Information sharing in contests*

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Abstract

We study the incentives to share private information ahead of contests, such as markets with promotional competition, procurement contests, or R&D. We consider the cases where firms have (i) independent values and (ii) common values of winning the contest. In both cases, when decisions to share information are made independently, sharing information is strictly dominated. With independent values, an industry-wide agreement to share information can arise in equilibrium. Expected effort is lower with than without information sharing. With common values, an industry-wide agreement to share information never arises in equilibrium. Expected effort is higher with than without information sharing.

Keywords: information sharing, contest, all-pay auction.

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1 Introduction

Competing firms can often commit to share relevant information with their competitors. Exchange of information not only takes place in joint ventures or cartels; one benefit of joining an industry association is better access to industry data. The incentives to share information have been extensively studied in the literature on imperfect competition. Competition in some oligopolistic markets is, however, best described as a contest or an all-pay auction, and the incentives to share information ahead of a contest appear to have not yet been explored. To date, the main focus of the literature has been on the implications of whether firms' decision variables are strategic substitutes or strategic complements. In the all-pay auction, however, these notions do not fit since the best replies may be nonmonotonic, involving either marginal overbidding, or spending zero effort.[1] The aim of this paper is to analyze this case.

The strategic interaction between firms in many markets has the characteristics of a contest. This is particularly true of markets with intense advertising or promotional competition (Schmalensee 1976, 1992), and in R&D races. Lichtenberg (1988) stresses the importance of 'design and technical competitions' for public procurement and points out that these competitions are best understood as contests. Dasgupta (1986) uses the all-pay auction with complete information as a model of R&D races and research tournaments. In a similar structure, Kaplan et al. (2003) analyze firms' innovation activities when potential gains are endogenous. See Konrad (2007) for a survey. This paper examines a popular type of contest, often used as a benchmark in the contest literature: an all-pay auction.[2] The all-pay auction

[1] In our analysis with continuous strategy spaces, formal best responses may not exist due to the open-endedness problem arising with discontinuous payoff functions. When referring to best replies in this context we are thinking of either $\varepsilon-$best replies or best replies in finite approximations of the continuous strategy space.

[2] The all-pay auction captures the notion that, conditional on expenditures, exogenous shocks do not play a significant role in determining a contest's outcome. Contests with

has been studied under a wide range of assumptions concerning the information possessed by competitors (Weber 1985; Hillman and Riley 1989; Baye et al. 1993, 1996; Amann and Leininger 1996; Krishna and Morgan 1997, Morath and Münster 2008a). Here we build on these results to investigate information sharing.

The literature on information sharing in oligopoly is extensive and we do not attempt to survey it here. Early contributions include Ponssard (1979), Novshek and Sonnenschein (1982), Vives (1984), and Gal-Or (1985). Raith (1996) presents a fairly general model that encompasses many of the known results; Vives (1999, Chapter 8) contains an overview. Most closely related to our paper are studies of information disclosure in R&D competition, going back to Bhattacharya and Ritter (1983). Gill (2008) and Jansen (2008) are recent contributions, and also include overviews of the literature. Our contribution to this literature is to focus on the two-player all-pay auction structure and on the incentives to reveal one's value of winning the contest to one's competitor. We also study the social efficiency of the decision to share information and find conditions under which a legal prohibition of information sharing or, alternatively, a requirement to share information, is welfare improving.

The paper is organized as follows. Section 2 sets out the model. Firms receive private information about the value they derive from winning the contest. If private information pertains to some firm-specific characteristic such as the cost structure, a model with private values may be appropriate; Section 3 considers this case. On the other hand, if the information is about some circumstances which are common to the firms such as demand conditions, we have common values; we study this case in Section 4. We summarize our findings in Section 5.

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exogenous noise, such as the Tullock (1980) or Lazear & Rosen (1981) models, generally require sufficient noise to ensure pure-strategy equilibria in the complete information game. Alcalde and Dahm (2008) and Che and Gale (2000) have recently shown that contests with "small" amounts of exogenous noise share many of the same properties as all-pay auctions.

2 The model

There are two firms $i = 1, 2$. At stage 1, each firm decides whether or not it will share information. In the literature, there are two approaches concerning how to model these decisions. We will describe each in detail below. Between stage 1 and stage 2, each firm receives a private signal s_i about its value v_i of winning the contest. We assume that the signals s_1 and s_2 are independent draws from a cumulative distribution function F with support $[s_l, s_h]$, $0 \leq s_l < s_h$. We assume that F is continuously differentiable. In the case of private values analyzed in Section 3 below, each firm's value of winning is equal to its signal. In Section 4, we investigate a common values environment in which each firm's value of winning equals a nonnegative continuously differentiable, strictly increasing, and symmetric function of the two signals. In stage 2, firms choose their outlays or efforts $x_i \in \mathbb{R}_+$. The higher effort wins; ties are broken randomly. Thus the probability that firm i wins is given by

$$p_i = \begin{cases} 0, & \text{if } x_i < x_j, \\ \frac{1}{2}, & \text{if } x_i = x_j, \\ 1, & \text{if } x_i > x_j. \end{cases}$$

The expected profit of firm i is equal to $p_i v_i - x_i$.

