The Extensive Margin of Exporting Goods: A Firm-level Analysis*

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Abstract

We examine three-dimensional panel data for Brazilian and Chilean exporters, their products and destinations. The data show that (i) the distribution of the exporters' number of goods (the exporter scope) is robust within destinations and approximately Pareto, (ii) exporter scope is positively associated with average sales per good within destinations but not across, and (iii) exports are concentrated in few top-selling goods by firm. We present a heterogeneous-firm model with product choice that implies these regularities and retains key predictions of previous trade models. At the country level, the model generates bilateral trade flows consistent with gravity-equation evidence. Across firms, the model explains regularities with convex product-entry costs that increase more than proportionally in scope on the distribution side. Within firms, variable product-entry costs, convex in scope, lead firms to concentrate sales in few goods.

Keywords: International trade; heterogeneous firms; multi-product firms; firm and product panel data; Brazil; Chile

JEL Classification: F12, L11, F14

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1 Introduction

Most exports are shipments by multiproduct firms.[1] We examine the extensive margin of introducing goods at export destinations to learn about determinants of trade flows and the nature of market-entry costs. We present a heterogeneous-firm model with product choice, and consistent empirical evidence, in which distribution-side costs determine exporter behavior at the extensive margin of adding goods. These distribution-side entry costs are repeated destination by destination and offer the key explanation for empirically observed regularities in exporter behavior.

We use three-dimensional panel data for Brazilian exporters, their destination markets and their products, and confirm the regularities with Chilean data.[2] We decompose total exports into the common extensive margin for the market entry of firms and sales per firm. Our data allow us to further decompose sales per firm into the extensive margin of product-entry with goods and the remaining intensive margin of sales per good. We focus our investigation on the novel extensive margin of product-entry with additional goods.

Three important regularities emerge from our data. First, the probability distribution of the exported number of goods per destination (the *exporter scope* at the destination) is remarkably robust across destination markets and resembles a Pareto distribution in the upper tail when firms are ranked by total sales. Second, the firms' average sales per good at the intensive margin (their *average product scale*) strictly increases with exporter scope worldwide, and destination by destination. We observe these two facts in the cross section of firms. Third, within firms, sales are concentrated in a few top goods by firm and destination.

We explain these regularities with a heterogenous-firm model where firms draw their productivity from a Pareto distribution, learn the ordering of their goods by competency and make choices at the three margins: entry by destination, exporter scope by destination, and product scale by good and destination. On the distribution side, firms incur variable product-entry costs—similar to Eckel and Neary (2006), Nocke and Yeaple (2006), Melitz and Ottaviano (2008) and Bernard, Redding and Schott

[1]Bernard, Jensen and Schott (2009) document for U.S. trade data in the year 2000, for instance, that firms that export more than five products at the HS 10-digit level make up 30 percent of exporting firms but account for 97 percent of all exports. In our Brazilian exporter data for 2000, 25 percent of all manufacturing exporters ship more than ten products at the HS 6-digit level and account for 75 percent of total exports.

[2]Evidence on Brazil is arguably informative for our understanding of a typical country's world export behavior. While Brazil ranks among the top 30 exporting countries in the world, its exports per capita are close to the world median. World trade flow (WTF) data for the year 2000, the final WTF year, show Brazil's total exports at the 88th percentile worldwide (top 27th out of 205). In terms of exports per capita, Brazil ranks at the 48th percentile (top 100th out of 192). Exporter behavior in Brazil is nevertheless strikingly similar to that in leading export countries such as France (Eaton, Kortum and Kramarz 2004) and the United States (Bernard, Redding and Schott 2007). Chilean data from (Álvarez, Faruq and López 2007) confirm the patterns.

(2008).[3] In addition, firms incur fixed market-entry costs, which vary by exporter scope at the destination. Nested consumer preferences over individual products and firms' product mixes give rise to monopolistic competition for a continuum of firms.[4]

The model is a tractable extension of the Melitz (2003) model to the multi-product setting and explains trade data at three levels. First, at the country level, the model generates a gravity equation for bilateral trade flows. Similar to Eaton and Kortum (2002), a single distribution parameter pins down the elasticity of bilateral trade with respect to trade costs. Second, in the cross section of firms, the model gives rise to Pareto distributed exporter scopes and average product scales. In the upper tail of the total sales distribution, the model resembles the Chaney (2008) version of Melitz (2003) and the Pareto shape parameter plays a similar role as the shape parameter in Eaton, Kortum and Kramarz (2008). Scope and scale are positively associated if and only if total entry costs increase more than proportionally in exporter scope, that is if and only if there are dis-economies of scope in total entry costs. The intuition is that a firm equates the marginal profit from introducing an additional good with the total additional costs of product entry. When total entry costs increase more than proportionally in exporter scope, only more productive firms choose wider exporter scope because only they command sufficiently profitable average sales per good. Third, exports are concentrated in a few top goods within firms.

We subject the model to several empirical tests. We query whether an exporter's global characteristics completely account for the firm's local performance relative to its source-country competitors and document that idiosyncratic deviations in local performance have no explanatory power for exports beyond a firm's global characteristics. We show that the relationship between a firms' local exports and their cumulative local percentile rank is log-linear, as the definition of the Pareto distribution requires. The goodness of fit in the according regressions for individual destinations is close to one so that deviations from the Pareto distribution are of minor importance for exporter distributions in the upper tail. A decomposition of the overall covariation between exporter scope and average product scale shows that scope and scale are positively associated within every destination market and firm, consistent with repeated local market-entry costs on the distribution side. But across destinations and firms the data show a negative scale-scope association. This decomposition finding is consistent with the interpretation that a firm faces repeated entry costs in each destination market but that firm-level marketing efficiency, in an extension of our model, allows wide-scope

[3] Bernard et al. (2008) offer a stochastic generalization by which firms endogenously adopt products of different competency levels.

[4] As Feenstra and Ma (2007) show in a related framework, a countable number of firms with product-market power would not lead to closed-form solutions. A countable number of firms, however, would allow for zero entry costs because widening exporter scope more than proportionally cannibalizes the sales of infra-marginal goods. In our setup, in contrast, the scope-elasticity of the cannibalization effect is constant and fails to limit exporter scope at a finite level so that we postulate entry costs.

firms to accept low product scales upon export-market entry.

Individual product sales within firms allow us to empirically separate the scope elasticity of variable product-entry costs and the scope-elasticity of fixed market-entry costs. Structural estimates suggest that the scope-elasticity of fixed market-entry costs is negative for Brazilian and Chilean manufacturers. So, manufacturers encounter economies of scope in fixed market-entry costs. Those economies of scope in fixed market-entry costs, however, are more than outweighed by dis-economies of scope in variable product-entry costs so that manufacturers face overall dis-economies of scope on the distribution side. Brazilian commercial intermediaries, in contrast, face dis-economies of scope in both fixed market-entry and variable product-entry costs. Since commercial intermediaries have no production, we view these findings as consistent with the idea that dis-economies of scope are related to the distribution of goods.

The paper has four more sections. Section 2 discusses the three-dimensional panel data on Brazilian and Chilean exporters, their products and destinations, and documents main facts. Section 3 presents our theoretical model. Section 4 turns to empirical analysis and evaluates predictions of the model. Section 5 concludes. Technical details on derivations and data sources are relegated to the Appendix.

2 Data

Our main data source are customs records that form a three-dimensional panel of Brazilian exporters, their respective destination countries, and their export products at the Harmonized-System 6-digit level in the year 2000. We complement these data with a similar panel data set for Chile in 2000. We establish four main stylized facts from tabulating and plotting the raw data, on which we build our theory.

2.1 Data sources

The Brazilian exporter data derive from the universe of customs declarations for merchandize exports in the year 2000, by any firm. The pristine Brazilian *NCM* product codes are 8-digit numbers, of which the first six digits coincide with the first six digits in the Harmonized System. We aggregate these exporter data to the firm, year and Harmonized-System 6-digit level. To assess the empirical regularities for an additional country, we use a three-dimensional panel of Chilean exporters from the universe of customs declarations by Chilean manufacturing firms in 2000 (Álvarez et al. 2007). For Chile, product codes are reported at the Harmonized System 8-digit level and we aggregate the information to the Harmonized-System 6-digit level for comparability. We combine the exporter data with worldwide bilateral trade information from outside sources (see Appendix E).

To relate our data to product-market information for destination countries and their sectors, we map the Harmonized System 6-digit codes to *ISIC revision 2* at the two-digit level and link our data to World Trade Flow (*WTF*) data for the year 2000 (Feenstra, Lipsey, Deng, Ma and Mo 2005).[5] The link between our data and *WTF* also provides us with an estimate of the coverage of Brazil's self-reported exports declarations. In 2000, our *SECEX* data for manufactured merchandize sold by Brazilian firms from any sector, including commercial intermediaries, covers 95.9 percent of Brazilian exports in *WTF*. The small discrepancy might be related to underreported Brazilian exports, which *WTF* may uncover as imports elsewhere, or to valuation differences because of differently reported exchange rate fluctuations and transportation costs.

For much of our analysis, we remove commercial intermediaries from the Brazilian data and keep manufacturing firms who report their direct export shipments. In our later regression analysis for market-access cost estimates, however, we also turn to the sample of commercial intermediaries that ship manufactured goods so we can compare the results to those for manufacturing firms with direct exports. The sample restriction to manufacturing firms and their manufactured products makes our findings most closely comparable to Eaton et al. (2004) and Bernard et al. (2008) but we lose many observations, mainly because of the importance of commercial intermediaries for export processing, partly because of missing sector information, and partly because of manufacturing firms' resales of non-manufactured goods. After restricting the sample to manufactured merchandize exported directly by Brazilian manufacturers, our sample covers 81.7 percent of the *WTF* manufactures exports.

At the firm level, the data exhibit market-presence patterns broadly similar to the French and U.S. firm-destination data. Similar to Eaton et al. (2004), for instance, the elasticity of the number of firms with respect to the number of export destinations is roughly -2.5, just as for French exporters. (We provide a comparison with French gravity regression results from Eaton et al. (2004) in Appendix F.) In contrast to the French and U.S. data, our Brazilian and Chilean data do not cover domestic sales. This restricts our analysis to export-market access. In regression analysis, however, we can completely control for the firms' domestic characteristics with firm-fixed effects in the cross section of firms' destination markets and products.

2.2 Sample characteristics

As Table 1 shows in column 4 (column 5), our Brazilian (Chilean) data include 10,215 (4,099) manufacturing firms with shipments of 3,717 (3,199) manufacturing goods at the 6-digit Harmonized System level to 170 (140) foreign destinations, and a total of 162,570 (37,183) exporter-destination-product observations.[6] Exporters shipping

[5] Our extended *SITC*-to-*ISIC* concordance is available at URL *econ.ucsd.edu/muendler/resource*.

[6] We remove export records with zero value from the Brazilian data, which include shipments of commercial samples but also potential reporting errors, and lose 408 of initially 162,978 exporter-destination-product observations. Our results on exporter scope do not materially change when including or excluding zero-shipment products from the product count. There are no reported shipments

Table 1: Sample Characteristics by Source and Destination

From source s	Brazil Manufacturers				Chile Manf.	Brazil Intm.
to destination d	USA (1)	Argentina (2)	OECD (3)	World (4)	World (5)	World (6)
# of Firms (M)	3,083	4,590	5,041	10,215	4,099	2,627
# of Destinations (N)	1	1	23	170	140	132
# of HS-6 goods (G)	2,144	2,814	2,772	3,535	3,199	2,777
# of Observations	10,775	21,623	36,359	162,570	37,183	35,960
Destination share in Total exp.	.257	.144	.559	1	1	1
Firm shares in Total exports						
Single-prod. firms	.123	.086	.142	.090	.041	.086
Multi-prod. firms' top product	.662	.555	.625	.597	.715	.595
Multi-prod. firms' other prod.	.215	.359	.233	.313	.243	.319
Median Total exports ($T_d(m)$)	.120	.068	.137	.089	.038	.041
Median Exporter scope ($G_d(m)$)	1	2	2	2	2	2
Median Avg. prod. scale ($z_d(m)$)	.068	.031	.070	.037	.014	.012
Mean Total exports ($\bar{t}_d$)	3.170	1.192	4.217	3.720	2.779	1.101
Mean Exporter scope ($\bar{G}_d$)	3.495	4.711	3.933	5.278	5.454	9.426
Mean Avg. prod. scale ($\bar{z}_d$)	.907	.253	1.072	.705	.510	.117

Sources: *SECEX* 2000 for Brazil, manufacturing firms and their manufactured products as well as commercial intermediaries and their manufactured products; Chilean customs data 2000 (Álvarez et al. 2007) for manufacturing firms.
Note: Aggregate regions (world, OECD) treated as single destinations, collapsing product shipments to different countries into single product shipment. Products at the Harmonized-System 6-digit level. Exports in US$ million fob. OECD includes all OECD members in 1990. The U.S. is Brazil's top export destination in 2000, Argentina second to top. Firms' mean product scale (z_d in US$ million fob) is the scope-weighted arithmetic mean of exporters' average product scales.

multiple goods dominate. They ship more than ninety percent of all exports both from Brazil and Chile, and their single top-selling products account for almost sixty percent of all Brazilian exports and more than seventy percent of all Chilean exports.

To analyze export behavior, we decompose a firm ϕ's total exports $t_{d\phi}$ from Brazil or Chile to destination market d into the firm's number of goods sold at d (the exporter scope) $G_{d\phi}$ and the firm's average sales per export good in d (the average product scale) $z_{d\phi} \equiv t_{d\phi}/G_{d\phi}$:

$$t_{d\phi} = \sum_{g=1}^{G_{d\phi}} p_{dg\phi} x_{dg\phi} = G_{d\phi}\, z_{d\phi},$$

where $p_{dg\phi}$ is the price of product g and $x_{dg\phi}$ its export quantity. To calculate summary medians and means of these variables for regional aggregates and the world as a whole in Table 1 (columns 3 to 6), we treat the aggregate as if it were a single destination and collapse all product shipments to different countries into a single product shipment.

with zero value in the Chilean data.

