Communication and Decision-Making in Committees: A Dynamic Approach*

Athanassios Pitsoulis[†], Gerrit Werthebach

June 13, 2007

Abstract

We model the processes of private and public communication between two factions in a committee or similar decision-making body as differential equation systems. What is communicated is information regarding the factions' leaders policy proposals. The models' parameters reflect different communication technologies as well as the availability of information regarding voter-preferences. It is possible to derive expected values for the development of several variables, like the number of active campaigners and votes, over time. We show that the dynamics of communication may lead to a previously unexplored voting paradox. Going beyond the models we explore Parliamentary Procedures and their effects on communication and decision-making.

JEL Classification D71; D74; D78

Keywords Economic Models of Political Processes; Conflict; Positive Analysis of Policy-Making; Elections, Legislatures, and Voting Behavior

1 Introduction

> Democracy means government by discussion, but it is only effective if you can stop people talking.
>
> Clement Atlee (1883-1967)

Democracy lives on making people talk and sometimes on making people stop talking. Our topic is the modeling of talk as transmission of information between groups of agents who are going to vote on passing a motion. We contribute to the literature on communication in a committee by proposing dynamic models of private and public communication, thus adding the dimension of time to

*Work in progress and incomplete. Please do not quote or reproduce without the authors' consent.

†Corresponding author. Address: Faculty III, Institute of Economics, Chair of Microeconomics, Technical University of Brandenburg, Konrad Wachsmann Allee 1, 03046 Cottbus, Germany. Phone: +49 355 69 2982. e-Mail: pitsouli@tu-cottbus.de.

the hitherto static models of committee interaction.[1] Our method combines elements derived from conflict models and epidemiological models.

Debates, discussion and talk can abstractly be interpreted as time-consuming processes of transmitting information between committee factions regarding their party leaders' policy proposals, in which the transmission of information is not always successful. The information-transmission probabilities can be interpreted as characteristics of media and technology, or argumentation and rhetoric. Both interpretations are possible. Seen from this point of view, communication serves all involved actors by reducing the incompleteness of information about what alternatives to the own leader's proposal exist. The chance of a successful transmission of information about a policy proposal may be time-independent (resulting in systems with constant coefficients), or time-dependent (resulting in systems with variable coefficients). We concentrate here on systems with constant coefficients.

It is plausible to start from the basic assumption that both the willingness to disseminate information and the willigness to vote are depending on individuals realizing whether their preferences are satisfied by the opposition's policy proposal or not. This means that some agents may form an interior opposition and take part in informing own group members about the fact that the opposition's proposal is a better alternative to the own proposal. The first imaginary picture of such a debate is that it must be linked to several factors: initial and actual group size, individual preferences, and information transmission probabilities. Some of these factors will be time-dependent, others not. They influence who communicates with whom and who votes for which side's proposal.

The models we propose are a tool for predicting several interconnected developments of communication processes, like the expected numbers of speakers and voters, inter and intra-party communication etc. They are a tool for picturing the technical aspects of communication, but do not cover the institutional background of the committee, like the Parliamentary Procedures, the discretionary powers of the speakers etc. The institutional background determines for instance whether Filibustering is possible, or whether the debate can be curtailed and votes be forced. Rational party leaders might exploit procedural rules with the strategic aim to tilt the number of votes in their favor.

We will show that the effect may be dramatic; the final result of the vote is as much a function of group size and communication as well as of the rules concerning how and how long to communicate. Democracy may be ineffective if no-one can stop people talking, but ineffective, too, *if* someone can stop people talking. This is why we add the dimension of rules and discretion in the dynamic context of the models.

[1] Instead of writing an overview of the existing literature we refer the reader to the comprehensive survey by Gerling et. al. (2005).

2 Committee Composition, Communication, and Voting

We will in following use the words “factions” and “parties” synonymously, as well as the expressions “policy proposals”, “motions” and “party programs”. Our aim is to formulate the models with a minimum set of as weak as possible assumptions.

Assumption 1. A collective decision is to be made by a committee C, which consists of two factions, $i = A, B$. Both factions are assumed to be led by a party leadership, L_i $(i = A, B)$, with conflicting policy proposals. Every faction consists of individuals and every individual has either a preference for the motion of the own leader or the other. Every individual is either actively informing others or not.

In order to economize on notation we propose the following language convention: Whenever we speak of a faction or party we shall use the capital letters A and B. Whenever we speak of numeric strength we shall use the expressions $A(t)$ and $B(t)$.

Definition 1. *Active campaigners* are defined as individuals which take part in informing other individuals about party programs. Let $A_i(t), B_i(t) \in \mathbb{Q}^+$ $(i = A, B)$ denote the expected number of active campaigners with a preference for the party program of L_i at each point in time $t \in \mathbb{R}^+$. Let $A(t), B(t) \in \mathbb{Q}^+$ denote the expected total number of active campaigners at each point in time $t \in \mathbb{R}^+$. We define $A(t) := A_A + A_B(t)$ and $B(t) := B_B + B_A(t)$, $A_A, A_B(0), B_B, B_A(0) \in \mathbb{N}$. The initial numeric size of the groups is given by $A(0) := A_0$ and $B(0) := B_0$. The size of the committee C is defined as $C_0 := A_0 + B_0$, $C_0 \in \mathbb{N}^*$.

We call A_A, B_B the “voter bases” and A_B, B_A the “swing voters”. Factions consist of two groups, the voter base members, which are always actively sending information, and the swing voters, which do so depending from the information received. It follows that $A_0 = A_A + A_B(0)$ and $B_0 = B_B + B_A(0)$. The variables $A(t)$ and $B(t)$ reflect the expected number of *inter-party*-campaigners, i.e. how many individuals are expected to communicate the party program to the other group at each point in time.

To keep things simple we assume the leaders play only a role as decision-makers.

Assumption 2. The factions's leaders L_A and L_B do neither take part in the dissemination of information, nor the vote. They take decisions solely with the aim not to loose the vote on their policy proposal.

We next introduce an assumption about who is expected to vote for which motion if a vote takes place.

Assumption 3. We assume the voter base as well as all uninformed swing voters would vote for the own leader's motion when a vote is called. So are those swing voters of the opposition which have successfully been informed.

This assumption reflects the idea of a party discipline under which group members will vote for the own leader's proposal in the absence of information about a better alternative.

The expected number of vote(r)s for each proposal at every point in time $t > 0$ equals the sum of the number of own campaigners and the number of opposition swing voters having been succesfully been informed. The latters' number can easily be calculated by subtracting the sum of the voter base and the still uninformed swing voters from the initial group size.

Definition 2. Let $V_i(t) \in \mathbb{Q}^+$ $(i = A, B)$ denote the *expected number of voters* for a the motion of L_i. At each point in time the expected number of voters is defined as $V_A(t) := A(t) + [B_0 - B(t)]$ and $V_B(t) := B(t) + [A_0 - A(t)]$.

It follows that $V_A(0) = A_0 = A_A + A_B(0)$ and $V_B(0) = B_0 = B_B + B_A(0)$. As $A(t) = A_A + A_B(t)$ and $B(t) = B_A + B_B(t)$ (by Definition 1) this means that the minimum number of votes at each point in time $t > 0$ for the motion of L_A is A_A and B_B for L_B.

