Cross-subsidizing and Profit-making Equilibria in a Competitive Insurance Market with Asymmetric Information*

Wanda Mimra and Achim Wambach†

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Abstract

We study market equilibria in a dynamic competitive insurance model with asymmetric information. The model extends the classic Rothschild-Stiglitz insurance model in the spirit of Wilson (1977) by introducing an additional stage in which initial contracts can be withdrawn after observation of competitor's contract offers but before consumers choose their insurance contract. We show that an equilibrium always exists where consumers obtain their respective Wilson-Miyazaki-Spence (WMS) contract, i.e. second-best efficiency can be achieved for any share of high-risk types in the population. However, jointly profit-making contracts can also be sustained as equilibrium contracts.

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†Address: University of Cologne, Department of Economics, Gyrhofstr. 8c, 50931 Cologne, Germany. Email: `mimra@wiso.uni-koeln.de, wambach@wiso.uni-koeln.de`

1 Introduction

Potential non-existence of equilibrium in competitive insurance markets under adverse selection has received much attention in the literature ever since the seminal work by Rothschild and Stiglitz (1976). In the Rothschild-Stiglitz model where firms set contracts in stage 1 and consumers choose contracts in stage 2, equilibrium fails to exist when the share of high risks is low as then single pooling contracts or cross-subsidizing contract pairs would be preferred by everyone over the candidate separating, separately zero-profit making Rothschild-Stiglitz (RS) contracts. Subsequent research has addressed the non-existence problem by considering mixed strategies (Dasgupta/Maskin 1986), introducing equilibrium concepts that differ from Nash-equilibrium (Wilson 1977; Riley 1979), extending the game structure by adding further stages (Hellwig 1987; Asheim/Nielssen (1996)) or introducing additional constraints like capacity constraints (Inderst/Wambach 2001).[1]

In Wilson's (1977) 'anticipatory equilibrium' concept, "each firm assumes that any policy will be immediately withdrawn which becomes unprofitable after that firm makes its own policy offer". Extending Wilson (1977) to contract menus, Miyazaki (1977) and Spence (1978) show that the anticipatory equilibrium concept results in cross-subsidizing, jointly zero-profit making contracts that are second-best efficient, the so-called Wilson-Miyazaki-Spence (WMS) contracts. However, so far with the exception of Asheim/Nielssen (1996) where firms can renegotiate contracts on a non-discriminatory basis, the second-best efficient WMS contracts lack game-theoretic foundation.

The present paper spells out the idea behind Wilson's anticipatory equilibrium by introducing an additional stage into the RS model in which firms can

[1] Other strands of the literature consider cooperative concepts (Lacker/Weinberg (1999)) or a general-equilibrium approach (Bisin/Gottardi (2006)).

withdraw contracts. In contrast to Hellwig (1987), where firms may decline to fulfil their contract after consumers have chosen contracts, we analyse a situation where contracts can be withdrawn (repeatedly) before consumers make their choice but after observing the contracts of the competitors. Thus, this involves no potential revelation of consumer type information as in Hellwig (1987) or Asheim/Nielssen (1996).[2]

In this set-up, we show that an equilibrium always exists where every consumer obtains her respective WMS contract. Intuitively, the possibility of contract withdrawal prevents cream-skimming deviations that upset the WMS contracts in the original RS set-up. However, we show that monopoly contracts can also be enforced as equilibrium contracts for certain parameter values as the possibility to retract contracts provides firms with adequate threat points. Consider setting monopoly contracts and WMS contracts in stage 1. If firms observe that every firm offers exactly this contract set, they withdraw WMS contracts in stage 2 and share monopoly profits. If, however, they observe deviation, the WMS contracts are not withdrawn and all firms make zero profits. This logic is viable as long as the threat with WMS contracts is credible, i.e. deviation leaves the remaining firms with zero profits even if they withdraw WMS contracts.

The rest of the paper is organized as follows: In Section 2, the model is introduced. In Section 3, an existence result for the WMS equilibrium is derived. Section 4 derives an equilibrium with monopoly contracts. We conclude in Section 5.