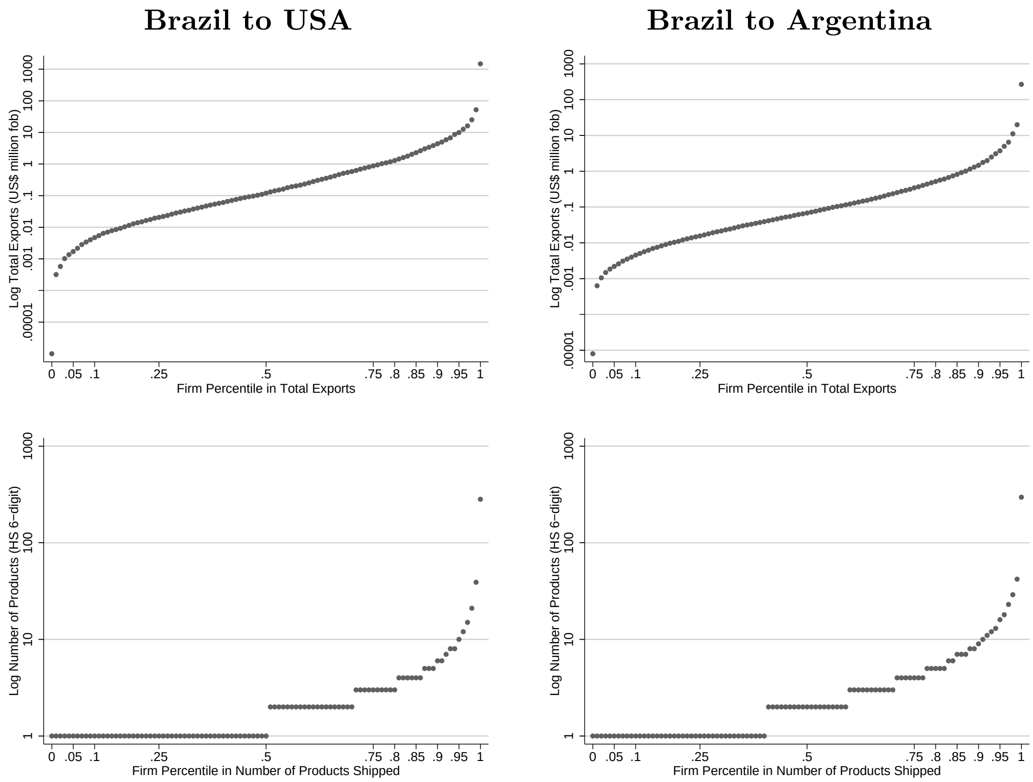
In all subsequent data treatments, in contrast, we will analyze these variables country by country, consistent with our main hypothesis that distribution-side determinants of trade matter repeatedly destination by destination.

The median exporter is a relatively small exporter, with sales to the rest of the world totalling around US$ 90,000 from Brazil (column 4) and US$ 40,000 from Chile (column 5). The mean Brazilian (Chilean) exporter, in contrast, sells around US$ 3.7 (2.8) million abroad, more than 40 (70) times as much as the median manufacturer. Exporter scope and average product scale exhibit similarly stark differences between mean and median. The median Brazilian manufacturer sells two products worldwide, but the mean scope per firm is 5.3 products. The median Brazilian manufacturer has a product scale of around US$ 40,000 per product, but the mean product scale per exporter is US$ 700,000, or around 20 times as high as that for the median firm.[7]

The importance of the top-selling product at multi-product exporters and the mean-median ratios repeat across destinations. To investigate the robustness across countries, we select Brazil's top two export destinations (United States and Argentina), as well as the OECD aggregate. Our theory emphasizes the importance of exporting behavior within destinations. Within single countries, the mean manufacturer's exports exceed the median manufacturer's exports by similarly large factors as in the aggregate, between 14 (in Argentina, column 2) and 26 (in the United States, column 1). In the OECD aggregate (column 3), exports of the mean firm exceed the exports of the median firm by a factor of about 30. Interestingly, the same mean-median ratio of about 30 prevails in the non-OECD aggregate (not reported).

To shed further light on exporting behavior within destinations, we look at Brazil's commercial intermediaries and their manufacturing-product shipments. The shares of single- and multi-product firms are strikingly similar for intermediaries selling manufactured goods (column 6) and the manufacturers shipping directly (column 4). The mean-median ratios vary more, however. While the mean Brazilian manufacturer sells more than 40 times as much as the median manufacturer, the mean Brazilian commercial intermediary sells only around 30 times as much as the median intermediary. This differences between Brazilian manufacturers and intermediaries is due to a smaller mean-median ratio for average scale at intermediaries than at manufacturers (10 times versus 20 times), whereas the mean intermediary sells more than nine products worldwide compared to the mean manufacturer with only five.

[7]The means in Table 1 are calculated as follows. A source country's total exports T_d are decomposed into $T_d = M_d \bar{G}_d \bar{z}_d$, where M_d is the number of exporters to destination d, $\bar{G}_d \equiv \sum_{\phi=1}^{M_d} G_d(\phi)/M_d$ is the exporters' mean exporter scope, and $\bar{z}_d \equiv \bar{t}_d/\bar{G}_d$ is their goods' mean product scale. Equivalently, $\bar{z}_d$ is the weighted arithmetic mean of $z_d(\phi)$ over all ϕ, with weights $G_d(\phi)$: $\bar{z}_d = \sum_{\phi=1}^{M_d} G_d(\phi)\, z_d(\phi)/[\sum_{\phi=1}^{M_d} G_d(\phi)] = \bar{t}_d/\bar{G}_d$. As the decomposition shows, scope weighting is necessary for the mean scope and the mean product scale to yield total exports when multiplied.



Source: *SECEX* 2000, manufacturing firms and their manufactured products.
Note: Products at the Harmonized-System 6-digit level.

Figure 1: **Total Sales and Exporter Scope Distributions**

2.3 Four stylized facts

We turn to exporter scope and its relation to other firm-destination characteristics.

Fact 1: Across firms, the total firm sales distribution is robust over destinations and approximately Pareto. The upper panel of Figure 1 plots firms' total exports against the firms' total-exports percentiles for Brazil's top two export destinations, the United States and Argentina. The plots are similar for most Brazilian destinations. Except for the small firms, total firm exports in the upper panel exhibit an approximate Pareto distribution. In this paper, we strive to explain the approximate Pareto shape of the distribution.[8]

Fact 2: Across firms, the exporter scope distribution is robust over destinations and approximately Pareto. The lower panel of Figure 1 plots firms'

[8]For an explanation of deviant small-firm behavior in the lower tail see Arkolakis (2008).

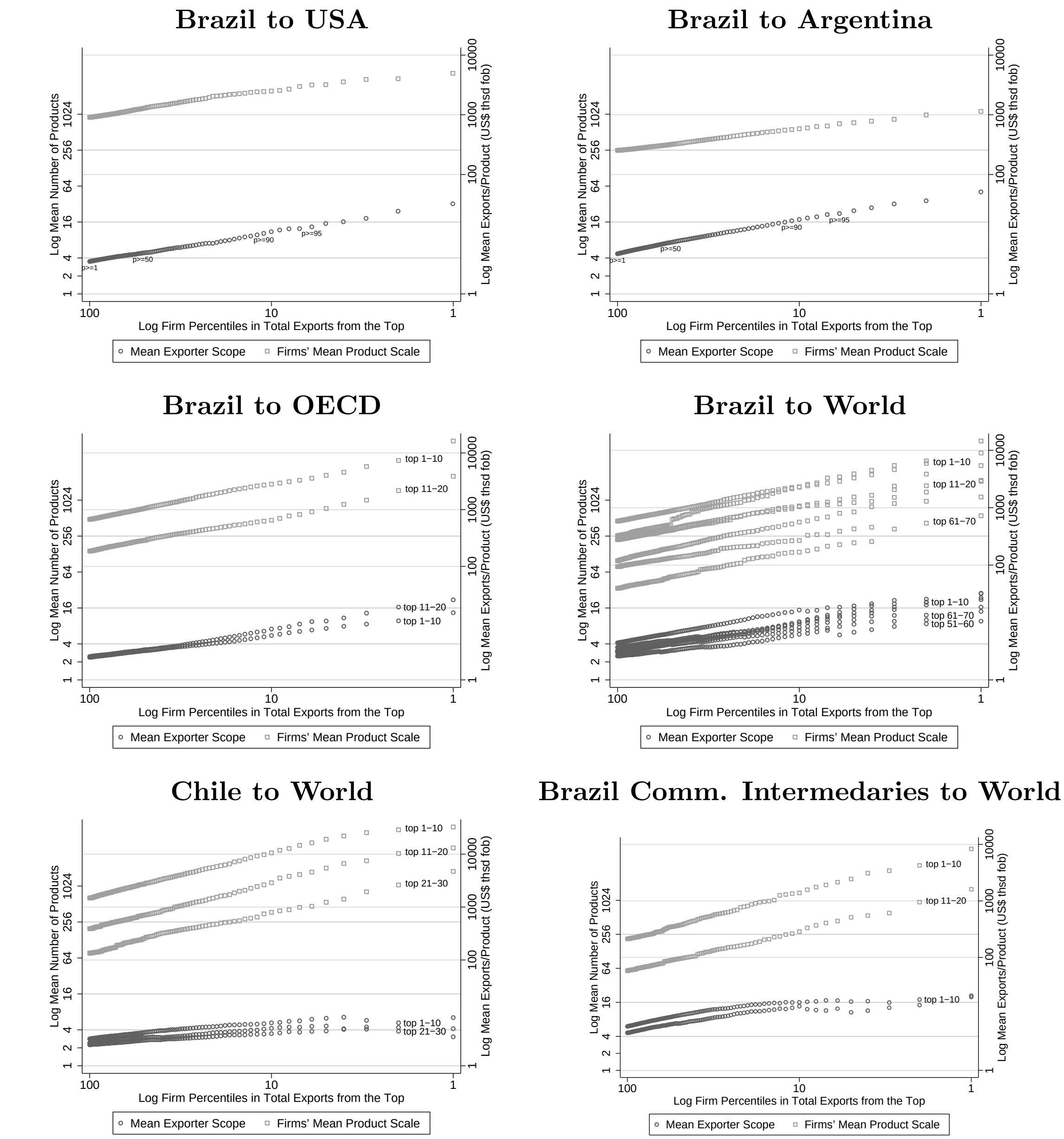
exporter scope against the firms' exporter-scope percentiles for Brazil's top two export destinations, the United States and Argentina. These plots too are similar for most Brazilian destinations. The median Brazilian exporter sells one or two products per destination (Table 1). Exporter scope is a discrete variable but the overall shape of the distributions approximately resembles the shape of continuous Pareto variables. The ratio of two Pareto distributed variables is Pareto distributed so mean product scale must be Pareto if total sales and exporter scope are Pareto. To query this implication, we assess the mean product scale distribution, along with exporter scope, in cumulative plots closer to the definition of a Pareto distribution.

Fact 3: Across firms, exporter scope and average product scale are positively associated. Figure 2 illustrates the relationship between the firms' rank in total exports on the one hand and the firms' mean scope and scale on the other hand, for the same six export markets as in Table 1. All axes have a logarithmic scale. On the horizontal axis, we now choose a grouping of firms that closely reflects the definition of a Pareto distribution: we cumulate firms that are at or above a given total exports percentile. At the origin of the horizontal axis, we cumulate all firms in the sample and plot the mean scope and mean product scale per firm, that is we plot the means that we also report in Table 1. Then we step one percentile to the right along the horizontal axis and restrict the sample to all those firms that are in the top 99 percentiles of total exports. The figure depicts the cumulative mean scope and the mean product scale for this higher-up exporter group.[9] We continue to move up in the total-exports ranking of firms, percentile by percentile, depicting the mean scope and the mean product scale for higher and higher-percentile groups of firms, until we reach the group of firms in the top (100th) percentile of total exports.

For a Pareto distributed variable, a curve as in Figure 2 must be linear. Indeed, both log mean scope and log mean scale increase roughly linearly with the log percentile in the two individual export markets United States and Argentina and, on average, in the aggregate regions (ten-country groups in OECD, world) both for Brazilian and Chilean manufacturers.[10] For Brazilian commercial intermediaries, there are deviations from linearity in exporter scope, but less so in average product scale. Regression analysis shows that a linear fit for Brazilian manufacturers explains more than 97 percent of the variation within destination countries, leaving little explanatory power for non-Pareto disturbances at manufacturers (see Table 3 below).

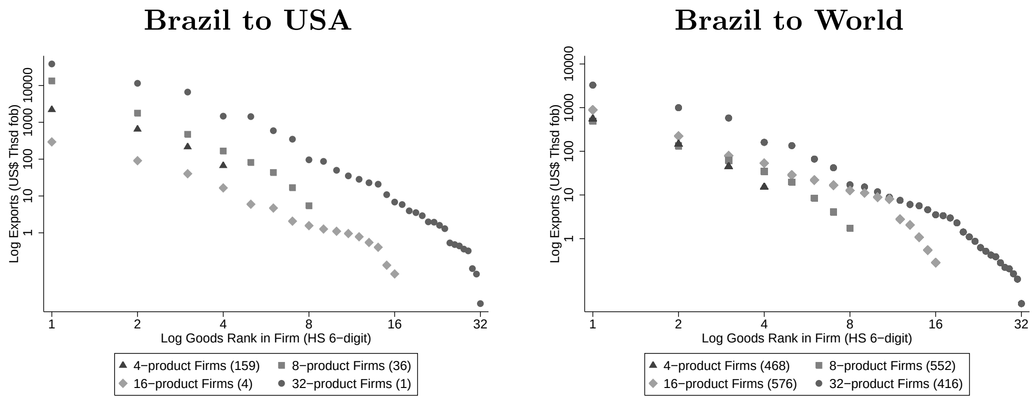
[9] Formally, we calculate total exports $t_d(\phi_1)$ for the top 99 percent of firms ϕ', whose exports exceed the first-percentile threshold $t_d(\phi') > t_d(\phi_1)$. For these M'_d firms in the top 99 percentiles, we calculate the mean scope $\sum_{\phi'=1}^{M'_d} G_d(\phi')/M'_d$ and the scope-weighted mean product scale $\sum_{\phi'=1}^{M'_d} G_d(\phi')z_d(\phi')/[\sum_{\phi'=1}^{M'_d} G_d(\phi')]$. By definition, scope weighting is necessary for the mean scope and the mean product scale to yield total exports when multiplied (see footnote 7). We repeat these calculations for total exports $t_d(\phi_p)$ at every percentile p.

[10] Deviant small-firm behavior in the lower tail weighs little after averaging over upper percentiles.



Sources: *SECEX* 2000 for Brazil, manufacturing firms as well as commercial intermediaries and their manufactured products; Chilean customs data 2000 (Álvarez et al. 2007) for manufacturing firms.
Note: Left-most observations are all exporters; at the next percentile are exporter observations with shipments in the top 99 percentiles; up to the right-most observations with exporters whose shipments are in the top percentile. Aggregate regions include only destinations with more than 100 firms; destinations ranked by total exports and lumped into groups of ten destinations for which unweighted means over distributions are shown (20 Oecd countries for Brazil, 70 worldwide for Brazil, 28 worldwide for Chile). Products at the Harmonized-System 6-digit level. Oecd includes all Oecd members in 1990. Firms' mean product scale ($\bar{z}_d$ in US$ thousand fob) is the scope-weighted arithmetic mean of exporters' average product scales.

Figure 2: **Scope, Average Scale and the Total Exports Distribution**



Source: *SECEX* 2000, manufacturing firms and their manufactured products.
Note: Products at the Harmonized-System 6-digit level. World average from pooling destinations to which firms in a given exporter-scope group ship.

Figure 3: **Within-firm Sales Distribution**

The simultaneous increase in both scope and scale with the firm's percentile in total exports implies that scope and scale are positively related. The regularity across markets is consistent with the idea that firms' choices of scope and scale are positively associated within every destination market.

Fact 4: Within firms, exports are concentrated in few top-selling goods. Within firms, we order goods by their rank from top-selling (rank 1) to least selling at a given destination. Figure 3 depicts Brazilian manufacturers that sell 4, 8, 16 or 32 goods and plots log sales per product for these firm groups against the good's log rank. The figure shows Brazil's top export destinations, the United States, and the worldwide average over all destinations. Plots are similar for most individual Brazilian destinations.