We next introduce an interesting element of 'viral marketing' by making the following assumption.

Assumption 4. A constant proportion of the swing voters of faction i $(i = A, B)$ is of a type that having successfully been informed about the alternative policy proposal will communicate that information to her *own* group.

This means a proportion of potential swing voters may be expected not only to vote according to their preference but also to take part in the dissemination of information.[2] We call individuals of this type "swing campaigners".

Definition 3. Let $\beta_i \in [0, 1]$ $(i = A, B)$ denote the proportion of faction i's swing voters that are *swing campaigners.*

The terms $\beta_A \cdot A_B(t)$ and $\beta_B \cdot B_A(t)$ reflect the expected number of potential *intra-party* campaigners. This concept is inspired by epidemiological models and reflects the idea of an 'infection' of a population. Accordingly, the terms $\beta_A[A_0 - A(t)]$ and $\beta_B[B_0 - B(t)]$ reflect the number of successfully informed, i.e. active, intra-party-campaigners. They form what we call the "interior opposition".

We next address communication in the committee.

Assumption 5. Active campaigners continuously engage in the transmission of information between groups and within groups via a medium (or technology) regarding the preferred policy proposal. Information transmission

[2] Although it is a highly interesting question whether β_i $(i = A, B)$ could depend on other factors (like the technology used, i.e. information transmission probabilities or costs) these parameters are for simplicity assumed here to be exogeneous.

is costless. The information transmission probability is constant and the same for all campaigners within a faction.

The assumption that there exists an information transmission probability makes sense because attempts to inform other individuals take time. This fundamental problem has to our knowledge not been addressed in the hitherto static approaches to communication in committees. If communication takes time, the expected number E of individuals informed during a certain time-interval Δt amounts to $E \cdot \Delta t > 0$. For large Δt the expected number can be larger than one, but this would conflict with our assumption. For sufficiently small Δt the expected value can however be approximated by an increasing function $P = E(\Delta t)$ with $P = 0$ for $\Delta t = 0$ and $P \leq 1$.[3]

We assume information can be transmitted either in private or in public.[4] Private communication reflects confidential talks typical for lobbying activities while public communication reflects plenary speeches typical for parliamentary debate.

Assumption 6. Communication is either public or private. Private communication is assumed to be made in such a way that information is exclusively sent to a randomly selected individual. Public communication is assumed to be made in such a way that information is sent to every member of the addressed group.

Information is assumed to be transmitted through noisy channels. These channels reflect different ways to disseminate information like media for forums.

Definition 4. An *information transmission technology* is defined by the probability of a successful transmission of information. Let $p_{ji} \in]0,1]$ $(i,j = A,B)$ denote the probability of a successful *private* transmission of information about the party program of L_i, i.e. from one member of faction i to one member of faction j. Let $q_{ji} \in]0,1]$ $(i,j = A,B)$ denote the probability of a successful *public* transmission of information about the party program of L_i from a member of faction i to all members of faction j.

A successful transfer of information from one committee member to another reveals the policy proposal. This in turn enables the receiver to decide for which motion she will vote. Swing voters will switch to vote for the opposition's motion, voter base members not.

The effect of communication crucially depends on the number of senders, receivers, transmission probabilities and technology. This shall be shortly illustrated. Imagine a group A with $A_0 = 2$ smokers and a group B with $B_0 = 2$

[3] The probability of a successful sending of information can alternatively be interpreted as (i) a probability of *persuasion* e.g. as in Yokoyama (1986) or (ii) the probability that an addressed individual *receives* information when it is addressed. This is consistent with the mathematical theory of communication, see e.g. Weaver (1964). A consequence is that the transmission probability may depend on the information sent. Some topics may be easier to communicate than others.

[4] Note that interaction does not necessarily need to be face-to-face. The underlying structure is the same for all kinds of media like TV, blogs, e-mail, leaflets, etc.

non-smokers are communicating about their conflicting policy proposals regarding smoking. Let us assume both smokers were of the type A_B, i.e. swing voters, and both non-smokers of the type B_B, i.e. members of the non-smoker voter base.

1. Private communication: The non-smokers make a total of two interactions, each with a success-chance q_{AB}. The group of non-smokers is thus expected to reveal the own proposal to $2q_{AB}$ smokers in the fist period. Suppose one smoker were successfully informed. In the second period the expected number of informed smokers is $2q_{AB}$, too. However, the expected number of 'converted' smokers relative to the number of addressed smokers is q_{AB} in the first period, but $2q_{AB}$ in the second period.

2. Public communication: The non-smokers essentially make a total of four interactions, each with a success chance p_{AB}. The non-smokers, as a group, are expected to inform $4p_{AB}$ smokers in the fist period. Let us again assume one smoker were successfully informed in the first period. The expected number of persuaded smokers is $4p_{AB}$ in the second period, too. However, the expected number of 'converted' smokers relative to the number of addressed smokers is $2p_{AB}$ in the first *and* second period.

Suppose now $p_{AB} = q_{AB}$. One might say that under these circumstances private communication is characterized by economies of scale caused by a greater relative group size. Public communication, on the other hand, is characterized by constant economies of scale. The reason is that private communication enables a faction to concentrate on the uninformed members of the opposition.

In the above example it was assumed that both smokers were swing voters. Suppose now one of the smokers were not a swing voter. If the identity of the swing voter were known to the non-smokers both could immeadiately concentrate their efforts on her without losing time by communicating with the other individual.[5] The effectivity of such a concentration of communication efforts is obviously depending on the availability of information. This observation leads us to the first hypotheses.

Proposition 1. *Suppose the identity of the swing voters of a faction A were common knowledge. Then the number of individuals to which L_B's policy proposal is revealed by private communication is proportional to* $B(t) + \beta_A[A_0 - A(t)]$.

[5]Concentrating on swing voters is most important in election campaigning. Campaigners try to concentrate their efforts by various means. One team recently came up with an idea to get voters to the polls for the democratic primary election candidate Joe Lieberman in the US Federal State of Connecticut. They hang "Vote Joe" signs on as many doorknobs as possible, and returned after a few hours. If the signs were gone by the time they returned they would go and concentrate their efforts on the voters having revealed they would vote for another candidate. Unfortunately for Joe Lieberman's campaigners it was a stormy day, and when they returned all the signs were gone with the wind (The Economist 2006: 33). This anecdote nicely demonstrates that information transmission probabilities may depend on exogeneous factors and therefore stochastic. To keep things simple we concentrate on constant transmission probablities.

Proof. The proof is straightforward. If the identity of the factions' swing voters is common knowledge to all individuals, both intra-party campaigners and inter-party campaigners can then use this knowledge to concentrate their efforts on the group of uninformed swing voters. It follows that at every point in time $t > 0$ the whole group of A's uninformed swing voters is exposed to information sent by the whole group of active opposition campaigners $B(t)$ and the whole interior opposition $\beta_A[A_0 - A(t)]$. (Q.E.D.)

Proposition 2. *Suppose the identity of the swing voters of a faction A were private knowledge of the individual members. Then the number of individuals to which a L_B's program is revealed by private communication is proportional to* $[A_B/A(t)] \cdot \{B(t) + \beta_A[A_0 - A(t)]\}$.