As noted above, there are two main approaches to information sharing in the literature: the decision whether or not to share information can be either unilateral, or a bilateral agreement. In the first approach, decisions to share information are taken simultaneously and independently. These decisions are binding commitments. Hence, if firm i has decided to share information, firm $j \neq i$ also learns the signal s_i before the efforts are chosen; otherwise, s_i is private information to firm i. In an alternative approach, the first stage decisions on information sharing are treated as an industry-wide agreement, where a firm shares its information before stage 2 if and only if the other firm does so as well. Here, in stage 1 both firms simultaneously indicate whether they would like an industry-wide agreement on information sharing. If both indicate that they want it, then all information is shared. If at least one firm indicates that it does not want to share, then no firm's information is shared. Note that in both approaches, 'sharing information' can be thought of as 'providing hard evidence that fully reveals the realization of one's signal'.[3]

Finally, we assume that social welfare depends on the firms' expected profits. Moreover, the firms' efforts may be socially desirable in themselves. For example, if x_i is innovative effort, it may provide positive spillovers to the rest of the economy. Thus, we assume that

$$W = \sum_{i=1,2} \left(p_i v_i - x_i + \kappa x_i \right).$$

Here, $\kappa \geq 0$ is a parameter that expresses the social value of the efforts not directly captured by the firms in the industry. Throughout, we analyze whether equilibrium information sharing is socially efficient. In particular, we study whether prohibiting information sharing, or forcing the firms to share information, increases welfare.

3 Private values

In this section, we assume that each firm's value of winning the contest coincides with its signal, $v_i = s_i$ for $i = 1, 2$. That is, each firm is privately informed about the value it derives from winning, and this value is independent of the other firm's value.

[3] It will become clear that, in both approaches, our findings are robust to a sequential timing of the decisions on information sharing where firm 1 decides first, and firm 2 decides after having observed the decision of firm 1.

3.1 Industry-wide agreements

We begin the analysis with the simpler case of industry-wide agreements. Here we only have to consider the symmetric situations in which either both firms share their information, or both keep their information secret. The corresponding subgame equilibria are well known.

Both firms share information If both firms share their information, the resulting subgames have complete information, and the all-pay auction has a unique equilibrium in mixed strategies (Baye, Kovenock, de Vries 1996).[4] Without loss of generality, let $s_1 \leq s_2$, which corresponds to $v_1 \leq v_2$. Firms play the following mixed strategies:

$$\begin{aligned} B_1(x) &= \frac{s_2 - s_1}{s_2} + \frac{x}{s_2} \text{ for } x \in [0, s_1], \\ B_2(x) &= \frac{x}{s_1} \text{ for } x \in [0, s_1]. \end{aligned}$$

To see this is an equilibrium, note that the expected profit of firm 1 from an effort $x_1 \in [0, s_1]$ equals

$$\frac{x_1}{s_1} s_1 - x_1 = 0.$$

A higher effort leads to a higher probability of winning, which is just outweighed by the increased costs; thus firm 1 is indifferent between all these effort levels. Moreover, choosing an effort $x_1 > s_1$ is suboptimal, since it leads to negative expected profits. Similarly, firm 2 is indifferent between all $x_2 \in (0, s_1]$, since they all give the same expected profit:

$$\left(\frac{s_2 - s_1}{s_2} + \frac{x_2}{s_2} \right) s_2 - x_2 = s_2 - s_1.$$

[4] There is a trivial technical complication if $s_l = 0$. In the event that $s_i = 0 < s_j$, firm i has a strictly dominant strategy to choose zero effort, hence firm j would like to choose the smallest strictly positive effort, which does not exist in a continuous strategy space. To fix this, we assume that in case $s_i = 0 < s_j$ and $x_i = x_j = 0$, firm j wins with probability one. A similar comment applies to Lemma 1 below.

To summarize, the expected profit of a firm i equals $\max\{s_i - s_j, 0\}$. Since firms decide on information sharing before they know their own value, this decision is based on the *ex ante expected profit,* i.e. the expectation at the beginning of stage 1. Firm i's ex ante expected profit from an agreement to share information is equal to

$$\int_{s_l}^{s_h}\int_{s_l}^{s_i}(s_i - s_j)\,dF(s_j)\,dF(s_i)\,. \tag{1}$$

No firm shares information If no firm shares information, then stage 2 is characterized by two-sided incomplete information. The equilibrium is in increasing strategies: a firm that receives a signal s chooses effort

$$\xi(s) = \int_{s_l}^{s} tdF(t)\,. \tag{2}$$

To see that this is an equilibrium, consider the expected profit of firm i, given that firm j follows this strategy.[5] Suppose firm i chooses an effort of $x_i \in \left[0, \int_{s_l}^{s_h} tdF(t)\right]$. Equivalently, firm i bids according to ξ but as if it had received a signal z such that $x_i = \xi(z)$. The expected profit of firm i equals

$$\begin{aligned}\Pr(\xi(s_j) < \xi(z))\,s_i - \xi(z) &= F(z)\,s_i - \int_{s_l}^{z} tdF(t) \\ &= \int_{s_l}^{z}(s_i - t)\,dF(t)\,.\end{aligned}$$

Since the integrand is positive if and only if s_i is greater than t, the optimal choice is $z = s_i$, and hence $x_i = \xi(s_i)$ as in (2).