[2]Hellwig (1987) only considers single contract offers by firms. He shows that the Wilson pooling contract corresponds to a stable equilibrium of the three-stage game. The WMS do not constitute equilibrium contracts as the H-type WMS contract would always be rejected in stage 3.

2 The model

There is a continuum of individuals with mass 1. Each individual faces two possible states of nature: In state 1, no loss occurs and the endowment is w_{01}, in state 2 a loss occurs and the endowment is w_{02} with $w_{01} > w_{02} > 0$. There are 2 types of individuals, an individual may be a high risk type (H) with loss probability p^H, or a low-risk type (L) with loss probability p^L, with $0 < p^L < p^H < 1$. Insurance is provided by firms in the set $F := \{1, ..., f, ...n\}$. Firms do not know, ex ante, any individual's type. If an individual buys insurance, then the endowment $w_0 = (w_{01}, w_{02})$ is traded for another state-contingent endowment $w = (w_1, w_2)$, we say the individual buys insurance contract w. The set of feasible contracts, W, is given by $W := \{(w_1, w_2) \,|(w_1 \geq w_2 > 0\}$ where $w_1 < w_2$ is ruled out for moral hazard considerations.

The expected utility of a J-type individual, $J \in \{H, L\}$ from chosing a contract $w \in W$ is abbreviated by $u^J(w) := (1 - p^J)v(w_1) + p^J v(w_2)$ where v is a strictly increasing, twice continuously differentiable and strictly concave von Neumann-Morgenstern utility function.

The timing of the game is as follows: First, firms set contracts simultaneously after which they observe their competitors' contract offers. Then, firms can withdraw contracts potentially repeatedly for several rounds whereby firms observe their competitors actions after each round. Contract withdrawal is possible as long as at least one contract was withdrawn by any firm in the previous round. After contract withdrawal ends, consumers make their contract choice.[3]

[3]The possibility to withdraw contracts prior to consumer selection of contracts can be motivated by several observations: First, insurance firms tend to have a better overview of the market than consumers and may therefore observe and react to new contract offers

Formally, the game proceeds as follows:

Stage 0: The risk type of each individual is chosen by nature. Each individual has a chance of γ_H, $0 < \gamma_H < 1$ to be a H-type, and of $(1 - \gamma_H)$ to be a L-type.

Stage 1: Each firm $f \in F$ offers a set of contracts $W_f^1 \subset W$. The offered sets are observed by all firms before the beginning of the next stage.

Stage 2: Stage 2 consists of $t = 1, 2, ...$ rounds. In each round t each firm $f \in F$ can withdraw a set $W_f^{2,t} \subseteq W_f^1 \setminus \bigcup_{i=1}^{t-1} W_f^{2,i}$. After each round, firms observe the remaining contract offers. If, for any t, $W_f^{2,t} = \emptyset \ \forall \ f \in F$, this stage ends.

Stage 3: Define $\hat{t}$ by $W_f^{2,\hat{t}} = \emptyset \ \forall \ f \in F$. Individuals choose among the remaining contracts $\bigcup \left\{ W_f^1 \setminus \bigcup_{i=1}^{\hat{t}-1} W_f^{2,i} \right\}$.

3 WMS contracts as equilibrium contracts

Let us first recall the Wilson-Miyazaki-Spence (WMS) contracts, which of all separating contracts that jointly break-even are the pair that is most preferred by the L-type. Formally, consider the following maximization problem:

$$\max_{w^L, w^H} \ u^L(w^L) \tag{1}$$

more quickly. Second, the above timing of the game can be interpreted as a stylized model of a market where consumers arrive sequentially in the market such that firms can alter contracts before the arrival of new consumers.

s.t.

$$u^H(w^H) \geq u^H(w^L) \tag{2}$$

$$u^H(w^H) \geq u^H(w^H_{RS}) \tag{3}$$

$$\gamma_H[(1-p_H)(w_{01}-w_1^H) - p_H(w_2^H - w_{02})] + \\ (1-\gamma_H)[(1-p_L)(w_{01}-w_1^L) - p_L(w_2^L - w_{02})] \geq 0 \tag{4}$$

where $w^H_{RS} = (w_{01} - p_H(w_{01} - w_{02}), w_{01} - p_H(w_{01} - w_{02}))$