Proof. See Proposition 1. If the identity of the swing voters is private knowledge, it is impossible to target the group of swing voters exclusively. According to Assumption 6 individuals will then be randomly selected out of the whole opposed faction. It follows that at every point in time the probability of selecting an uninformed swing voter must equal the proportion of uninformed swing voters in the whole faction, $A_B(t)/A(t)$. (Q.E.D.)

Proposition 3. *Suppose the identity of the swing voters of A were faction-specific (i.e. common knowledge to the own party members but unknown to any member of the opposition). Then the number of individuals to which L_B's program is revealed by private communication is proportional to* $[A_B(t)/A(t)] \cdot B(t) + \beta_A[A_0 - A(t)]$.

Proof. See Propositions 1 und 2. If the identity of the swing voters is faction-specific information, it is only possible to target the group of swing voters for A's interior opposition but not for B's campaigners. (Q.E.D.)

Private communication between the committee factions will thus lead to different outcomes in terms of votes depending on the availability of voter-type knowledge. Public communication is not affected by the availability of voter-type information, as every member of a communicating group sends policy information to every member of the opposition anyway.

Proposition 4. *Irrespective of the information structure regarding the identity of swing voters of a faction A the number of individuals to which L_B's policy proposal is revealed by private communication is proportional to* $A(t) \cdot B(t) + A(t) \cdot \beta_A[A_0 - A(t)]$.

Proof. The proof is straightforward. Public communication means that every active campaigner sends information to every potential receiver. At every point in time $t > 0$ L_B's program is revealed to $A(t)$ opposition members by $B(t)$ own active inter-party campaigners and additionally by the proportion β_A of successfully informed members of A that are swing campaigners. (Q.E.D.)

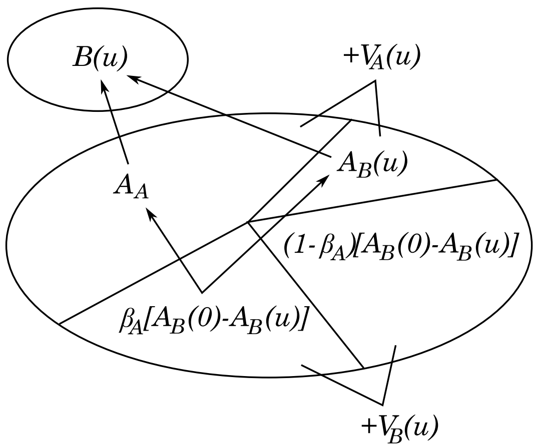
The communication process is assumed to lead towards a collective decision. We need an assumption regarding the social choice rule.

Assumption 7. One simple majority vote may be taken on the conflicting motions at some point in time. The communication process is assumed to end at this point. Each individual is assumed to have one and only one vote.

Definition 5: Let Q denote $C_0/2$ and η denote a 'small' number of voters larger than unity. The *quorum* necessary to win a simple majority vote is defined as $Q + \eta$.

Having established the basic framework the composition of the factions shall be illustrated by means of a picture. Figure 1 shows the composition of faction A at some point in time $t = u$. The arrows show the communication out of and within the faction. Faction A consists of several sub-groups, namely the voter base A_A, the potential swing voters $A_B(u)$, and the individuals that have successfully been informed by B. Their number is $A_0 - A_B(u)$. This group consists of two subgroups, the proportion $(1 - \beta_A)$ of 'silent' swing voters for L_B's policy proposal and the proportion β_A of the active swing voters. Would a vote be taken at $t = u$, A_A and $A_B(u)$ would join the ranks $V_A(u)$ of voters for L_A's program and $A_0 - A_B(u)$ individuals would join the ranks $V_B(u)$ of voters for L_B's proposal.

Figure 1: Composition of faction A at some point in time $t = u$



3 The Dynamics of Communication in Committees

We propose one differential equation model for public and one for private communication. The models are sufficiently general to cover several interesting special cases. They basically describe a "playing tag"-like mechanism, where agents try to catch other agents and where those caught in turn will (or not) join the group of the catchers.

3.1 Private Communication

Each group faces an expected loss of active campaigners which is proportional to the number of opposed agents if communication is private (e.g. organized as confidential talks, typical for lobbying). We propose the linear system of differential equations

$$\begin{aligned} \dot{A}(t) &= -q_1 r_1 B(t) - q_2 r_2 \beta_A [A_0 - A(t)], & (1)\\ \dot{B}(t) &= -q_3 r_3 A(t) - q_4 r_4 \beta_B [B_0 - B(t)] & (2) \end{aligned}$$

as a model of private communication between the two factions in the committee C. The model is sufficiently general to cover voter-type information structures and technology structures.

The variables r_k ($k = 1, \ldots, 4$) reflect the availability of voter-type information. According to Propositions 1–3 they take the following values: (i) If the voter type is common knowledge $r_k = 1$. (ii) If the voter type is private information $r_1 = r_2 = A_B(t)/A(t)$ and $r_3 = r_4 = B_A(t)/B(t)$. (iii) If the voter type is party-specific $r_1 = A_B(t)/A(t)$, $r_2 = B_A(t)/B(t)$ and $r_3 = r_4 = 1$.

The variables q_k ($k = 1, \ldots, 4$) reflect different technology structures: (i) If the interior oppositions use their own group's technology $q_1 = q_2 = q_{AB}$ and $q_3 = q_4 = q_{BA}$. (ii) If the interior opposition uses the opposition's medium $q_1 = q_4 = q_{AB}$ and $q_2 = q_3 = q_{BA}$.

For $A_A, B_B > 0$ system (1),(2) can alternatively be formulated in decay rates (i.e. negative growth rates) by dividing the first equation through $A(t)$ and the second equation through $B(t)$:

$$\begin{aligned} \frac{d}{dt}[\ln A(t)] &= -q_1 r_1 \frac{B(t)}{A(t)} - q_2 r_2 \beta_A \left(\frac{A_0}{A(t)} - 1 \right), & (3)\\ \frac{d}{dt}[\ln B(t)] &= -q_3 r_3 \frac{A(t)}{B(t)} - q_4 r_4 \beta_B \left(\frac{B_0}{B(t)} - 1 \right). & (4) \end{aligned}$$

Equation (3) shows—metaphorically spoken—the expected rate of decay of the cake-piece $A_A + A_B$ in Figure 1. Under the assumptions made here it can not shrink to a size smaller than A_A. Equation (4) shows the same for the other committee faction.

By inserting the general solutions for initial value combinations (A_0, B_0) or their numerical spproximations into $V_A(t)$ and $V_B(t)$ as by Definition 2 the expected number of voters for each $t > 0$ can be calculated. Note that a general solution can only be derived with standard methods for the case of complete information where (1),(2) is linear.

Proposition 5. *The general solution of (1),(2) consists of linear combinations of exponential functions.*

Proof. See Appendix.