Interim expected profits, conditional on s_i, equal

$$F(s_i)\,s_i - \int_{s_l}^{s_i} tdF(t) = \int_{s_l}^{s_i}(s_i - s_j)\,dF(s_j)$$

[5] The equilibrium was first derived in Weber (1985). Uniqueness follows from Amann and Leininger (1996).

and ex ante expected profits are

$$\int_{s_l}^{s_h} \int_{s_l}^{s_i} (s_i - s_j)\, dF(s_j)\, dF(s_i)\,. \tag{3}$$

Comparing (1) and (3) shows that expected profit when both firms share information is equal to expected profit when no firm shares information.[6]

Proposition 1 *Consider information sharing as an industry-wide agreement, where a firm shares information if and only if the rival shares information. With private values, both information sharing and no information sharing can arise in equilibrium. Firms' profits are identical in the two cases. Expected efforts are higher without information sharing.*

Proof. The equivalence of firms' profits in the two cases has been shown above. Therefore, if firm i proposes to share information, firm j is indifferent whether or not to agree. Thus both cases can arise in equilibrium. It remains to consider the implications for expected efforts. Suppose no firm shares information and denote expected effort of firm i by $E\left(x_i^{NN}\right)$. Then

$$E\left(x_i^{NN}\right) = \int_{s_l}^{s_h} \xi(s_i)\, dF(s_i) = \int_{s_l}^{s_h} \int_{s_l}^{s_i} t dF(t)\, dF(s_i)\,.$$

Now suppose both firms share information. Conditional on s_1 and s_2, expected effort of firm i is equal to $s_j/2$ if $s_i > s_j$, and equal to $s_i^2/(2s_j)$ if $s_i < s_j$. Therefore, the ex ante expected effort of firm i equals

$$E\left(x_i^{SS}\right) = \int_{s_l}^{s_h} \left(\int_{s_l}^{s_i} \frac{s_j}{2} dF(s_j) + \int_{s_i}^{s_h} \frac{s_i^2}{2s_j} dF(s_j) \right) dF(s_i)\,.$$

[6] This payoff equivalence has first been shown in Morath and Münster (2008b), who use a different method of proof than the one given here. The result holds not only for the all-pay auction but also for the first-price auction and the second-price auction. It is also more general than presented here in that it does not rely on there being only two players. Moreover, the result also holds for discrete probability distributions.

The difference is

$$\begin{aligned}
E\left(x_i^{NN}\right)-E\left(x_i^{SS}\right) &= \int_{s_l}^{s_h}\int_{s_l}^{s_i}\frac{s_j}{2}dF\left(s_j\right)dF\left(s_i\right)-\int_{s_l}^{s_h}\int_{s_i}^{s_h}\frac{s_i^2}{2s_j}dF\left(s_j\right)dF\left(s_i\right) \\
&= \int_{s_l}^{s_h}\int_{s_j}^{s_h}\frac{s_j}{2}dF\left(s_i\right)dF\left(s_j\right)-\int_{s_l}^{s_h}\int_{s_i}^{s_h}\frac{s_i^2}{2s_j}dF\left(s_j\right)dF\left(s_i\right),
\end{aligned}$$

where the second equality uses Fubini's theorem. Renaming the variables of integration in the first term by exchanging i and j, we get

$$E\left(x_i^{NN}\right)-E\left(x_i^{SS}\right)=\int_{s_l}^{s_h}\int_{s_i}^{s_h}\frac{s_i}{2}\left(1-\frac{s_i}{s_j}\right)dF\left(s_j\right)dF\left(s_i\right)>0.$$

■

Proposition 1 indicates that an industry-wide agreement to share information may occur in equilibrium, but depresses effort. In the context of a procurement contest or an R&D race, for example, where it might be expected that effort has positive spillover effects, banning industry-wide information sharing would result in a Pareto improvement. Since profits of the firms are unchanged, but the efforts are higher, when $\kappa>0$, welfare is higher. The amount by which the efforts increase is exactly equal to the gain in allocative efficiency: without information sharing, the firm with the higher value wins the contest with probability one, whereas with information sharing the equilibrium is in mixed strategies and thus the firm with the lower value sometimes wins.

3.2 Independent commitments to share information

In this section, we turn to the two stage model, where firms independently decide whether or not to commit to share information. Here, if exactly one firm shares its information, an asymmetric situation arises at the contest stage: the signal, and hence the value, of one firm is common knowledge, while the value of the other firm is its private information.

Suppose, without loss of generality, that firm 1 has committed to share information, whereas 2 has committed not to share information. The equilibrium will then exhibit a mixture of the properties of the equilibria in the two symmetric cases discussed above. Firm 1, whose value is common knowledge, will randomize continuously according to a cumulative distribution function which we denote by B_1. Firm 2, on the other hand, will choose effort as an increasing function of its privately known signal. Firm 1 may choose zero effort with a positive probability, which we denote by $B_1(0) \in [0,1)$. Similarly, there may be a signal s' such that firm 2 chooses zero effort for all signals $s_2 \leq s'$. Hence $F(s') \in [0,1)$ is the ex ante probability that firm 2 chooses an effort of zero.