Definition 1. The Wilson-Miyazaki-Spence contracts, denoted w^H_{WMS} and w^L_{WMS}, are the unique solution to the above maximization problem.[4]

Note that the WMS contracts are second-best efficient.[5] Denote by w^H_{RS} and w^L_{RS} the H-type and L-type RS contracts. w^H_{RS} specifies full coverage while the expected zero-profit condition for insurers on this contract holds,

$$w^H_{RS} = (w_{01} - p_H(w_{01} - w_{02}), w_{01} - p_H(w_{01} - w_{02})),$$

while w^L_{RS} is pinned down by maximizing $u^L(w)$ subject to the expected zero-profit condition for the insurer on this contract and the H-type incentive compatibility constraint, which both become binding. Note that the WMS contracts correspond to the RS contracts when (3) is binding. When (3) is not binding, WMS contracts are such that the fully-insured H-types are subsidized by the partially insured L-types. We will focus on this more interesting case for the remainder of this paper.[6]

[4]Dionne and Fombaron (1996) show that the solution is unique for any type of risk averse utility function despite non-convexities of the possibility frontier in the income-state space.

[5]This was shown by Crocker/Snow (1985).

[6]This is precisely when equilibrium fails to exist in the RS set-up when firms are allowed to offer contract menues. Our results hold trivially for the case that the WMS contracts correspond to the RS contracts.

The WMS and RS contracts are shown below.

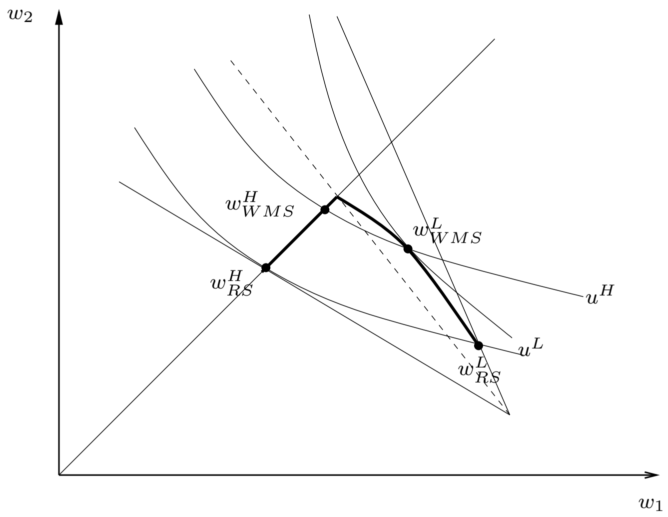

Figure 1: WMS contracts

We can now state our result.

Proposition 1. *There exists a symmetric equilibrium where every individual obtains her respective WMS contract in stage 3.*

Proof. Consider the partition $\Omega = (W_{WMS}, W_{RS}, W_{CS}, W_{LR}, W_H, W_R)$ of the feasible set W with

$$
\begin{aligned}
W_{WMS} &:= \left\{w^H_{WMS}, w^L_{WMS}\right\} \\
W_{RS} &:= \left\{w^H_{RS}, w^L_{RS}\right\} \\
W_{CS} &:= \left\{w \in W \,\middle|\, u^L(w) \geq u^L(w^L_{WMS}) \text{ and } u^H(w) \leq u^H(w^L_{WMS}), w \neq w^L_{WMS}\right\} \\
W_{LR} &:= \Big\{w \in W \,\Big|\, u^L(w) < u^L(w^L_{WMS}), u^L(w) > u^L(w^L_{RS}) \\
&\qquad \text{and} (1-p_L)(w_{01}-w_1) - p_L(w_2 - w_{02}) = 0\Big\} \\
W_H &:= \left\{w \in W \,\middle|\, u^H(w) \leq u^H(w^H_{WMS}), w \notin W_{CS} \cup W_{WMS} \cup W_{RS} \cup W_{LR}\right\} \\
W_R &:= \{w \in W \setminus \{W_{RS} \cup W_{LR} \cup W_{CS} \cup W_{WMS} \cup W_H\}\}
\end{aligned}
$$

Note that due to the single crossing property, this indeed constitutes a partition of W. We claim that one possible equilibrium strategy is the following: firm $f \in F$ sets $W_f^1 = W_{WMS} \cup W_{RS} \cup W_{LR}$.

