Carbon leakage, the green paradox and perfect future markets*

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Abstract

Policies of lowering carbon demand may aggravate rather than alleviate climate change. This possibility is called the 'green paradox' (Sinn 2008). In a two-period three-country general equilibrium model with profit-maximizing suppliers of non-renewable fossil fuel we analyze carbon leakage and the green paradox. One country supplies fossil fuel and the other two consume it. Only one of the latter enforces (and tightens) an emissions cap either in the first or in the second period. The extent of carbon leakage turns out to depend on the interaction of various parameters and elasticities. All determinants of carbon leakage resulting from tightening the first-period cap work in opposite direction when the second-period cap is tightened. No unambiguous support is found for the proposition that tightening the second-period cap necessarily leads to the green paradox.

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Key words: carbon leakage, green paradox, emissions cap

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1 The problem

Growing scientific evidence (IPCC 2007) suggests that we cannot stabilize the world climate at safe levels unless we substantially slow down the world emissions of greenhouse gases. A number of countries have already increased their efforts to curb emissions, notably the (Annex 1) countries that committed to emissions reductions in the Kyoto Protocol. Yet many small and large countries still refrain from taking (strong) action. That raises the question what the chances are of a subset of abating countries to bring down world emissions to safe levels.

We will address this issue by restricting our focus on carbon dioxide that is the most important greenhouse gas. Carbon dioxide emissions (emissions, for short) are generated almost in proportion to burning fossil energy resources (fossil fuel, for short) which are still the dominant source of energy consumption. Any national policy of curbing emissions is bound to raise domestic energy costs and thus enables firms in non-abating countries to expand. For that reason, the effort of abating countries will be offset to some extent by increasing emissions in non-abating countries. That phenomenon has come to be known as *carbon leakage*. Since it is the aggregate world emissions that determine the damage from climate change, the net emissions cutback by a group of abating countries is smaller than that group's gross emissions reductions. It is conceivable that the induced emissions increase in non-abating countries is equal to or even greater than the gross emissions reduction achieved by the group of abating countries. The extreme case in which demand-reducing measures of abating countries increase rather than reduce aggregate world emissions, as compared to their level in the absence of abatement efforts, is referred to as *green paradox* by Sinn (2008).

High rates of carbon leakage would cast serious doubt on the effectiveness of any subglobal abatement strategy as represented, e.g., by the Kyoto approach. Since it is unclear at present whether an effective post-Kyoto agreement will be reached over the next years mandating strong action for *all* major carbon emitting countries, it is important to have a good understanding of the key mechanisms underlying carbon leakage.

The bulk of research on carbon leakage has been carried out in (large-scale) CGE analyses. According to Burniaux and Martins (2000) the estimates of such models range from leakage rates of 20% to lower bound estimates of 2% to 5%. The IPCC (2007) estimates the leakage effect in about the same range for the climate policy based on the Kyoto Protocol. Burniaux and Martins (2000) conclude from their extensive sensitivity analysis (ibidem, p. 13) that "... carbon leakages are small for the range of parameters most frequently quoted in the literature ...", and they emphasize that this assessment strongly relies on the assumption that the supply of coal is fairly elastic over the medium term. Felder and Rutherford (1993), Paltsev (2001), Babiker (2005), Gerlagh and Kuik (2007), Marschinski et al. (2008) and others provide further informative insights into various channels and determinants of carbon leakage.

The prevailing view of relatively modest leakage rates is challenged by a line of research in the area of (intertemporal) theory of nonrenewable natural resources that takes as its point of departure an extraction path of fossil fuel that is suboptimally steep in laissez-faire e.g. because of the global warming externality (Sinn 1982; Sinclair 1994; Hoel and Kverndokk 1996; Farzin 1996; Rubio and Escriche 2001; Sinn 2008). In models that differ with respect to their assumptions on market power and strategic behavior the question is addressed as to what the potential is of various kinds of taxation to restore efficiency by flattening the extraction path. Under various qualifications a major though not undisputed result is that carbon taxes tend to have little impact on the time profile of extraction and that the extraction path is steepened if tax rates rise in time. Accordingly, Sinn's (2008, p. 360) verdict is that "... if suppliers feel threatened by a gradual greening of (demand-reducing) policies in the Kyoto countries that would damage their future prices; they will extract their stocks more rapidly, thus accelerating global warming" (Sinn 2008, p. 360). From this perspective the prevailing view on the effectiveness of demand-reducing policies is flawed because the public and academic discourse (including the Stern Review 2006) has largely neglected the close link between the economics of global change and the economics of non-renewable resources and has therefore failed to account for the supply side of the problem in an appropriate way.

The supply-side-literature aggregates all fossil-fuel consuming countries into a single country which amounts to presupposing full cooperation of all countries. Yet the very notion of carbon leakage as introduced above requires distinguishing abating and non-abating countries since carbon emissions leak from the former to the latter, after all.[1] We are not aware of analytical studies that model intertemporal wealth maximizing resource supply and consider, at the same time, the leakage of carbon from the group of abating countries to non-abating countries.

The present paper aims at examining the determinants of carbon leakage and the green paradox in a simple two-period general-equilibrium model that explicitly considers the intertemporal use of fossil fuel as a non-renewable resource in finite supply. We map the

[1]This is not to say that approaches focusing on global centralized emissions control policies are irrelevant for the topic of the present paper. A green paradox can certainly be said to occur when a global policy intending to flatten the extraction path results in steepening it. For the link between that literature and the present paper see also Section 4.0.

prevailing real-world scenario in a stylized way by constructing a three-country economy consisting of a fossil-fuel exporting country, an abating country and a non-abating country. The abating country represents the coalition of countries that have committed to observe binding national emissions caps à la Kyoto and the non-abating country stands for the rest of the world (except the fuel exporting countries) assumed to refrain from taking (strong) action to curb emissions. To keep a clear focus on leakage, we do not deal with environmental damage from carbon emissions and optimal corrective policies.

In that general equilibrium framework we explore the conditions for carbon leakage and the green paradox. We investigate by means of comparative static analysis how much carbon leaks into the non-abating country when the abating country tightens its emissions cap and when the resource supplier follows a (simplified) Hotelling rule. We find that carbon leakage is unavoidable and the green paradox may occur depending on the interplay of demand conditions, in particular the elasticity of intertemporal substitution in demand, and supply conditions, especially the price elasticities of fuel demand. There are parameter constellations under which the green paradox may occur when the emissions constraint is tightened *either* in the first *or* in the second period. The proposition which ties the green paradox to the "gradual greening of (demand-reducing) policies" therefore does not receive unambiguous support from our analysis.[2] When more countries join the coalition of abating countries less carbon tends to leak into the non-abating countries. The incidence of reducing emissions (= tightening the emissions cap) *either* in the first *or* in the second period is shown to be mirror-symmetric: Essentially, parameter constellations under which the green paradox is avoided, when the first-period cap is tightened, tend to generate a green paradox, when the second-period cap is tightened, and vice versa.

The paper is organized as follows. Section 2 sets up the model. Section 3 investigates the determinants of carbon leakage and the green paradox when the abating country tightens its first-period emissions cap. Section 4 explores the effects of enlarging the group of abating countries. In Section 5 the same issues as in Section 3 are addressed for the case that the abating country tightens its second-period emissions cap. Section 6 concludes.

2 The model

Consider a two-period model with three (groups of) countries A, N and F, where A is the abating country, N is the non-abating country and F is the fossil-fuel exporting country.

[2] See also Ulph and Ulph (1994) who show in a different analytical framework that the optimal carbon tax need not necessarily be falling.

Except for their carbon emissions control (see below) the economies of the countries A and N are alike. In period $t = 1, 2$ each country $i = A, N$ produces the output x_{it}^s of the consumption good X with the input e_{it} of fossil fuel according to the increasing and strictly concave production function[3]

$$x_{it}^s = X^i(e_{it}) \qquad i = A, N. \tag{1}$$

The countries A and N import all fossil fuel from country F that is endowed with a stock of fossil fuel, $\bar{e}$. Country F does not produce good X but rather buys that good from the countries A and N paying for those imports with the revenues from exporting fossil fuel.

The representative consumer of country i derives utility from consumption x_{i1} in period 1 and from x_{i2} in period 2 according to the intertemporal utility function

$$u_i = U^i(x_{i1}, x_{i2}) \equiv U(x_{i1}, x_{i2}) \qquad i = A, F, N, \tag{2}$$

which is increasing in both arguments and quasi-concave. The elasticity of intertemporal substitution (in consumption), defined as

$$\sigma_i := \frac{\mathrm{d}\left(\frac{x_{i2}}{x_{i1}}\right)}{\frac{x_{i2}}{x_{i1}}} \cdot \frac{\frac{U_{x_{i1}}}{U_{x_{i2}}}}{\mathrm{d}\left(\frac{U_{x_{i1}}}{U_{x_{i2}}}\right)} \in [0, \infty[,$$

is a property of the utility function that will turn out to play an important role in the subsequent analysis.

In each period, good X and fossil fuel are traded on perfectly competitive world markets (comprising all three countries) at prices p_{xt} and p_{et}, respectively. For $t = 1, 2$ the market clearing conditions are

$$x_{At}^s + x_{Nt}^s = x_{At} + x_{Nt} + x_{Ft}, \tag{3}$$

$$e_{Ft} = e_{At} + e_{Nt}, \tag{4}$$

where e_{Ft} is the fossil fuel supply of country F in period t. Obviously, the supplies e_{Ft} for $t = 1, 2$ need to satisfy the intertemporal constraint

$$\bar{e} = e_{F1} + e_{F2}. \tag{5}$$

The countries A and N differ with respect to their carbon emissions regulation. We envisage an international agreement on reducing carbon emissions like the Kyoto protocol that does not encompass all countries in the world. Country N represents the group of

[3]In (1) the superscript s indicates quantities supplied. Upper case letters denote functions and subscripts attached to them indicate first partial derivatives.

fuel-consuming countries that do not participate in the agreement and hence do not constrain their carbon emissions at all.[4] Country A then represents the group of participating countries. Denote by $\bar{e}_{At}$ the emissions cap country A imposes in period $t = 1, 2$. Country A may cap its emissions either in both periods or in one of the periods only. At present there is already a group of countries capping their emissions. Therefore, we will restrict our focus on the scenarios (i) where a binding cap $\bar{e}_{A1}$ exists but no cap in period 2, i.e.

$$e_{A1} \leq \bar{e}_{A1} \quad \text{and} \quad e_{A2} \text{ unconstrained,} \tag{6a}$$

or (ii) where binding caps exist in both periods, i.e.

$$e_{A1} \leq \bar{e}_{A1} \quad \text{and} \quad e_{A2} \leq \bar{e}_{A2}. \tag{6b}$$

The caps can *either* be imposed directly, *or* through a tax-and-standard scheme *or* through an emissions trading system. Given the high level of abstraction of our model all these policies of implementing an emissions cap are equivalent. To simplify the exposition we refer to emissions trading only in the subsequent analysis assuming that the emissions permits are auctioned at some price $\pi_t, t = 1, 2$, that is determined endogenously.