Lemma 1 *(Morath and Münster 2008a) Suppose that only firm* 1 *shares its private information. In the unique equilibrium of stage* 2*, firm* 2 *plays the following pure strategy:*

$$\xi_2(s_2) = \begin{cases} 0 & \text{for} \quad s_2 \in [s_l, s') \\ F(s_2)\, s_1 - F(s')\, s_1 & \text{for} \quad s_2 \in [s', s_h] \end{cases} . \tag{4}$$

Firm 1 *randomizes according to*

$$B_1(x_1) = \int_0^{x_1} \frac{1}{\xi_2^{-1}(z)} dz + B_1(0) \quad \text{for} \quad x_1 \in [0, (1 - F(s'))\, s_1] . \tag{5}$$

$B_1(0)$ *and* s' *are uniquely defined by* $\min\{B_1(0), F(s')\} = 0$ *and* $B_1(\xi_2(s_h)) = 1$.

Proof. Here we only show that this is an equilibrium; for the proof of uniqueness see Morath and Münster (2008a). Consider firm 1 and suppose that firm 2 follows the strategy in (4). Firm 1's profit from an effort $x_1 \in (0, (1 - F(s'))\, s_1]$ equals

$$\Pr(\xi_2(s_2) < x_1)\, s_1 - x_1 = F\left(F^{-1}\left(\frac{x_1 + F(s')\, s_1}{s_1}\right)\right) s_1 - x_1 = F(s')\, s_1.$$

Thus firm 1 is indifferent between all these efforts. Higher efforts are clearly suboptimal, since they might be lowered without decreasing the chances to win. Whenever $s' > s_l$, an effort of zero is also suboptimal, since it involves the risk of losing the contest even in case that firm 2 chooses zero effort. When $s' = s_l$, any $x_1 \in [0, s_1]$ gives a profit of zero and thus firm 1 is indifferent between these efforts.

Now consider firm 2 and suppose that firm 1 follows (5). The profit of firm 2 from an effort $x_2 \in (0, (1 - F(s')) s_1]$ equals

$$\left(\int_0^{x_2} \frac{1}{\xi_2^{-1}(z)} dz + B_1(0) \right) s_2 - x_2.$$

Since ξ_2^{-1} is strictly increasing, the profit of firm 2 is strictly concave in x_2. The maximum is thus unique and described by the first order condition

$$\begin{aligned} \frac{1}{F^{-1}\left(\frac{x_2 + F(s') s_1}{s_1}\right)} s_2 - 1 &\leq 0 \\ x_2 &\geq 0, \end{aligned}$$

together with the complementary slackness condition. If $s' < s_2$, we have an interior solution with

$$x_2 = F(s_2) s_1 - F(s') s_1.$$

Otherwise, an effort of zero is optimal.

It remains to show that $B_1(0)$ and s' are uniquely determined. Note first that firm 1 won't choose an effort that is higher than the highest possible effort of firm 2, and thus $B_1(\xi_2(s_h)) = 1$. With the substitution $\xi_2^{-1}(z) = s$, the boundary condition $B_1(\xi_2(s_h)) = 1$ can be written as

$$\int_{s'}^{s_h} \frac{s_1}{s} dF(s) + B_1(0) = 1. \tag{6}$$

The first term is continuous and strictly decreasing in s'; moreover, it would vanish is s' were equal to s_h. It follows that $B_1(\xi_2(s_h)) = 1$ has a unique solution that fulfills $\min\{B_1(0), F(s')\} = 0$. ■

As in the case where both firms share information, the distribution of efforts of firm 2, considered from the point of view of firm 1, is a uniform distribution, with possibly a mass point at zero. Moreover, the slope is just $1/s_1$. For firm 1, a higher effort leads to a greater chance of winning, which is just outweighed by the higher cost. Thus firm 1 is indifferent between the efforts it randomizes over.

It is straightforward to see that, in equilibrium, at least one of the mass points $B_1(0)$ and $F(s')$ must be zero. Suppose to the contrary that $B_1(0) > 0$ and $F(s') > 0$. Then firm 1 chooses an effort of zero with strictly positive probability. But choosing a sufficiently small but strictly positive effort ε gives a higher profit: the probability of winning increases discretely at an arbitrarily small cost, contradicting equilibrium. Thus, at least one of the mass points is zero. Whether firm 1 or firm 2 has a mass point at zero depends, in general, both on the distribution function F and on the realization of the signal s_1.

For future reference, note that (6) together with $\min\{B_1(0), F(s')\} = 0$ implies $s' < s_1$ for all $s_1 > s_l$. To understand the economics behind this, suppose to the contrary that $s' \geq s_1$. Then firm 2 has zero profit for any signal $s_2 \leq s_1$. Firm 1 has a profit of $F(s')\,s_1$. Therefore, the effort of firm 1 can be no greater than $(1 - F(s'))\,s_1$. But this implies that, whenever $s_2 > (1 - F(s'))\,s_1$, firm 2 can guarantee itself a strictly positive profit by bidding slightly more than $(1 - F(s'))\,s_1$, contradiction. Hence $s' < s_1$.

In contrast to the case of industry wide agreements, information sharing cannot arise in equilibrium when decisions on information sharing are taken independently.