If, at the end of stage 1, firm f observes any contract $w_j \in W_{CS}$, then

- $W_f^{2,1} = W_{WMS} \cup \left\{ w \in W_{LR} \left| u^H(w) > max\ u^H(w_j') \ \forall w_j' \in W_H \right. \right\}$ and
- if, for any $t > 1$, $W_j^{2,t-1} \neq \emptyset$, then
  $W_f^{2,t} = \left\{ w \in W_{LR} \left| u^H(w) > max\ u^H(w_j'') \ \forall w_j'' \in W_H \cup W_{LR} \right. \right\}$, otherwise $W_f^{2,t} = \emptyset$.

If, at the end of stage 1, firm f does not observe any contract $w_j \in W_{CS}$, then

- if, for any t, there exists a $j \in F$ for which $w_{WMS}^L \in W_j^1 \setminus \bigcup_{i=1}^{t-1} W_j^{2,i}$ and $w_{WMS}^H \notin W_j^1 \setminus \bigcup_{i=1}^{t-1} W_j^{2,i}$, then
  $W_f^{2,t} = W_{WMS} \cup \left\{ w \in W_{LR} \left| u^H(w) > max\ u^H(w_j') \ \forall w_j' \in W_H \right. \right\}$ and
- if, for any t, $W_j^{2,t-1} \neq \emptyset$, then
  $W_f^{2,t} = \left\{ w \in W_{LR} \left| u^H(w) > max\ u^H(w_j'') \ \forall w_j'' \in W_H \cup W_{LR} \right. \right\}$, otherwise, $W_f^{2,t} = \emptyset$.

It remains to show that there is no profitable deviation. Consider the case that all firms follow the above strategy apart from firm $\overline{f}$. We consider three cases:

Case 1: We begin with the simplest case, a pure cream-skimming deviation. Let $W_{\overline{f}}^1 \subseteq W_{CS}$. Then $W_f^{2,1} = W_{WMS} \ \forall f \in F, f \neq \overline{f}$. It follows that at the end of $t = 1$, firm $\overline{f}$ has the best contract offer for both types, i.e. it would attract all consumers. However, by construction of the WMS contracts, serving the whole market with a contract $w \in W_{CS}$ is loss making. Subsequently setting $W_{\overline{f}}^{2,t} = W_{\overline{f}}^1$ for any t leaves firm $\overline{f}$ with zero profits as no consumer is served, hence, this deviation is not provitable.

Case 2: We now consider deviation by either only setting the L-type WMS contract in stage 1 or behaving according to equilibrium strategy in stage 1, but withdrawing the H-type WMS contract in stage 2. Let $W^1_{\overline{f}} \subseteq W_{WMS} \cup W_{RS} \cup W_{LR}$ and for some $\underline{t} \geq 1$, $w^H_{WMS} \notin W^1_{\overline{f}} \setminus \bigcup_{i=1}^{t-1} W^{2,i}_{\overline{f}}$. Then $W^{2,\underline{t}}_f = W_{WMS}\ \forall f \in F, f \neq \overline{f}$. It follows that at the end of $\underline{t}$, firm $\overline{f}$ offering w^L_{WMS} has the best contract offer for both types, i.e. it would attract all consumers. However, by construction of the WMS contracts, serving the whole market with the contract w^L_{WMS} is not profitable. Subsequently withdrawing w^L_{WMS} leaves firm $\overline{f}$ with zero profits as either no consumer is served or $\overline{f}$ offers zero-profit RS contracts, hence, this is not a profitable deviation.