It is helpful to summarize some properties of system (1),(2):

1. $\dot{A}(t), \dot{B}(t) \leq 0$ for all $A(t), B(t) \geq 0$, $t > 0$.
2. $\lim_{t\to\infty} A(t) = -\infty$ and $\lim_{t\to\infty} B(t) = -\infty$ with A_A and B_B being the lower limits of $A(t)$ and $B(t)$.
3. $\lim_{t\to\infty} A_B(t)/A(t) = 0$ and $\lim_{t\to\infty} B_A(t)/B(t) = 0$.
4. For $t \to \infty$ system (1),(2) reduces to

$$\begin{aligned} \dot{A}(t) &= -q_1 r_1 B_B - q_2 r_2 \beta_A A_B, \quad (5) \\ \dot{B}(t) &= -q_3 r_3 A_A - q_4 r_4 \beta_B B_A. \quad (6) \end{aligned}$$

   (a) If the voter-type is common knowledge (5),(6) reduces to

$$\begin{aligned} \dot{A}(t) &= -q_1 B_B - q_2 \beta_A A_B, \\ \dot{B}(t) &= -q_3 A_A - q_4 \beta_B B_A \end{aligned}$$

   for $t \to \infty$.

   (b) If the voter-type is party-specific (5),(6) reduces to

$$\begin{aligned} \dot{A}(t) &= -q_2 \beta_A A_B, \\ \dot{B}(t) &= -q_4 \beta_B B_A \end{aligned}$$

   for $t \to \infty$.

   (c) If the voter-type is private information it follows $\dot{A}(t) = \dot{B}(t) = 0$ for $t \to \infty$.

5. $\dot{V}_A(t), \dot{V}_B(t) \gtreqless 0$ for all $V_A(t), V_B(t) \geq 0$, $t > 0$.
6. $\lim_{t\to\infty} V_A(t) = \pm\infty$ and $\lim_{t\to\infty} V_B(t) = \mp\infty$ with $A_A + B_A$ and $A_B + B_B$ being the lower and upper limits of $V_A(t)$ and $V_B(t)$.

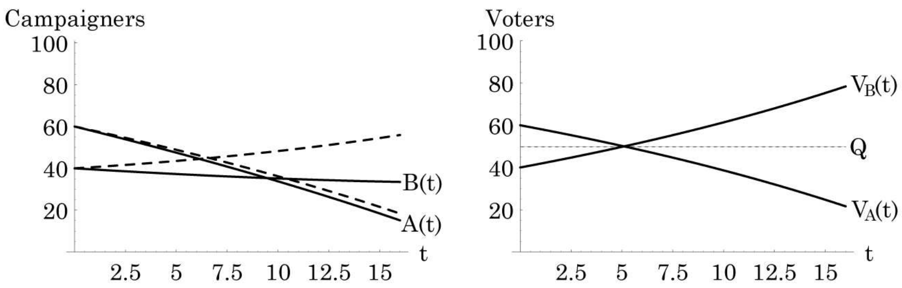
One important property can immeadiately be derived from the general solution.

Proposition 6. *The functions $A(t)$ and $B(t)$ exhibit no cycles.*

Proof. See the proof of Proposition 5 in the Appendix. Cyclical behaviour does only appear when the Eigenvalues of the System Matrix M, i.e. if the exponential functions forming part of the solution of (1),(2), have complex arguments. As by the assumptions made regarding the parameters the Eigenvalues of M are real the general solution of (1),(2) are non-periodic. (Q.E.D.)

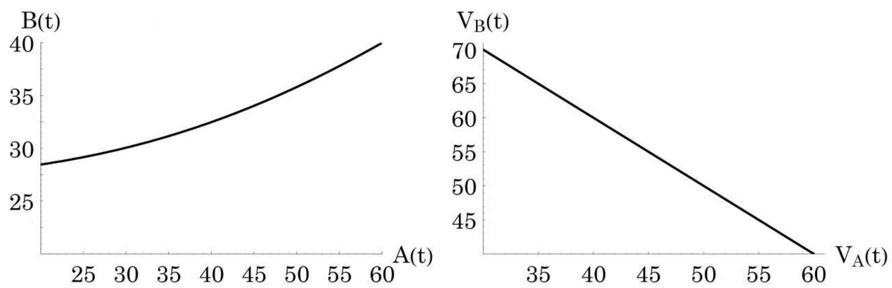
We assume $A_A = 20$, $A_B = 40$, $q_{BA} = 0.01$, $\beta_A = 0.5$ and $B_B = 30$, $B_A = 10$, $q_{AB} = 0.06$, $\beta_B = 0.5$ for a numerical example. Interior oppositions use the own faction's medium ($q_1 = q_2 = q_{AB}$, $q_3 = q_4 = q_{BA}$) and voter-type-information is complete ($r_1 = r_2 = r_3 = r_4 = 1$). The graphs are shown in Figure 2. The dashed curves in the left diagram show the graphs of the *total* expected number of active campaigners, i.e. inter-party campaigners plus informed opposition intra-party campaigners. The interpretation is intuitive. The communication process obviously has an effect on the number of active campaigners and the number of voters. The process can be decomposed into two phases: In the first phase L_A has a single majority for her policy proposal. Voting in $t \simeq 5.15$ would result in a draw. In the second phase L_B has a majority for her program.

Figure 2: Private Communication with Complete Voter-Type Information



The parametric plots of the combinations $A(t), B(t)$ and $V_A(t), V_B(t)$ shown in Figure 3 help to demonstrate the effects of communication on the number of campaigners and voters. They show what might be called the 'transformation-curves' of the communication process. The graph of $B(t)$ as a function of $A(t)$ starts from the intial-value combination $(A_0, B_0) = (60, 40)$ and reaches the combination $(A_A, B_B) = (20, 30)$. The graph of $V_B(t)$ as a function of $V_A(t)$ starts from the intial-value combination $(V_A(0), V_B(0)) = (60, 40)$ and reaches the combination $(30, 70)$.

Figure 3: Parametric Plots



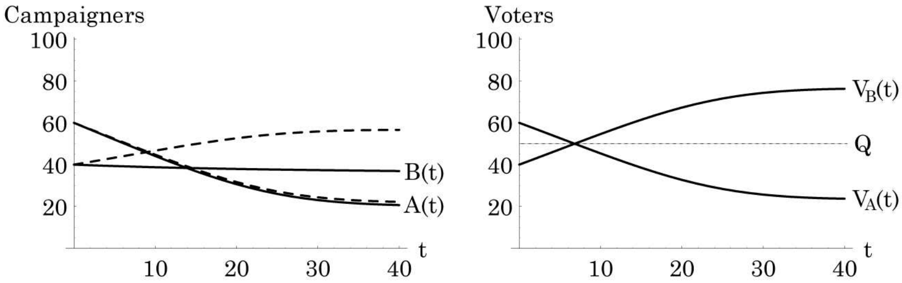
For better interpretation some variations of the parameters β_A, β_B and q_{AB} are shown in Figure 9 (see Appendix).

The effect of of private information regarding the identity of the swing-voters, i.e. $r_1 = r_2 = A_B(t)/A(t)$ and $r_3 = r_4 = B_A(t)/B(t)$, is shown in Figure 4.