Proposition 2 *Consider independent decisions on information sharing. With private values, sharing information is strictly dominated.*

Proof. We show that, for any $s_i > s_l$, sharing information is strictly worse than not sharing if the rival firm shares information (step 1), and similarly if the rival firm does not share information (step 2). Therefore, the ex ante profit of firm i is strictly higher if i does not share information.

Step 1. Suppose that firm j shares its information. We first argue that for all realizations of s_i and s_j, the profit of firm i is *weakly* lower if it shares information than if it does not. If firm i shares its information, given s_i and s_j its profit equals $\max\{0, s_i - s_j\}$. Suppose that firm i does not share information. Any effort $x_j > s_j$ is strictly dominated for firm j. Moreover, firm j chooses $x_j = s_j$ with probability zero. Therefore, by choosing $x_i = 0$ if $s_i \leq s_j$, and $x_i = s_j$ if $s_i > s_j$, firm i can guarantee itself a profit of $\max\{0, s_i - s_j\}$, and its equilibrium profit cannot be lower.

It remains to show that firm i's interim expected profit is *strictly* higher if it does not share information. Suppose i does not share information. As argued above, for any $s_j > s_l$ the corresponding critical signal s' is strictly smaller than s_j.[7] Thus, for any $s_i > s_l$, if s_j happens to be equal to s_i, the corresponding s' is strictly smaller than s_i; hence firm i chooses strictly positive effort and has a strictly positive profit. By continuity, this is still true if $s_j \in (s_i, s_i + \delta)$ for some $\delta > 0$. On the other hand, if firm i shares information, it gets zero profit whenever $s_j \geq s_i$. It follows that whenever $s_j \in [s_i, s_i + \delta)$, firm i 's profit is strictly higher if firm i does not share information. Together with the last paragraph, this implies that firm i's interim expected profit is strictly higher if it does not share information.

Step 2. Now suppose that firm j does not share information. We focus on an interim perspective and show that given any signal $s_i > s_l$, the profit of firm i is strictly higher if it does not share information. (If $s_i = s_l$, the profit of firm i is zero whether or not it shares information.)

[7] Here s' is defined in Lemma 1, replacing subscript 1 by j and subscript 2 by i. Remember that here firm j shares information, whereas in Lemma 1 firm 1 shares information. Similarly, the firm that does not share is firm i here and firm 2 in Lemma 1.

If firm i with signal s_i does not share information, its profit is

$$\int_{s_l}^{s_i} (s_i - s_j)\, dF(s_j). \tag{7}$$

If firm i shares information, by Lemma 1 (replacing subscript 1 by i and subscript 2 by j) firm i gets a profit of

$$\int_{s_l}^{s'} s_i dF(s_j) \tag{8}$$

which is equal to the probability that j has a signal lower than s' and thus chooses zero effort, multiplied by i's value s_i. If $s' = s_l$, we are done since the profit of i equals zero if it shares its information, while the profit of i is strictly positive if i does not share its information. Therefore, suppose in the following that $s' > s_l$. Then the critical signal s' is determined such that

$$\int_{s'}^{s_h} \frac{s_i}{s} dF(s) = 1. \tag{9}$$

As argued above, $s' < s_i$. For notational convenience, let Δ denote the difference between the profits (7) and (8):

$$\Delta := \int_{s_l}^{s_i} (s_i - s_j)\, dF(s_j) - \int_{s_l}^{s'} s_i dF(s_j).$$

Straightforward manipulations show that

$$\begin{aligned}\Delta \;=\; & \int_{s_l}^{s'} (s' - s_j)\, dF(s_j) + \int_{s'}^{s_i} \left(s_i - s_j + s' - s'\frac{s_i}{s_j}\right) dF(s_j) \\ & - \int_{s_l}^{s_i} s' dF(s_j) + \int_{s'}^{s_i} s'\frac{s_i}{s_j} dF(s_j).\end{aligned}$$

Rewriting the last term and using (9) gives

$$\begin{aligned}\int_{s'}^{s_i} s' \frac{s_i}{s_j} dF(s_j) &= s' \left(\int_{s'}^{s_h} \frac{s_i}{s_j} dF(s_j) - \int_{s_i}^{s_h} \frac{s_i}{s_j} dF(s_j) \right) \\ &= s' \left(1 - \int_{s_i}^{s_h} \frac{s_i}{s_j} dF(s_j) \right).\end{aligned}$$

Thus

$$\begin{aligned}\Delta &= \int_{s_l}^{s'} (s' - s_j) dF(s_j) + \int_{s'}^{s_i} (s_j - s') \frac{s_i - s_j}{s_j} dF(s_j) \\ &\quad + s' \left(1 - \int_{s_l}^{s_i} dF(s_j) - \int_{s_i}^{s_h} \frac{s_i}{s_j} dF(s_j) \right).\end{aligned}$$

The first and the second term are both strictly positive since $s_l < s' < s_i$, and the third term is nonnegative. Thus $\Delta > 0$. ■

If an industry-wide agreement on information sharing is not possible, there is a unique equilibrium where firms do not share their information. Independently of the rival's decision, they prefer to keep their own information secret.[8] In the asymmetric situation where only firm 1 shares its signal, the interim profit of firm 1 is increasing in s_1. Thus one might conjecture that a firm with a high signal may have an incentive to share its information. However, the interim profit in the case where no firm shares its information is also increasing in one's own type. In fact, if F is the uniform distribution on the unit interval, Δ is monotonically increasing in s_1: the higher one's signal, the higher is the benefit from hiding keeping it hidden. In general,