Case 3: We now consider the case that firm $\overline{f}$ follows a more sophisticated strategy by setting a cream-skimming contract and a contract (set) that would make a positive profit if all consumers were served with this contract (set), using the cream-skimming contract as a threat point to force firms to withdraw the WMS contracts.
Let $W^1_{\overline{f}} = \{w^{cs}, w^p\}$ with $w^{cs} \in W_{CS}$ and $w^p \in W_H$,

$$\gamma_H[(1-p_H)(w_{01}-w^p_1) - p_H(w^p_2 - w_{02})] + \\ (1-\gamma_H)[(1-p_L)(w_{01}-w^p_1) - p_L(w^p_2 - w_{02})] > 0.$$

Then $W^{2,1}_f = W_{WMS} \cup \{w \in W_{LR} \,|u^H(w) > u^H(w^p)\}\ \forall f \in F, f \neq \overline{f}$. As was argued in *Case 1*, it cannot be profitable for $\overline{f}$ not to withdraw w^{cs}. It remains to be shown that $\overline{f}$ cannot make a positive profit by withdrawing w^{cs} but leaving w^p on offer. We distinguish two cases: First, if $u^H(w^p) \leq u^H(w^H_{RS})$, then $\bigcup W^1_f \setminus W^{2,1}_f = W_{RS}$ and $\overline{f}$ does not serve any consumer. Second, if $u^H(w^p) > u^H(w^H_{RS})$, due to the single crossing property there always exists a contract $w^* \in \bigcup W_f^1 \setminus W_f^{2,1}$ satisfying $u^H(w^*) < u^H(w^p)$, $u^L(w^*) > u^L(w^p)$, i.e. w^p would only be chosen by H-types. However, as $u^H(w^p) > u^H(w^H_{RS})$, $\overline{f}$ would make a loss on w^p. Note that the prescribed strategy for firms $f \in F, f \neq \overline{f}$ is a best response in the subgame as serving the L-types with w^* when w^p is on offer does not yield negative profits. Note furthermore that although only the case of a profit-making pooling contract was considered, the same logic applies to a profit-making contract set, the proof is therefore ommited. Note also that a similar deviating strategy with only setting the L-type WMS contract as threat point instead of a cream-skimming contract is unprofitable following exactly the same reasoning, therefore this variant is omitted as well.

Any other deviation not considered explicitly is either not profitable as no consumer is served, the whole market is served at a loss, or a variant of the former cases with an analogous reasoning applies. □

The reasoning behind Proposition 1 works as follows: The possibility to withdraw contracts prevents simple cream-skimming deviations that upset the WMS contracts in the original RS set-up as cream-skimming contracts become unprofitable if the remaining firms withdraw the WMS contracts. With the same logic deviation by only setting the L-type WMS contract and making a positive profit on this contract is prevented. However, the possibility of contract withdrawal also allows more sophisticated deviating strategies: A deviator could force the other firms to withdraw their WMS contracts by setting a cream-skimming contract, withdrawing this contract subsequently and making a positive profit on a pooling contract or contract menu that will be the best offer available for both types after WMS contracts have been withdrawn. However, this deviation is prevented by leaving exactly those contracts on the L-type fair insurance line that attract away only L-types from any such contract set.

The above Proposition provides an existence result for an equilibrium that is second-best efficient. However, in the next section it will be shown that there also exists an (inefficient) equilibrium where firms share monopoly profits.

4 Monopoly contracts as equilibrium contracts

Monopoly contracts in insurance markets with adverse selection have the following characteristics: contracts are separating, high risks receive complete coverage and low risks receive partial coverage or no insurance and are always indifferent between purchasing insurance or remaining uninsured.[7] More precisely, depending on the share of H-types in the population, a monopolist serves only the H-types and extracts all consumer surplus involved in the reduction of risk from the H-types, or serves both types and extracts all consumer surplus from the L-types, whereby it is possible that a loss is incurred with the H-types.

We will show that if a monopolist would only offer a H-type contract, the monopoly contract can be sustained as equilibrium contract in a symmetric equilibrium for any number of firms in the market. Formally, let

$$\overline{r} = \left(\frac{v'(w_{02})}{v'(w_{01})} - 1\right)\left(\frac{v'(w_2^H)}{v'(w_{02})}\right)\frac{p_L(1-p_L)}{p_H(1-p_L) - p_L(1-p_H)}$$

with $u^H(w^H) = u^H(w_0)$.