Each country hosts a representative firm generating the profit

$$\Pi^A := \sum_t \left[p_{xt} X^A(e_{At}) - (p_{et} + \pi_t) e_{At} \right], \tag{7}$$

$$\Pi^N := \sum_t \left[p_{xt} X^N(e_{Nt}) - p_{et} e_{Nt} \right], \tag{8}$$

$$\Pi^F := \sum_t p_{et} e_{Ft}, \tag{9}$$

where $\pi_2 \equiv 0$ if (6a) applies. In (7) - (9) we have not discounted the second-period profits, because in the absence of capital investment the market rate of interest is zero. Moreover, the firm in country F does not incur any extraction costs. While this assumption is not realistic[5] it is not central for the qualitative conclusions to be derived.

The first-order conditions of maximizing (7), (8) and (9) read, respectively,

$$\pi_1 = p_{x1} X^A_{e_{A1}} - p_{e1} > 0 \quad \text{and} \quad p_{x2} X^A_{e_{A2}} = p_{e2}, \tag{10}$$

$$p_{x1} X^N_{e_{N1}} = p_{e1} \quad \text{and} \quad p_{x2} X^N_{e_{N2}} = p_{e2}, \tag{11}$$

$$p_{e1} = p_{e2}. \tag{12}$$

[4]When climate is treated as a global public good, the business-as-usual scenario is commonly modeled as a Nash equilibrium where each country's emissions-reduction policy is the best reply to the other countries' abatement efforts. For the resultant "free-rider leakage" in such an approach see e.g. Carraro and Siniscalco (1993). In our model governments do not play Nash. Instead they do or do not take action depending on their (non)commitment in a Kyoto-type international agreement.

[5]In fact, zero extraction costs tend to favor carbon leakage because it makes the supply of fossil fuel perfectly elastic. For the consideration of stock-dependent extraction costs see Sinn (2008).

We assume $\pi_1 > 0$ in (10) because we consider an emissions cap $\bar{e}_{A1}$ that is strictly binding in the relevant range of equilibrium prices.[6] As noted above, $\pi_2 \equiv 0$ if (6a) applies and $\pi_2 > 0$ if the relevant constraints are given by (6b). In case of $p_{e1} \neq p_{e2}$ the fossil-fuel selling firm would sell all fossil fuel either in the first or in the second period generating an excess demand in that period in which its supply is zero. Hence equation (11) represents a necessary (arbitrage) condition for equilibrium.[7] In equilibrium, the firm is indifferent between selling its fossil fuel in period 1 or 2.

The consumer maximizes utility (2) subject to her budget constraint[8]

$$\sum_t p_{xt} x_{it} = \begin{cases} = \Pi^{A*} + \pi_1 \bar{e}_{A1} & \\ = \Pi^{i*} & \text{for } i = F, N, \end{cases} \tag{13}$$

which yields

$$\frac{U_{x_{i1}}}{U_{x_{i2}}} = \frac{p_{x1}}{p_{x2}} \quad \text{for } i = A, F, N. \tag{14}$$

We have thus completed the description of the model and are ready for studying the impact of policy changes in country A. In the next Section 3 we will consider the policy scenario (6a) and investigate the allocative effects when country A tightens its emissions cap $\bar{e}_{A1}$ ($\mathrm{d}\bar{e}_{A1} < 0$). Section 4 explores the effects of enlarging the group of abating countries and after that we will turn to the scenario (6b) in Section 5 and investigate the impact of the policy changes ($\mathrm{d}\bar{e}_{A1} < 0$ and $\mathrm{d}\bar{e}_{A2} = 0$) as well as ($\mathrm{d}\bar{e}_{A1} = 0$ and $\mathrm{d}\bar{e}_{A2} < 0$).

3 Tightening the emissions cap in the first period

Consider a competitive equilibrium in the three-country model (1) - (5), (6a), (7) - (14) in which the constraint (6a) is strictly binding and suppose the emissions cap $\bar{e}_{A1}$ is tightened:[9] $\hat{\bar{e}}_{A1} := \frac{\mathrm{d}\bar{e}_{A1}}{\bar{e}_{A1}} < 0$. *Carbon leakage* is said to occur if $\hat{e}_{N1}/\hat{\bar{e}}_{A1} < 0$. Carbon leakage is particularly severe, if the reduction of carbon emissions in country A is overcompensated by the (induced) increase in carbon emissions in country N, i.e. if $\hat{e}_{F1}/\hat{\bar{e}}_{A1} < 0$. In such

[6] Sufficient for (10) and (11) are the regularity conditions $\lim_{e_{jt} \to 0} X^j_{e_{jt}} = \infty$ and $\lim_{e_{jt} \to \infty} X^j_{e_{jt}} = 0$ for $j = A, N$ and $t = 1, 2$ which we assume to hold.

[7] The Hotelling rule requires the market rate of interest to equal the rate of increase in the price of the natural resource. Since in our model the market rate of interest is zero by assumption, (12) is a simplified version of the Hotelling rule.

[8] In (12) Π^{i*} is the maximum profit of the firm in country i. The budget constraints can be rearranged to $\sum_t [p_{xt} (x^s_{it} - x_{it}) - p_{et} e_{it}]$ for $i = A, N$ and $\sum_t (p_{et} e_{Ft} - p_{xt} x_{Ft}) = 0$ which turn out to be the countries' intertemporal trade balances.

[9] Throughout the paper the 'hat variables' are defined as $\hat{y} = \mathrm{d}y/y$.

a situation tightening the emissions control in country A increases total carbon emissions in period 1, which is called 'green (policy) paradox'. Country A's effort of fighting global warming actually turns out to promote global warming.[10]

We aim at investigating the conditions under which the green paradox occurs in the analytical framework developed in Section 2. For that purpose we first determine the displacement effect of $\hat{\bar{e}}_{A1} \neq 0$ on the intertemporal market for fossil fuel:[11,12]

$$\hat{e}_{F1} \cdot e_{F1} = \underbrace{\bar{e}_{A1}\hat{\bar{e}}_{A1}}_{[1]} - \underbrace{\frac{p_{e1}e_{N1}\left|\eta_{N1}\right|\bar{e}_{A1}}{\gamma_p - p_{e1}e_{N1}\eta_{N1}}}_{[2]}\hat{\bar{e}}_{A1} - \underbrace{\frac{\gamma_p e_{N1}\left|\eta_{N1}\right|}{\gamma_p - p_{e1}e_{N1}\eta_{N1}}}_{[3]}\hat{p}_{x2} \tag{15a}$$

$$= \frac{\gamma_p \bar{e}_{A1}}{\gamma_p - p_{e1}e_{N1}\eta_{N1}}\hat{\bar{e}}_{A1} - \frac{\gamma_p e_{N1}\left|\eta_{N1}\right|}{\gamma_p - p_{e1}e_{N1}\eta_{N1}}\hat{p}_{x2}. \tag{15b}$$

In (15), $\eta_{N1} := \frac{X^N_{e_{N1}}}{e_{N1}X^N_{e_{N1}e_{N1}}} < 0$ is country $N's$ price elasticity of demand for fossil fuel in period 1 and $\gamma_p := -p_{e1}(e_{A2}\eta_{A2} + e_{N2}\eta_{N2}) > 0$. We are in the position to show

Proposition 1. *$\hat{p}_{x2}/\hat{\bar{e}}_{A1} > 0$, $\hat{p}_{e1}/\hat{\bar{e}}_{A1} > 0$ and $\mathrm{d}e_{F1}/\mathrm{d}\bar{e}_{A1} < 1$ if the utility function is homothetic.*[13]

Proof. Contrary to the ascertion suppose that $\hat{p}_{x2}/\hat{\bar{e}}_{A1} < 0$. In that case (15b) yields $\hat{e}_{F1}/\hat{\bar{e}}_{A1} > 0$ and $\hat{e}_{F2}/\hat{\bar{e}}_{A1} < 0$ (due to $e_{F1}\hat{e}_{F1} + e_{F2}\hat{e}_{F2} = 0$). Differentiation of (1), (4), $q^s := \frac{x^s_{A1}+x^s_{N1}}{x^s_{A2}+x^s_{N2}}$ and then using (10) - (12) yields

$$\hat{x}^s_{A1}x^s_{A1} + \hat{x}^s_{N1}x^s_{N1} = p_{e1}(\hat{\bar{e}}_{A1}\bar{e}_{A1} + \hat{e}_{N1}e_{N1}) + \pi_1\hat{\bar{e}}_{A1}\bar{e}_{A1}, \tag{16}$$

$$\hat{x}^s_{A2}x^s_{A2} + \hat{x}^s_{N2}x^s_{N2} = \frac{p_{e2}}{p_{x2}}(\hat{e}_{A2}e_{A2} + \hat{e}_{N2}e_{N2}), \tag{17}$$

$$e_{Ft}\hat{e}_{Ft} = \hat{e}_{At}e_{At} + \hat{e}_{Nt}e_{Nt} \quad t = 1, 2, \tag{18}$$

$$\hat{q}^s = \frac{\hat{x}^s_{A1}x^s_{A1} + \hat{x}^s_{N1}x^s_{N1}}{x^s_{A1} + x^s_{N1}} - \frac{\hat{x}^s_{A2}x^s_{A2} + \hat{x}^s_{N2}x^s_{N2}}{x^s_{A2} + x^s_{N2}}. \tag{19}$$

Taking advantage of $\hat{e}_{F1}/\hat{\bar{e}}_{A1} > 0$, $\hat{e}_{F2}/\hat{\bar{e}}_{A1} < 0$, and (18) in (16) and (17), we get $(\hat{x}^s_{A1}x^s_{A1} + \hat{x}^s_{N1}x^s_{N1})/\hat{\bar{e}}_{A1} > 0$ and $(\hat{x}^s_{A2}x^s_{A2}+\hat{x}^s_{N2}x^s_{N2})/\hat{\bar{e}}_{A1} < 0$ and hence $\hat{q}^s/\hat{\bar{e}}_{A1} > 0$ from (19). Consider now the demand side and observe that (14) implies

$$\widehat{\left(\frac{x_{i1}}{x_{i2}}\right)} =: \hat{q}^d_i = \sigma_i\hat{p}_{x2} \quad \text{for } i = A, F, N. \tag{20}$$

[10] In view of (5) we have sign $(\hat{e}_{F1}/\hat{\bar{e}}_{A1}) = -\text{sign}\,(\hat{e}_{F2}/\hat{\bar{e}}_{A1})$. As the goal of climate policy is to delay the consumption of fossil fuel, tightening the emissions cap $\hat{\bar{e}}_{A1}$ promotes that goal only if $\hat{e}_{F1}/\hat{\bar{e}}_{A1} > 0$.

[11] (15a) is derived in the Appendix.

[12] Throughout the rest of the paper good X in peroid 1 is chosen as numeraire ($p_x \equiv 1$).

[13] A function is homothetic, if it can be written as an increasing transform of a linear homogeneous function. The class of homothetic functions encompases CES functions, Leontief functions, and isoelastic functions. Isoelastic utility functions (see (27) below) are often applied in empirical studies and, e.g., in the Stern review (2006).