[8] Proposition 2 goes through if the signals are drawn from a discrete distribution function and the number of possible signals is strictly larger than 2. The case of a binary distribution is an exteme case in the sense that, given that firm j does not share information, i's profit are the same whether or not i shares information, and information sharing is only weakly dominated (for the equilibrium see Konrad 2007). However, if j does share information, i is strictly better off if it keeps its information secret. Thus, there are three equilibria: one equilibrium where both firms do not share information, and two equilibria where exactly one firm shares information and the other does not share.

however, Δ is not monotone.[9]

The result that information sharing is dominated holds not only if this decision has to be taken before the firms receive their signals. Since the proof of Proposition 2 considers an interim perspective, firm i has a dominant strategy not to share information for each possible signal i can obtain. Hence, if the decisions on information sharing were taken only after having received the signal, Proposition 2 would in a sense still hold. Equivalently, firms do not have an incentive to reconsider their decision and, for instance, inform the rival in case they have a high value of winning the contest.

To be more precise, consider the following game:

1. Firms privately receive their signals.

2. Firms decide independently whether or not to share their signals. As above, sharing information means providing hard evidence that fully reveals the realization of one's signal.

3. The contest takes place.

We argue that this game has a perfect Bayesian equilibrium where no firm ever shares its information. In this equilibrium, the beliefs of a firm about the signal of its rival are as follows. If firm i does not reveal its signal, firm j believes that s_i is distributed according to the ex ante distribution F. If firm i deviates and reveals its signal, the belief of j is pinned down by the hard evidence, that is, firm j knows s_i.

Now suppose that firm j never reveals its signal. Consider whether firm i wants to reveal its signal on stage 2. This is exactly the comparison we did in the proof of Proposition 2, step 2: for any $s_i > s_l$, firm i is strictly better off if it does not reveal its signal.[10]

[9] To give an example where Δ is not monotone, suppose that signals are distributed according to $F(s) = s^3$ on the unit interval.

[10] The game may have other equilibria. We are exploring this issue in ongoing research.

4 Common values

In the previous section, we assumed that the firms' values v_i are private. In many environments, however, it is reasonable to assume that the values of winning depend on the other firm's signal as well. This section studies common values where

$$v_1(s_1, s_2) = v_2(s_1, s_2) = v(s_1, s_2).$$

We assume that v is nonnegative, continuously differentiable, strictly increasing in s_1 and s_2, and symmetric, i.e. $v(s_1, s_2) = v(s_2, s_1)$ for all (s_1, s_2).

4.1 Industry-wide agreements

Both firms share information Here, at the contest stage, the values of winning v_1 and v_2 are commonly known. As before, under complete information, there is a unique equilibrium in mixed strategies. With $v_1 = v_2 = v$, we have complete rent dissipation, and expected profits are zero.

The sum of the expected efforts of both firms is equal to $v(s_1, s_2)$. Ex ante expected efforts are equal to the expected value of winning

$$\int_{s_l}^{s_h} \int_{s_l}^{s_h} v(s_1, s_2)\, dF(s_2)\, dF(s_1).$$

No firm shares information Here, firm i knows s_i but not s_j. Krishna and Morgan (1997) have shown that there is a symmetric equilibrium in strictly increasing strategies $x_i = \xi(s_i)$ where

$$\xi(s_i) = \int_{s_l}^{s_i} v(t, t)\, dF(t).$$

To see that this is an equilibrium, suppose firm j follows the strategy ξ. If i chooses its effort according to ξ but as if its signal was z, it gets a profit of

$$\int_{s_l}^{z} \left(v\left(s_i, t\right) - v\left(t, t\right)\right) dF\left(t\right).$$

Since v is strictly increasing in its arguments, the integrand is strictly positive for all $t < s_i$ and strictly negative for all $t > s_i$, and it is optimal for i to choose $z = s_i$.[11]

Ex ante expected profits are equal to

$$\int_{s_l}^{s_h} \int_{s_l}^{s_i} \left(v\left(s_i, t\right) - v\left(t, t\right)\right) dF\left(t\right) dF\left(s_i\right) > 0, \tag{10}$$

and hence higher than if both firms share their information.

Proposition 3 *Consider industry-wide agreements to share information about a common value where firm i shares its information if and only if firm j does. Then there will be no information sharing in equilibrium.*

Proof. Not sharing information leads to a positive expected profit given in (10), but sharing information gives zero profit. ■

Contrary to the case of private values of winning, the firms' profits are higher if they do not share their information with their rival, and thus an agreement on industry-wide information sharing can not arise in equilibrium.

4.2 Independent commitments to share information

We now consider information sharing with independent decisions. As above, this necessitates considering the case where only one firm shares its informa-

[11] Due to our assumption that the signals are independent, the condition for existence of the equilibrium given in Krishna and Morgan (1997) is automatically fulfilled. The equilibrium is unique: the uniqueness proof in Amann and Leininger (1996) carries over since a firm's expectation about the signal of the rival does not depend on its own signal.

tion. Again, the equilibrium exhibits properties of each of the two symmetric cases, where either both firms share information or no firm does.

