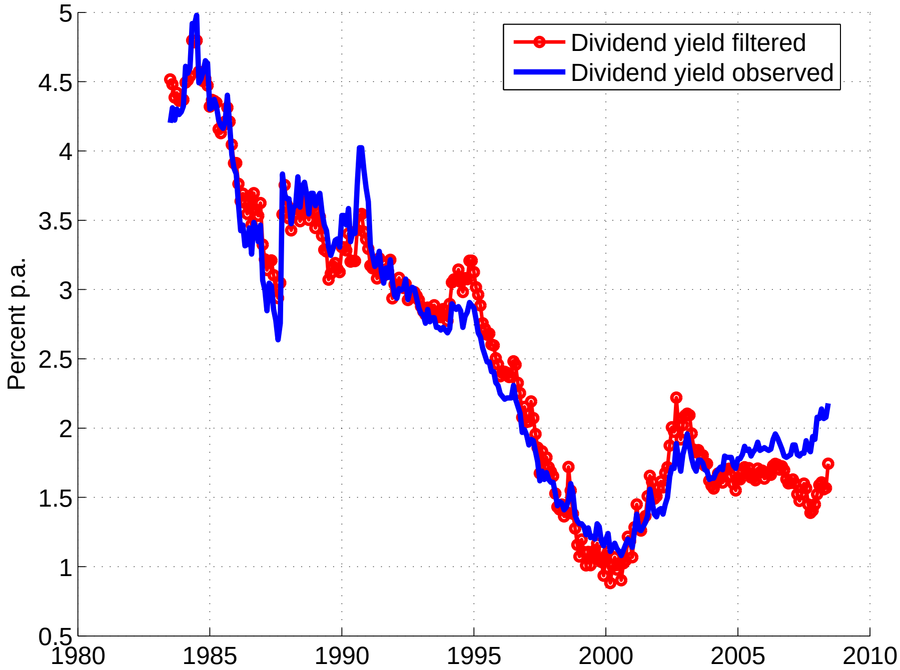

Figure 2: Dividend yield: Measured and filtered model-implied.

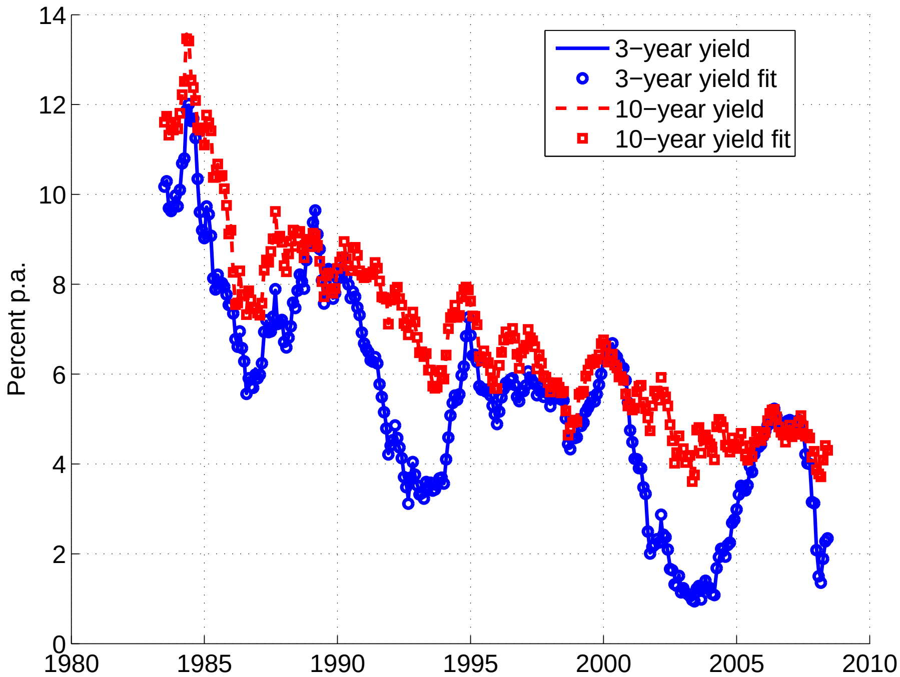

Figure 3: Observed bond yields for three- and ten-year maturities and model-implied counterparts based on filtered states.