For shipments to the United States, the top-selling good (rank 1) sells on average between US$ 300,000 (by 16-good firms) and US$ 38 million (by 32-good firms). So, sales of the top-selling good differ by a factor of roughly 100 between narrow-scope and wide-scope firms. The least-selling good, in contrast, sells at a scale that is magnitudes smaller at wide-scope firms than at narrow-scope firms. The least-selling good of 32-goods exporters to the United States sells for merely US$ 12 in 2000 (rank 32) and 16-goods exporters ship just US$ 76 of their least-selling good (rank 16). In contrast, the least-selling good of 8-goods exporters (rank 8) sells US$ 5,000 and the least-selling good of 4-goods exporters (rank 4) sells US$ 67,000. So, sales of the least-selling goods differ by a factor of roughly 1,000 between narrow-scope and wide-scope firms.

Wide-scope firms tolerate a much longer tail of small-scale products than narrow-scope firms. But wide-scope firms are also firms with high average scale (fact 3). So, we would expect wide-scope firms to command higher scales for their top-selling goods on average. Indeed, the intercepts of the within-firm sales distributions with the vertical axis are typically higher for wide-scope firms (16-goods firms happen to be an outlier for the United States). In other words, exports are concentrated in few top-selling goods within firms. In the two panels of Figure 3, the elasticity of individual product sales with respect to the rank of the good is about -2.8 in the United States and -2.6 on average worldwide.

We now turn to a model of exporting that is consistent with these four facts, and then revisit the data to evaluate the implied relationships.

3 A Model of Exporter Scope and Product Scale

Our model has heterogeneous firms that sell one or multiple products and is designed to explain trade data at three levels. At the country level, the model generates bilateral trade flows consistent with gravity-equation evidence. In the cross section of firms, the model gives rise to Facts 1 through 3 from above. Within firms in the cross section of their goods, the model matches Fact 4. Firms incur destination-specific variable and fixed market entry costs that depend on the number of products offered at a destination. Consumers in every country have nested preferences over firms' product mixes and the firms' individual products.

3.1 Consumers

There are N countries. When we consider an export destination, we label the country with d. The source country of an export shipment receives the label s. There is a measure of L_d consumers at destination d. Consumers have symmetric preferences with a constant elasticity of substitution σ over a continuum of varieties. In our multi-product setting, a conventional "variety" becomes the product composite

$$X_{isd}(\omega) \equiv \left[\int_{\underline{g}}^{G_{isd}(\omega)} x_{isdg}(\omega)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \mathrm{d}g$$

that a seller ω from industry i in source country s offers for sale at destination d. In marketing terminology, the product composite is a firm's *product mix*. The elasticity of substitution ε is constant across goods within the product mix and possibly different from σ. G_{isd} is the measure of a seller ω's goods, and $\underline{g}$ is the index number for any seller's first good (such as $\underline{g} = 1$ similar to discrete product space). We assume that every product mix is uniquely offered by a single firm, but a firm may ship different product mixes to different destinations.

The set of product mixes shipped from industry i in source country s to destination d for consumption is $\mathbf{\Omega}_{isd}$. So the consumer's utility at destination d is

$$\left(\sum_{s=1}^{N}\sum_{i=1}^{I(s)}\int_{\omega\in\mathbf{\Omega}_{isd}}\left[\int_{\underline{g}}^{G_{isd}(\omega)} x_{isdg}(\omega)^{\frac{\varepsilon-1}{\varepsilon}}\,\mathrm{d}g\right]^{\frac{\varepsilon}{\varepsilon-1}\,\frac{\sigma-1}{\sigma}}\mathrm{d}\omega\right)^{\frac{\sigma}{\sigma-1}} \quad \text{where } \varepsilon>1,\ \sigma>1, \varepsilon\neq\sigma. \tag{1}$$

A similar nested utility function is also used by Allanson and Montagna (2005) to study implications of the product life-cycle for market structure, and by Agur (2007) to analyze trade patterns in a two-country heterogeneous-firm model with a continuum of products. The specification generalizes monopolistic-competition models of trade (such as Krugman 1980).[11] For preferences to be well defined, we require that $\varepsilon > 1$ and $\sigma > 1$. Subsequent derivations do not materially differ if we assume that products within a product mix are more substitutable among each other than with outside goods ($\varepsilon > \sigma > 1$), or less substitutable ($\sigma > \varepsilon > 1$). Evidence in Broda and Weinstein (2007) suggests that products are stronger substitutes within firms than across firms so we consider $\varepsilon > \sigma > 1$ our main case. (Their preferred estimates for ε and σ within and across domestic U.S. brand modules are 11.5 and 7.5.)

Every consumer receives per-capita income $y_d = w_d + \Pi_d$, where w_d is the wage for labor, inelastically supplied to producers in country d, and $\Pi_d \equiv \sum_{i=1}^{I}\sum_{\varsigma=1}^{N}\pi_{id\varsigma}/L_d$ is the consumer's share in total profits earned by the country's producers. Country d's total income is $y_d L_d$. The consumer's first-order conditions of utility maximization imply a product demand

$$x_{isdg}(\omega) = \left(\frac{p_{sdg}(\omega)}{P_{isd}(\omega; G_{isd})}\right)^{-\varepsilon} X_{isd}(\omega; G_{isd}), \tag{2}$$

[11]There is a counterpart to (1) in discrete product space, where consumers at d have preferences

$$\left(\sum_{s=1}^{N}\sum_{i=1}^{I(s)}\int_{\omega\in\mathbf{\Omega}_{isd}}\left[\sum_{g=1}^{G_{isd}(\omega)} x_{isdg}(\omega)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}\,\frac{\sigma-1}{\sigma}}\mathrm{d}\omega\right)^{\frac{\sigma}{\sigma-1}} \quad \text{where} \quad \varepsilon>1,\ \sigma>1, \varepsilon\neq\sigma,$$

and $\mathbf{\Omega}_{isd}$ is the set of product mixes shipped from s to d. Atkeson and Burstein (2007) adopt a similar nested CES form in a heterogeneous-firms model of trade but do not consider multi-product firms. A discrete number of products closely relates to empirical work with product-level data, and we have re-derived several results for the discrete case. To compare our model's implications directly to related heterogeneous-firm models, however, we present the continuous version of the product space. We could make the elasticities of substitution country-specific (ε_d and σ_d), and all our results would continue to apply. We keep the elasticities the same across destinations to simplify notation and to emphasize that our results do not depend on preference assumptions.

given the firm's product mix $X_{isd}(\omega; G_{isd})$ and the product-mix price $P_{isd}(\omega; G_{isd})$:

$$X_{isd}(\omega; G_{isd}) \equiv \left[\int_{\underline{g}}^{G_{isd}(\omega)} x_{isdg}(\omega)^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}g \right]^{\frac{\varepsilon}{\varepsilon-1}} \text{ and } P_{isd}(\omega; G_{isd}) \equiv \left[\int_{\underline{g}}^{G_{isd}(\omega)} p_{sdg}(\omega)^{-(\varepsilon-1)} \mathrm{d}g \right]^{-\frac{1}{\varepsilon-1}}.$$

The first-order conditions also imply a product-mix demand

$$X_{isd}(\omega; G_{isd}) = \left(\frac{P_{isd}(\omega; G_{isd})}{P_d} \right)^{-\sigma} X_d, \tag{3}$$

where

$$X_d \equiv \left[\sum_{s=1}^{N} \sum_{i=1}^{I(s)} \int_{\omega \in \mathbf{\Omega}_{isd}} X_{isd}(\omega; G_{isd})^{\frac{\sigma-1}{\sigma}} \mathrm{d}\omega \right]^{\frac{\sigma}{\sigma-1}} \text{and } P_d \equiv \left[\sum_{s=1}^{N} \sum_{i=1}^{I(s)} \int_{\omega \in \mathbf{\Omega}_{isd}} P_{isd}(\omega; G_{isd})^{-(\sigma-1)} \mathrm{d}\omega \right]^{-\frac{1}{\sigma-1}}.$$

So, the demand for firm ω's product g, produced in source country s and industry i and sold to destination country d, is

$$x_{isdg}(\omega) = \left(\frac{p_{sdg}(\omega)}{P_{isd}(\omega; G_{isd})} \right)^{-\varepsilon} \left(\frac{P_{isd}(\omega; G_{isd})}{P_d} \right)^{-\sigma} \frac{y_d L_d}{P_d} \tag{4}$$

with $P_d \, X_d = y_d L_d$.

This is an important relationship and gives rise to a cannibalization effect for $\varepsilon > \sigma$ (our main case consistent with Broda and Weinstein (2007)). Since $P_{isd}(\omega; G_{isd})$ strictly decreases in exporter scope for $\varepsilon > 1$, wider exporter scope diminishes infra-marginal sales and reduces $x_{isdg}(\omega)$ for $\varepsilon > \sigma$. In other words, if products within a product mix are more substitutable among each other than with outside goods, widening exporter scope is costly to the exporter because it diminishes the sales of infra-marginal products. (For the converse case with $\sigma > \varepsilon$, however, wider exporter scope boosts infra-marginal sales and raises $x_{isdg}(\omega)$.) As long as the product-entry costs are sufficiently convex in the firm's offered scope, our model will generate results consistent with the stylized facts, irrespective of whether infra-marginal sales decline, remain constant ($\sigma = \varepsilon$), or increase with scope. The only difference between the case of a decline in infra-marginal sales and the converse case is that product-entry cost need to be a more convex function of scope for a well defined optimum to exist if there is no decline in infra-marginal sales.

3.2 Firms

There is a continuum of firms that differ ex ante only in their worldwide unique productivity scalar ϕ. Each firm is located in a single source country s and belongs to an industry i. Compared to Feenstra and Ma (2007), who consider a countable number of firms that have product-market power, our setup retains the assumption of monopolistic competition with an infinite number of firms. An advantage of monopolistic competition is that we obtain closed-form solutions for aggregate exports to generate gravity-equation predictions and to characterize cross-firm distributions in market equilibrium, whereas Feenstra and Ma (2007) need numerical simulations. Under monopolistic competition, however, constant elasticities of substitution imply that the cannibalization effect is never strong enough to limit the optimal number of goods at a finite level so that we need to assume variable product-entry or fixed market-entry costs beyond any cannibalization effect, in contrast with Feenstra and Ma (2007) who do not need such costs.

Firm ϕ manufactures every one of its products with a potentially different productivity and then ships the product to destination d at a variable cost that potentially varies by destination d: $q_{isdg}(\phi_{idg}) = \phi_{idg}\,\ell$, independent of s but different for each good g, where ℓ is employment contracted at the source country's wage w_s. Upon entry, a firm both learns its unique productivity parameter ϕ and gets to know the firm-specific ordering of the goods $g \in [\underline{g}, \infty)$ for every destination, where the good with the smallest index $\underline{g}$ is the firm's most productive and the highest-index good is the firm's least productive at a given destination. The ordering is potentially different for each firm and destination market. When exported, a product incurs a standard iceberg trade cost so that $\tau_{sd} > 1$ units must be shipped from s for one unit to arrive at destination d. We assume that $\tau_{ss} = 1$ for domestic sales. Iceberg trade cost are common to all firms shipping from s to d whereas the productivity ordering of goods can differ across firms.

We call an exporter's measure of goods G_{isd} shipped to destination d the *exporter scope* at destination d. We call the sales $p_{sdg}\,x_{isdg}$ of a firm's individual product g the *product scale* at destination d, where p_{sdg} is the product price. A firm maximizes its profits by choosing its scope G_{isd} for every destination d and the scale $p_{sdg}\,x_{isdg}$ for every product g at destination d.

As the firm widens its exporter scope, it faces two types of costs. First, the exporter incurs a *variable product-entry cost*. Under our convention that a firm's lowest-index good is its most productive good, we write a good g's productivity as

$$\phi_{idg} \equiv \frac{\phi}{h_d(g)} \qquad \text{with} \quad h'_d(g) \geq 0,\; g \in [\underline{g}, G_{isd}(\phi)] \tag{5}$$

for a marginal-cost schedule $h_d(g)$ that is potentially destination-specific. An implication of a strictly increasing marginal-cost schedule $h'_d(g) > 0$ is that a firm will enter export market d with the most productive product first and then expand its scope moving up the marginal-cost ladder good by good. We allow for the possibility that a firm expands scope with a different sequence of products destination by destination. For $h'_d(g) = 0$, however, any sequence of introducing products is equally profitable. We define the *average productivity of goods* for firm ϕ at d as

$$H_d(G_{isd}(\phi)) \equiv \int_{\underline{g}}^{G_{isd}(\phi)} h_d(g)^{-(\varepsilon-1)} \mathrm{d}g \geq 0$$

so that the product-mix price becomes $P_{isd}(\phi; G_{isd})^{-(\varepsilon-1)} = [\eta_{isd}\tau_{sd}w_s/\phi]^{-(\varepsilon-1)} H_d(G_{isd})$.

It is a common assumption in multi-product models of exporters that the firm faces a drop in productivity for each additional good as its exporter scope widens. Related models include Eckel and Neary (2006), who call the lowest-index good the firm's core competency, Nocke and Yeaple (2006) and Melitz and Ottaviano (2008). Bernard et al. (2008) offer a stochastic generalization of this framework, in which firms endogenously adopt products of different competency levels. The dependence of the h_d schedule on destination d is a testable generalization.

Second, the exporter incurs a destination-specific *fixed market-entry cost* that is non-zero at zero scope and weakly increases in exporter scope[12]

$$F_{id}(G_{isd}) \qquad \text{with} \quad F_{id}(0) > 0 \text{ and } F'_{id}(G_{isd}) \geq 0. \tag{6}$$

Marginal market-entry cost must be strictly positive for a well-defined optimum to exist.

Under these assumptions, firms with a given productivity ϕ from a given country s face an identical optimization problem. The firm's profit in a given destination market d is

$$\pi_{isd}(\phi) = \max_{G_{isd}, P_{isd}, \{p_{sdg}\}_{g=1}^{G_{isd}}} \int_{\underline{g}}^{G_{isd}} \left(p_{sdg} - \tau_{sd} \frac{w_s h_d(g)}{\phi} \right) (p_{sdg})^{-\varepsilon} \frac{(P_{isd})^{\varepsilon-\sigma}}{(P_d)^{-(\sigma-1)}} y_d L_d \, \mathrm{d}g - F_{id}(G_{isd}) \tag{7}$$

by demand (4), where $\pi_{isd}(\phi)$ denotes maximized profits. In general, the product-mix price $P_{isd}(\phi; G_{isd})$ is a function of G_{isd} so that a firm takes into account how every single product competes with other products in the firm's product mix at a destination. For constant elasticities of substitution and monopolistic competition, however, the optimal markup does not depend on exporter scope.[13]

The firm's profit maximization problem can be broken into two steps. First is the choice of the price index P_{isd} along with the individual product prices p_{sdg} to maximize (7), given G_{isd} and subject to the constraint that $(P_{isd})^{-(\varepsilon-1)} = \int (p_{sdg})^{-(\varepsilon-1)} \mathrm{d}g$

[12]In continuous product space with nested CES utility, market-entry costs must be non-zero at zero scope because a firm would otherwise export to all destinations worldwide. No fixed market-entry cost is needed in the discrete product-space version of our model where the firm's first good causes a nontrivial fixed market-entry cost.

[13]Even in discrete product space, the optimal markup does not vary with exporter scope for constant elasticities of substitution under monopolistic competition.

by (2).[14] The first-order conditions with respect to the product-mix price P_{isd} and individual prices p_{sdg} imply product prices $p_{sdg}(\phi;g) = \eta\,\tau_{sd}\,w_s\,h_d(g)/\phi$ with an identical markup over marginal cost $\eta \equiv \sigma/(\sigma-1) > 1$ for $\sigma > 1$ (see Appendix A).