Lemma 2 *Consider the case of a common value $v(s_1, s_2)$. Suppose the signal of firm* 1 *is commonly known, whereas s_2 is private information of firm* 2*. The equilibrium is unique. Firm* 2 *plays a pure strategy*

$$\xi_2(s_2) = \int_{s_l}^{s_2} v(s_1, t)\, dF(t). \tag{11}$$

Firm 1 *randomizes according to*

$$B_1(x_1) = F\left(\xi_2^{-1}(x_1)\right). \tag{12}$$

Proof. First, we show that this is an equilibrium. Consider firm 1 and suppose it chooses an effort $x_1 \in \left[0, \int_{s_l}^{s_h} v(s_1, t)\, dF(t)\right]$. Higher efforts are obviously suboptimal since they can be lowered without changing the probability of winning. Let $z = \xi_2^{-1}(x_1)$. The profit of firm 1 equals

$$\int_{s_l}^{\xi_2^{-1}(x_1)} v(s_1, s_2)\, dF(s_2) - x_1 = \int_{s_l}^{z} v(s_1, s_2)\, dF(s_2) - \xi_2(z) = 0.$$

Therefore, firm 1 is indifferent between all these efforts.

Now consider firm 2. Its profit is

$$B_1(x_2)\, v(s_1, s_2) - x_2 = F\left(\xi_2^{-1}(x_2)\right) v(s_1, s_2) - x_2.$$

Suppose firm 2 chooses effort as if its signal were z. Then it gets

$$F(z)\, v(s_1, s_2) - \xi_2(z) = \int_{s_l}^{z} \left(v(s_1, s_2) - v(s_1, t)\right) dF(t)$$

Since the integrand is strictly positive whenever $t < s_2$, and strictly negative whenever $t > s_2$, the optimal choice is $z = s_2$.

Observe that from the point of view of firm 1, the expected value of winning is $E_{s_2}[v(s_1,s_2)]$, and firm 2's value of winning is distributed on $[v(s_1,s_l), v(s_1,s_h)]$. Thus, uniqueness can be established along the lines in Morath and Münster (2008a) with minor adaptions necessary to account for the common values assumed here. ■

If exactly one firm shares its information, ex ante expected efforts are the same for both firms. In fact, the distribution of the effort of the firm that shares information,

$$\Pr(x_1 \leq z) = F\left(\xi_2^{-1}(z)\right),$$

is the same as the distribution of the efforts of the firm that does not share information:

$$\Pr(x_2 \leq z) = \Pr(\xi_2(s_2) \leq z) = F\left(\xi_2^{-1}(z)\right).$$

Using the equilibrium of the contest in case only one firm shares information, we can derive the incentives for information sharing with independent decisions.

Proposition 4 *Consider the case of common values and independent decisions about information sharing. Regardless of the choice of strategy of its rival, for any $s_j > s_l$, firm j earns an expected profit of zero by sharing information and a strictly positive expected profit by not sharing information. Thus sharing information is strictly dominated.*

Proof. Suppose firm i shares information. If firm j also shares information, it earns zero expected profits, as has been shown in the proof of Lemma 2. If j does not share information, i randomizes its contest effort as in (12) and j chooses an effort as in (11). Hence, given s_i, firm j's expected profit

$$\int_{s_l}^{s_j} (v(s_i,s_j) - v(s_i,t))\, dF(t)$$

is strictly positive for any $s_j > s_l$. A fortiori, ex ante expected profit is strictly positive, and the best reply is not to share information.

Suppose that firm i does not share information. If firm j shares information, j randomizes its contest effort as in (12). Its expected profit is zero. If it does not share information, it gets a strictly positive profit by (10). Thus j strictly prefers not to share information. ■

Note that the proof of Proposition 4 also establishes that sharing information is dominated from an interim perspective. Hence, if the decisions on information sharing were taken only after having received the signal, there is still no incentive to share information, just as in the case of private values (see the discussion following Proposition 2).

We now compare expected profits and efforts across the different information structures. Due to the common value, the winner of the contest has the same value as the loser, and there cannot be an allocative inefficiency. Ex post, the sum of profits is

$$\sum_{i=1}^{2} \left(p_i v\left(s_1, s_2\right) - x_i\right) = v\left(s_1, s_2\right) - \sum_{i=1}^{2} x_i,$$

and the sum of profits and efforts is always $v\left(s_1, s_2\right)$. Consequently, the sum of expected profits and expected efforts has to be the same in all information structures. Therefore, the ranking of expected efforts is just the opposite of the ranking of expected profits.

If both firms share information, expected profits are zero; otherwise the sum of expected profits is strictly positive. Therefore, expected efforts are highest if both firms share information. The comparison between the remaining cases, however, depends on the function v.[12]

Remark 1 *Suppose that firm j does not share information. Whenever v is supermodular, j's profit is higher if i shares information than if i does not*

[12] The proof is in the appendix. Kim (2008) obtains a similar result for the first price auction with common values.

share. If v is modular, j's profit is the same in both cases. If v is submodular, j's profit is lower if i shares information than if no firm shares.