Since all utility functions are assumed to be identical we have $\sigma_i = \sigma$ and $q_i^d = q^d$ for $i = A, F, N$. Under this condition (20) implies $\hat{q}^d/\hat{\bar{e}}_{A1} = \sigma \cdot (\hat{p}_{x2}/\hat{\bar{e}}_{A1}) < 0$ for $\hat{p}_{x2}/\hat{\bar{e}}_{A1} < 0$. $(\hat{q}^s - \hat{q}^d)/\hat{\bar{e}}_{A1} > 0$ follows. However, $(\hat{q}^s - \hat{q}^d)/\hat{\bar{e}}_{A1} = 0$ is a necessary equilibrium condition. This contradiction proves the claim $\hat{p}_{x2}/\hat{\bar{e}}_{A1} > 0$. $\hat{p}_{e1}/\hat{\bar{e}}_{A1} > 0$ is straightforward from $\hat{p}_{x2}/\hat{\bar{e}}_{A1} > 0$ and (A30), and $\mathrm{d}e_{F1}/\mathrm{d}\bar{e}_{A1} < 1$ follows from (15b). ■

Proposition 1 conveys the important messages that if country A tightens its first-period emissions cap the world market price of fossil fuel (in terms of first-period consumption) falls and first-period consumption becomes more expensive relative to second-period consumption.[14] For both reasons it is profitable for the firms in country N to expand their output and hence their fossil fuel consumption.

Equation (15a) specifies the first-period emissions reduction induced by tightening the cap $\bar{e}_{A1}$ ($\hat{\bar{e}}_{A1} < 0$). [1] is the (partial) direct effect which would imply zero leakage in the absence of market adjustments. The terms [2] and [3] represent leakage effects. [2] captures the increase in e_{F1} caused by the drop in p_{e1} if p_{x2} is (hypothetically) kept constant. The increase in e_{F1} due to [2] is the smaller the more price elastic the fuel demand of the countries A and N are in period 2 (γ_p larger) and the more price elastic the fuel demand of country N is in period 1 ($|\eta_{N1}|$ larger). Observe that the effect [2] generates carbon leakage but never leads to the green paradox since the term $\frac{\gamma_p}{\gamma_p - p_{e1}e_{N1}\eta_{N1}}$ in (15b) is positive but less than one. However, the effect [3] exacerbates carbon leakage (since $\hat{p}_{x2}/\hat{\bar{e}}_{A1} > 0$) and creates the possibility of the green paradox which will then occur if and only if the effect [3] is sufficiently strong. As response to $\hat{\bar{e}}_{A1} < 0$, the increase in e_{F1} due to [3] is larger the more price elastic the aggregate fuel demand is in period 2 (γ_p larger), the more price elastic the fuel demand of country N is in period 1 ($|\eta_{N1}|$ larger), and the greater is $\hat{p}_{x2}/\hat{\bar{e}}_{A1}$. The role of effect [3] is to equilibrate the markets for the consumption good in both periods while maintaining the equilibrium in the markets for fossil fuel through an appropriate reduction in p_{e1}. This observation highlights that the effect [3] emerges in our model because it contains a full set of competitive (future) markets all of which are required to clear.

The effects of tightening the cap $\bar{e}_{A1}$ are illustrated in Figure 1. Let AG be the first-period fuel demand curve of country A when no cap is applied and let $ABCD$ be the aggregate first-period demand curve (where country N's demand is horizontally added to country A's demand). The line ACS is the aggregate first-period demand for fossil fuel when country A's emissions cap is $\bar{e}_{A1}^0$. With that cap in place, aggregate demand is still unconstrained in the segment ABC. The segment CD, however, is now replaced by the

[14]This observation is clearly equivalent to the statement that second-period consumption becomes less expensive relative to first-period consumption which we have chosen as numeraire.

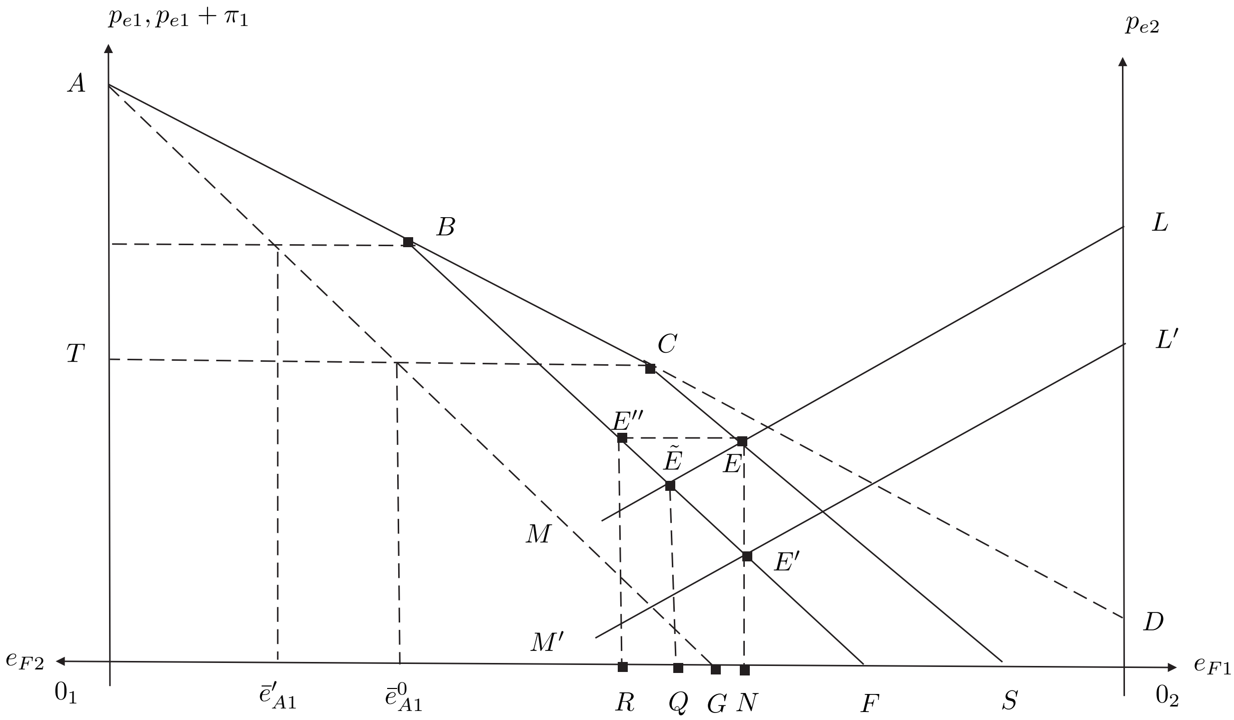

Figure 1: Impact of tightening the emissions cap in period 1

line CS. For fossil-fuel prices less than 0_1T country A's fuel demand is constant at $\bar{e}^0_{A1}$ while country N's demand expands with sinking prices along CS.

According to (10) and (11) the aggregate second-period demand for fossil fuel can be depicted in Figure 1 only for some predetermined price p_{x2}. Suppose, p^0_{x2} is the equilibrium value of p_{x2} when country A has fixed the emissions cap $\bar{e}^0_{A1}$ and let LM represent the aggregate fuel demand in period 2 when $p_{x2} = p^0_{x2}$ prevails and when second-period emissions are not capped. According to (12) point E in Figure 1 then characterizes the equilibria in the periods 1 and 2 of the world markets for fossil fuel where $p^0_{e1} = p^0_{e2} = NE$.

Suppose now country A tightens its emissions cap from $\bar{e}^0_{A1}$ to $\bar{e}'_{A1}$, $\bar{e}'_{A1} < \bar{e}^0_{A1}$. As a consequence, the curve of the first-period aggregate fuel demand shifts from ACS to ABF. In Figure 1, the emissions reduction $\bar{e}^0_{A1} - \bar{e}'_{A1}$ is given by $EE'' = NR$ representing the partial effect [1] in equation (15a). If the second-period demand curve LM remained unchanged (which would be the case if and only if p_{x2} remained unchanged) the equilibrium shifts from E to $\tilde{E}$. The resultant increase in e_{F1} by QR (from 0_1R to 0_1Q) corresponds to the partial effect [2] in equation (15a). In point $\tilde{E}$ in Figure 1 the markets for fossil fuel are cleared in both periods but the commodity markets are still in disequilibrium. We know from Proposition 1 that $\hat{p}_{x2}/\hat{\bar{e}}_{A1} > 0$ is necessary to clear the commodity markets. As an implication, tightening $\bar{e}_{A1}$ reduces p_{x2} which, in turn, shifts downward the demand curve LM to e.g. $L'M'$. The new equilibrium point is E' and the increase NQ in e_{F1} involved in the move from $\tilde{E}$ to E' corresponds to the partial effect [3] in equation (15a).[15]

In their first-period production plan myopic firms in country N would have ignored the reduction in p_{x2}. However, with their two-period planning horizon and perfect future markets the firms in our model account for $\hat{p}_{x2}/\hat{\bar{e}}_{A1} > 0$ and they also anticipate the subsequent reduction in p_{e1} (Hotelling rule). When the input price p_{e1} falls and the output price remains constant ($p_{x1} \equiv 1$), it is profitable for the firms in country N to expand their output and fossil fuel input. This is why the effect [3] from (15a) works counter country A's cap tightening.

According to (15b) and Proposition 1 the price change $\hat{p}_{x2}$ is the key determinant for the green paradox because - as we have illustrated with the help of Figure 1 - it is crucial how large the reduction in p_{x2} must be to bring about the necessary equilibrium condition $\hat{q}^d = \hat{q}^s$. To better understand the relation between $\hat{p}_{x2}$ and $\hat{q}^d$ we resort to the class of CES utility functions that are homogeneous of degree $b > 0$:

$$U(x_{i1}, x_{i2}) = \left(a_1 x_{i1}^{-e} + a_2 x_{i2}^{-e}\right)^{-\frac{b}{e}}, \tag{21}$$

where $a_1 > 0, a_2 > 0$ and $e := (1-\sigma)/\sigma$. When combined with (14), standard calculations lead to

$$q^d = \left(\frac{a_2 p_{x2}}{a_1}\right)^{\sigma} =: Q^d(p_{x2}, \sigma). \tag{22}$$

The equilibrium condition is

$$Q^d\left(\underset{(+)}{p_{x2}}, \underset{(+)}{\sigma}\right) = Q^s\left(\underset{(+)}{\bar{e}_{A1}}, \underset{(-)}{p_{x2}}\right), \tag{23}$$

where the function Q^s is implicitly determined in (19).[16] The equilibrium price p_{x2} is uniquely determined by (23) and it obviously depends on both σ and $\bar{e}_{A1}$.

In Figure 2 we have plotted the graphs of Q^d for alternative values of σ: $\sigma_{\mathrm{I}} = 0$, $\sigma_{\mathrm{II}} = 1$ and σ_{III} very large. Figure 2 also contains the graphs of the function Q^s for $\bar{e}_{A1} = \bar{e}_{A1}^0$ and for $\bar{e}_{A1} = \bar{e}'_{A1} < \bar{e}_{A1}^0$. Starting from an initial equilibrium[17] E_0 and tightening the emissions cap from $\bar{e}_{A1}^0$ to $\bar{e}'_{A1}$ leads to new equilibria $E_0, E_{\mathrm{I}}, E_{\mathrm{II}}$ or E_{III}. The abscissa shows that the lower the substitution elasticity the greater is the reduction in the price p_{x2}.

[15] The shift of the second-period demand curve from LM to $L'M'$ happens to be chosen such that carbon leakage exactly offsets country A's emissions reduction $\bar{e}_{A1}^0 - \bar{e}'_{A1}$.

[16] The signs of the derivatives of the function Q^s are shown in the Appendix.

[17] We have demonstrated in (23) that the equilibrium values of p_{x2} and q depend on the parameters $\bar{e}_{A1}$ and σ. The only reason for taking E_0 in Figure 2 as one and the same equilibrium point for *alternative* values of σ is to ease the exposition.