[7] For a detailed analysis see Stiglitz (1977).

Proposition 2. *Monopoly contracts can be sustained as equilibrium contracts in a symmetric equilibium for any* $n \geq 1$ *if* $\frac{\gamma_H}{1-\gamma_H} > \overline{r}$.

Proof. See Appendix.

The proof of the proposition works as follows: Consider offering the monopoly contract and the WMS contracts in stage 1. If only those contracts are observed, the WMS contracts are withdrawn (sequentially) in stage 2. If any deviating, stand-alone profit-making contracts are observed, the WMS contracts are not withdrawn. This intuition works as it is credible for firms not to withdraw the WMS contracts and make zero profits on WMS contracts when they observe deviation.

5 Conclusion

We modify the standard Rothschild-Stiglitz insurance model in the spirit of Wilson (1977) by introducing an additional stage in which firms can withdraw contracts after observation of competitor's contract offers but before consumers choose their insurance contract. It is shown that an equilibrium always exists where consumers obtain their respective Wilson- Miyazaki-Spence (WMS) contract, i.e. second-best efficiency can be achieved for any share of high-risk types in the population. However, it is also shown that an equilibrium exists where firms share monopoly profits.

We did not fully characterize all possible equilibria of the game. It is easy to show that any pooling contract as well as separating contracts which jointly are not loss making and where either the H-type or L-type contract is on the corresponding type-specific zero-profit line can be sustained as equilibrium contracts with strategies analogous to those decribed in the proof of Proposition 2. It remains to be shown that monopoly contracts can be enforced when the share of H-types is such that a monopolist would serve both types.

6 Appendix

Proof of Proposition 2.

Proof. Denote by w_M^H the (H-type) monopoly contract. Consider the partition $\Omega = (W_M, W_{WMS}, W_{RS}, W_{CS}, W_{LR}, W_H, W_R)$ of the feasible set W with

$$\begin{aligned}
W_M &:= \{w_M^H\} \\
W_{WMS} &:= \{w_{WMS}^H, w_{WMS}^L\} \\
W_{RS} &:= \{w_{RS}^H, w_{RS}^L\} \\
W_{CS} &:= \{w \in W \,|u^L(w) \geq u^L(w_{WMS}^L) \text{ and } u^H(w) \leq u^H(w_{WMS}^L), w \neq w_{WMS}^L\} \\
W_{LR} &:= \{w \in W \,|u^L(w) < u^L(w_{WMS}^L), u^L(w) > u^L(w_{RS}^L) \\
&\qquad \text{and} (1-p_L)(w_{01}-w_1) - p_L(w_2 - w_{02}) = 0\} \\
W_H &:= \{w \in W \,|u^H(w) \leq u^H(w_{WMS}^H), w \notin W_M \cup W_{CS} \cup W_{WMS} \cup W_{RS} \cup W_{LR}\} \\
W_R &:= \{w \in W \setminus \{W_M \cup W_{RS} \cup W_{LR} \cup W_{CS} \cup W_{WMS} \cup W_H\}\}
\end{aligned}$$

We claim that one possible equilibrium strategy is the following: firm $f \in F$ sets $W_f^1 = W_M \cup W_{WMS} \cup W_{RS} \cup W_{LR}$. Then,

- if $\bigcup W_j^1 \subseteq W_M \cup W_{WMS} \cup W_{RS} \cup W_{LR}$, $W_f^{2,1} = w_{WMS}^H$. If at the end of $t = 1$, $w_{WMS}^H \notin \bigcup \{W_j^1 \setminus W_j^{2,1}\}$, then $W_f^{2,2} = W_{RS} \cup W_{LR} \cup w_{WMS}^L$, otherwise $W_f^{2,2} = \emptyset$.