This implies that, whenever v is modular or submodular, (1) the sum of expected profits is lower, and (2) the sum of expected efforts is higher if exactly one firm shares information than if no firm shares information. If v is supermodular, expected efforts may be higher if no firm shares information.

In the common values environment, firms prefer to keep their information secret, whether or not an industry wide agreement on information sharing is possible. The ranking of the expected efforts show that they are highest if both firms share their information with their rival. Therefore, contrary to the case of private values, agreements on information sharing about a common value can be desirable from a welfare point of view if the investments in the contest are socially valuable. In fact, if the value of the efforts to society is higher than their cost to the firms (i.e. $\kappa > 1$), then a legal requirement to share information is welfare improving.

5 Conclusion

This paper considered incentives to share information ahead of competition in markets that are described by an all-pay auction. We first considered private values. We found that, with industry-wide agreements, firms are indifferent between sharing and not sharing information. Thus, an industry-wide agreement on information sharing may emerge in equilibrium. Aggregate efforts, however, are higher without information sharing. In such a situation, a ban on industry-wide agreements on information sharing is a Pareto improvement whenever effort generates positive spillovers outside of the contest as, for example,may be the case in a procurement contest or a R&D race. With independent decisions whether or not to share information, however, sharing information is strictly dominated.

Second, we considered a common values framework, where the true value of winning is a continuously differentiable, strictly increasing, and symmetric function of the firms' private signals. Here, efforts are highest if both firms share information. Information sharing will not arise in equilibrium - firms are strictly better of if they do not share information, no matter whether they decide individually, where information sharing is a strictly dominated strategy, or consider an industry-wide agreement. When the effort generates positive spillovers outside of the contest, information sharing may be inefficiently low. Thus, whereas there may be too much information sharing with private values, there may be too little information sharing with common values.

A Appendix

A.1 Proof of Remark 1

We compare the interim profit of firm 2 in the asymmetric setting where only firm 1 shares information, which we denote by $\pi_2^{SN}(s_2)$ (the first superscript indicates firm 1 does share information, the second says that firm 2 does not shares information), with the interim profit of firm 2 if no firm shares, denoted by $\pi_2^{NN}(s_2)$. We have

$$\begin{aligned}
\pi_2^{SN}(s_2) &= \int_{s_l}^{s_h} \int_{s_l}^{s_2} \left(v(s_1, s_2) - v(s_1, t)\right) dF(t)\, dF(s_1) \\
\pi_2^{NN}(s_2) &= \int_{s_l}^{s_2} \left(v(s_2, t) - v(t, t)\right) dF(t) \\
&= \int_{s_l}^{s_2} \left(v(t, s_2) - v(t, t)\right) dF(t)
\end{aligned}$$

where the last line uses the symmetry of v. If $s_2 = s_l$, firm 2 chooses an effort of zero in both cases, and

$$\pi_2^{SN}(s_l) = \pi_2^{NN}(s_l) = 0.$$

Moreover,

$$\begin{aligned}
\frac{\partial}{\partial s_2}\pi_2^{SN}(s_2) &= \int_{s_l}^{s_h}\int_{s_l}^{s_2} \frac{\partial v(s_1,s_2)}{\partial s_2} dF(t)\, dF(s_1) \\
&= F(s_2)\int_{s_l}^{s_h} \frac{\partial v(s_1,s_2)}{\partial s_2} dF(s_1) \\
&= F(s_2)\, E_{s_1}\left(\frac{\partial v(s_1,s_2)}{\partial s_2}\right),
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial}{\partial s_2}\pi_2^{NN}(s_2) &= \int_{s_l}^{s_2} \frac{\partial v(t,s_2)}{\partial s_2} dF(t) \\
&= F(s_2)\, E_{s_1}\left(\frac{\partial v(s_1,s_2)}{\partial s_2} | s_1 \le s_2\right).
\end{aligned}$$

Hence

$$\begin{aligned}
&\frac{\partial}{\partial s_2}\pi_2^{SN}(s_2) - \frac{\partial}{\partial s_2}\pi_2^{NN}(s_2) \\
= &\ F(s_2)\left(E_{s_1}\left(\frac{\partial v(s_1,s_2)}{\partial s_2}\right) - E_{s_1}\left(\frac{\partial v(s_1,s_2)}{\partial s_2} | s_1 \le s_2\right)\right)
\end{aligned}$$

which is strictly positive if $\frac{\partial v(s_1,s_2)}{\partial s_2}$ increases in s_1. It follows that $\pi_2^{SN}(s_2) > \pi_2^{NN}(s_2)$ for all $s_2 > s_l$ whenever $v(\cdot)$ is supermodular. Similarly, $\pi_2^{SN}(s_2) < \pi_2^{NN}(s_2)$ for all $s_2 > s_l$ if $v(\cdot)$ is submodular, and $\pi_2^{SN}(s_2) = \pi_2^{NN}(s_2)$ for all $s_2 > s_l$ if $v(\cdot)$ is modular.

Since firm 1's profit is zero if it shares information, the sum of the profits is strictly lower in (S,N) than in (N,N) whenever v is modular or submodular. Correspondingly, the sum of expected efforts is higher in (S,N) than in

(N, N) if v is (sub)modular. If v is supermodular, the sum of expected profits may be lower, and the sum of expected efforts may be higher in (N, N) than in (S, N).

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