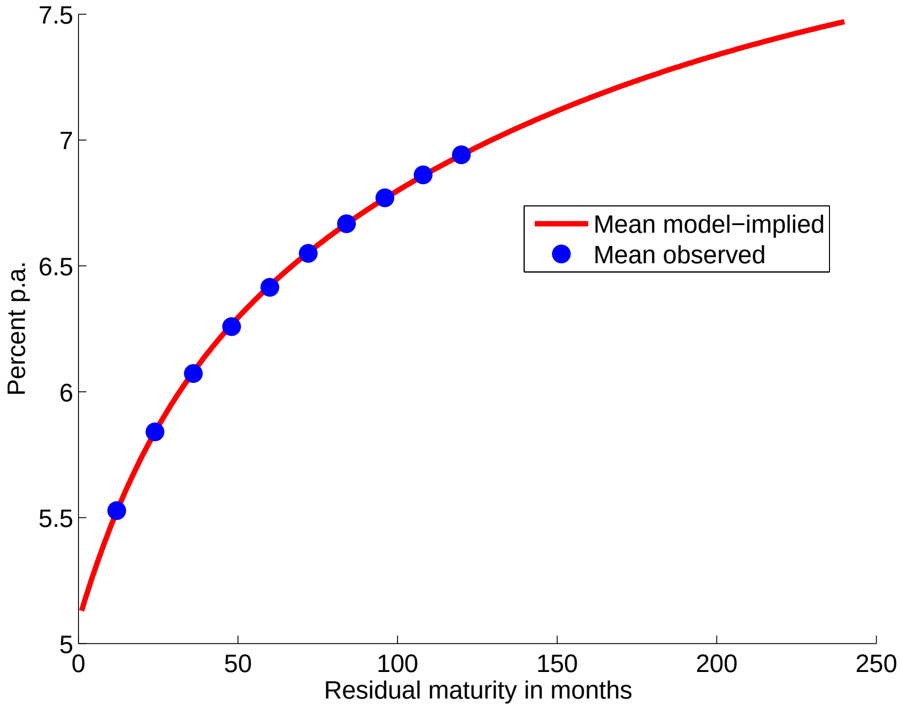

Figure 4: Average of model-implied term structure of nominal bond yields and averages of observed (Gurkaynak-Sack-Swanson) yields. Note that only 1-, 3-, 5- and 10-year yields are used for estimation.

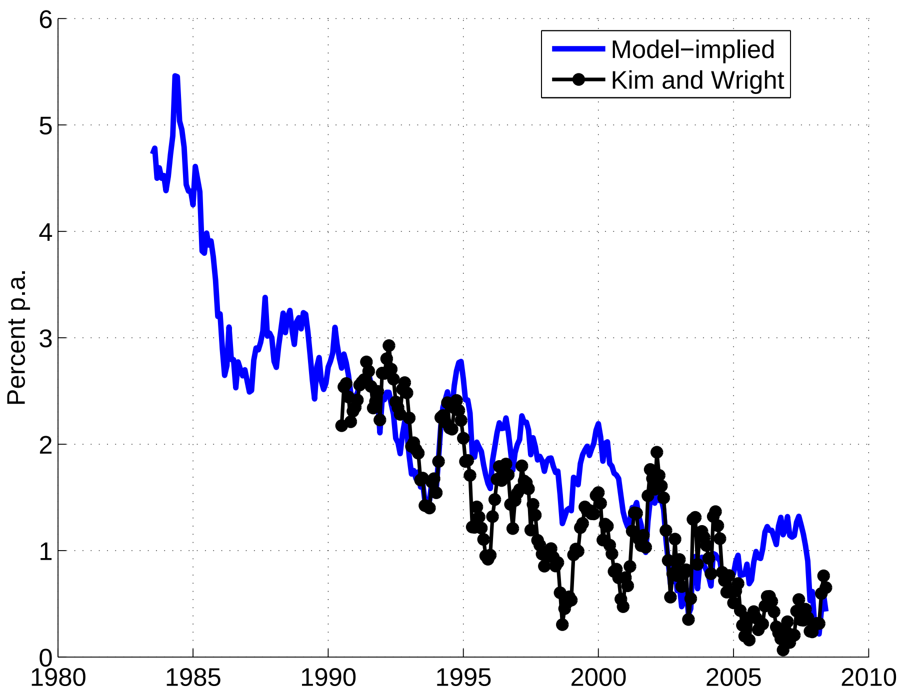

Figure 5: Ten-year nominal term premium: i) model-implied and ii) estimated by Kim and Wright (2005).