Second is the choice of exporter scope G_{isd}, given optimal price index $P_{isd}(\phi;G_{isd})$ and optimal individual product prices $p_{sdg}(\phi;g)$. A firm's choice of optimal prices implies optimal product scale for product g

$$p_{sdg}(\phi;g)\,x_{isdg}(\phi;g) = \frac{y_d L_d}{H_d(G_{isd})^{1-\bar{\sigma}}}\left(\frac{\phi\,P_d}{\eta\,\tau_{sd}\,w_s}\right)^{\sigma-1} h_d(g)^{-(\varepsilon-1)} \quad \text{with } \bar{\sigma} \equiv \frac{\sigma-1}{\varepsilon-1}. \tag{8}$$

The composite elasticity term $\bar{\sigma} \equiv (\sigma-1)/(\varepsilon-1)$ satisfies $\bar{\sigma} < 1$ iff $\varepsilon > \sigma > 1$ (that is iff products within the product mix are closer substitutes among each other than with outside goods so that widening scope cannibalizes inframarginal sales).

Integrating (8) over the firm's products at destination d, firm ϕ's optimal total exports to destination d are

$$t_{isd}(\phi) = \int_{\underline{g}}^{G_{isd}} p_{sdg}(\phi;g)\,x_{isdg}(\phi;g)\,\mathrm{d}g = H_d(G_{isd})^{\bar{\sigma}}\,y_d L_d\left(\frac{\phi\,P_d}{\eta\,\tau_{sd}\,w_s}\right)^{\sigma-1}. \tag{9}$$

Given identical constant markups over marginal cost, scale-optimized profits from exports to destination d are therefore

$$\pi_{isd}(\phi) = \max_{G_{isd}} \frac{t_{isd}(\phi)}{\sigma} - F_{id}(G_{isd}) = \max_{G_{isd}} \frac{H_d(G_{isd})^{\bar{\sigma}} y_d L_d}{\sigma}\left(\frac{\phi\,P_d}{\eta\,\tau_{sd}\,w_s}\right)^{\sigma-1} - F_{id}(G_{isd}).$$

Taking the first derivative of the profit function with respect to G_{isd} and setting it to zero, optimal exporter scope $G_{isd}(\phi)$ is implicitly given by the first-order condition

$$\frac{\bar{\sigma}\,h_d(G_{isd}(\phi))^{-(\varepsilon-1)}\;y_d L_d}{\sigma\,H_d(G_{isd}(\phi))^{1-\bar{\sigma}}}\left(\frac{\phi\,P_d}{\eta\,\tau_{sd}\,w_s}\right)^{\sigma-1} = F'_{id}(G_{isd}(\phi)). \tag{10}$$

For a well-defined profit-maximum to exist, profits must be concave in G_{isd} at the optimal $G_{isd}(\phi)$.[15] By the second-order condition, the total of fixed market-entry costs and variable product-entry costs must be sufficiently convex in exporter scope. Otherwise, firms would choose an infinite exporter scope because $H_d(G_{isd})$ strictly increases in G_{isd}.

[14] We thank Elhanan Helpman for pointing out this Lagrangian solution strategy.

[15] The second-order condition for optimal product scope is equivalent to

$$\frac{F''_{id}(G_{isd}(\phi))}{F'_{id}(G_{isd}(\phi))} > -(1-\bar{\sigma})\frac{h_d(G_{isd}(\phi))^{-(\varepsilon-1)}}{H_d(G_{isd}(\phi))} - (\varepsilon-1)\frac{h'_d(G_{isd}(\phi))}{h_d(G_{isd}(\phi))},$$

after using the first-order condition to substitute terms.

Optimal product scale for product g is

$$p_{sdg}(\phi;g)\,x_{isdg}(\phi;g)=\frac{\sigma}{\bar{\sigma}}\,F'_{id}(G_{isd}(\phi))\left(\frac{h_d(g)}{h_d(G_{isd}(\phi))}\right)^{-(\varepsilon-1)}\qquad\text{for } g\leq G_{isd}(\phi),\qquad(11)$$

using (10) in (8). This relationship is consistent with Fact 4: product scale strictly drops in a good's rank g within firm ϕ.

We now seek to express the first-order condition for optimal scope more intuitively. Firm ϕ exports from s to d iff $\pi_{isd}(\phi)\geq 0$. This break-even condition is equivalent to the condition that exporter scope weakly exceed a minimum profitable scope $G_{isd}(\phi)\geq G^*_{id}$ and to the condition that the firm's productivity weakly exceed a productivity threshold $\phi\geq\phi^*_{isd}$. Using first-order condition (10) in the profit function and restricting profits to zero, the minimum profitable scope G^*_{id} is implicitly given by

$$\frac{F'_{id}(G^*_{id})}{F_{id}(G^*_{id})}=\bar{\sigma}\,\frac{H'_d(G^*_{id})}{H_d(G^*_{id})}.\qquad(12)$$

Equivalently, reformulating the break-even condition and using the above expression for minimum profitable scope, the productivity threshold for exporting from s to d is

$$\phi^*_{isd}\equiv\left(\frac{\sigma}{\bar{\sigma}}\frac{F'_{id}(G^*_{id})\,H_d(G^*_{id})^{1-\bar{\sigma}}}{H'_d(G^*_{id})}\frac{1}{y_d L_d}\right)^{\frac{1}{\sigma-1}}\;\frac{\eta\,\tau_{sd}\,w_s}{P_d}.\qquad(13)$$

Using the definitions of minimum profitable scope (12) and minimum productivity (13), we restate the first-order condition for optimal scope (10) more succinctly as

$$\frac{F'_{id}(G_{isd}(\phi))\,H_d(G_{isd}(\phi))^{1-\bar{\sigma}}}{H'_d(G_{isd}(\phi))}=\frac{F'_{id}(G^*_{id})\,H_d(G^*_{id})^{1-\bar{\sigma}}}{H'_d(G^*_{id})}\left(\frac{\phi}{\phi^*_{isd}}\right)^{\sigma-1}.\qquad(14)$$

Note that the left-hand side of (14) must strictly increase in $G_{isd}(\phi)$ by the second-order condition and that the right-hand side of (14) strictly increases in ϕ. So, $G_{isd}(\phi)$ strictly increases in ϕ.

Using the above results, a firm ϕ's optimal total exports to destination d are

$$t_{isd}(\phi)=\frac{\sigma}{\bar{\sigma}}F'_{id}(G_{isd}(\phi))\frac{H_d(G_{isd}(\phi))}{H'_d(G_{isd}(\phi))}\qquad(15)$$

and firm ϕ's optimal average product scale is

$$z_{isd}(\phi)\equiv\frac{t_{isd}(\phi)}{G_{isd}(\phi)}=\frac{\sigma}{\bar{\sigma}}\frac{F'_{id}(G_{isd}(\phi))}{G_{isd}(\phi)}\frac{H_d(G_{isd}(\phi))}{H'_d(G_{isd}(\phi))},\qquad(16)$$

both conditional on the firm's exporting from s to d.

Proposition 1 *Suppose the total of market-entry and product-entry costs is sufficiently convex so that the second-order condition for optimal exporter scope is satisfied. Then for all* $s, d \in \{1, \ldots, N\}$

- *there exists a productivity threshold* $\phi^*_{isd} > 0$ *such that exporter scope* $G_{isd}(\phi) > 0$ *and shipments* $x_{isdg}(\phi) > 0$ *for all products* $g \in [\underline{g}, G_{isd}(\phi)]$ *iff* $\phi \geq \phi^*_{isd}$*;*

- *exporter scope* $G_{isd}(\phi)$ *strictly increases in* ϕ *for* $\phi \geq \phi^*_{isd}$*;*

- *total firm exports* $t_{isd}(\phi)$ *strictly increase in* $G_{isd}(\phi)$ *and in* ϕ *for* $\phi \geq \phi^*_{isd}$*;*

- *average product scale* $z_{isd}(\phi)$ *strictly increases in* $G_{isd}(\phi)$ *and in* ϕ *for* $\phi \geq \phi^*_{isd}$ *iff market-entry and product-entry costs satisfy*

$$\frac{F''_{id}(G_{isd}(\phi))}{F'_{id}(G_{isd}(\phi))} > \frac{1}{G_{isd}(\phi)} - \left[\frac{h_d(G_{isd}(\phi))^{-(\varepsilon-1)}}{H_d(G_{isd}(\phi))} + (\varepsilon-1)\frac{h'_d(G_{isd}(\phi))}{h_d(G_{isd}(\phi))}\right].$$

Proof. The former two statements follow directly from the discussion above. For a proof of the third statement take the first derivative of (15) with respect to G_{isd} and note that (15) strictly increases in G_{isd} iff

$$\frac{F''_{id}(G_{isd}(\phi))}{F'_{id}(G_{isd}(\phi))} > - \left[\frac{h_d(G_{isd}(\phi))^{-(\varepsilon-1)}}{H_d(G_{isd}(\phi))} + (\varepsilon-1)\frac{h'_d(G_{isd}(\phi))}{h_d(G_{isd}(\phi))}\right],$$

which must be satisfied by the second-order condition. Similarly, for a proof of the fourth statement take the first derivative of (16) with respect to G_{isd}. ■

Whereas total exports and optimal scope increase with the firm's productivity, optimal scale per product increases in productivity only if market-entry costs are not too concave relative to product-entry costs. The statements of Proposition 1 resemble Facts 1 through 3 but do not address the evidence that all three variables behave like Pareto variables in the upper tail. To match this distributional behavior under Pareto distributed firm productivity ϕ, a necessary and sufficient condition is that the average productivity of goods and the marginal market-entry costs converge to power functions as exporter scope widens (see Appendix B for a formal proof).

3.3 Predictions for bilateral trade at the country level

To put the model to the data, consider a firm's productivity ϕ to be drawn from a Pareto distribution with a source-country dependent location parameter b_s and a worldwide shape parameter θ over the support $[b_s, +\infty)$ for $s = 1, \ldots, N$.

At the country level, we turn to the limiting case where differences between within-firm and cross-firm elasticities of substitution matter little so that $\bar{\sigma}$ is arbitrarily close to one. Consider average total exports $\bar{t}_{isd}$ from source country s and industry i to destination d. Integrating (15) over all firms from source country s and industry i that ship to destination d, and weighting by their Pareto density, yields

$$\begin{aligned} \bar{t}_{isd} &\equiv \int_{\phi^*_{isd}}^{\infty} \frac{\sigma}{\bar{\sigma}} \frac{F'_{id}(G_{isd}(\phi))\, H(G_{isd}(\phi))}{H'(G_{isd}(\phi))} \frac{\theta\, (\phi^*_{isd})^{\theta}}{(\phi)^{\theta+1}}\, \mathrm{d}\phi \\ &= \frac{\sigma}{\bar{\sigma}} \frac{\theta}{\theta-(\sigma-1)} \frac{F'_{id}(G^*_{id}) H(G^*_{id})}{H'(G^*_{id})} \int_{\phi^*_{isd}}^{\infty} \frac{H(G_{isd}(\phi))^{\bar{\sigma}}}{H(G^*_{id})^{\bar{\sigma}}} \frac{[\theta-(\sigma-1)]\, (\phi^*_{isd})^{\theta-(\sigma-1)}}{(\phi)^{\theta-(\sigma-1)+1}}\, \mathrm{d}\phi, \end{aligned} \tag{17}$$

where the second line follows from first-order condition (14). To generate gravity-equation predictions in the spirit of Eaton and Kortum (2002), we need average total exports to be independent of the source country. So, the integral in (17) needs to collapse to a constant that does not depend on ϕ^*_{isd}.

Proposition 2 *Suppose the total of market-entry and product-entry costs is sufficiently convex so that the second-order condition for optimal exporter scope is satisfied, ϕ is Pareto distributed, and $\bar{\sigma}$ is arbitrarily close to one. Then for all $d \in \{1, \ldots, N\}$ average total sales $\bar{t}_{isd} = \bar{t}_{id}$ are independent of the source country iff average productivity $H(G_{isd})$ and market-entry costs $F(G_{isd})$ are power functions of exporter scope.*

Proof. If $H(G)$ is a power function with $H(G) = c_H + a_H(G)^{b_H}$ (for real numbers a_H, b_H, c_H), then $H(\cdot)^{\bar{\sigma}}$ is a power function of exporter scope plus a constant for $\bar{\sigma} \to 1$; moreover, if $H(G)$ and $F(G)$ are power functions then G is a power function of (ϕ^*_{isd}/ϕ) for $\bar{\sigma} \to 1$ by (14) and the integral collapses to a constant that does not depend on ϕ^*_{isd}. Conversely, if the integral does not depend on ϕ^*_{isd} then, for $\bar{\sigma} \to 1$, $H(\cdot)^{\bar{\sigma}}$ must be a power function of (ϕ^*_{isd}/ϕ) plus a constant so $H(G)$ and $F'(G)$ must be power functions by (14). ■

Under Pareto distributed ϕ, the total measure of firms from source country s and industry i exporting to destination d is $M_{isd} = J_{is} b_s^{\theta} / (\phi^*_{isd})^{\theta}$. So, the market share of country s's exports in country d's total consumption of industry i goods is

$$\lambda_{isd} \equiv \frac{M_{isd}\, \bar{t}_{id}}{\sum_{\varsigma=1}^{N} M_{i\varsigma d}\, \bar{t}_{id}} = \frac{J_{is}\, (b_s/\phi^*_{isd})^{\theta}}{\sum_{\varsigma} J_{i\varsigma} \left(b_{\varsigma}/\phi^*_{i\varsigma d}\right)^{\theta}} = \frac{J_{is}\, (b_s)^{\theta}\, (\tau_{sd}\, w_s)^{-\theta}}{\sum_{\varsigma} J_{i\varsigma}\, (b_{\varsigma})^{\theta}\, (\tau_{\varsigma d}\, w_{\varsigma})^{-\theta}}. \tag{18}$$

It is remarkable that the elasticity of trade with respect to trade costs is $-\theta$, as in Eaton and Kortum (2002).[16] The isomorphism of our model in terms of aggregate bilateral trade flows with the Eaton and Kortum (2002) framework ensures that our predictions are consistent with the gravity equation. Moreover, the model is reminiscent of the finding in Eaton et al. (2004) that France's exports to destinations are largely explained by the measure of active exporters $J_{is}\, (b_s/\phi^*_{isd})^{\theta}$. We find a similar pattern for Brazilian and Chilean exporters (Appendix F).

[16]In our model, the elasticity of trade with respect to trade costs is the (negative) Pareto shape parameter, whereas it is the (negative) Fréchet shape parameter in Eaton and Kortum (2002).

3.4 Predictions for the cross section of firms

We now turn to specific functional forms. Though not necessary for Pareto distributions in the upper tail (see Appendix B for necessary conditions), our parametrization allows us to obtain closed-form expressions for the cross section of firms and for sales of goods within firms.

Assumption 1 *Fixed market-entry costs have an invariant destination-specific component and a scope-dependent component with*

$$F_{id}(G_{isd}) = \gamma_i \kappa_d \, w_d + \gamma_i \gamma_d \, w_d \, (G_{isd})^{\delta}/\delta \qquad \textit{where} \quad \gamma_i\kappa_d, \gamma_i\gamma_d > 0,\ \delta > 0, \tag{19}$$

in terms of destination-country labor units.