3.2 Public Communication

Let us now explore the effect of communication between the factions of the committee when communication is public (e.g. organized as plenary debate). It

Figure 4: Private Communication with Private Voter-Type Information



is irrelevant whether the identity of swing voters is common knowledge or not. Variables reflecting different information structures are therefore not needed. According to Proposition 4 each group faces continuous expected losses of campaigners proportional to the product of the communicating groups' expected numeric size as long as the debate lasts. We propose the quadratic system of differential equations

$$\dot{A}(t) = -p_1 A(t)B(t) - p_2 A(t)\beta_A[A_0 - A(t)], \tag{7}$$

$$\dot{B}(t) = -p_3 B(t)A(t) - p_4 B(t)\beta_B[B_0 - B(t)] \tag{8}$$

as a model of public communication in the committee. The technology structures are: (i) $p_1 = p_2 = p_{AB}$, $p_3 = p_4 = p_{BA}$ and (ii) $p_1 = p_4 = p_{AB}$, $p_2 = p_3 = p_{BA}$. System (7),(8) can be expressed in attrition rates:

$$\frac{d}{dt}[\ln A(t)] = -p_1 B(t) - p_2\beta_A[A_0 - A(t)],$$

$$\frac{d}{dt}[\ln B(t)] = -p_3 A(t) - p_4\beta_B[B_0 - B(t)].$$

Unfortunately the quadratic system (7),(8) is rather deep and a general solution can not be found with standard methods. Numerical approximations do however suffice for our purposes.

The model is similar to (1),(2) but differs in some important points which will be taken up later. We summarize the properties of system (7),(8):

1. $\dot{A}(t), \dot{B}(t) \leq 0$ for all $A(t), B(t) \geq 0$, $t > 0$.
2. $\lim_{t\to\infty} A(t) = A_A$ and $\lim_{t\to\infty} B(t) = B_B$.
3. $\dot{V}_A(t), \dot{V}_B(t) \gtreqless 0$ for all $V_A(t), V_B(t) \geq 0$ and $t > 0$.
4. $\lim_{t\to\infty} V_A(t) = A_A + B_A$ and $\lim_{t\to\infty} V_B(t) = B_B + A_B$.
5. System (7),(8) reduces to

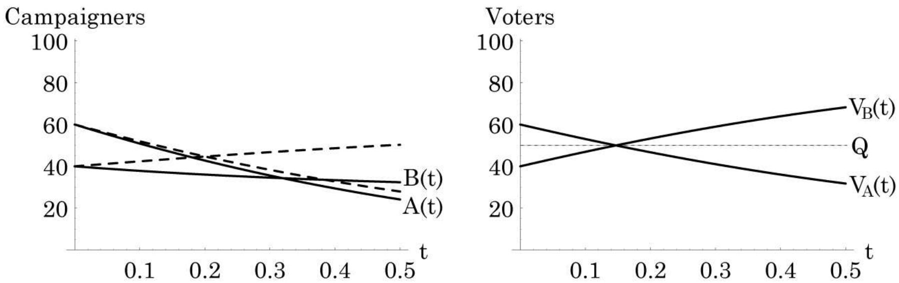
$$\dot{A}(t) = -p_1 A_A B_B - p_2\beta_A A_A A_B,$$

$$\dot{B}(t) = -p_3 B_B A_A - p_4\beta_B B_B B_A$$

for $t \to \infty$.

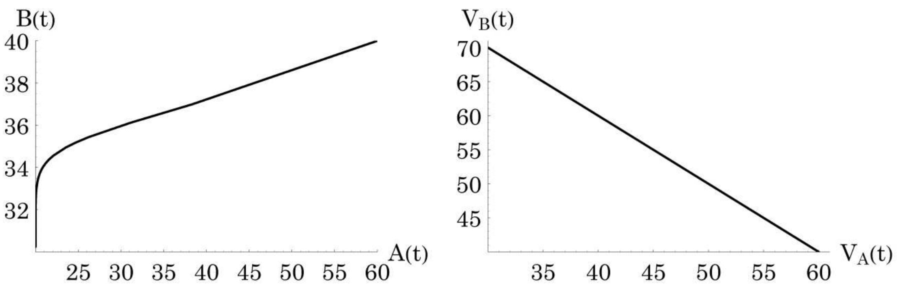
The graphs for $A_A = 20$, $A_B = 40$, $p_{BA} = 0.01$, $\beta_A = 0.5$, $B_B = 30$, $B_A = 10$, $p_{AB} = 0.04$, $\beta_B = 0.5$ and $p_1 = p_2 = p_{AB}$, $p_3 = p_4 = p_{BA}$ are shown in Figure 5.

Figure 5: Public Communication



Parametric plots of the combinations $A(t), B(t)$ and $V_A(t), V_B(t)$ are shown in Figure 6.

Figure 6: Public Communication: Parametric Plots



Parameter-variations of β_A, β_B and p_{AB} are shown in Figure 10 (see Appendix).

3.3 Special Cases and Properties

We first briefly explore some special cases of the models.

1. $A_B, B_A = 0$: All party members belong to their leaders' voter base. The number of votes can not be influenced by communication and is constant for all $t \geq 0$. This case arises in reality when the voters' preferences are perfectly homogeneous or the leaders impose a strict party discipline.

2. $A_A, B_B = 0$: All party members are potential swing voters. This case reflects an improbable situation where all members of a group are basically in the 'wrong' committee faction. It arises if the information transmission probability is interpreted as a persuasion probability. Under these circumstances the long-run behavior of (1),(2) and (7),(8) is unrealistic.

3. $\beta_A, \beta_B = 0$: No potential swing voter is a swing campaigner. This case reflects a flexible party discipline allowing individuals to vote for the opposition leader's proposal but not to actively take part in informing other members of the own party.[6]

4. $\beta_A, \beta_B = 1$: All potential swing voters are swing campaigners. This case reflects a weak party discipline allowing individuals not only to vote for the opposition leader's proposal but also to actively inform own party members.

5. $p_{ji}, q_{ji} = 1$ $(i, j = A, B;\, i \neq j)$: Public or private communication is always successful. This case reflects the absence of any noise. It is more realistic to assume $p_{ji}, q_{ji} < 1$. Transmission probabilities may however be close to unity when C_0 is small and the policy proposals are very easy to communicate.

6. $p_{ji} = q_{ji}$ $(i, j = A, B;\, i \neq j)$: Public and private information transmission have the same success probability. This case arises when both methods of communication use the same noisy channels. It may reflect speech-based communication under modern conditions.

It is helpful to formulate the following proposition.

Proposition 7. *The relative slope of the functions $V_i(t)$ $(i = A, B)$, whenever defined, equals -1 for every $t > 0$.*

Proof. Differentiating $V_A(t) = A(t) + [B_0 - B(t)]$ and $V_B(t) = B(t) + [A_0 - A(t)]$ for t yields $\dot{V}_A(t) = \dot{A}(t) - \dot{B}(t)$ and $\dot{V}_B(t) = \dot{B}(t) - \dot{A}(t)$. The relative slope $\dot{V}_A(t)/\dot{V}_B(t)$ must accordingly always equal -1 for $\dot{V}_A(t), \dot{V}_B(t) \neq 0$. (Q.E.D.)

Proposition 7 says in other words that the curves V_A and V_B are mirror-images of each other. It follows directly that Q is the horizontal mirror-axis. This means communication leads to a 'migration' of voters. Depending on the initial values and the 'migration balance' the models may exhibit two states at some $t \geq 0$: (i) there is a majority for one faction or (ii) both factions have the same number of votes (we call this a 'stalemate'). Communication may lead to a transformation of a minority into a majority or end a stalemate.