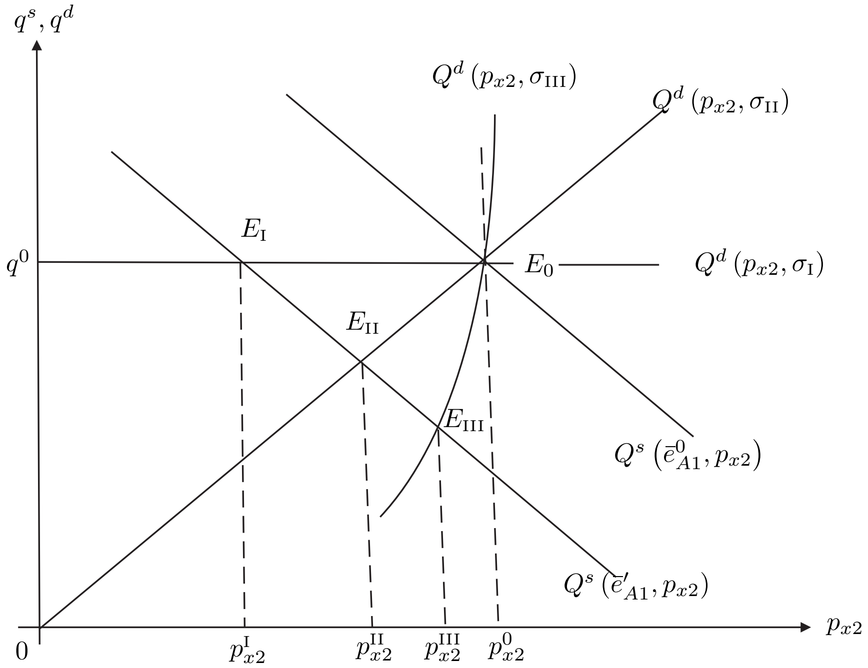

Figure 2: Changes in p_{x2} when the cap is tightened from $\bar{e}^0_{A1}$ to $\bar{e}'_{A1}$ depending on the size of σ ($\sigma_{\mathrm{I}} = 0$, $\sigma_{\mathrm{II}} = 1$, σ_{III} very large)

For homothetic utility functions (not restricted to CES functions) we calculate the impact on p_{x2} of tightening the cap $\bar{e}_{A1}$ in full-scale comparative statics analysis (Appendix) as

$$\frac{\hat{p}_{x2}}{\hat{\bar{e}}_{A1}} = \frac{\gamma_p \bar{e}_{A1}(p_{e1} + \lambda\pi_1) - \lambda\pi_1\bar{e}_{A1}p_{e1}e_{N1}\eta_{N1}}{D}, \tag{24}$$

where

$$\begin{aligned} D &:= (\gamma_p - p_{e1}e_{N1}\eta_{N1})\lambda\sigma\left(x^s_{A1} + x^s_{N1}\right) - \gamma_p p_{e1}e_{N1}\eta_{N1} > 0, \\ \gamma_p &:= -p_{e1}(e_{A2}\eta_{A2} + e_{N2}\eta_{N2}) > 0 \quad \text{and} \quad \lambda := \frac{p_{x2}}{p_{x2} + H(p_{x2})} > 0, \end{aligned}$$

which is positive - as we already know from Proposition 1. To make further progress in exploring the conditions for the green paradox we insert (24) into (15b) to obtain, after some rearrangement of terms,

$$\frac{\hat{e}_{F1}}{\hat{\bar{e}}_{A1}} = \frac{\bar{e}_{A1}\gamma_p\lambda}{e_{F1}D}\left[\sigma \cdot \left(x^s_{A1} + x^s_{N1}\right) + \pi_1 e_{N1}\eta_{N1}\right]. \tag{25}$$

In (25), the term $x^s_{A1} + x^s_{N1}$ is positive and the term $\pi_1 e_{N1}\eta_{N1}$ is negative. Therefore, $\sigma = 0$ implies $\hat{e}_{F1}/\hat{\bar{e}}_{A1} < 0$ (the green paradox occurs!) while for $\sigma > 0$ the sign of $\hat{e}_{F1}/\hat{\bar{e}}_{A1}$ is ambiguous. We conclude from (25) that

$$\frac{\hat{e}_{F1}}{\hat{\bar{e}}_{A1}} \gtreqless 0 \quad \Longleftrightarrow \quad \sigma \cdot \left(x^s_{A1} + x^s_{N1}\right) \gtreqless -\pi_1 e_{N1}\eta_{N1}. \tag{26}$$

Proposition 2. *Suppose the utility function is homothetic and the cap $\bar{e}_{A1}$ is tightened. Then the green paradox occurs if and only if $\sigma < \bar{\sigma} := -\frac{\pi_1 e_{N1}\eta_{N1}}{x^s_{A1}+x^s_{N1}}$.*

For Leontief utility functions ($\sigma = 0$) the green paradox occurs but it does not occur in the case of utility functions exhibiting sufficiently large substitution elasticities. If we consider the class of isoelastic utility functions

$$U(x_{i1}, x_{i2}) = \begin{cases} \frac{\mu x_{i1}^{1-\eta}}{1-\eta} + \frac{1}{1+\rho} \cdot \frac{\mu x_{i2}^{1-\eta}}{1-\eta} & \text{for} \quad \mu > 0, \eta \neq 1, \\ \ln x_{i1} + \frac{1}{1+\rho} \ln x_{i2} & \text{for} \quad \eta = 1, \end{cases} \tag{27}$$

where ρ is a positive pure rate of time preference, we find that $\eta = \frac{1}{\sigma}$ and hence $\sigma \to \infty$ if and only if $\eta \to 0$. No doubt, isoelastic utility functions with $\eta \to 0$ are unrealistic as well as Leontief utility functions. Yet these polar cases are relevant in that they prove the existence and non-existence, respectively, of the green paradox.[18] The main message of Proposition 2 in combination with Figure 2 is that the lower the substitution elasticity the greater is the price effect $\hat{p}_{x2}/\hat{\bar{e}}_{A1}$ and the more likely is the green paradox.

Proposition 2 focusses on the intertemporal substitution elasticity as a determinant of the green paradox. Note, however, that the size of the threshold value $\bar{\sigma}$ defined in Proposition 2 also depends on the size of the price elasticity of demand for fossil fuel, η_{N1}, which is entirely technology-determined. To get more information on the interaction of demand and supply conditions yielding the green paradox we parametrize the production function by

$$X^i(e_{it}) = e_{it}^{\theta_{it}}, \quad \theta_{it} \in]0,1[\quad \text{for}\, i = A, N. \tag{28}$$

For production functions (28), the equivalence (26) simplifies to

$$\frac{\hat{e}_{F1}}{\hat{\bar{e}}_{A1}} \gtreqless 0 \quad \Longleftrightarrow \quad \sigma \cdot (1 - \theta_{N1}) \gtreqless \frac{\pi_1 e_{N1}}{x^s_{A1} + x^s_{N1}} =: \gamma_{\theta 1}. \tag{29}$$

From (29) we infer

Proposition 3. *Suppose the utility function is homothetic, the production function $X^N(e_{N1})$ from (1) is specified by (28) and the cap $\bar{e}_{A1}$ is tightened.*

(i) Then the green paradox occurs if and only if $\sigma \cdot (1 - \theta_{N1}) < \gamma_{\theta 1}$.

(ii) The green paradox does not occur, either

(a) if $p_{e1} \geq \pi_1$ and $\sigma \cdot (1 - \theta_{N1}) \geq 1$ or

[18] It is also worth mentioning that the condition for the green paradox does not hinge upon the pure rate of time preference.

(b) *if* $p_{e1} \geq \pi_1, \bar{e}_{A1} \geq e_{N1}$ *and* $\sigma \cdot (1-\theta_{N1}) \geq \frac{1}{3}$.

Proof of Proposition 3(ii). Total first-period profits are

$$\Pi^1 := x^s_{A1} + x^s_{N1} - (p_{e1} + \pi_1)\bar{e}_{A1} - p_{e1}e_{N1},$$

which are positive according to (10) and (11) (and footnote 6). Using that definition of Π^1 we rewrite $\gamma_{\theta 1}$ from (29) as

$$\gamma_{\theta 1} := \frac{\pi_1 e_{N1}}{x^s_{A1} + x^s_{N1}} = \frac{\pi_1 e_{N1}}{\Pi^1 + \pi_1 \bar{e}_{A1} + p_{e1} e_{F1}} = \frac{1}{\frac{\Pi^1}{\pi_1 e_{N1}} + \frac{\bar{e}_{A1}}{e_{N1}}\left(1 + \frac{p_{e1}}{\pi_1}\right) + \frac{p_{e1}}{\pi_1}}.$$

Obviously, $\gamma_{\theta 1} < 1$ if $p_{e1} \geq \pi_1$ and $\gamma_{\theta 1} < \frac{1}{3}$, if $p_{e1} \geq \pi_1$ *and* $\bar{e}_{A1} \geq e_{N1}$. ■

Proposition 3 highlights the relevance for the green paradox of the production technology in country N and period 1 and of the interaction of supply and demand conditions. Proposition 3 conforms with our intuition that a highly elastic demand for fossil fuel in country N is conclusive to the green paradox. That elasticity is the higher the closer to one is the production parameter θ_{N1}, i.e. the more the production function tends to be linear. Yet even if θ_{N1} is small the green paradox occurs according to Proposition 3(i) if σ is sufficiently small. On the other hand, Proposition 3(ii) states conditions to avoid the green paradox. Proposition 3(ii) does not imply, however, that avoiding the green paradox requires $\sigma > 1$, because the conditions for Proposition 3ii to hold are sufficient but not necessary. To sum up, according to the Propositions 2 and 3 the green paradox depends on the order of magnitude of the parameters σ and θ_{N1}. This calls for a thorough discussion of the empirical estimates of those parameters which is, however, beyond the scope of the present paper.

4 Enlarging the group of abating countries

Up to now we have not made any explicit assumption about the size of the abating country A compared to the size of the non-abating country N. We will do so now in the simplest possible way by introducing a fixed world endowment of an immobile (internationally non-tradable) factor (e.g. land), $\bar{\ell} = 1$, where $\ell \in]0,1[$ and $1-\ell$, respectively, are the inputs of land employed in the countries A and N. To further simplify the exposition, suppose the production functions are Cobb-Douglas such that for $t = 1, 2$

$$x^s_{At} = e^{\theta}_{At}\ell^{1-\theta} \quad \text{and} \quad x^s_{Nt} = e^{\theta}_{Nt}(1-\ell)^{1-\theta}, \tag{30}$$

where $\theta \in]0,1[$. Note first that in the absence of emissions capping the aggregate demand functions for fossil fuel (in either period) are independent of ℓ. This is easily verified by combining (30) with the profit maximizing condition $X^i_{e_{it}} = p_t$ (with $p_1 = p_{e1}$ and $p_2 = p_{e2}/p_{x2}$) to calculate the countries' fuel demand functions as

$$e_{At} = \ell \left(\frac{\theta}{p_t}\right)^{\frac{1}{1-\theta}} \quad \text{and} \quad e_{Nt} = (1-\ell)\left(\frac{\theta}{p_t}\right)^{\frac{1}{1-\theta}}.$$

Adding up these equations shows that for any given p_t the sum $e_{At}+e_{Nt}$ remains unchanged when ℓ is varied. We interpret an increase in ℓ as new countries joining the group of abating countries which we continue to address as "country A". Differentiating country A's fuel demand function yields

$$\hat{p}_t = (\hat{\ell} - \hat{\bar{e}}_{At})(1-\theta).$$

If we would increase ℓ and would keep constant the emissions cap $\bar{e}_{A1}$ we would combine enlarging the group with tightening the cap for all members of the group. To avoid such mixed strategy we will consider $\hat{\ell} = \hat{\bar{e}}_{At} > 0$, a scenario, that appears plausible since it implies $\hat{p}_{e1} = 0$ so that the new countries entering the abatement coalition commit to the same constraint as the old members.