Variable product-entry costs are determined by the marginal-cost schedule

$$h(g) = g^{\alpha} \qquad \textit{for} \quad \alpha > 1/(\varepsilon-1) \textit{ and } \underline{g} = 1, \tag{20}$$

with the related average productivity of goods

$$H(G_{isd}) \equiv \int_{1}^{G_{isd}} g^{-\alpha(\varepsilon-1)} \, dg = \frac{1}{\bar{\alpha}} \left(1 - \frac{1}{(G_{isd})^{\bar{\alpha}}} \right) \quad \textit{for } \bar{\alpha} \equiv \alpha(\varepsilon-1) - 1 > 0. \tag{21}$$

In order to ship the first unit of the first product to a destination, a firm has to incur a beachhead market-entry cost $\gamma_i \kappa_d \, w_d > 0$ in terms of destination-country labor units.[17] There is an industry-specific component γ_i and a destination-specific component κ_d. In addition, a firm must pay a scope-dependent market-entry cost at every sales destination. Market-entry cost strictly increase ($\delta > 0$) in an exporter's offered scope. That part of market-entry cost has the same industry-specific component γ_i as the beachhead cost and a potentially different destination-specific component γ_d.

Under the marginal-cost schedule, the rank of a good may vary destination by destination. There are several general antiderivatives to the marginal-cost schedule other than $H(G_{isd})$ in (21).[18] But the requirements that $\alpha(\varepsilon-1) \approx 2.6$ by (8) and Fact 4, and that $\varepsilon \approx 11.5$ given the evidence in Broda and Weinstein (2007), narrow the set of permissible antiderivatives to functions of the form (21) with $\underline{g} > 0$.

Assumption 1 implies that the elasticity of total market-entry and product-entry costs with respect to exporter scope is $\delta + \bar{\alpha}$ (*scope elasticity of total entry costs*), where δ is the scope elasticity of fixed market-entry costs and $\bar{\alpha}$ is the scope elasticity of variable product-entry costs. The second-order condition for optimal exporter scope requires that the scope elasticity of total entry costs $(\delta + \bar{\alpha})$ plus the scope elasticity of profit losses from the cannibalization effect $(1-\bar{\sigma})\bar{\alpha}/(G^{\bar{\alpha}}-1)$ (for $\bar{\sigma} < 1$) strictly exceed zero, $\delta + \bar{\alpha} + (1-\bar{\sigma})\bar{\alpha}/(G^{\bar{\alpha}}-1) > 0$. Note that the cannibalization effect becomes

[17] For the case of discrete goods, $\gamma_i \kappa_d \, w_d \geq 0$ suffices.

[18] Examples include the antiderivative for $\alpha(\varepsilon-1) - 1 \leq 0$ and alternative choices of $\underline{g}$.

negligibly small as scope grows (contrary to Feenstra and Ma (2007)). There are dis-economies of scope in entry if the scope elasticity of total entry costs exceeds one, $\delta + \bar{\alpha} > 1$, because then entry costs increase more than proportionally when firms widen their exporter scope (convex total entry costs in scope).

With the parametrization under Assumption 1, the Pareto assumption for firm productivity ϕ allows us to restate cross-sectional predictions for empirical work.

Proposition 3 *Suppose market-entry and product-entry costs are given by Assumption 1, the total of market-entry and product-entry costs is sufficiently convex so that the second-order condition for optimal exporter scope is satisfied, and ϕ is Pareto distributed with shape parameter θ. Then for all $d \in \{1, \ldots, N\}$*

- *exporter scope at destination d is implicitly given by*

$$\frac{G_{isd}(\phi)}{G_d^*} \left(\frac{1 - (G_{isd}(\phi))^{-\bar{\alpha}}}{1 - (G_d^*)^{-\bar{\alpha}}} \right)^{\frac{1-\bar{\sigma}}{\delta+\bar{\alpha}}} = \left(\frac{\phi}{\phi_{isd}^*} \right)^{\frac{\sigma-1}{\delta+\bar{\alpha}}} \tag{22}$$

  and Pareto distributed in the upper tail with shape $\theta\,(\delta + \bar{\alpha})/(\sigma - 1)$, where the productivity threshold for exporting from s to d is

$$\phi_{isd}^* \equiv \left[\frac{\sigma}{\bar{\sigma}} \frac{\gamma_i\,\gamma_d\,w_d}{y_d L_d} \left(\frac{(G_d^*)^{\bar{\alpha}} - 1}{\bar{\alpha}} \right)^{1-\bar{\sigma}} (G_d^*)^{\delta+\bar{\sigma}\bar{\alpha}} \right]^{\frac{1}{\sigma-1}} \frac{\eta\,\tau_{sd}\,w_s}{P_d} \tag{23}$$

  and minimum profitable scope is implicitly given by

$$(G_d^*)^{\delta} \left[(G_d^*)^{\bar{\alpha}} - 1 - \frac{\bar{\sigma}\bar{\alpha}}{\delta} \right] = \bar{\sigma}\bar{\alpha}\frac{\kappa_d}{\gamma_d}; \tag{24}$$

- *total firm exports to destination d are*

$$t_{isd}(\phi) = \frac{\sigma}{\bar{\sigma}} \frac{\gamma_i \gamma_d\, w_d}{\bar{\alpha}}\, G_{isd}(\phi)^{\delta+\bar{\alpha}} \cdot \left[1 - (G_{isd}(\phi))^{-\bar{\alpha}} \right] \tag{25}$$

  and Pareto distributed in the upper tail with shape $\theta/(\sigma - 1)$;

- *average product scale at destination d is*

$$z_{isd}(\phi) \equiv \frac{t_{isd}(\phi)}{G_{isd}(\phi)} = \frac{\sigma}{\bar{\sigma}} \frac{\gamma_i \gamma_d\, w_d}{\bar{\alpha}}\, G_{isd}(\phi)^{\delta+\bar{\alpha}-1} \cdot \left[1 - (G_{isd}(\phi))^{-\bar{\alpha}} \right] \tag{26}$$

  and Pareto distributed in the upper tail with shape $\theta\,(\delta + \bar{\alpha})/[(\sigma - 1)(\delta + \bar{\alpha} - 1)]$ if there are dis-economies of scope in total entry costs ($\delta + \bar{\alpha} > 1$).

Proof. To obtain (22), use the functional forms of Assumption 1 in (14) and note that the second factor on the left-hand side changes with a scope elasticity of $[1/(\delta+\bar{\alpha})][\bar{\alpha}(1-\bar{\sigma})/(G^{\bar{\alpha}}-1)]$, which approaches zero as G grows (it is strictly positive iff $\bar{\sigma}<1$). So the left-hand side grows unboundedly and approximately proportionally (with an elasticity of one) as scope G widens. This implies that the distribution of exporter scope G approximates a Pareto distribution in the upper tail of the ϕ distribution. Using the functional forms in (12) and (13) yields the solutions for ϕ^*_{isd} and G^*_d.

To obtain (25) and (26), use the functional forms in (15) and (16), respectively, and note that the last factor on the right-hand side changes with a scope elasticity of $\bar{\alpha}/(G^{\bar{\alpha}}-1)$, which approaches zero as G grows. So the right-hand side of (25) changes with an elasticity of approximately $\delta + \bar{\alpha}$ and the right-hand side of (26) changes with an elasticity of approximately $\delta + \bar{\alpha} - 1$ in the upper tail of the ϕ distribution. This implies that the distributions of total firm exports and average product scale approximate Pareto distributions in the upper tail.

The Pareto shape parameters follow because for a Pareto distributed random variable ϕ with shape parameter θ and location parameter ϕ^*_{isd}, the transformed random variable $x = A\,(\phi)^B$ is Pareto distributed with shape θ/B and location $A\,(\phi^*_{isd})^B$. To see this, apply the change of variables theorem to $\phi(x) = (x/A)^{1/B}$ and the Pareto probability density function $\mu(\phi)$ to find that $\int_a^b \mu(\phi)\,\mathrm{d}\phi = \int_{x(a)}^{x(b)} \mu(\phi(x))\phi'(x)\,\mathrm{d}x = \int_{x(a)}^{x(b)} (\theta/B)\,[A(\phi^*_{isd})^B]^{\theta/B}/(x)^{\theta/B+1}\,\mathrm{d}x$. ∎

Proposition 3 matches Facts 1 through 2 for specific parameters. $G_{isd}(\phi)$ approaches a Pareto distribution in the upper tail because the second factor on the left-hand side of (22) converges to a constant as $G_{isd}(\phi)$ grows. If $\bar{\sigma}$ approaches one, that is if products within the product mix become just as substitutable among each other as with outside goods, the second factor on the left-hand side of (22) vanishes and exporter scope is Pareto distributed at any level. With $G_{isd}(\phi)$ being Pareto distributed in the upper tail, total firm exports are approximately Pareto distributed in the upper tail.

For total firm exports, the shape parameter $\theta/(\sigma-1)$ in the upper tail is the same power as in Eaton et al. (2008). In our model, $G_{isd}(\phi)$ is Pareto distributed in the upper tail with shape $\theta\,(\delta+\bar{\alpha})/(\sigma-1)$ and total firm exports are proportional to $G_{isd}(\phi)^{\delta+\bar{\alpha}}$ in the upper tail so that the Pareto shape parameter for total firm exports becomes $\theta/(\sigma-1)$. The Pareto distribution of total firm exports also resembles the model by Chaney (2008).

For exporter scope, if the market-entry cost parameter for the first shipment $\gamma_i\kappa_d$ and the scope-dependent market-entry cost parameter $\gamma_i\gamma_d$ vary proportionally across destinations so that κ_d/γ_d is roughly constant then the median exporter scope varies little across markets, just as Table 1 suggests (also see Appendix F).

Proposition 3 also matches Fact 3 for a sufficiently convex total of market-entry and product-entry costs. Average product scale $z_{isd}(\phi)$ and exporter scope $G_{isd}(\phi)$ are positively associated iff $\delta + \bar{\alpha} > 1$, that is iff there are dis-economies of scope in total entry cots. The intuition is that a firm equates the marginal profit from introducing an additional good with the marginal costs of product entry. Under a constant elasticity of substitution, the marginal profit of an additional good is a constant fraction $1/\sigma$ of the good's sales. So, all firms would have identical average sales per product if marginal product entry costs were constant. When the total of marginal product-entry costs and fixed market-entry costs increases in exporter scope, which is the case for $\delta + \bar{\alpha} > 1$, only more productive firms will choose wider exporter scope because only they command sufficiently profitable average sales per good in equilibrium.

Statements in proposition 3 are closely related to Figure 2. The shape parameters of average product scale and exporter scope differ by a factor of exactly $\delta+\bar{\alpha}-1$ so that the ratio of the slopes of the respective cumulative Pareto curves in Figure 2 provides an estimate for $\delta+\bar{\alpha}-1$. By the properties of the Pareto distribution, the probability that an active firm is at the $(1-\mathrm{Pr})$-th percentile in the productivity distribution or above, attaining a productivity of at least ϕ_0, is

$$1-\mathrm{Pr} = 1 - F_{isd}(\phi_0) = \left(\frac{\phi_0}{\phi^*_{isd}}\right)^{-\theta}.$$

For the upper tail of the exporter scope distribution, this implies that the mean exporter scope $\bar{G}_{sd}(1-\mathrm{Pr})$ of firms at the $(1-\mathrm{Pr})$-th percentile or above is

$$\bar{G}_{sd}(1-\mathrm{Pr}) = A_d \left(1-\mathrm{Pr}\right)^{-\frac{\sigma-1}{\theta(\delta+\bar{\alpha})}} \quad \text{with } A_d \equiv G^*_d \left[1-(G^*_d)^{-\bar{\alpha}}\right]^{\frac{1-\bar{\sigma}}{\delta+\bar{\alpha}}} \tag{27}$$

by (22) because the second factor on the left-hand side of (22) converges to a constant as $G_{isd}(\phi)$ grows. Figure 2 plots average exporter scope $\bar{G}_{sd}(1–\mathrm{Pr}) = A_d\,(1–\mathrm{Pr})^{-1/\theta_G}$ in logs, after reverting the horizontal axis, where the power is the negative of the inverse of the Pareto shape parameter $\theta_G = \theta\,(\delta+\bar{\alpha})/(\sigma-1)$. Similar relationships hold for the total exports and average product scale distributions.

3.5 Predictions for the sales of goods within firms

Using the functional forms under Assumption 1 in (11) yields

$$p_{sdg}(\phi; g)\, x_{isdg}(\phi; g) = \frac{\sigma\, \gamma_i \gamma_d\, w_d}{\bar{\sigma}} \left[G_{isd}(\phi)\right]^{\delta+\bar{\alpha}} g^{-(1+\bar{\alpha})} \quad \text{for } g \leq G_{isd}(\phi). \tag{28}$$

The relationship matches Fact 4 ($\bar{\alpha} > 0$).

3.6 Marketing heterogeneity

We have so far limited our attention to firm-level manufacturing productivity as a single source of firm heterogeneity. We now broaden the view in preparation of empirical analysis and allow for a second source of heterogeneity. So far, a firm learns upon entry its unique productivity parameter ϕ and gets to know the individual ordering of its goods for every destination. In addition, we now let a firm also learn its individual marketing productivity μ_ϕ, which reduces market-entry costs. The subscript indicates that the firm's stochastic marketing productivity μ_ϕ may be (positively) correlated with its manufacturing productivity ϕ.

Concretely, let fixed and scope-dependent market-entry cost be

$$F_{id}(G_{isd}) = \frac{\gamma_i \kappa_d\, w_d}{\mu_\phi} + \frac{\gamma_i \gamma_d\, w_d}{\delta\, \mu_\phi} \left(G_{isd}\right)^\delta \qquad \text{where} \quad \gamma_i \gamma_d > 0$$

in terms of destination-country labor units at destination d. Then average product scale at destination d becomes

$$z_{isd}(\phi, \mu_\phi) \equiv \frac{t_{isd}(\phi, \mu_\phi)}{G_{isd}(\phi, \mu_\phi)} = \frac{\sigma\, \gamma_i \gamma_d\, w_d}{\bar{\sigma}\, \bar{\alpha}} \left[G_{isd}(\phi, \mu_\phi)\right]^{\delta + \bar{\alpha} - 1} \cdot \left[1 - \left(G_{isd}\left(\phi, \mu_\phi\right)\right)^{-\bar{\alpha}}\right] \frac{1}{\mu_\phi} \tag{29}$$

and sales of goods within firms become

$$p_{sdg}(\phi, \mu_\phi; g)\, x_{isdg}(\phi, \mu_\phi; g) = \frac{\sigma\, \gamma_i \gamma_d\, w_d}{\bar{\sigma}} \left[G_{isd}(\phi, \mu_\phi)\right]^{\delta + \bar{\alpha}} g^{-(1+\bar{\alpha})} \cdot \frac{1}{\mu_\phi}. \tag{30}$$

The former relationship implies that the positive scale-scope association (Fact 3) is weaker for firms with strong marketing productivity. If manufacturing productivity and marketing productivity are positively correlated, then the scale-scope association could turn negative in firm-level data (unless firm-fixed effects control for μ_ϕ in the regression). The intuition is that, even if product- and market-entry costs increase more than proportionally as exporter scope widens (so that only firms with high manufacturing productivity choose wider exporter scope in equilibrium), simultaneously high marketing productivity may overturn the positive association and allow firms of any production scale per product to seek wide exporter scope.