Definition 6. Let ε be a finite time-interval. *Majority switches* are said to occur at some $t = u$ $(u > 0)$ if $V_A(u - \varepsilon) > V_B(u - \varepsilon)$ and $V_A(u + \varepsilon) < V_B(u + \varepsilon)$ or vice versa.

Every majority switch necesarily involves a point in time where a stalemate occurs.

[6] The combination of cases 2 and 3 reduce the models (1),(2) and (7),(8) to the two basic versions of the so-called Lanchester combat model. For more information the reader is referred to the large literature on the Lanchester model (see e.g. Coleman 1982). For an economics perspective see Hirshleifer (2002).

Definition 7. A *stalemate* is said to occur if $V_A(u) = V_B(u) = V_A(u+\varepsilon) = V_B(u+\varepsilon)$.

A stalemate may exist if the migration of voters results in a dynamic equilibrium of the kind that the migration balance is zero for some $t > 0$. If the voter migration balance is zero $V_A(t)$ and $V_B(t)$ will be parallel lines. If $A_0 = B_0$ there will be a stalemate.

If a stalemate ends or a majority switch occurs we shall speak of a 'majority development'. We now explore under which conditions majority developments do occur.

Proposition 8. *For a given* $0 < p_{ji}, q_{ji} \leq 1$ $(i, j = A, B;\ i \neq j)$ *a majority development will occur at some* $t = u$ $(u > 0)$ *if* $A_0 \geq B_0$ *and* $A_A < B_B$ *or* $A_0 > B_0$ *and* $A_A \leq B_B$.

Proof. $A_0 \geq B_0$ can be written as

$$\begin{aligned} V_A(0) &\geq V_B(0) \\ \Leftrightarrow A_A + A_B(0) &\geq B_B + B_A(0). \end{aligned}$$

We now ask: Supposed $A_A < B_B$, does this necessarily imply a majority development? A majority development occurs at some $t = u$ $(u > 0)$ if $V_A(u-\varepsilon) \geq V_B(u-\varepsilon)$ and $V_A(u+\varepsilon) < V_B(u+\varepsilon)$. To simplify notation we define $u - \varepsilon := 0$ and $u + \varepsilon := v$. $V_A(u+\varepsilon) < V_B(u+\varepsilon)$ can then be written as

$$A_A + A_B(v) - B_A(v) + B_A(0) < B_B + B_A(v) - A_B(v) + A_B(0). \qquad (9)$$

$A_B(v) - B_A(v)$ is the swing voter migration balance between the two factions. Two things can be observed: (i) As $\lim_{v\to\infty} A_B(v) = \lim_{v\to\infty} B_A(v) = 0$ (9) is fulfilled in the long run for $A_A < B_B$. (ii) The swing voter migration may be zero for some $t \in [0, u[$. Under these circumstances $V_A(0 \leq t < u) = A_0$ and $V_B(0 \leq t < u) = B_0$. If $A_A < B_B$ $B(t)$ must reach its lower limit of inter-party campaigners first. Supposed the level B_B is reached in $t = u$. Then $B_A(v)$ must necessarily be zero while $A_B(v)$ is non-zero. It follows the swing voter migration balance an not be zero for $t > v$ until A_A has been reached, too. The same is true for the other cases. (Q.E.D.)

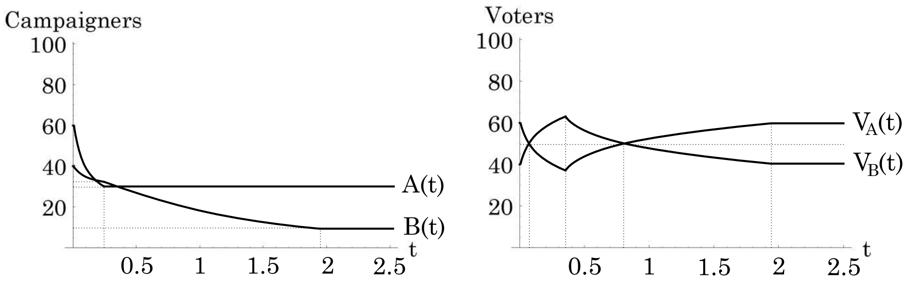
Proposition 8 says in other words that a majority development must be expected to occur sooner or later if a faction of equal or larger inital size has the smaller voter base, the information transmission probabilities are positive and both factions communicate. A permanent stalemate is ruled out under these conditions. Communication must always result in reaching the state where all committee members are informed.

This is not trivial, as communication may lead to an interesting phenomenon. It has, to our knowledge, not been explored in the earlier literature: majority switches can happen more than once although $A(t)$ and $B(t)$ do not behave cyclically as shown by Proposition 6. This can be classified as a new voting paradox, based on the Nurmi-criterion that a voting paradox arises whenever "the relationship between the voting result and the voter preferences is counter-intuitive or unreasonable in some sense" (Nurmi 1998: 335).

Definition 8. A *"multiple switch" voting paradox* is said to occur if more than one majority switch occur.

Such a voting paradox arises e.g. for $A_A = 30$, $A_B = 30$, $p_{BA} = 0.02$ and $B_B = 10$, $B_A = 30$, $p_{AB} = 0.08$ in the public communication case with $p_1 = p_2 = p_{AB}$, $p_3 = p_4 = p_{BA}$ (we have assumed $\beta_A = \beta_B = 0$ for simplicity). The graphs are shown in Figure 7. Two majority switches occur, the first in $t \simeq 0.083$ and the second in $t \simeq 0.80$. This is caused by the fact that A reaches its voter base-level first, which in turn alters the dynamic of communication until B reaches the voter base-level.

Figure 7: A Double Majority Switch Voting Paradox



4 Parliamentary Procedures and Strategic Decisions

Communication serves the important function of enabling the committee to realize the 'true' preference of the majority—if there is one. However, if the *need* to communicate long enough is satisfied by a *right* to communicate as long as one wants the latter can be exploited with the aim to obstruct decision-making by preventing a vote. Government by discussion may clearly be ineffective if no-one can stop people talking. The right to communicate has therefore to be checked somehow.

Two approaches are possible: rules and discretion. Both institutional safeguards against obstructionism suffer from a rules-versus-discretion-problem: discretionary powers, on the one hand, can be exploited in order to distort the outcome of the vote. 'Objective' rules, on the other hand, automatically distort the outcome of the vote, as they do not take into account the nature of the debate. It may thus be ineffective, too, *if* someone or something can stop people talking. In this section we shall shortly explore the potentially adverse effects of institutional safeguards for orderly communication in a committee.

If at least one majority switch occurs winning the vote invariably depends on the timing of the vote. This in turn is a matter of *Parliamentary Procedure*:

1. If the President or Chair(wo)man of C has the discretionary power to fix the timing of the vote her faction will necessarily win the vote. Suppose a majority switch is expected to occur at $t = u$. The majority leader prefers the vote to take place earlier than u. The minority prefers a vote later than u.