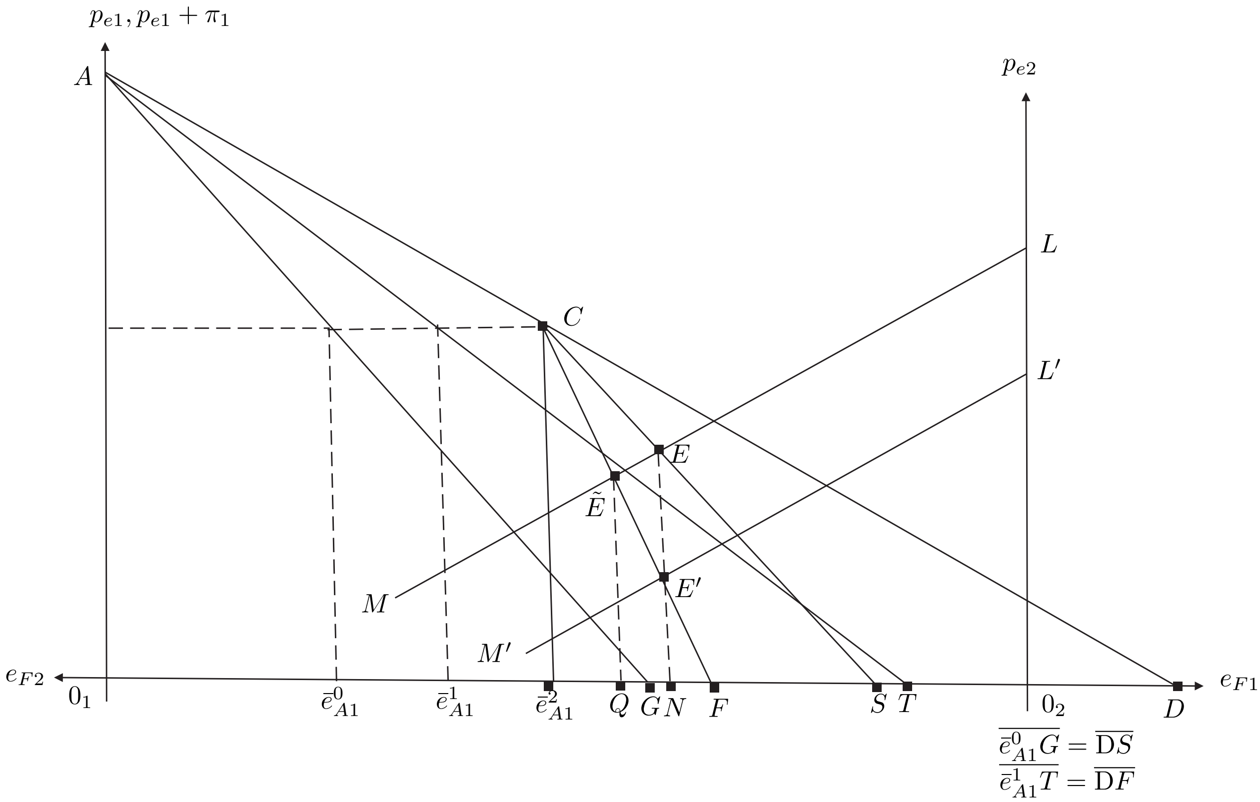

Figure 3: Enlarging the coalition of abating countries

Figure 3 illustrates the scenario $\hat{\ell} = \hat{\bar{e}}_{A1} > 0$. The initial situation is as in Figure 1: AG is country A's fuel demand curve[19], ACD is the aggregate demand curve without

[19]For Cobb-Douglas production functions the associated demand functions are hyperbolic. For sake of simplicity the demand curves in Figure 3 are drawn as straight lines.

cap and ACS is the aggregate demand curve when the emissions cap $\bar{e}^0_{A1}$ is implemented in country A. The line CS is constructed such that the segments $\bar{e}^0_{A1}G$ and DS are equal in length. With the aggregate second-period demand being LM the initial equilibrium is assumed to be attained in point E. Suppose now country A's demand curve shifts from AG to AT ($\hat{\ell} > 0$) leaving the aggregate demand curve ACD unchanged. $\hat{\ell} = \hat{\bar{e}}_{A1}$ is illustrated in Figure 3 by moving from $\bar{e}^0_{A1}$ to $\bar{e}^1_{A1}$. The aggregate first-period demand curve associated to $\bar{e}^1_{A1}$ is now given by ACF where the line segment CF is constructed such that $\bar{e}^1_{A1}T = DF$ is satisfied. Since the demand curve AT is flatter than AG the segment $\bar{e}^0_{A1}G$ is smaller than $\bar{e}^1_{A1}T$ implying that the line CF is steeper than the line CS.

The conclusions are qualitatively similar to those we elaborated in the context of Figure 1 and are briefly summarized as follows: If the second-period aggregate demand curve would remain unchanged (which will not be the case) one would have carbon leakage but no green paradox. Yet p_{x2} must shrink causing the LM curve to shift downward to a curve such as $L'M'$. How far the curve LM shifts down depends on the determinants elaborated in the previous Section 3. A green paradox occurs when the demand conditions require a drop in the price p_{x2} which is so strong that the second-period demand curve LM is forced to shift below the line $L'M'$. Intuitively speaking, this is the less likely, however, the steeper is the line segment CF. This line segment is the steeper, in turn, the closer is ℓ to $\ell = 1$. To see this suppose we start from the equilibrium E' in Figure 3 (with $\bar{e}_{A1} = \bar{e}^1_{A1}$) and successively raise ℓ until $\ell = 1$. Since we continue to require $\hat{\ell} = \hat{\bar{e}}_{A1}$, $\ell \to 1$ obviously implies $\bar{e}^1_{A1} \to \bar{e}^2_{A1}$ in Figure 3. When $\ell = 1$ is reached the aggregate first-period fuel demand curve is $AC\bar{e}^2_{A1}$. In that case no leakage occurs anymore no matter how strong the downward shift of the curve LM may be.

$\ell = 1$ means that there is a *global* coalition in which *all* countries commit to reduce emissions. In that case our model turns out to be a very simple version of Sinn's (2008) model who considers a single 'aggregate' fuel-demanding country representing the global coalition of all fuel-demanding countries. In that case carbon leakage is trivially absent. However, as we know from the Kyoto protocol the global coalition is far from being a realistic scenario.

5 Tightening the emissions cap in the second period

In this section we assume that country A regulates emissions not only in the first period, but also in the second. The model now consists of the equations (1) - (5), (6b), (7) - (14), and a green paradox is said to occur, if $\hat{e}_{F1}/\hat{\bar{e}}_{A1} < 0$ or if $\hat{e}_{F1}/\hat{\bar{e}}_{A2} < 0$. Let us first consider the impact of $\hat{\bar{e}}_{A1} < 0$ and $\hat{\bar{e}}_{A2} = 0$:

Proposition 4.
Consider the policy $\hat{\bar{e}}_{A1} < 0$ and $\hat{\bar{e}}_{A2} = 0$ of country A. In qualitative terms, the conditions for carbon leakage and the green paradox are the same as under the policy $\hat{\bar{e}}_{A1} < 0$, when e_{A2} is unconstrained.

As shown in the Appendix, the only change necessary is replacing $\gamma_p > 0$ by $\tilde{\gamma}_p := -p_{e2}e_{N2}\eta_{N2} > 0$. With this slight modification the equations (15b), (24) and (25) continue to hold and hence the Propositions 1 through 3 apply.

Next, we explore the policy $\hat{\bar{e}}_{A2} < 0$ and $\hat{\bar{e}}_{A1} = 0$. The displacement effects of $\hat{\bar{e}}_{A2} \neq 0$ on $\hat{e}_{F1}$ are formally given by

$$\hat{e}_{F1}e_{F1} = \underbrace{\frac{e_{N1}\eta_{N1}p_{e1}\bar{e}_{A2}}{\tilde{\gamma}_p - p_{e1}\eta_{N1}e_{N1}}\hat{\bar{e}}_{A2}}_{[1]} + \underbrace{\frac{\tilde{\gamma}_p e_{N1}\eta_{N1}}{\tilde{\gamma}_p - p_{e1}\eta_{N1}e_{N1}}\hat{p}_{x2}}_{[2]}. \tag{31}$$

Proposition 5. $\hat{p}_{x2}/\hat{\bar{e}}_{A2} < 0$, $\hat{p}_{e1}/\hat{\bar{e}}_{A2} < 0$ *and* $\mathrm{d}e_{F1}/\mathrm{d}\bar{e}_{A2} > -1$ *if the utility function is homothetic.*

Proof. Contrary to the ascertion suppose that $\hat{p}_{x2}/\hat{\bar{e}}_{A2} > 0$. In that case we obtain $\hat{e}_{F1}/\hat{\bar{e}}_{A2} > 0$ and $\hat{e}_{F1}/\hat{\bar{e}}_{A2} < 0$. Using the same arguments as in the proof of Proposition 1 the last inequalities translate into $(\hat{x}^s_{A1}x^s_{A1} + \hat{x}^s_{N1}x^s_{N1})/\hat{\bar{e}}_{A2} < 0$ and $(\hat{x}^s_{A2}x^s_{A2} + \hat{x}^s_{N2}x^s_{N2})/\hat{\bar{e}}_{A2} > 0$ or $\hat{q}^s/\hat{\bar{e}}_{A2} < 0$. On the demand side we get $\hat{q}^d/\hat{\bar{e}}_{A2} = \sigma \cdot (\hat{p}_{x2}/\hat{\bar{e}}_{A2}) > 0$ for $\hat{p}_{x2}/\hat{\bar{e}}_{A2} > 0$ which implies $(\hat{q}^s - \hat{q}^d)/\hat{\bar{e}}_{A2} < 0$. This condition contradicts the necessary equilibrium condition $(\hat{q}^s - \hat{q}^d)/\hat{\bar{e}}_{A2} = 0$, which proves $\hat{p}_{x2}/\hat{\bar{e}}_{A2} < 0$. $\hat{p}_{e1}/\hat{\bar{e}}_{A2} < 0$ follows from $\hat{p}_{x2}/\hat{\bar{e}}_{A2} < 0$ and (A54), and $\mathrm{d}e_{F1}/\mathrm{d}\bar{e}_{A2} > -1$ follows from (31). ■

We proceed by considering the price effect (derived in the Appendix)

$$\frac{\hat{p}_{x2}}{\hat{\bar{e}}_{A2}} = -\frac{(1-\lambda)\bar{e}_{A2}\pi_2(\tilde{\gamma}_p - p_{e1}e_{N1}\eta_{N1}) - p^2_{e1}\bar{e}_{A2}e_{N1}\eta_{N1}}{\tilde{D}}, \tag{32}$$

where

$$\begin{aligned} \tilde{D} &:= (\tilde{\gamma}_p - p_{e1}e_{N1}\eta_{N1}) + \sigma\lambda\,(x^s_{A1} + x^s_{N1}) - \tilde{\gamma}_p p_{e1}e_{N1}\eta_{N1} > 0, \\ \tilde{\gamma}_p &:= -p_{e2}e_{N2}\eta_{N2} > 0. \end{aligned}$$

Insert (32) into (31) to obtain, after some rearrangement of terms,

$$\frac{\hat{e}_{F1}}{\hat{\bar{e}}_{A2}} = \frac{\bar{e}_{A2}p_{e1}e_{N1}\eta_{N1}(1-\lambda)}{e_{F1}\tilde{D}}\left[\sigma p_{x2}\cdot(x^s_{A2} + x^s_{N2}) + \pi_2 e_{N2}\eta_{N2}\right] \tag{33}$$

implying

$$\frac{\hat{e}_{F1}}{\hat{\bar{e}}_{A2}} \gtreqless 0 \quad \Longleftrightarrow \quad \sigma p_{x2} \cdot (x^s_{A2} + x^s_{N2}) \lesseqgtr -\pi_2 e_{N2} \eta_{N2}. \tag{34}$$

Comparing (34) and (26) reveals a striking symmetry. The right side of both equivalences differs in two respects: the index 1 in (26) is replaced by the index 2 in (34) *and* the set of inequalities in (26) is reversed[20] in (34).