4 Empirical Evaluation of Model Implications

We evaluate four main aspects of the model. First, we test the prediction that an exporter's global characteristics fully account for the firm's local exporter scopes. Second, we test how closely the assumption of Pareto distributed exporter scope and product scale in the upper tail fits the data. This test also provides us with a first estimate for the elasticity of total market-entry and product-entry costs with respect to scope. Third, we turn to the pristine three-dimensional firm data, without imposing any ranking on the exporters as in the Pareto test, and use the relationship between average product scale and exporter scope to estimate the scope elasticity of total entry costs. This third evaluation also affords us with a test of the prediction that exporter scope

Table 2: Log Exporter Scope and Local Total-Exports Percentile Correlations

Log # Products	Brazil			Chile		
estimator	OLS	OLS	Firm FE	OLS	OLS	Firm FE
controls		Dest eff.	Dest. eff.		Dest eff.	Dest. eff.
	(1)	(2)	(3)	(4)	(5)	(6)
Log Local total-exp. percentile	.393	.394	5.11e-25	.247	.244	-2.32e-26
	(.006)	(.006)	(9.00e-15)	(.011)	(.011)	(3.36e-15)
Observations	68,055	68,055	68,055	12,423	12,423	12,423
Firm panels			10,209			4,091
R^2 (within for FE)	.054	.118	0	.040	.085	0

Sources: Brazilian *SECEX* 2000 and Chilean customs data 2000 (Álvarez et al. 2007), manufacturing firms and their manufactured products.
Note: Aggregation to exports by firm and destination. Products at the Harmonized-System 6-digit level. R^2 is within fit for FE regressions in columns 3 and 6. Standard errors in parentheses.

and average product scale are associated within every destination market because of repeated local market-entry costs. Finally, we use the pristine three-dimensional data at the product level to estimate the relationship between an individual product's scale, the firm's exporter scope and the good's sales rank within the firm and destination. The final exercise also allows us to obtain individual estimates for the scope elasticity of variable product-entry on the one side and the scope elasticity of fixed market-entry costs on the other side.

Prediction 1: Firm-level determinants of exporter scope are identical across destinations. We take the natural log of exporter scope (27) and augment the relationship by a disturbance $\epsilon_{d\phi}$,

$$\ln G_{d\phi} = \ln A_d + \tfrac{\sigma-1}{\theta(\delta+\bar{\alpha})} \ln(1-\Pr_{d\phi}) + \epsilon_{d\phi}.$$

To estimate the relationship, we pool all firms' log exporter scopes $\ln G_{d\phi}$ and log percentiles $\ln(1-\Pr_{d\phi})$ over destinations for one source country at a time. A firm's percentile $(1-\Pr_{d\phi})$ is its local rank in total exports at the destination across all industries. The model implies that an exporter's global characteristic (ϕ/ϕ^*_{isd}), that is the exporter's global ranking relative to its competitors from the same industry and source country (Brazil or Chile), fully explains all local rankings of the firm relative to its source-country competitors. We use destination indicators to estimate $\ln A_d$.

Table 2 shows in columns 1 and 4 the coefficient estimates from a regression of local log exporter scope on the firm's local log rank and a constant, omitting destination indicators in this first exercise. The coefficient on the firm's local log rank is positive as the model predicts and statistically significant. Including destination indicators in the regression does not alter this finding as a comparison to columns 2 and 5 shows. Destination-fixed effects themselves improve the goodness fit, but the low overall R^2

documents that much idiosyncratic variation unrelated to destination attributes or the firm's local export ranking remains. So suppose, contrary to our model's setup, that it is a firm's destination-specific appeal to consumers ϕ_d that determines its local rank $\ln(\phi_d/\phi^*_{isd})$ and that the firm's global characteristic ϕ cannot explain much local variation. Then the positive association between log scope and destination-specific log rank would remain statistically significant also in a regression that conditions on firm-fixed effects. That, however, is not the case empirically. Table 2 shows in columns 3 and 6 that a global firm-fixed effect completely wipes out any predictive power of the firm's local log ranks. So, the firm's local exporter scopes are completely predicted by a firm's global characteristic such as ϕ.

The predictive power of a global firm-fixed effect is not perfect, however. The Spearman rank correlation coefficient between the firm's global rank in total exports and all its local ranks is only .577 (.596) for Brazil (Chile). So a good part of a firm's local ranks is not explained by the firm's global rank. Munch and Nguyen (2008) document for Danish exporters to Sweden and Germany in a related exercise that a firm's home-market rank in product sales is not a perfect predictor of the firms' destination-market rank. The regression results in Table 2 show for Brazilian and Chilean exporters, however, that the unexplained deviation in a firm's local rank is not systematically related to the level of its local exporter scope. Similar regressions for total exports and average product scale lead to the same conclusion. In this sense, idiosyncratic deviations in local ranks have no explanatory power for trade patterns beyond a firm's global characteristic.

Prediction 2: Log average product scale and log exporter scope linearly increase in firms' cumulative log total-exports percentiles. We take the natural logs of (27) and the according relationship for (26) in the upper tail and estimate equations

$$\begin{aligned} \ln \bar{G}_{d,(1-\mathrm{Pr})} &= \ln A_d - \tfrac{\sigma-1}{\theta(\delta+\bar{\alpha})} \ln(1-\mathrm{Pr}) + \epsilon^G_{d,(1-\mathrm{Pr})}, \\ \ln \bar{z}_{d,(1-\mathrm{Pr})} &= \ln \tfrac{\sigma\bar{\gamma}\gamma_d\, w_d(\delta+\bar{\alpha}-1)}{\bar{\sigma}\bar{\alpha}} A_d - \tfrac{(\sigma-1)(\delta+\bar{\alpha}-1)}{\theta\,(\delta+\bar{\alpha})} \ln(1-\mathrm{Pr}) + \epsilon^z_{d,(1-\mathrm{Pr})}, \end{aligned}$$

where $\epsilon^G_{d,1-\mathrm{Pr}}$ and $\epsilon^z_{d,1-\mathrm{Pr}}$ are potentially percentile-specific disturbances, using ordinary least squares destination by destination on one hundred percentile observations each. A firm's percentile $(1-\mathrm{Pr})$ is its local rank in total exports at the destination across all industries. Note that the ratio between the log-percentile coefficient in the first and second regression is $1/(\delta + \bar{\alpha} - 1)$ and provides an estimate of the scope elasticity of total entry costs.

Table 3 reports the coefficient estimates. We fit curves for individual countries as in Figure 2, and an average relationship for the worldwide aggregate. The goodness of fit is close to one in regressions for individual destinations, the United States and Argentina here (columns 1, 2, and 4, 5). It is a definition of the Pareto distribution that the

Table 3: Log-Linear Fits of Cumulative Scope and Average Scale Distributions

From source s to destination d	Brazil			Chile		
	USA (1)	Argentina (2)	World (3)	USA (4)	Argentina (5)	World (6)
Log # Products						
Log Percentile $(1 - \text{Pr})$	-.479	-.540	-.417	-.175	-.273	-.145
	(.002)	(.003)	(.006)	(.008)	(.008)	(.010)
R^2	.998	.996	.402	.828	.917	.076
Log exports/product						
Log Percentile $(1 - \text{Pr})$	-.422	-.357	-.469	-.733	-.594	-.752
	(.006)	(.002)	(.013)	(.009)	(.004)	(.027)
R^2	.979	.996	.165	.985	.996	.223
Implied scope elasticity of total entry cost $(\delta + \bar{\alpha})$	1.882	1.661	2.123	5.179	3.180	6.187

Sources: Brazilian *SECEX* 2000 and Chilean customs data 2000 (Álvarez et al. 2007), manufacturing firms and their manufactured products.
Note: Ordinary-least-squares regressions of firms' mean scope at given percentile or above and firms' mean product scale (the scope-weighted arithmetic mean of exporters' average product scales $\bar{z}_d$ in US$ thousand fob) at given percentile or above on log percentile $\ln(1-\text{Pr})$ and a constant, using one hundred percentile observations per destination (Figure 2). World includes only destinations with more than 100 source-country firms (70 countries for Brazil, 28 for Chile); destination observations weighted by total exports. Products at the Harmonized-System 6-digit level. Standard errors in parentheses.

relationships should be linear. In regressions with higher-order terms (not reported), deviations from linearity are statistically significant. But these deviations are of little economic relevance. A goodness of fit of close to one indicates that deviations from the Pareto distribution are of minor importance for predicting exporter distributions.

For Brazilian manufacturers, the coefficient in the exporter-scope regression is around -.5 and implies that the Pareto shape parameter of exporter scope is around 2. Similarly, the coefficient in the average-product-scale regression of about -.4 implies that the Pareto shape parameter of average product scale is around 2.5 for Brazil. The ratio of the two regression coefficients implies a scope elasticity of total entry costs $(\delta + \bar{\alpha})$ of between 1.7 and 1.9 (2.1 in the world aggregate). So, overall there are dis-economies of scope in entry (total entry costs are convex in scope). The degree of convexity is remarkably similar across destinations for Brazil. The relationships are less robust for Chile. In all cases, however, the estimate for $(\delta + \bar{\alpha})$ exceeds one.

Prediction 3: The average-product-scale and exporter-scope association is explained by within-firm and within-destination variation. Figure 2 illustrates, and results in Table 3 document, that average product scale and exporter scope are positively associated. The stable scope elasticity of total entry costs across desti-

Table 4: Decomposition of Product Scale and Exporter Scope Correlations

Log Exp./prd.	Firms[a]	Firm-destination data[b]				Firm-destination-product data[c]		
estimator	OLS	OLS	OLS	Ind. FE	Firm FE	Firm FE	OLS	Firm FE
controls			Dest.	Dest.	Dest.	Dest.	Dest., prd.	Dest., prd.
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Brazilian Producers exporting Manufactures								
Log # Prod.	.341	-.160	-.068	.072	.260	1.180	.336	.977
	(.022)	(.011)	(.011)	(.011)	(.013)	(.014)	(.014)	(.014)
Obs.	10,215	46,208	46,208	46,208	46,208	76,964	76,964	76,964
Panels				259	10,215	10,,215		10,215
R^2 (within)	.023	.004	.091	.074	.131	.133	.237	.229
Corr. coeff. Firm FE, Log # Prod.					-.234	-.309		-.279
Scope elasticity of total entry cost $(\delta + \bar{\alpha})$					1.260			
Chilean Producers exporting Manufactures								
Log # Prod.	.135	-.303	-.111		.226	.840	.338	.792
	(.035)	(.024)	(.025)		(.027)	(.028)	(.030)	(.028)
Obs.	4,099	12,777	12,777		12,777	21,142	21,142	21,142
Panels					4,099	4,099		4,099
R^2 (within)	.004	.012	.102		.124	.082	.294	.176
Corr. coeff. Firm FE, Log # Prod.					-.235	-.218		-.182
Scope elasticity of total entry cost $(\delta + \bar{\alpha})$					1.226			
Brazilian Commercial Intermediaries exporting Manufactures								
Log # Prod.	-.109	-.419	-.301	-.184	.055	.845	.344	.757
	(.032)	(.024)	(.025)	(.025)	(.034)	(.024)	(.027)	(.025)
Obs.	2,627	6,265	6,265	6,265	6,265	14,781	14,781	14,781
Panels				70	2,627	2,627		2,627
R^2 (within)	.004	.045	.146	.097	.100	.121	.267	.196
Corr. coeff. Firm FE, Log # Prod.					-.250	-.256		-.244
Scope elasticity of total entry cost $(\delta + \bar{\alpha})$					1.055			

[a] Aggreg.: Firm ($\ln z_{\cdot\phi} = \ln(\sigma\,\bar{\gamma}\gamma_{\cdot}\,w_{\cdot}/\bar{\sigma}\bar{\alpha}) + (\delta{+}\bar{\alpha}{-}1)\ln G_{\cdot\phi} + \epsilon_{\cdot\phi}$).
[b] Aggreg.: Firm and destination ($\ln z_{d\phi} = \ln(\sigma\,\gamma_i\gamma_d\,w_d/\bar{\sigma}\bar{\alpha}) - \ln\mu_\phi + (\delta{+}\bar{\alpha}{-}1)\ln G_{d\phi} + \epsilon_{d\phi}$).
[c] Aggreg.: Firm, dest. and prod. group ($\ln z_{dh\phi} = \ln(\sigma\,\gamma_h\gamma_d\,w_d/\bar{\sigma}\bar{\alpha}) - \ln\mu_\phi + (\delta{+}\bar{\alpha}{-}1)\ln G_{dh\phi} + \epsilon_{dh\phi}$).

Sources: Brazilian *SECEX* 2000 and Chilean customs data 2000 (Álvarez et al. 2007), manufacturing firms as well as commercial intermediaries shipping manufactured products.
Note: Products at the Harmonized-System 6-digit level; product-group fixed effects at the Harmonized-System 2-digit level. Industry fixed effects at the *CNAE* two-digit level for Brazil. Constant, destination fixed and product fixed effects not reported. R^2 is within fit for FE regressions (columns 4, 5, 6 and 8). Correlation coefficient between firm fixed effects and log number of products. Standard errors in parentheses.

nations raises the empirical concern, however, that the regularity may not be driven by repeated convex market-entry costs, destination market by destination market as in the model, but in fact by firm-wide determinants such as production characteristics. To query this issue, we decompose the covariation between average product scale and exporter scope into cross-firm, cross-destination, and within destination-and-firm covariation.

We take the natural log of optimal average product scale (29) as a function of optimal exporter scope in the upper tail and augment the relationship by a disturbance $\epsilon_{d\phi}$:

$$\ln z_{d\phi} = \ln \tfrac{\sigma\,\gamma_i\gamma_d\,w_d}{\bar{\sigma}\,\bar{\alpha}} - \ln \mu_\phi + (\delta+\bar{\alpha}-1)\ln G_{d\phi} + \epsilon_{d\phi}.$$

We estimate the relationship with linear regression, and let industry, destination and firm fixed effects capture $\ln \gamma_i$, $\ln w_d\gamma_d$ and $\ln \mu_\phi$.[19]

We use different levels of aggregation for the analysis. First, we aggregate the three-dimensional firm-destination-product data up to single-dimensional data for firms worldwide. Table 4 shows the regression result at this worldwide aggregate level in column 1. Similar to Bernard et al. (2007) for U.S. data, the regression coefficient for log scale on scope is significantly positive in the Brazilian manufacturer data (upper panel). This is the expected sign given our evidence above (Fact 3). To query the source of this positive covariation between log scale and scope, we turn to two-dimensional firm-destination data. In the absence of any decomposition, the overall correlation between average product scale and exporter scope turns significantly negative (column 2).

We remove destination-specific components that are possibly responsible for a covariation between destination-mean product scale and exporter scope. This raises the coefficient estimate (and the fit) slightly (column 3). The removed negative destination bias suggests the interpretation that destinations with lower product scale $\ln z_{\cdot\phi}$ are typically also destinations with wider exporter scope $\ln G_{\cdot\phi}$, consistent with lower scope-dependent market-entry costs $\gamma_i\gamma_d$ at destinations where beachhead market-entry cost $\gamma_i\kappa_d$ are slightly higher. The destination effects in the scale-scope relationship are in turn correlated with common gravity-equation predictors in an expected way (see Table 6 in the appendix).