2. Suppose the expectation by the minority leader is that the opposition's majority can not be overturned. If procedures allow for Filibustering (i.e. if an unchecked right to communicate exists) motions can still literally be 'talked out' of the decision-making process.

3. If there is a mechanism for terminating debate ("cloture") the question arises who exerts control over it. In the US Senate a rule allowing for the termination of the debate was first introduced in 1917. The requirement was a successful vote with a two-thirds majority (67 votes). In 1975 the cloture rule was revised so that a three-fifths majority was needed (60 votes).

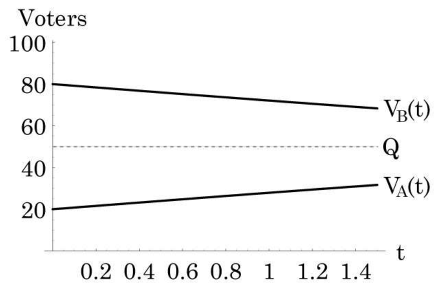
4. If the time for communication is strictly limited by procedural rules Filibusters can not occur. The timing of the vote is then either exactly predetermined or the vote is supposed to happen during a certain time-interval. In the Canadian House of Commons, for instance, the "time allocation rule" allows for specific lengths of time to be set aside for the consideration of a bill. It can be used by the government to impose strict limits on the time for debate (see e.g. Marleau/Montpetit 2000, Stanford 1995). In most cases, time is allocated in terms of sitting days or hours. When there is no agreement between the parties, the amount of time allocated may not be less than one day.

Having said this it becomes clear that the outcome of the communication process and thus the collective decision crucially depends on the time-factor, which is subject to institutional constraints. Apart from the time-factor another matter which is subject to Parliamentary Procedure is the *choice of the technologies of communication.* Even if the time for communication is absolutely limited the strategic choice between available technologies of communication may be used to circumvent institutional constraints. *Adjournments*, *session breaks* and *confidential meetings* can therefore be strategically employed in order to switch from public to private communication. This can be very effective if the voter-type is common knowledge, as private communication enables to to isolate opposed swing voters.

Consider the following scenario, which is based on an extension of the numerical example for the public communication model: in the run-up to a plenary debate L_B sends only her $B_B = 30$ base voters to sway faction A's 40 swing voters. Supposed there is sufficient time so that $A_B(0)$ is reduced to zero when the public debate starts in $t = 0$. The new 'balance of power' is $V_A(0) = 20$

and $V_B(0) = 80$. In the ensuing public communication phase L_B faces only 20 active campaigners. The effect is dramatic (see Figure 8): L_B enjoys a majority for all $t > 0$ and a majority switch does not occur. Moreover, Filibusters by L_A are made impossible as L_B has a supermajority of more than sixty votes all the time. L_B would win under any time allocation rule and indeed even if L_A had discretionary powers to fix the timing of the vote.

Figure 8: The Effect of Strategic Private Communication



5 A Concluding Remark

We proposed two dynamic models of communication-processes between committee factions, combining elements of inter-party as well as intra-party campaigning. Although they are prototypes the models can be used to predict the most likely outcome of a vote when the committee members communicate with each other. The prediction must always take Parliamentary Procedures and the discretionary powers of the committee's president into account. Rules and discretion determine whether Filibustering is possible, or whether the debate can be curtailed. Rational party leaders will take Parliamentary Procedures into account and exploit decision-making rights with the strategic aim to influence voting results.

The scope for further research is indeed large. The next step from an economics perspective is certainly to include costs of communication and voting. It is interesting, too, to deal with the strategic aspects of agenda-setting in the dynamic context. Especially a formalization of Parliamentary Procudures would help to understand the depence of voting results on institutional constraints of communication. From a more technical point of view an interesting next step is the re-formulating of the models with nonconstant coefficients, e.g. in order to reflect learning-curve-effects. Apart from this, it would be interesting to derive information transmission probabilities from classroom experiments and to deal with the ethical questions, e.g. by formulating an axioms of productive debate and compare them to those proposed in the literature on discursive ethics.

References

[1] Coleman, C.S., 1982, Combat Models, in W.J. Lucas (ed.), Differential Equation Models, New York: 109-31.

[2] Gerling, K., H.P. Gruener, A. Kiel, E. Schulte, 2005, Information Acquisition and Decision Making in Committees, European Journal of Political Economy 21: 563-597.

[3] Hirshleifer, J., 2000, The Macrotechnology of conflict. Journal of Conflict Resolution 44(6): 773-92.

[4] Marleau, R., C. Montpetit (eds.), 2000, House of Commons Procedure and Practice, Ottawa.

[5] Nurmi, H., 1998, Voting Paradoxes and Referenda, Social Choice and Welfare 15: 333-50.

[6] Stanford, G., 1995, Bourinot's Rules of Order: A Manual on the Practices and Usages of the House of Commons of Canada and on the Procedure at Public Assemblies, Ottawa.

[7] Sturgis, A., 2000, The Standard Code of Parliamentary Procedure, 4th ed., New York.

[8] The Economist, 2006, The Democrats and Iraq: An anti-war centre begins to hold, August 12: 33-4.

[9] Weaver, W., 1964, Recent Contributions to the Mathematical Theory of Communication, in Shannon, C.E., W. Weaver (eds), The Mathematical Theory of Communication, Urbana, III: 6-8.

[10] Yokoyama, A., 1991, An economic theory of Persuasion. Public Choice 71: 101-15.

Appendix

Proof of Proposition 5

System (1),(2) in the case of complete voter-type information reduces to

$$\begin{aligned}\dot{A}(t) &= -q_1 B(t) - q_2\beta_A\big[A_0 - A(t)\big],\\ \dot{B}(t) &= -q_3 A(t) - q_4\beta_B\big[B_0 - B(t)\big].\end{aligned}$$

$$\iff \quad \begin{array}{rcllll}\dot{A} &=& -q_2\beta_A A_0 & +q_2\beta_A A & & -q_1 B,\\ \dot{B} &=& -q_4\beta_B B_0 & & -q_3 A & +q_4\beta_B B.\end{array}$$

In order to economize on notation as much as possible we reformulate the system and introduce some new variables:

$$\boldsymbol{x}(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} = \begin{pmatrix} A(t) \\ B(t) \end{pmatrix}, \quad \boldsymbol{x}_0 = \begin{pmatrix} x_{10} \\ x_{20} \end{pmatrix} = \begin{pmatrix} A_0 \\ B_0 \end{pmatrix},$$

$$M = \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} q_2\beta_A & -q_1 \\ -q_3 & q_4\beta_B \end{pmatrix},$$

$$\boldsymbol{y} = -\begin{pmatrix} a\,x_{10} \\ d\,x_{20} \end{pmatrix} = -\begin{pmatrix} q_2\beta_A A_0 \\ q_4\beta_B B_0 \end{pmatrix}.$$

M is the System Matrix. The differential equation system to be solved can now be written as

$$\begin{aligned}\dot{x}_1 &= y_1 \; + a\,x_1 \; + b\,x_2,\\ \dot{x}_2 &= y_2 \; + c\,x_1 \; + d\,x_2,\end{aligned}$$

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{y} + M\boldsymbol{x}(t), \quad \text{or} \quad \dot{\boldsymbol{x}} - M\boldsymbol{x} = \boldsymbol{y}.$$

We suppose from now on that M is regular, i.e. $\operatorname{Det} M \neq 0$. A first observation can be made: as the Eigenvalues of M are real the general solutions of (1),(2) are non-periodic and will not show any cycles or chaotic behavior.