Closer inspection of (34) leads to

Proposition 6. *Suppose the utility function is homothetic and the cap $\bar{e}_{A2}$ is tightened. Then the green paradox does not occur if and only if $\sigma < \tilde{\sigma} := -\frac{\pi_2 e_{N2} \eta_{N2}}{p_{x2}(x^s_{A2}+x^s_{N2})}$.*

The impact of σ established in Proposition 2 is reversed in Proposition 6. More specifically, if we tighten the first-period cap ($\hat{\bar{e}}_{A1} < 0$) and either have the second-period cap constant $\left(\hat{\bar{e}}_{A2} = 0\right)$ or do not implement a second-period cap (e_{A2} free) we can exclude the green paradox for sufficiently large values of σ. In contrast, under the policy $\hat{\bar{e}}_{A2} < 0$ and $\hat{\bar{e}}_{A1} = 0$ the green paradox is excluded for sufficiently small values of σ.

Following the procedure in Section 3 we now invoke the parametric function (28) again to complement our findings of Proposition 6. With the production function (28) the equivalence (34) turns into

$$\frac{\hat{e}_{F1}}{\hat{\bar{e}}_{A2}} \gtreqless 0 \quad \Longleftrightarrow \quad \sigma \cdot (1 - \theta_{N2}) \lesseqgtr \frac{\pi_e e_{N2}}{p_{x2}(x^s_{A2} + x^s_{N2})} =: \gamma_{\theta 2}. \tag{35}$$

The information contained in (35) is summarized in

Proposition 7. *Suppose the utility function is homothetic, the production function $X^N(e_{N2})$ from (1) is specified by (28) and the cap $\bar{e}_{A2}$ is tightened.*

(i) Then the green paradox occurs if and only if $\sigma \cdot (1 - \theta_{N2}) < \gamma_{\theta 2}$.

(ii) The green paradox does not occur, either

(a) if $p_{e2} \geq \pi_2$ and $\sigma \cdot (1 - \theta_{N2}) \geq 1$ or

(b) if $p_{e2} \geq \pi_2, \bar{e}_{A2} \geq e_{N2}$ and $\sigma \cdot (1 - \theta_{N2}) \geq \frac{1}{3}$.

Summarizing, the impact on carbon leakage of σ and θ as established in the Propositions 2 and 3 is reversed in the Proposition 6 and 7. All parameter changes that make the green

[20] This reversal is exclusively due to the fact that we focus on the impact of $\hat{\bar{e}}_{A2}$ on e_{F1} rather than on e_{F2} (because we are interested in total emissions in period 1). If we had explored the effect of $\hat{\bar{e}}_{A2}$ on e_{F2} we could have simply referred to Section 3 interchanging the indexes 1 and 2 everywhere. This observation can be easily verified by carrying out such a swap of indexes in Figure 1.

paradox more *likely* when the cap is tightened in period 1 work in opposite direction when the cap is tightened in period 2. In particular, the green paradox will *likely* occur when the emissions control is strengthened in the second period, if it does *not* result from tightening the emissions cap in the first period and vice versa.[21]

It is informative to highlight from a different perspective our finding that tightening the first-period and the second-period caps, respectively, tends to affect leakage in opposite directions. To this end suppose that a cap effectively constrains the second-period emissions in country A while first-period emissions are unconstrained. Suppose further that analogous to our exercise in Section 4 the coalition of abating countries becomes larger and tends toward the global coalition ($\ell \to 1$). If $\ell = 1$, total emissions in period 2 are strictly capped, say at $\bar{e}_{F2}$. The smaller $\bar{e}_{F2}$ the greater are necessarily total first-period emissions and the more pronounced is the green paradox.

6 Concluding remarks

Following Ockham's razor, we have abstracted from many real-world complexities such as extending the time horizon beyond two periods, or including stock-dependent extraction costs, capital accumulation and insecure property rights. Without doubt, all of these aspects are empirically relevant but they do not appear to be at the core of the green-paradox phenomenon. To remain focused we also refrained from getting involved in the controversial though important debate on normative 'social' discounting, time preference and intertemporal inequality aversion. Our use of the demand parameters is meant to be descriptive which does not exclude extending the analysis to normative issues, of course.

We have applied the economics of intertemporal allocation of non-renewable natural resources in its simplest form and have been able to show how this theory drives the results. As is well known, in a perfectly competitive world with a full set of future markets a necessary equilibrium condition is that resource extracting and supplying firms are indifferent between selling the resource today or at any other period in the future (Hotelling rule in the wide sense; here: $p_{e1} = p_{e2}$). The requirement of clearing the market for the consumption good in both periods combined with the Hotelling rule tends to exacerbate carbon leakage when the first-period emissions cap is tightened in the abating countries. An interesting re-

[21] This result has an important implication for policies of tightening the emissions caps in both periods simultaneously, because the net impact on carbon leakage of simultaneous reductions in the caps of both periods is the result of "forces" working in opposite direction. More precisely, it can be shown that if $\hat{\bar{e}}_{A1}$ and $\hat{\bar{e}}_{A2}$ are tightened proportionally $\left(\text{i.e. } \hat{\bar{e}}_{A1} = \beta \frac{\bar{e}_{A2}}{\bar{e}_{A1}} \hat{\bar{e}}_{A2} \text{ where } \beta := \frac{e_{N2}\lambda[\sigma \cdot (x^s_{A1} + x^s_{N1}) + \pi_1 e_{N1}\eta_{N1}]}{e_{N1}(1-\lambda)[\sigma p_{x2} \cdot (x^s_{A2} + x^s_{N2}) + \pi_2 e_{N2}\eta_{N2}]}\right)$ then there is no impact on total emissions in period 1 at all $\left(\text{i.e. } \frac{\hat{e}_{F1}}{\hat{\bar{e}}_{A2}} = 0 \text{ if } \hat{\bar{e}}_{A1} = \beta \frac{\bar{e}_{A2}}{\bar{e}_{A1}} \hat{\bar{e}}_{A2}\right)$.

sult is also that the impact of strengthened emissions control depends crucially on whether that policy is carried out in the first or in the second period. All determinants of carbon leakage resulting from tightening the first-period cap work in opposite direction when the second-period cap is tightened. However the extent of carbon leakage is determined by the interaction of various parameters and elasticities. Our model gives no unambiguous support to the proposition that tightening the second-period cap necessarily leads to the green paradox and we cannot confirm either that the green paradox results from tightening the second-period cap, *if and only if* it does not occur when the first-period cap is tightened.

Our analysis suggests that apart from specific characteristics of consumer preferences and production technologies it is the general equilibrium approach in a model with a complete set of perfectly competitive markets and the corresponding account of interdependence effects of markets across countries (space) and time which determines the allocation of resources including the extent of carbon leakage. Such an approach is certainly satisfactory from an intellectual viewpoint because of its consistency. However, one also needs to know how empirically relevant it is to model economic agents and policy makers who anticipate in their plans - and trade on - perfect markets from the presence into the far future. Addressing that issue is beyond the scope of the present paper. But as fighting global change is an urgent empirical policy issue, assessing the reliability of theoretical guidance ought to be high on the agenda of future research, in particular, because many contributions to this issue do not integrate their economics of global change into the established intertemporal theory of nonrenewable resources.

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Appendix

Emissions cap in the first period

The competitive equilibrium of the model is characterized by the following equations

$$\begin{aligned}
X^A_{e_{A1}} - p_{e1} - \pi_1 &= 0, && && \text{(A1)}\\
X^N_{e_{N1}} - p_{e1} &= 0, && && \text{(A2)}\\
p_{x2} X^i_{e_{i2}} - p_{e2} &= 0, && i = A, N, && \text{(A3)}\\
p_{e1} - p_{e2} &= 0, && && \text{(A4)}\\
e_{A1} - \bar{e}_{A1} &= 0, && && \text{(A5)}\\
e_{Ft} - e_{At} - e_{Nt} &= 0, && t = 1, 2, && \text{(A6)}\\
\bar{e} - e_{F1} - e_{F2} &= 0, && && \text{(A7)}\\
X^A(e_{A1}) - x_{A1} - p_{e1} e_{A1} + p_{x2} \left[X^A(e_{A2}) - x_{A2}\right] - p_{e2} e_{A2} &= 0, && && \text{(A8)}\\
X^N(e_{N1}) - x_{N1} - p_{e1} e_{N1} + p_{x2} \left[X^N(e_{N2}) - x_{N2}\right] - p_{e2} e_{N2} &= 0, && && \text{(A9)}\\
p_{e1} e_{F1} - x_{F1} + p_{e2} e_{F2} - p_{x2} x_{F2} &= 0, && && \text{(A10)}\\
X^A(e_{A2}) + X^N(e_{N2}) - x_{A2} - x_{N2} - x_{F2} &= 0, && && \text{(A11)}\\
\frac{U_{x_{i2}}}{U_{x_{i1}}} - p_{x2} &= 0, && i = A, F, N. && \text{(A12)}
\end{aligned}$$