Removing the covariation between the industry's mean product scale across its destinations $\ln z_{d\cdot}$ and the industry's mean scope across destinations $\ln G_{d\cdot}$ from the regression raises the coefficient estimate into the positive range (column 4). So industries with wide exporter scope worldwide are typically also industries with low product scale worldwide. The removed negative industry bias is consistent with the interpretation that a source of the positive scale-scope covariation worldwide (in column 1) is the repeated positive scale-scope relationship by destination and by industry (column 4).

Finally, we control for the firm-fixed effect $-\ln \mu_\phi$. So we remove the covariation between the firm's worldwide mean product scale across its destinations and the firm's

[19] The last factor on the right-hand side of (29) converges to one in the upper tail.

worldwide mean scope across destinations. This raises the coefficient estimate even further into the positive range (column 5). The removed negative firm-level bias is consistent with the interpretation that firms that face low product scales over all destinations worldwide are firms that command wide scopes over all destinations worldwide (because of a high marketing productivity μ_ϕ say, which allows wide-scope firms to accept low product scales on average upon export-market entry).[20] The correlation coefficient between a firm's worldwide product scale (the firm-fixed effect) and the firm's scope is $-.234$.

Controlling for destination and firm fixed effects documents that, within destinations and within firms, there is a repeated positive scale-scope association that gives rise to the aggregate positive scale-scope association (column 5), but across destinations and firms the data show a negative scale-scope association (column 2 where cross-destination and cross-firm variation is not decomposed). This decomposition finding is consistent with the interpretation that a firm faces repeated entry costs in each destination market. Moreover, the coefficient in column 5 of around .3 implies a scope elasticity of total entry costs $\delta+\bar{\alpha}$ of around 1.3. So total entry costs are convex in scope also by this measure but the degree of convexity is somewhat smaller than in the regressions of Table 3 for Brazil, where the ranking of firms by total exports imposed additional structure.

A remaining empirical concern is that firms may more frequently adopt product types that offer high product scale so that the positive scale-scope association would be driven by product selection and not by convex market-entry costs. We therefore decompose the covariation between log scale and scope further into sources of covaration within product groups (at the Harmonized System 2-digit level) and across product groups. Columns 5 through 7 in Table 4 show that the positive association between log scale and scope becomes even more pronounced. The convexity in entry costs is significantly stronger within product groups (column 7) than across (column 4 where across and within product-group variation is not decomposed). The product-group effects in the scale-scope relationship are in turn correlated with measures for the products' degree of differentiation in an expected way (see Table 7 in the appendix). So it is not the firms' adoption of high-scale products across product groups but a strong positive scale-scope association within product groups that accounts for the positive scale-scope relationship. The positive covariation within destinations, firms and their product groups in the data is fully consistent with a parametrization of our model where repeated convex market-entry costs destination by destination and firm by firm determine exporter behavior. These patterns are similar for Chile (middle panel).

[20] The expected difference between the ordinary least squares estimate (column 2) and the firm-fixed effects estimator (column 5) is proportional to the correlation between the firm-fixed effects and the mean explanatory variables: $\mathbb{E}[\hat{\beta}^{\mathrm{OLS}} - \hat{\beta}^{\mathrm{FE}}] = \mathbb{E}[(X'X)^{-1}X'A\,\alpha]$, where X is the data matrix, A is a matrix of firm indicators, and α is the coefficient vector of firm-fixed effects.

As a final robustness check, we present in the bottom panel of Table 4 the covariance decomposition for Brazilian commercial intermediaries that export manufactured goods. If our interpretation is correct that dis-economies of scope destination by destination determine exporter behavior, then commercial intermediaries should exhibit a similar correlation decomposition. Indeed, the sign of the scale-scope correlation flips as before from significantly negative (in column 3) to positive when firm-fixed effects are included (in column 5). The magnitude of the within-firm within-destination correlation (column 5) is weaker for commercial intermediaries than for producers, however, and the positive correlation is only significant at the 10-percent level in the much smaller intermediary sample. A possible interpretation for the less pronounced convexity of market-entry costs is that commercial intermediaries may face lower costs of offering wide product ranges of resold goods than producers so that market-entry costs increase less pronouncedly for intermediaries even within product groups (columns 6 to 8).

Prediction 4: Individual product sales increase in exporter scope and drop more than proportionally with the product's rank within the firm. We take the natural log of (30) and augment the relationship by a disturbance $\epsilon_{d\phi g}$:

$$\ln p_{d\phi g}\, x_{d\phi g} = \ln \tfrac{\sigma\, \gamma_i \gamma_d\, w_d}{\bar{\sigma}} - \ln \mu_\phi + (\delta + \bar{\alpha}) \ln G_{d\phi} - (1 + \bar{\alpha}) \ln g + \epsilon_{d\phi g}.$$

Figure 3 illustrates the variation that identifies coefficients in this equation. The scope-elasticity of variable product-entry costs $\bar{\alpha}$ is determined by the slope of the graph. Given $\bar{\alpha}$, the scope-elasticity of fixed market-entry costs δ is pinned down by the intercept. The functional form of (30) restricts the relationship between product sales and the product's rank to be log linear. Figure 3 exhibits some curvature, however. In order to fit the regression equation to the higher-ranked and more important goods, we drop product sales with a value below US$100 from the sample.

Table 5 reports the results for the same succession of fixed-effect decompositions as before. Progressing from column 1 through column 4, the coefficient on log exporter scope increases. This covariance decomposition documents again that destination-specific components (column 2), industry-fixed components (column 3) and a firm-fixed effect (column 4) are negatively correlated with exporter scope on average. This is consistent with the interpretation that destinations, industries and firms with wide exporter scope are typically also destinations, industries and firms with low product scale. Irrespective of decomposition, individual product sales significantly increase in exporter scope.

In the individual product scales regression, the coefficient on log exporter scope is an estimate for the scope-elasticity of total entry cost. Results are similar to the estimates from the scale-scope regressions above: the scope-elasticity of total entry cost for Brazilian manufacturers is 1.205 in column 4 (compared to 1.260 in Table 4), 1.054 for Chilean manufacturers (1.226), and 1.055 for Brazilian commercial intermediaries

Table 5: Individual Product Sales Regressions

Log Exp.		Firm-destination-product data[a]			
	estimator	OLS	OLS	Ind. FE	Firm FE
	controls		Dest.	Dest.	Dest.
		(1)	(2)	(3)	(4)
Brazilian Producers exporting Manufactures					
Log # Products		.827	.889	1.015	1.205
		(.007)	(.007)	(.007)	(.008)
Log Product Rank		-1.911	-1.964	-2.070	-2.179
		(.007)	(.007)	(.007)	(.008)
Obs.		141,163	141,163	141,163	141,163
Panels				259	10,197
R^2 (within)		.370	.433	.433	.526
Scope elast. of total entry cost ($\delta+\bar{\alpha}$)					1.205
Scope elast. of market-entry cost (δ)					.026
Scope elast. of product-entry cost ($\bar{\alpha}$)					1.179
Chilean Producers exporting Manufactures					
Log # Products		.600	.723		1.054
		(.016)	(.016)		(.016)
Log Product Rank		-1.814	-1.832		-2.088
		(.016)	(.016)		(.016)
Obs.		34,024	34,024		34,024
Panels					4,090
R^2 (within)		.328	.375		.501
Scope elast. of total entry cost ($\delta+\bar{\alpha}$)					1.054
Scope elast. of market-entry cost (δ)					-.033
Scope elast. of product-entry cost ($\bar{\alpha}$)					1.088
Brazilian Commercial Intermediaries exporting Manufactures					
Log # Products		.740	.777	.882	1.055
		(.012)	(.012)	(.011)	(.014)
Log Product Rank		-1.518	-1.566	-1.612	-1.764
		(.012)	(.012)	(.011)	(.014)
Obs.		31,326	31,326	31,326	31,326
Panels				70	2,616
R^2 (within)		.389	.458	.461	.645
Scope elast. of total entry cost ($\delta+\bar{\alpha}$)					1.055
Scope elast. of market-entry cost (δ)					.291
Scope elast. of product-entry cost ($\bar{\alpha}$)					.764

[a]Agg.: Firm, dest. and prod. ($\ln p_{d\phi g} x_{d\phi g} = \ln \sigma\gamma_i\gamma_d w_d/\bar{\sigma} - \ln \mu_\phi + (\delta{+}\bar{\alpha}) \ln G_{d\phi} - (1{+}\bar{\alpha}) \ln g + \epsilon_{d\phi g}$).

Sources: Brazilian *SECEX* 2000 and Chilean customs data 2000 (Álvarez et al. 2007), manufacturing firms as well as commercial intermediaries shipping manufactured products, dropping product sales with a value below US$100.
Note: Products at the Harmonized-System 6-digit level. Industry fixed effects at the *CNAE* two-digit level for Brazil. Constant and destination fixed effects not reported. R^2 is within fit for FE regressions (columns 3 and 4). Standard errors in parentheses. F tests on implied scope elasticities (δ and α) show significance at the 1-percent level in all cases.

(coinciding with 1.055 in Table 4). So, firms encounter overall dis-economies of scope market by market.

The coefficient on the log product rank exceeds one in absolute value. So, individual product sales drop more than proportionally with the product's rank within the firm. This also validates our parametrization of the antiderivative of the marginal-cost schedule (the average productivity of goods) for $\bar{\alpha} > 0$. The coefficient estimate for the log product rank finally permits a separation between the scope elasticity of variable product-entry cost and the scope-elasticity of fixed market-entry cost.

For Brazilian manufacturers and commercial intermediaries, the scope-elasticity of fixed market-entry costs δ is clearly below one. So, the exporter encounter economies of scope in fixed market-entry costs, and repeatedly so destination by destination. For Chilean exporters, the scope-elasticity estimate is slightly below zero, a finding not consistent with our theory. In ongoing reseearch, we are extending our theory to allow for a relationship between product sales and the product's rank that is not log linear but exhibits the observed curvature (Figure 3); we will estimate the extended functional form with non-linear least squares.

The economies of scope in fixed market-entry costs, however, are more than outweighed by dis-economies of scope in variable product-entry costs ($\bar{\alpha} > 1$) for manufacturers. Brazilian commercial intermediaries, in contrast, face economies of scope in both fixed market-entry and variable product-entry costs individually, but the total scope-elasticity $\delta + \bar{\alpha}$ exceeds one and thus amounts to overall dis-economies of scope. Commercial intermediaries have no production of their own so that the strictly positive scope-elasticity of their variable product-entry costs can be viewed as consistent with the idea that the dis-economies of scope in product-entry costs are at least partly related to the distribution of goods.

5 Conclusion

The extensive margin of introducing additional export goods offers new insight into exporter behavior. Data on the universe of exporters in Brazil, their sales destinations and individual products, document that an exporter's number of products (the exporter scope) and the exporter's average sales per product (the product scale) are jointly Pareto distributed in the upper tail and positively associated destination by destination.

We introduce a heterogeneous-firm model where exporters face variable product-entry costs and fixed market-entry costs. The model preserves main predictions of prior heterogeneous-firm models of trade, such as a single elasticity of exporting and the extensive margin of firm entry. Structural estimates suggest that manufacturers encounter economies of scope in fixed market-entry costs but even stronger dis-economies of scope in variable product-entry costs so that manufacturers face overall dis-economies of scope in each destination market. This explains the repeated positive scale-scope association destination by destination. Commercial intermediaries, in contrast, face dis-economies of scope both in fixed market-entry and variable product-entry costs. Since intermediaries have no production, these findings suggest that dis-economies of scope occur in the distribution of goods. The empirical evidence and the model suggest that, beyond production-side explanations, distribution-side features are important determinants of exporter behavior.

Appendix

A Optimal markup choice

Maximizing the firm's constrained objective function

$$\max_{P_{isd},\{p_{sdg}\}_{g=1}^{G_{isd}}} \pi_{isd}(\phi) + \lambda \left(P_{isd}(\phi; G_{isd}) - \left[\int_0^{G_{isd}} p_{sdg}(\phi)^{1-\varepsilon} \mathrm{d}g \right]^{\frac{1}{1-\varepsilon}} \right),$$

where profit $\pi_{isd}(\phi)$ is given by (7), yields the first order conditions

$$p_{sdg}(\phi) = \frac{\varepsilon\, y_d L_d}{(\varepsilon - 1)\, y_d L_d + \lambda\, (P_d)^{1-\sigma}\, P_{isd}(\phi; G_{isd})^{\sigma}} \frac{\tau_{sd}\, w_s}{\phi} \tag{31}$$

for individual product prices p_{sdg} and the first-order condition

$$\lambda(\phi) = -(\varepsilon - \sigma)\, G_{isd} \left(p_{sdg}(\phi)^{1-\varepsilon} - \tau_{sd} \frac{w_s}{\phi}\, p_{sdg}(\phi)^{-\varepsilon} \right) \frac{P_{isd}(\phi; G_{isd})^{\varepsilon-\sigma-1}}{(P_d)^{1-\sigma}}\, y_d L_d \tag{32}$$

for the product-mix price P_{isd}, where λ is the Lagrange multiplier on the constraint ($\lambda < 0$ iff $\varepsilon > \sigma$) and G_{isd} is given. Note that prices $p_{sdg}(\phi)$ are identical in optimum for all products of a firm by (31). To solve out for optimal individual prices, guess that $p_{sdg}(\phi) = \eta(\cdot)\, \tau_{sd}\, w_s/\phi$ for some markup $\eta(\cdot)$, which could be a function of any variable or parameter of the model. Using (32), $\eta(\cdot)$ and the fact that $P_{isd}(\phi; G_{isd}) = (G_{isd})^{1/(1-\varepsilon)} p_{sdg}(\phi)$ in (31) implies that

$$\eta(\cdot) = \frac{\sigma}{\sigma - 1} \quad \text{so that} \quad p_{sdg}(\phi) = \frac{\sigma}{\sigma - 1} \frac{\tau_{sd}\, w_s}{\phi}.$$

B Necessary and sufficient conditions for Pareto distributions in upper tail

Consider a firm's productivity ϕ to be drawn from a Pareto distribution with a source-country dependent location parameter b_s and a worldwide shape parameter θ over the support $[b_s, +\infty)$ for $s = 1, \ldots, N$. To match the distributional evidence, it is necessary to restrict the limiting behavior of average productivity and marginal market-entry costs. We require that the average productivity of goods and the marginal market-entry costs converge to power functions in the limit.

Assumption 2 *Average productivity of goods $H_d(G)$ and marginal market-entry costs $F'_{id}(G)$ converge to exponential functions as G grows:* $\lim_{G\to\infty} H_d(G) = c_H + a_H(G)^{b_H}$ *and* $\lim_{G\to\infty} F'_{id}(G) = a_F(G)^{b_F}$ *for real numbers a_H, a_F, b_H, b_F, c_H.*

The conditions in Assumption 2 are implied by the specific functional forms in Assumption 1.