In order to solve the general, inhomogeneous, linear first-order differential equation with constant coefficients but variable inhomogeneity,

$$\dot{\boldsymbol{x}}(t) - M\boldsymbol{x}(t) = \boldsymbol{y}(t),$$

we proceed as following: we assign the corresponding homogeneous equation $\dot{\boldsymbol{x}}_h(t) - M\boldsymbol{x}_h(t) = 0$, with the subscript h denoting its solution. If $\boldsymbol{x}_h(t)$ is a general solution and a function $\boldsymbol{x}_p(t)$ fulfills the condition

$$\dot{\boldsymbol{x}}_p(t) - M\boldsymbol{x}_p(t) = \boldsymbol{y}(t)$$

we call $\boldsymbol{x}_p(t)$ a particular integral, i.e. some solution of the inhomogeneous equation. It follows that the *general solution* of the inhomogeneous differential equation is:

$$\boldsymbol{x}(t) = \boldsymbol{x}_p(t) + \alpha\,\boldsymbol{x}_h(t), \quad \alpha = \text{const.} \in \mathbb{R}$$

because

$$\begin{aligned}\dot{\boldsymbol{x}}(t) - M\boldsymbol{x}(t) &= \dot{\boldsymbol{x}}_p(t) + \alpha\,\dot{\boldsymbol{x}}_h(t) - M\big(\boldsymbol{x}_p(t) + \alpha\,\boldsymbol{x}_h(t)\big)\\ &= \dot{\boldsymbol{x}}_p(t) - M\boldsymbol{x}_p(t) + \alpha\big(\dot{\boldsymbol{x}}_h(t) - M\boldsymbol{x}_h(t)\big) = \boldsymbol{y}(t).\end{aligned}$$

We now ask whether $\boldsymbol{x}_p$ is a particular integral. Be $\boldsymbol{x}_p = -M^{-1}\boldsymbol{y}$; then $\dot{\boldsymbol{x}}_p = 0$ and we have that

$$\dot{\boldsymbol{x}}_p - M\boldsymbol{x}_p = -M\big(-M^{-1}\boldsymbol{y}\big) = MM^{-1}\boldsymbol{y} = \boldsymbol{y}.$$

It follows that $\boldsymbol{x}_p$ is a particular integral.

Skipping the subscript h we write the homogeneous equation in extensive form:

$$\begin{aligned}\dot{x}_1 &= a\,x_1 + b\,x_2,\\ \dot{x}_2 &= c\,x_1 + d\,x_2.\end{aligned}$$

We now multiply the first equation with c and the second equation with $(-a)$. Adding the second equation to the first and differentiating the second equation once for t yields

$$\begin{aligned}c\,\dot{x}_1 - a\,\dot{x}_2 &= \phantom{c\,\dot{x}_1} - (ad-bc)\,x_2,\\ \ddot{x}_2 &= c\,\dot{x}_1 + d\,\dot{x}_2.\end{aligned}$$

Solving the second equation for $c\,\dot{x}_1$ and inserting this term into the first equation yields

$$\ddot{x}_2 - (a+d)\,\dot{x}_2 = -(ad-bc)\,x_2, \quad \ddot{x}_2 - \mathrm{Tr}\,M\,\dot{x}_2 + \mathrm{Det}\,M\,x_2 = 0.$$

The resulting equation is a common second order differential equation. It can be solved with the method of exponential functions:

$$y(t) = \mathrm{e}^{\lambda\,t}, \quad \ddot{y} - 2S\,\dot{y} + D\,y = 0 \quad \Rightarrow \quad \big(\lambda^2 - 2\lambda S + D\big)\,\mathrm{e}^{\lambda\,t} = 0,$$

$$(\lambda - S)^2 - (S^2 - D) = (\lambda - S)^2 - \Delta^2 = 0, \quad \Rightarrow \quad \lambda_{1,2} = \frac{\mathrm{Tr}\,M}{2} \mp \Delta,$$

with $S := \mathrm{Tr}\,M/2,\ D := \mathrm{Det}\,M$ and $\Delta := \sqrt{S^2 - D} = \frac{1}{2}\sqrt{(\mathrm{Tr}\,M)^2 - 4\,\mathrm{Det}\,M}$. Accordingly, the general solution is:

$$x_2(t) = c_1\,\mathrm{e}^{\lambda_1\,t} + c_2\,\mathrm{e}^{\lambda_2\,t}.$$

Note that the coefficients are fixed by the initial value assumptions. Inserting the general solution into the remaining differential equation $\ddot{x}_2 = c\,\dot{x}_1 + \ d\,\dot{x}_2$ and integrating once over t yields the solution for $x_1(t)$. (Q.E.D.)

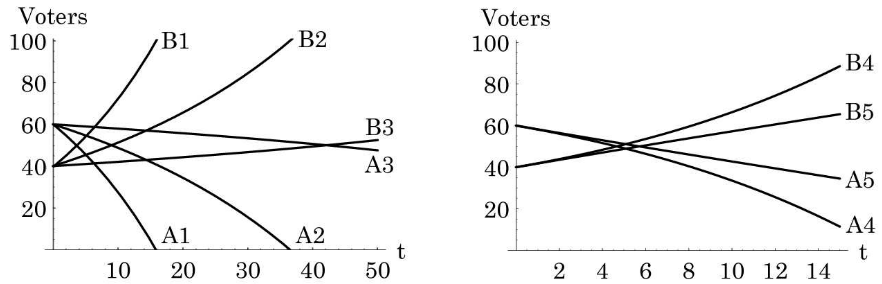
Graphical Representations of Parameter-Variations

Private Communication, Complete Voter-Type Information

The initial values are $A_0 = 60$, $B_0 = 40$, $A_A = B_B = 0$ and $q_1 = q_2 = q_{AB}$, $q_3 = q_4 = q_{BA}$, $r_1 = r_2 = r_3 = r_4 = 1$. The curves shown correspond to the following cases (ceteris paribus):

- A1: $V_A(t)$ and B1: $V_B(t)$ for $q_{AB} = 0.08$, $q_{BA} = 0.01$.
- A2: $V_A(t)$ and B2: $V_B(t)$ for $q_{AB} = 0.04$, $q_{BA} = 0.01$.
- A3: $V_A(t)$ and B3: $V_B(t)$ for $q_{AB} = 0.02$, $q_{BA} = 0.01$.
- A4: $V_A(t)$ and B4: $V_B(t)$ for $\beta_A = 1$, $\beta_B = 0$.
- A5: $V_A(t)$ and B5: $V_B(t)$ for $\beta_A = 0$, $\beta_B = 1$.

Figure 9: Parameter Variations



Public Communication

The initial values are $A_0 = 60$, $B_0 = 40$, $A_A = B_B = 0$ and $p_1 = p_2 = p_{AB}$, $p_3 = p_4 = p_{BA}$. The curves shown correspond to the same cases as above.

Figure 10: Public Communication: Parameter Variations