Without loss of generality good X in period $t = 1$ is chosen as numeraire ($p_{x1} \equiv 1$). The variables determined by (A1)-(A12) are $e_{i1}, e_{i2}, x_{i1}, x_{i2}$ for $i = A, F, N$, p_{e1}, p_{e2}, p_{x2} and π_1. The emissions cap $\bar{e}_{A1}$ is treated here as an exogenous parameter. Total differentiation of (A1) - (A12) yields, after some rearrangement of terms,

$$\begin{aligned}
e_{A1} X^A_{e_{A1} e_{A1}} \hat{e}_{A1} - p_{e1} \hat{p}_{e1} - \pi_1 \hat{\pi}_1 &= 0, && && \text{(A13)}\\
\frac{\hat{e}_{N1}}{\hat{p}_{e1}} - \eta_{N1} &= 0, && && \text{(A14)}\\
\frac{\hat{e}_{i2}}{\hat{p}_{e2} - \hat{p}_{x2}} - \eta_{i2} &= 0, && i = A, N && \text{(A15)}\\
\hat{p}_{e1} - \hat{p}_{e2} &= 0, && && \text{(A16)}\\
\hat{e}_{A1} - \hat{\bar{e}}_{A1} &= 0, && && \text{(A17)}\\
e_{Ft} \hat{e}_{Ft} - e_{At} \hat{e}_{At} - e_{Nt} \hat{e}_{Nt} &= 0, && t = 1, 2, && \text{(A18)}\\
e_{F1} \hat{e}_{F1} + e_{F2} \hat{e}_{F2} &= 0, && && \text{(A19)}\\
(X^A_{e_{A1}} - p_{e1}) e_{A1} \hat{e}_{A1} - x_{A1} \hat{x}_{A1} - p_{x2} x_{A2} \hat{x}_{A2} & && && \\
-(e_{A1} + e_{A2}) p_{e1} \hat{p}_{e1} + \left[X^A(e_{A2}) - x_{A2}\right] p_{x2} \hat{p}_{x2} &= 0, && && \text{(A20)}\\
-x_{N1} \hat{x}_{N1} - p_{x2} x_{N2} \hat{x}_{N2} - (e_{N1} + e_{N2}) p_{e1} \hat{p}_{e1} & && && \\
+ \left[X^N(e_{N2}) - x_{N2}\right] p_{x2} \hat{p}_{x2} &= 0, && && \text{(A21)}
\end{aligned}$$

$$
\begin{aligned}
\bar{e}p_{e1}\hat{p}_{e1} - x_{F1}\hat{x}_{F1} - p_{x2}x_{F2}(\hat{x}_{F2} + \hat{p}_{x2}) &= 0, && \text{(A22)}\\
X^A_{e_{A2}}e_{A2}\hat{e}_{A2} + X^N_{e_{N2}}e_{N2}\hat{e}_{N2} - x_{A2}\hat{x}_{A2} - x_{N2}\hat{x}_{N2} - x_{F2}\hat{x}_{F2} &= 0, && \text{(A23)}\\
\hat{x}_{i2} - \hat{x}_{i1} + \hat{p}_{x2}\sigma_i &= 0, \quad i = A, F, N, && \text{(A24)}
\end{aligned}
$$

where $\eta_{N1} := \frac{X^N_{e_{N1}}}{e_{N1}X^N_{e_{N1}e_{N1}}} < 0$, $\eta_{i2} := \frac{X^i_{e_{i2}}}{e_{i2}X^i_{e_{i2}e_{i2}}} < 0$, $\sigma_i := \frac{\mathrm{d}\left(\frac{x_{i2}}{x_{i1}}\right)}{\frac{x_{i2}}{x_{i1}}} \cdot \frac{\frac{U_{x_{i1}}}{U_{x_{i2}}}}{\mathrm{d}\left(\frac{U_{x_{i1}}}{U_{x_{i2}}}\right)} \geq 0$ for $i = A, F, N$.

Derivation of (15a) and (24):

Making use of (A24) in (A20), (A21) and (A22), respectively, yields

$$
\begin{aligned}
\hat{x}_{A2} &= \frac{p_{x2}\Delta x_{A2} - x_{A1}\sigma_A}{y_A}\hat{p}_{x2} - \frac{p_{e1}e_A}{y_A}\hat{p}_{e1} + \frac{(X^A_{e_{A1}} - p_{e1})e_{A1}}{y_A}\hat{e}_{A1}, && \text{(A25)}\\
\hat{x}_{N2} &= \frac{p_{x2}\Delta x_{N2} - x_{N1}\sigma_N}{y_N}\hat{p}_{x2} - \frac{p_{e1}e_N}{y_N}\hat{p}_{e1}, && \text{(A26)}\\
\hat{x}_{F2} &= \hat{p}_{e1} - \frac{p_{x2}x_{F2} + x_{F1}\sigma_F}{y_F}\hat{p}_{x2}, && \text{(A27)}
\end{aligned}
$$

where $\Delta x_{i2} := (x^s_{i2} - x_{i2})$ for $i = A, F, N$, $x^s_{F2} \equiv 0$, $y_i = x_{i1} + p_{x2}x_{i2}$ for $i = A, F, N$, $e_i = e_{i1} + e_{i2}$ for $i = A, N$.
Making use of (A15) and (A16) in (A23), one gets

$$
\gamma_p(\hat{p}_{x2} - \hat{p}_{e1}) = p_{x2}x_{A2}\hat{x}_{A2} + p_{x2}x_{N2}\hat{x}_{N2} + p_{x2}x_{F2}\hat{x}_{F2}, \tag{A28}
$$

where $\gamma_p := -p_{e1}\left(e_{A2}\eta_{A2} + e_{N2}\eta_{N2}\right) > 0$.
Inserting (A25) - (A27), (A1) in (A28) and rearranging terms yields

$$
\gamma_x\hat{p}_{x2} - \gamma_e\hat{p}_{e1} = \lambda_A e_{A1}\pi_1\hat{\bar{e}}_{A1}, \tag{A29}
$$

where

$$
\begin{aligned}
\gamma_x &:= \gamma_p + \lambda_F(p_{x2}x_{F2} + x_{F1}\sigma_F) - \sum_{j=A,N}\lambda_j(p_{x2}\Delta x_{j2} - x_{j1}\sigma_j)\\
\gamma_e &:= \gamma_p + p_{x2}x_{F2} - \sum_{j=A,N}\lambda_j p_{e1}e_j,
\end{aligned}
$$

where $\lambda_i := \frac{p_{x2}x_{i2}}{x_{i1}+p_{x2}x_{i2}}$ for $i = A, F, N$. Solving (A18) with respect to $\hat{e}_{Ft}$, inserting this term in (A19) and making use of (A14) - (A17) we obtain

$$
-(\gamma_p - p_{e1}e_{N1}\eta_{N1})\hat{p}_{e1} + \gamma_p\hat{p}_{x2} = -p_{e1}\bar{e}_{A1}\hat{\bar{e}}_{A1}. \tag{A30}
$$

We solve (A30) for $\hat{p}_{e1}$ and insert this term in $\hat{e}_{F1}e_{F1} = \bar{e}_{A1}\hat{\bar{e}}_{A1} + e_{N1}\eta_{N1}\hat{p}_{e1}$ which follows from (A14), (A17) and (A18), to obtain (15a) after some rearrangement of terms.
(A29) and (A30) jointly determine $\hat{p}_{x2}$ and $\hat{p}_{e1}$. These equations read in matrix notation

$$
\begin{bmatrix} \gamma_e & -\gamma_x \\ (\gamma_p - p_{e1}e_{N1}\eta_{N1}) & -\gamma_p \end{bmatrix}
\begin{bmatrix} \hat{p}_{e1} \\ \hat{p}_{x2} \end{bmatrix}
=
\begin{bmatrix} -\lambda_A\bar{e}_{A1}\pi_1\hat{\bar{e}}_{A1} \\ p_{e1}\bar{e}_{A1}\hat{\bar{e}}_{A1} \end{bmatrix}. \tag{A31}
$$

Solving the equation system (A31) by using Cramer's rule yields

$$\frac{\hat{p}_{x2}}{\hat{\bar{e}}_{A1}} = \frac{\gamma_e p_{e1}\bar{e}_{A1} + \lambda_A \bar{e}_{A1}\pi_1(\gamma_p - p_{e1}e_{N1}\eta_{N1})}{D}, \tag{A32}$$

where $D := -\gamma_e\gamma_p + \gamma_x(\gamma_p - p_{e1}e_{N1}\eta_{N1})$.

Lemma 1 . *If the utility function is homothetic, then*

(i) $\lambda_i = \frac{p_{x2}}{H(p_{x2})+p_{x2}} =: \lambda$ *for all* $i = A, F, N$*;*

(ii) $\sigma_i = \frac{H_{p_{x2}}}{H(p_{x2})}p_{x2} =: \sigma$ *for all* $i = A, F, N$*;*

(iii) $\gamma_x = \gamma_p + \lambda\sigma(x^s_{A1} + x^s_{N1}) > 0$*;*

(iv) $\gamma_e = \gamma_p > 0$*;*

(v) $D = (\gamma_p - p_{e1}e_{N1}\eta_{N1})\lambda\sigma(x^s_{A1} + x^s_{N1}) - \gamma_p p_{e1}e_{N1}\eta_{N1} > 0.$

Proof:

(i) Using $x_{i1} = H(p_{x2})x_{i2}$, which holds for homothetic utility functions, in $\lambda_i = \frac{p_{x2}x_{i2}}{x_{i1}+p_{x2}x_{i2}}$ yields $\lambda_i = \frac{p_{x2}}{H(p_{x2})+p_{x2}}$.

(ii) Total differentiation of $x_{i1} = H(p_{x2})x_{i2}$ gives us $\hat{x}_{i1} = \frac{H_{p_{x2}}}{H(p_{x2})}p_{x2}\hat{p}_{x2} + \hat{x}_{i2}$. Comparing this term with (A24) establishes $\sigma_i = \frac{H_{p_{x2}}}{H(p_{x2})}p_{x2}$.

(iii) Verify that

$$\begin{aligned}\gamma_x &= \gamma_p + \lambda_F(p_{x2}x_{F2} + x_{F1}\sigma_F) - \sum_{j=A,N}\lambda_j(p_{x2}\Delta x_{j2} - x_{j1}\sigma_j)\\ &= \gamma_p + \lambda\sum_{j=A,F,N}(p_{x2}x_{j2} + x_{j1}\sigma) - \lambda\sum_{j=A,N}p_{x2}x^s_{j2}\\ &= \gamma_p + \lambda\sigma\sum_{j=A,F,N}x_{j1} = \gamma_p + \lambda\sigma(x^s_{A1} + x^s_{N1}).\end{aligned} \tag{A33}$$

(iv) Making use of (A9), (A10) and (3) we obtain

$$\begin{aligned}\gamma_e &= \gamma_p + p_{x2}x_{F2} - \lambda\sum_{j=A,N}p_{e1}e_j\\ &= \gamma_p + p_{x2}x_{F2} - \lambda\left[\Delta x_{A1} + \Delta x_{N1} + p_{x2}\left(\Delta x_{A2} + \Delta x_{N2}\right)\right]\\ &= \gamma_p + p_{x2}x_{F2} - \lambda\left(x_{F1} + p_{x2}x_{F2}\right).\end{aligned} \tag{A34}$$

Inserting $\lambda = \frac{p_{x2}x_{F2}}{x_{F1}+p_{x2}x_{F2}}$ in (A34) establishes $\gamma_e = \gamma_p$.

(v) follows from using Lemma 1 (iii) and 1 (iv) in the definition of D and rearranging terms.