Proposition 4 *Suppose the total of market-entry and product-entry costs is sufficiently convex so that the second-order condition for optimal exporter scope is satisfied. Then for all* $s, d \in \{1, \dots, N\}$ *exporter scope* $G_{isd}(\phi)$*, total sales* $t_{isd}(\phi)$ *and average product scale* $z_{isd}(\phi)$ *are Pareto distributed in the upper tail iff Assumption 2 holds and* ϕ *is Pareto distributed.*

Proof. Note that for a Pareto distributed random variable ϕ with shape parameter θ and location parameter ϕ^*_{isd}, the transformed random variable $x = A\,(\phi)^B$ is Pareto distributed with shape θ/B and location $A\,(\phi^*_{isd})^B$. To see this, apply the change of variables theorem to $\phi(x) = (x/A)^{1/B}$ and the Pareto probability density function $\mu(\phi)$ to find that $\int_a^b \mu(\phi)\,\mathrm{d}\phi = \int_{x(a)}^{x(b)} \mu(\phi(x))\phi'(x)\,\mathrm{d}x = \int_{x(a)}^{x(b)} (\theta/B)\,[A(\phi^*_{isd})^B]^{\theta/B}/(x)^{\theta/B+1}\,\mathrm{d}x$. So, under Assumption 2 on the functional forms of $H(G)$ and $F'(G)$, $G_{isd}(\phi)$ is Pareto distributed by (14). For $G_{isd}(\phi)$ being Pareto, $t_{isd}(\phi)$ and $z_{isd}(\phi)$ are Pareto by (15) and (16). Conversely, inspection of (14) shows that $G_{isd}(\phi)$ is Pareto distributed only if $H(G)$ and $F'(G)$ satisfy Assumption 2. ■

Together, propositions 1 and 4 match Facts 1 through 3.

C Exports data for Brazil

The Brazilian customs office *SECEX* (*Secretaria de Comércio Exterior*) collects and compiles export reports by product code at the plant, month and *NCM* (*Nomenclatura Comum do Mercosul*) level. *NCM* coincides with the Harmonized System at the 6-digit level. We use the year 2000 only and aggregate the data to the firm, destination and Harmonized-System 6-digit level. This facilitates comparisons to other Brazilian and international data sources.[21]

We map destination information from Brazilian country codes into the international ISO system. Product codes at the 6-digit level in the Brazilian data include codes in the 999000s, for which there exist no corresponding Harmonized System entries. These codes are not closely related to traded merchandize and relate to entries such as on-board aircraft consumption of combustibles or merchandize for non-financial rental. We remove the codes from the data. To compare our data to sector-level product-market information by destination country, we map the Harmonized System 6-digit codes to *ISIC revision 2* at the two-digit level.[22]

[21] The aggregation is comparable to export-country studies at the six-digit Harmonized System level such as Feenstra (1994) or Hummels and Klenow (2005), and to firm-level studies such as Eaton et al. (2004).

[22] Our novel concordance will become available at URL *econ.ucsd.edu/muendler/brazil* shortly.

D Exports data for Chile

The Chilean data for the year 2000 are courtesy of Álvarez et al. (2007) and derive from the universe of Chilean customs declarations for merchandize exports, similar to the Brazilian *SECEX* data. The Chilean customs authorities collect the reports by firm and Harmonized System eight-digit code. We aggregate the pristine eight-digit Harmonized System information to information by exporting firm at the six-digit Harmonized System level. This ensures comparability to the Brazilian data (and international sources, as mentioned above).

We map destination country names into the international ISO system. The sector affiliation of Chilean exporters is reported at *ISIC revision 2* three-digit level. We use the *ISIC revision 2* for the export firm from the original data. Robustness checks using product-level information for sector affiliates from the Harmonized System six-digit level and using the *ISIC revision 2* product code of the top selling product for the firm do not yield substantively different results.

E Auxiliary data for Brazil and Chile

Trade flow data by industry and destination. We link the firm-level product and destination information for Brazil and Chile to *WTF* (*World Trade Flow*) data for the year 2000 (Feenstra et al. 2005). We extract sector-level trade flow statistics in current US$ for Brazil's and Chile's export destination markets. For Brazil, we map the *SITC Rev. 2* four-digit sector information to the *SITC Rev. 2* two-digit level, and then to the two-digit *ISIC revision 2* level for combination with *SECEX*. For Chile, we map the *ISIC revision 2* information at the three-digit level to the two-digit *ISIC revision 2* level for combination exports data.

Output data by industry and destination. We obtain manufacturing output by destination country and manufacturing industry for 2000 from the *Unido* Industrial Statistics Database at the two-digit *ISIC revision 2* level in current US$ (UNIDO 2005). We map the Harmonized System six-digit codes to *ISIC revision 2* at the two-digit level for this purpose.

Country and geographic data by destination. National accounts information for host-country regressors comes from the World Bank's World Development Indicators and the IMF's International Financial Statistics (population, GDP, consumption expenditure and household consumption expenditure in current US$). We use CEPII bilateral geographic data;[23] the data include the mean distance between Brasília or Santiago de Chile on the one hand and foreign capital cities (km) on the other hand,

[23]From *www.cepii.fr/anglaisgraph/bdd/distances.htm.*

common borders with Brazil or Chile, and a common language with Brazil (Portuguese-speaking Angola, China Macão SAR, Guinea Bissau, Mozambique and Portugal) or Chile (Spanish speaking countries).

Goods data. We calculate Balassa (1965) comparative-advantage measures for Brazilian and Chilean goods from UN Comtrade trade data for the year 2000 at the *ISIC rev. 2* four-digit level. Good h's Balassa advantage is

$$BADV_h \equiv \frac{X_h^{\text{Brazil}} / \sum_k X_{k,t}^{\text{Brazil}}}{X_h^{\text{World}} / \sum_k X_{k,t}^{\text{World}}},$$

where X_h are exports. Note that this index measures revealed comparative advantage from international comparisons of exports data, and is blind to possible sources of advantage. Any explanation of comparative advantage is consistent with this measure. We first map the *ISIC rev. 2* information to the Harmonized System six-digit level and then aggregate to the Harmonized System two-digit level by taking the unweighed average across six-digit goods in the Brazilian data.

We use the Rauch (1999) classification of goods by degree of differentiation under Rauch's conservative definition.[24] We first map Rauch's *SITC Rev. 2* four-digit sector information to the Harmonized System six-digit level and then aggregate to the Harmonized System two-digit level by taking the unweighed average across six-digit goods in the Brazilian data.

We reuse the *WTF* data for the year 2000 (Feenstra et al. 2005) to obtain goods-level measures of typical import destinations. For this purpose, we drop Brazilian or Chilean exports and imports from the *WTF* data and calculate for the rest of the world the number of destinations to which goods at the *SITC Rev. 2* four-digit level (Brazil) or the *ISIC rev. 2* three-digit level (Chile) ship, and what import values they exhibit worldwide, in the OECD and Mercosur (Argentina, Paraguay, Uruguay). For Brazil, we map the *SITC Rev. 2* four-digit sector information to the Harmonized System six-digit level and then aggregate to the Harmonized System two-digit level by taking the unweighed average across six-digit goods. For Chile, we just aggregate from the Harmonized System six-digit level to the Harmonized System two-digit level by taking the unweighed average across six-digit goods.

F Comparison to French firm data

A comparison to well known facts about French exporters shows several similarities in market access patterns. Following Eaton et al. (2004), total exports T_{sd} from source country s (France, Brazil) to destination market d can be decomposed into: $M_{sd}\bar{t}_{isd}$,

[24]We use Rauch's revision 2 from 2007 (available at *www.econ.ucsd.edu/∼jrauch/intltrad*)

where M_{sd} is the number of source country s's exporters with shipments to destination country d, and $\bar{t}_{isd}$ are these exporters' average sales in destination country d. The same total exports T_{sd} can also be decomposed into: $\lambda_{sd}T_d$, where λ_{sd} is the market share of source country s's exports in destination d, and T_d is the market size of destination country d (manufacturing absorption). By definition, $M_{sd}\bar{t}_{isd} = \lambda_{sd}T_d$. We regress the log number of firms on the log of $\lambda_{sd}T_d$ to inspect how these market characteristics are associated with the market presence of additional firms M_{sd} (as opposed to additional sales per firm $\bar{t}_{isd}$):[25]

$$\ln M_{sd} = \underset{(1.108)}{-5.710} + \underset{(.065)}{.719} \ln \lambda_{sd} + \underset{(.043)}{.626} \ln T_d.$$

The R^2 is 0.833 (standard errors in parentheses).[26] Firm presence explains most of the variation in Brazilian exports, but is somewhat less important than in France. Given market size and industry bias, a higher Brazilian (French) market share λ_{sd} in a destination typically reflects 72 (88) percent more firms selling there and 28 (12) percent more sales per firm. Given market share, larger market size T_d is associated with 63 (62) percent more firms and 37 (38) percent more sales per firm.

Consider a further decomposition of the left-hand side variable into our main variable of interest, the exporters' mean exporter scope, and the then remaining residual category (the exporters' mean product scale per good). Concretely, $\bar{t}_{isd} = \bar{G}_d\,\bar{z}_d$, where $\bar{G}_d$ is the exporters' mean exporter scope and $\bar{z}_d \equiv \bar{t}_d/\bar{G}_d$ is their goods' mean product scale. We regress the log mean exporter scope $\bar{G}_d$ on the log of $\lambda_{sd}T_d$ to examine how market characteristics are related to exporter scope:

$$\ln \bar{G}_d = \underset{(.676)}{2.324} + \underset{(.057)}{.087} \ln \lambda_{sd} - \underset{(.046)}{.058} \ln T_d.$$

The R^2 is 0.281 (standard errors in parentheses). Neither market share nor market size are statistically significant predictors of exporter scope at conventional levels.[27] So, most of the variation in firms' exports to a market is due to variation in their mean scale per product. Our theory will address the similarity of exporter scope across destinations. Although exporter scope has little explanatory power for exports in descriptive regressions, its association with scale by destination provides important insight into market access costs.

[25]We aggregate the *SECEX* exporter data to 16 SIC industries as in Eaton et al. (2004) for this purpose and link the data to destination information from *WTF* (Feenstra et al. 2005) and *Unido* Industrial Statistics (UNIDO 2005). The regression sample contains 67 destinations in the Brazilian data (excluding the domestic Brazilian market), whereas there are 113 destinations in the French data (including the domestic French market). Gomes and Ellery Jr. (2007) present similar regressions for a sub-sample of *SECEX* exporters linked to firm survey data (*Pesquisa Industrial Anual*) in 1999.

[26]Because of the identity connecting the variables, a regression of $\ln \bar{t}_{isd}$ on the log of $\lambda_{sd}T_d$ yields coefficients of 1 minus the ones reported above.

[27]The R^2 drops to .212 when including $\ln M_{sd}$ and industry-fixed effects but coefficients become statistically significant at conventional levels except for market size, while magnitudes change little.

Table 6: Correlates of Destination Effects on Product Scale and Exporter Scope

	Destination Eff. on Prod. Scale from Log Exp./prod. regressions		Destination Eff. on Exp. Scope from Log # Products regressions	
	(1)	(2)	(3)	(4)
	Brazil	Chile	Brazil	Chile
	Firm & dest. FE, & scope	Firm & dest. FE, & scope	Firm & dest. FE, & scale	Firm & dest. FE, & scale
Mean Log Market size	-.042	-.002	.043	-.003
	(.031)	(.010)	(.058)	(.013)
Log Population	.348	.032	.287	.033
	(.048)	(.014)	(.085)	(.018)
Log GDP per cap.	.287	.028	.291	.005
	(.044)	(.014)	(.086)	(.019)
Log Distance	-.331	-.236	-.462	-.155
	(.138)	(.044)	(.304)	(.057)
Common borders	-.171	.227	-.282	.255
	(.276)	(.081)	(.630)	(.138)
Common language	-.078	.048	.007	.099
	(.300)	(.092)	(.380)	(.085)
Const.	-8.354	1.907	-8.124	1.278
	(1.302)	(.434)	(2.980)	(.562)
Obs.	106	102	94	94
R^2	.56	.574	.401	.396

Sources: Brazilian *SECEX* 2000 and Chilean customs data 2000 (Álvarez et al. 2007), manufacturing firms and their manufactured products, linked to *Cepii* distance data (Mayer and Zignago 2006) and *Unido* Industrial Statistics (UNIDO 2005).
Note: Aggregation to exports by firm and destination. Regressions of destination fixed effects on destination-level predictors, where destination fixed effects on product scale are from a destination fixed effects regression controlling for scope and firm fixed effects (see column 3 in Table 4). Destination fixed effects on exporter scope are from a destination fixed effects regression controlling for scale and firm fixed effects. Mean log market size is average sectoral absorption over *ISIC rev. 2* industries at destination level. Standard errors in parentheses.

Table 7: Correlates of Product Effects on Product Scale and Exporter Scope

	Destination Eff. on Prod. Scale from Log Exp./prod. regressions		Destination Eff. on Exp. Scope from Log # Products regressions	
	(1)	(2)	(3)	(4)
	Brazil	Chile	Brazil	Chile
	Firm, dest. & prd. FE, & scope	Firm, dest. & prd. FE, & scope	Firm, dest. & prd. FE, & scale	Firm, dest. & prd. FE, & scale
Comparative adv.	.186	.010	.012	.004
	(.119)	(.020)	(.037)	(.006)
Reference priced	-2.964	.062	-1.594	.072
	(.881)	(.151)	(.833)	(.142)
Differentiated	-2.031	.125	-1.858	-.007
	(.813)	(.139)	(.761)	(.130)
Log ww. # Dest.	-1.765	.253	-.883	.212
	(.856)	(.147)	(.899)	(.153)
No OECD imp.	21.525	3.034	-6.657	-.991
	(47.204)	(8.093)	(11.326)	(1.929)
Log OECD Imp.^{a}	.544	.012	.234	.108
	(.254)	(.044)	(.228)	(.039)
No Mercosur imp.	-1.661	-.109	1.180	.132
	(2.090)	(.358)	(2.204)	(.376)
Log Mercos. Imp.^{a}	.083	.013	-.042	-.009
	(.213)	(.037)	(.216)	(.037)
Const.	5.304	-.644	-1.826	-.822
	(4.546)	(.779)	(4.824)	(.822)
Obs.	91	91	94	94
R^2	.202	.25	.167	.37

^{a}Log of nonzero imports × indicator.

Sources: Brazilian *SECEX* 2000 and Chilean customs data 2000 (Álvarez et al. 2007), manufacturing firms and their manufactured products, linked to *WTF* (Feenstra et al. 2005) and *Unido* Industrial Statistics (UNIDO 2005).
Note: Aggregation to exports by firm, destination, product group (Harmonized System 2-digit level). Regressions of product fixed effects at the Harmonized-System 2-digit level on product-level predictors, where product fixed effects on product scale are from a product fixed effects regression controlling for scope as well as destination and firm fixed effects (see column 6 in Table 4). Product fixed effects on exporter scope are from a product fixed effects regression controlling for scale as well as destination and firm fixed effects. Balassa (1965) comparative-advantage for Brazil from UN Comtrade trade data for 2000 at the *ISIC Rev. 2* level: product h's comparative advantage is $BADV_h \equiv [T_h^{\text{Brazil}} / \sum_k T_k^{\text{Brazil}}]/[T_h^{\text{World}} / \sum_k T_k^{\text{World}}]$, where T_h are worldwide exports. Goods classification by degree of differentiation from Rauch (1999), conservative definition, revision 2 (2007): share of Harmonized-System 6-digit goods at the Harmonized-System 2-digit level; omitted benchmark category is homogeneous goods (traded on an organized exchange). Worldwide product-group imports exclude Brazil as importer and exporter. Standard errors in parentheses.

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