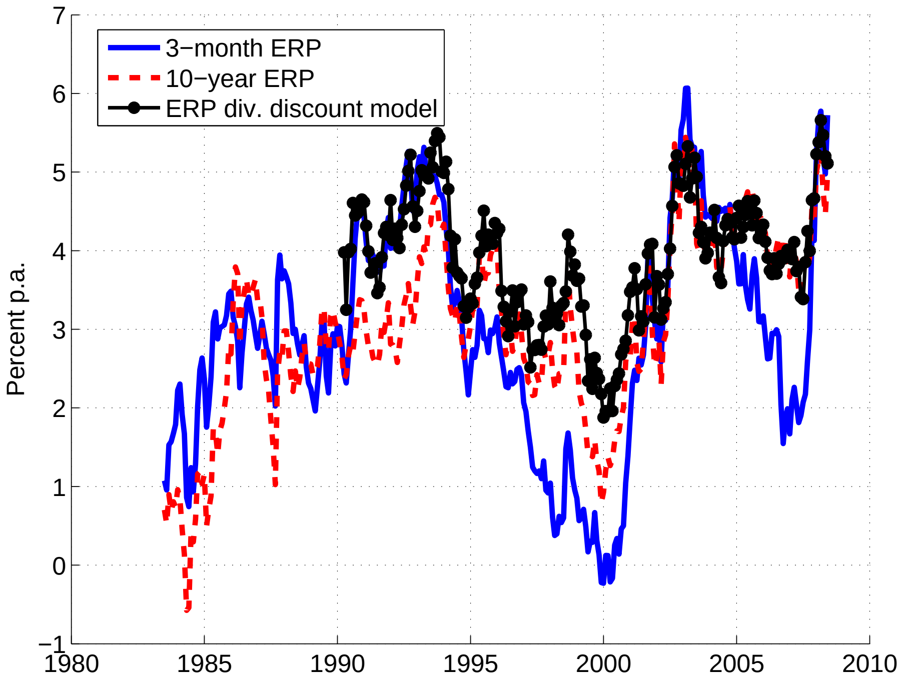

Figure 6: Model-implied equity risk premia for 3-month and ten-year horizons and equity risk premium implied by three-stage dividend discount model.

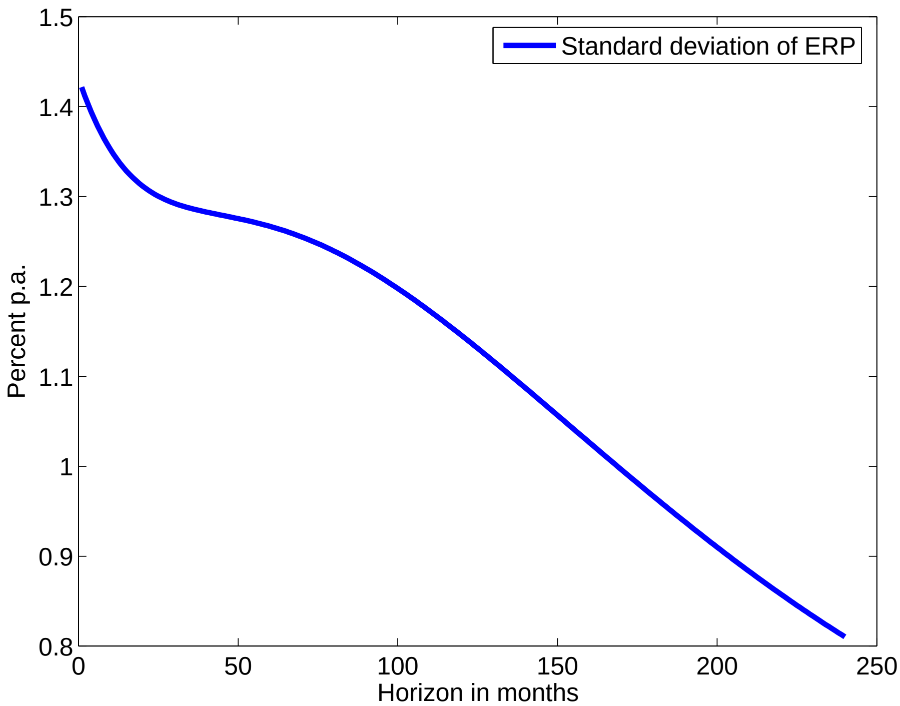

Figure 7: Standard deviation of estimated equity risk premia for different horizons.