$\blacksquare$

Finally, we use the information of Lemma 1 in (A32) to obtain

$$\frac{\hat{p}_{x2}}{\hat{\bar{e}}_{A1}} = \frac{\gamma_p \bar{e}_{A1}(p_{e1} + \lambda \pi_1) - \lambda \pi_1 \bar{e}_{A1} p_{e1} e_{N1} \eta_{N1}}{(\gamma_p - p_{e1} e_{N1} \eta_{N1}) \lambda \sigma (x^s_{A1} + x^s_{N1}) - \gamma_p p_{e1} e_{N1} \eta_{N1}}. \tag{A35}$$

(A35), in turn, is inserted in (15b) to get, after some rearrangements of terms,

$$\frac{\hat{e}_{F1}}{\hat{\bar{e}}_{A1}} = \frac{\bar{e}_{A1} \gamma_p \lambda}{e_{F1} D} \left(\sigma (x^s_{A1} + x^s_{N1}) + \pi_1 e_{N1} \eta_{N1} \right). \tag{A36}$$

The function $Q^s(\bar{e}_{A1}, p_{x2})$ and its derivatives

We start at equation (A30) which can be rearranged to

$$\hat{p}_{e1} = \frac{p_{e1} \bar{e}_{A1} \hat{\bar{e}}_{A1} + \gamma_p \hat{p}_{x2}}{\gamma_p - p_{e1} e_{N1} \eta_{N1}} \tag{A37}$$

and

$$\hat{p}_{e1} - \hat{p}_{x2} = \hat{p}_{e2} - \hat{p}_{x2} = \frac{p_{e1} \bar{e}_{A1} \hat{\bar{e}}_{A1} + p_{e1} e_{N1} \eta_{N1} \hat{p}_{x2}}{\gamma_p - p_{e1} e_{N1} \eta_{N1}}. \tag{A38}$$

Differentiation of (1) yields

$$\hat{x}^s_{it} = \frac{X^i_{e_{it}}}{x^s_{it}} e_{it} \hat{e}_{it} \qquad i = A, N, \quad t = 1, 2. \tag{A39}$$

Making use of (A14) - (A17), (A37), (A38) in (A39) we get

$$\hat{x}^s_{A1} = X^A_{e_{A1}} \bar{e}_{A1} \hat{\bar{e}}_{A1}, \tag{A40}$$

$$\hat{x}^s_{N1} = X^N_{e_{N1}} \bar{e}_{N1} \eta_{N1} \cdot \frac{p_{e1} \bar{e}_{A1} \hat{\bar{e}}_{A1} + \gamma_p \hat{p}_{x2}}{\gamma_p - p_{e1} e_{N1} \eta_{N1}}, \tag{A41}$$

$$\hat{x}^s_{A2} = X^A_{e_{A2}} e_{A2} \eta_{A2} \cdot \frac{p_{e1} \bar{e}_{A1} \hat{\bar{e}}_{A1} + p_{e1} e_{N1} \eta_{N1} \hat{p}_{x2}}{\gamma_p - p_{e1} e_{N1} \eta_{N1}}, \tag{A42}$$

$$\hat{x}^s_{N2} = X^N_{e_{N2}} e_{N2} \eta_{N2} \cdot \frac{p_{e1} \bar{e}_{A1} \hat{\bar{e}}_{A1} + p_{e1} e_{N1} \eta_{N1} \hat{p}_{x2}}{\gamma_p - p_{e1} e_{N1} \eta_{N1}}. \tag{A43}$$

From (A40) - (A43) we readily infer

$$\frac{\hat{x}^s_{A1}}{\hat{p}_{x2}} < 0, \quad \frac{\hat{x}^s_{N1}}{\hat{p}_{x2}} < 0, \quad \frac{\hat{x}^s_{A2}}{\hat{p}_{x2}} > 0, \quad \frac{\hat{x}^s_{N2}}{\hat{p}_{x2}} > 0 \tag{A44}$$

and hence in view of (19) we get $\frac{\hat{q}_s}{\hat{p}_{x2}} < 0$ and $Q^s_{p_{x2}} < 0$, respectively. To verify $Q^s_{\bar{e}_{A1}} > 0$ we totally differentiate the function Q^s to obtain

$$\mathrm{d}q^s = Q^s_{p_{x2}} \, \mathrm{d}p_{x2} + Q^s_{\bar{e}_{A1}} \mathrm{d}\bar{e}_{A1}. \tag{A45}$$

From Proposition 1 and its proof we know that $\frac{\mathrm{d}p_{x2}}{\mathrm{d}\bar{e}_{A1}} > 0$ and $\frac{\mathrm{d}q^s}{\mathrm{d}\bar{e}_{A1}} = \frac{\mathrm{d}q^d}{\mathrm{d}\bar{e}_{A1}} > 0$. In view of (A45), $\frac{\mathrm{d}q^s}{\mathrm{d}\bar{e}_{A1}} > 0$ can only be satisfied for $\frac{\mathrm{d}p_{x2}}{\mathrm{d}\bar{e}_{A1}} > 0$ and $Q^s_{p_{x2}} < 0$ if $Q^s_{\bar{e}_{A1}} > 0$.

Emissions caps in both periods

The competitive equilibrium of the model is characterized by (A1), (A2), (A3) for $i = N$, (A4) - (A12)

$$\begin{aligned} p_{x2}X^A_{e_{A2}} - p_{e2} - \pi_2 &= 0, & \text{(A46)} \\ e_{A2} - \bar{e}_{A2} &= 0. & \text{(A47)} \end{aligned}$$

Total differentiation of these equations yields (A13), (A14), (A15) for $i = N$, (A16) - (A19), (A21) - (A24),

$$p_{x2}e_{A2}X^A_{e_{A2}}\hat{e}_{A2} - p_{x2}X^A_{e_{A2}}\hat{p}_{x2} - p_{e2}\hat{p}_{e2} - \pi_2\hat{\pi}_2 = 0 \qquad \text{(A48)}$$

$$\hat{e}_{A2} - \hat{\bar{e}}_{A2} = 0, \qquad \text{(A49)}$$

$$\begin{aligned} &\left(X^A_{e_{A1}} - p_{e1}\right) e_{A1}\hat{e}_{A1} + \left(p_{x2}X^A_{e_{A2}} - p_{e2}\right) e_{A2}\hat{e}_{A2} - x_{A1}\hat{x}_{A1} - p_{x2}x_{A2}\hat{x}_{A2} \\ &\quad - \left(e_{A1} + e_{A2}\right) p_{e1}\hat{p}_{e1} + \left[X^A(e_{A2}) - x_{A2}\right] p_{x2}\hat{p}_{x2} = 0. \qquad \text{(A50)} \end{aligned}$$

Making use of (A24) in (A50), (A21) and (A22) yields

$$\hat{x}_{A2} = \frac{p_{x2}\Delta x_{A2} - x_{A1}\sigma_A}{y_A}\hat{p}_{x2} - \frac{p_{e1}e_A}{y_A}\hat{p}_{e1} + \frac{\pi_1\bar{e}_{A1}}{y_A}\hat{\bar{e}}_{A1} + \frac{\pi_2\bar{e}_{A2}}{y_A}\hat{\bar{e}}_{A2}, \qquad \text{(A51)}$$

(A26) and (A27).

Next, we insert (A15) and (A16) and (A49) in (A23) to obtain

$$\tilde{\gamma}_p\left(\hat{p}_{x2} - \hat{p}_{e1}\right) + (p_{e2} + \pi_2)\bar{e}_{A2}\hat{\bar{e}}_{A2} = p_{x2}x_{A2}\hat{x}_{A2} + p_{x2}x_{B2}\hat{x}_{B2} + p_{x2}x_{F2}\hat{x}_{F2}, \qquad \text{(A52)}$$

where $\tilde{\gamma}_p := -e_{N2}\eta_{N2}p_{e2} > 0$.

Inserting (A51), (A26), (A27) in (A52) and rearranging terms we get

$$\tilde{\gamma}_x\hat{p}_{x2} - \tilde{\gamma}_e\hat{p}_{e1} = \lambda_A\pi_1\bar{e}_{A1}\hat{\bar{e}}_{A1} + \left[\lambda_A\pi_2 - (p_{e2} + \pi_2)\right] e_{A2}\hat{\bar{e}}_{A2}, \qquad \text{(A53)}$$

where

$$\begin{aligned} \tilde{\gamma}_x &:= \tilde{\gamma}_p + \lambda_F(p_{x2}x_{F2} + x_{F1}\sigma_F) - \sum_{j=A,N}\lambda_j(p_{x2}\Delta x_{j2} - x_{j1}\sigma_j), \\ \tilde{\gamma}_e &:= \tilde{\gamma}_p + p_{x2}x_{F2} - \sum_{j=A,N}\lambda_j p_{e1}e_j. \end{aligned}$$

Solving (A18) with respect to $\hat{e}_{Ft}$, inserting this term in (A19) and making use of (A14) - (A17) and (A47) we obtain

$$-(\tilde{\gamma}_p - p_{e1}e_{N1}\eta_{N1})\hat{p}_{e1} + \tilde{\gamma}_p\hat{p}_{x2} = -p_{e1}\bar{e}_{A1}\hat{\bar{e}}_{A1} - p_{e2}\bar{e}_{A2}\hat{\bar{e}}_{A2}. \qquad \text{(A54)}$$

We solve (A54) for $\hat{p}_{e1}$ and insert this term into $\hat{e}_{F1}e_{F1} = e_{N1}\eta_{N1}\hat{p}_{e1}$ to establish after some rearrangement of terms (31).

Next, solving (A53) and (A54) yields

$$\frac{\hat{p}_{x2}}{\hat{\bar{e}}_{A1}} = \frac{\tilde{\gamma}_p\bar{e}_{A1}(p_{e1} + \lambda\pi_1) - \lambda\pi_1\bar{e}_{A1}p_{e1}e_{N1}\eta_{N1}}{\tilde{D}}, \tag{A55}$$

$$\frac{\hat{p}_{x2}}{\hat{\bar{e}}_{A2}} = -\frac{(1-\lambda)\bar{e}_{A2}\pi_2(\tilde{\gamma}_p - p_{e1}e_{N1}\eta_{N1}) - p_{e1}^2\bar{e}_{A2}e_{N1}\eta_{N1}}{\tilde{D}}, \tag{A56}$$

where $\tilde{D} := -\tilde{\gamma}_e\tilde{\gamma}_p + \tilde{\gamma}_x(\tilde{\gamma}_p - p_{e1}e_{N1}\eta_{N1})$. Using the same arguments as in Lemma 1 one can show that

$$\tilde{D} = (\tilde{\gamma}_p - p_{e1}e_{N1}\eta_{N1})\lambda\sigma\left(\sum_{j=A,F,N} x_{j1}\right) - \tilde{\gamma}_p p_{e1}e_{N1}\eta_{N1} > 0.$$

Inserting (A55) in (15b) and (A56) in (31) we obtain

$$\frac{\hat{e}_{F1}}{\hat{\bar{e}}_{A1}} = \frac{\tilde{\gamma}_p\bar{e}_{A1}\lambda}{e_{F1}\tilde{D}}\left(\sigma\sum_{j=A,F,N} x_{j1} + \pi_1 e_{N1}\eta_{N1}\right), \tag{A57}$$

$$\frac{\hat{e}_{F1}}{\hat{\bar{e}}_{A2}} = \frac{p_{e1}e_{N1}\eta_{N1}\bar{e}_{A2}}{e_{F1}\tilde{D}}\left(\lambda\sigma\sum_{j=A,F,N} x_{j1} + (1-\lambda)\pi_2 e_{N2}\eta_{N2}\right). \tag{A58}$$

Finally, we rearrange (A58) with the help of $\lambda x_{j1} = (1-\lambda)p_{x2}x_{j2}$ to

$$\frac{\hat{e}_{F1}}{\hat{\bar{e}}_{A2}} = \frac{(1-\lambda)p_{e1}e_{N1}\eta_{N1}\bar{e}_{A2}}{e_{F1}\tilde{D}}\left(\sigma\sum_{j=A,F,N} p_{x2}x_{j2} + \pi_2 e_{N2}\eta_{N2}\right). \tag{A59}$$