- if, at the end of stage 1, firm f observes any contract $w_j \in W_H$ with $u^H(w_j) > u^H(w_M^H)$, and if firm f does not observe any contract $w_j' \in$

W_{CS}, then $W_f^{2,1} = \emptyset$.

- If, at the end of stage 1, firm f observes any contract $w_j \in W_{CS}$, then
  - $W_f^{2,1} = W_{WMS} \cup \left\{ w \in W_{LR} \left| u^H(w) > max\ u^H(w_j') \ \forall w_j' \in W_H \right. \right\}$ and
  - if, for any $t > 1$, $W_j^{2,t-1} \neq \emptyset$, then $W_f^{2,t} = \left\{ w \in W_{LR} \left| u^H(w) > max\ u^H(w_j'') \ \forall w_j'' \in W_H \cup W_{LR} \right. \right\}$, otherwise $W_f^{2,t} = \emptyset$.
- otherwise, $W_f^{2,1} = W_{RS} \cup W_{LR} \cup W_{WMS}$.

It remains to show that there is no profitable deviation. Consider the case that all firms follow the above strategy apart from firm $\underline{f}$. We consider three cases:

Case 1: We begin with the simplest case, a deviation aimed at attracting H-types with a contract that is preferred over the monopoly contract. Let $W_{\underline{f}}^1 \subseteq W_H$ and there is at least one contract $w' \in W_{\underline{f}}^1$ for which $u^H(w') > u^H(w_M^H)$. Then $W_f^{2,1} = \emptyset \ \forall f \in F, f \neq \underline{f}$. It follows that at the end of $t = 1$, firm $\underline{f}$ does not serve any consumer. Note that it is a best response for firms $f \in F, f \neq \underline{f}$ not to withdraw the WMS contracts (and W_{RS} and W_{LR}) in this subgame as they would not serve any consumer if they withdrew these contracts.

Case 2: We now consider a deviation aimed at attracting H-types with a contract that is preferred over the monopoly contract, and additionally setting a contract that cream-skims on the L-type WMS contract in order to force firms to withdraw the WMS contracts. Let $w^{cs}, w^p \in W_{\underline{f}}^1$ with $w^{cs} \in W_{CS}$ and $w^p \in W_H$, $u^H(w^p) > u^H(w_M^H)$, $u^H(w^p) \geq u^H(w') \ \forall w' \in W_{\underline{f}}^1$. Then $W_f^{2,1} = W_{WMS} \cup \left\{ w \in W_{LR} \left| u^H(w) > u^H(w^p) \right. \right\} \ \forall f \in F, f \neq \underline{f}$. As $\underline{f}$

makes a loss if it does not withdraw w^{cs}, it remains to be shown that $\underline{f}$ cannot make a positive profit by withdrawing w^{cs} but leaving w^p on offer. Note that this is analogous to *Case 3* of Proposition 1, the proof is therefore omitted. Note again that it is a best response for firms $f \in F, f \neq \underline{f}$ in this subgame not to withdraw W_{RS} and all contracts $w \in W_{LR}$ with $u^H(w) < u^H(w^p)$ as they would not serve any consumer if they withdrew these contracts.

Case 3: We now consider deviation by behaving according to equilibrium strategy in stage 1, but not withdrawing the H-type WMS contract in $t = 1$ in order to make a positive profit by offering the H-type WMS contract as a pooling contract for both types. Let $W^1_{\underline{f}} = W_M \cup W_{WMS} \cup W_{RS} \cup W_{LR}$. Then, $W^{2,1}_f = w^H_{WMS}$ $\forall f \in F, f \neq \underline{f}$. If $w^H_{WMS} \in W^1_{\underline{f}} \setminus W^{2,1}_{\underline{f}}$, then $W^{2,2}_f = \emptyset$ $\forall f \in F, f \neq \underline{f}$, i.e., if $\underline{f}$ does not withdraw w^H_{WMS} subsequently, $\underline{f}$ would serve all H-types but only a share of $1/n$ of L-types. However, by construction of the WMS contracts, this would be loss making.

Any other deviation not considered explicitly is either a variant of the former cases where an analogous reasoning applies, not profitable as no consumer is served, or the whole market is served at a loss. □

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