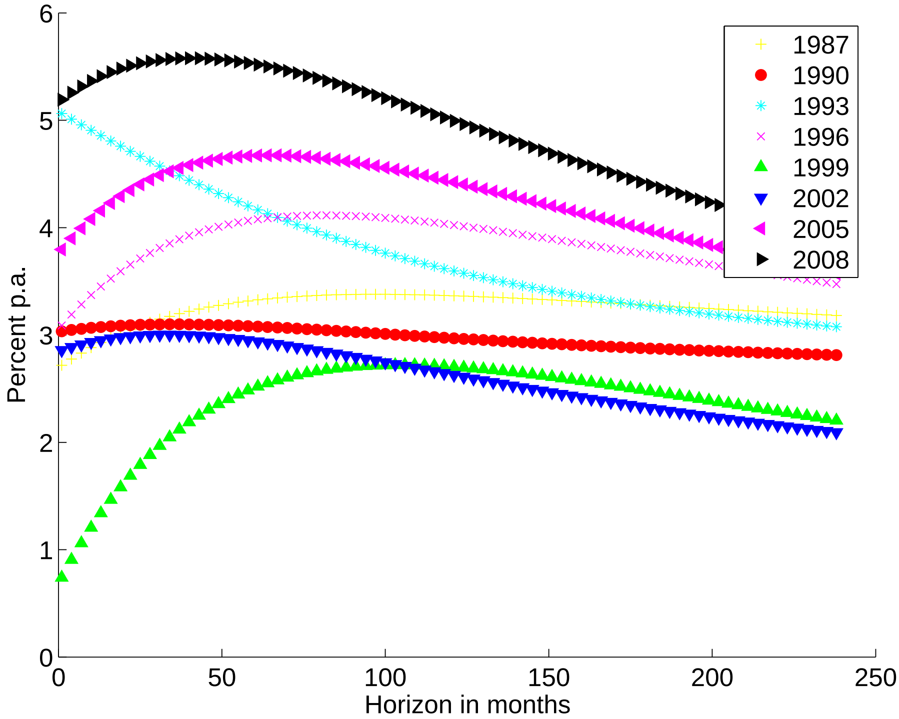

Figure 8: Equity premia for different horizons ('Term structure of ERPs') recorded in January of the indicated years.

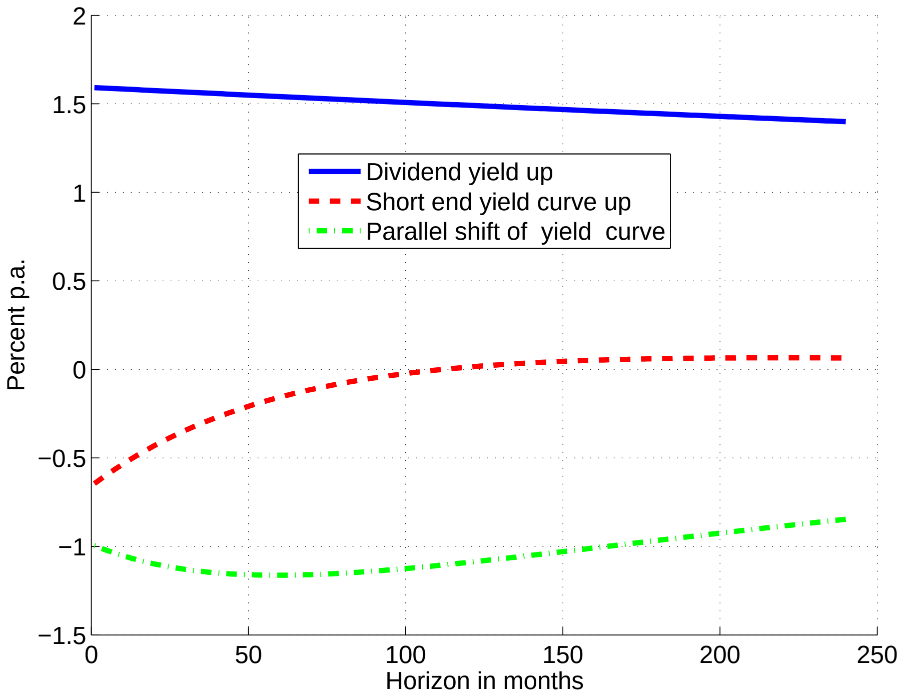

Figure 9: Contemporaneous impact of i) an increase of the dividend yield by one percentage point, ii) a shift in the short end of the term structure of real interest rates, lifting the three-month real rate by one percentage point and keeping the ten-year rate constant, and iii) an almost parallel shift of the real term structure, increasing both the three-month and ten-year rate by one percentage point – on the term structure of equity risk premia.