Two-stage rent-seeking contests: USA vs. German elections or Olympic Games vs. World Cup placing*

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Abstract

Two variations of a two-stage rent-seeking contest model are considered. We compare the degree of rent dissipation in the Between Group and the Semi-Final model. The contribution of this paper is to extend these two models by introducing rent-seekers with negatively interdependent preferences. Focusing on the relationship between contest-structure and preference-type we find that rent dissipation is generally lower in the Between-Group model than in the Semi-Finals model given that the players maximize absolute payoffs. When assuming relative preferences, we can find the reverse.

Keywords: two-stage contests; interdependent preferences; elections

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1 Introduction

Many economic, political and social interactions can be viewed as rent-seeking contests. Some prominent examples are political elections, R&D competition, and lobbying for government support, such as subsidies, relaxed regulations, or tax reductions. An important feature of most of these real life competitions is that the competition take place over more than one period until the price is granted. This chapter examines rent-seeking contests in which rent-seeking activities take place in two stages.[1]

As noted by Gradstein and Konrad (1999), in certain contexts the contest rule is exogenously determined. In other applications it is the outcome of a judicious design intended to attain a variety of objectives. According to experiences in political rent-seeking, politicians who allocate rents through a contest value high outlays. Would they have the choice to design a contest they would choose the one that results in outlay maximization. Therefore, one objective is the maximization of the outlays by the contestants in a rent-seeking contest. To achieve the objective best possible, we analyze the outlays of players under two different contest rules and compare the resulting rent dissipation.

In the first stage of the Between-Group (BG) model two groups of homogeneous players compete for a single fixed rent V. Aggregate group effort determines the probability that the group wins the rent. No predetermined distribution rule exists in the groups like in Nitzan (1991). Instead, the members of the winning group compete in a second stage for the prize. Individual spending on this second stage determines the probability that a particular individual receives the rent. Afterwards, we compare the rent dissipation of the Between-Group model with the rent dissipation of a Semi-Finals (SF) contest. In the Semi-Finals model one player is chosen from each of the two groups through competing for the position and then these two players compete for the rent by expending effort in a second stage.

Earlier work in the literature for two-stage rent-seeking is provided by Katz and Tokatlidu (1996). Modeling symmetric rent valuation in a Between-Group con-

[1] For an overview of the literature on rent-seeking and contests see Tollison (1982), Nitzan (1994), Konrad (2007) or Corchon (2007).

test, they investigate the relation between group size and aggregate rent-seeking. They show that the rent dissipation does not only depend on total number of players but also on the distribution of players across groups. In this case group size asymmetry facilitates the reduction of rent dissipation. Built upon the model of Katz and Tokatlidu, Stein and Rapoport (2004) compare the Between-Group model with the Semi-Finals model, like we do. They consider more than two groups and focus on asymmetries between groups and players. They find that the percentage of rent dissipation - the ratio of total expenditures to the expected value of the rent - is higher under the Semi-Finals than under the Between-Group model.

The result on rent dissipation in two-stage contests provided by Stein and Rapoport (2004) is point of departure for this chapter. We also compare rent dissipation in the two variations of a two-stage contest. However, we go a step further by allowing for an alternative preference structure of the players, i.e. relaxing the common assumption of purely self-interested preferences. In order to allow for status seeking interests we introduce the concept of negatively interdependent (relative) preferences. Using this concept we can show that the Semi-Finals model does not always lead to higher rent dissipation.

Relative, interdependent preferences have been shown to play an important role in both individual and group decision making. Porac et al. (1999) and Alexopolos and Sapp (2006) analyze the impact on the observed economic behavior of firms. Executive compensation in firms is based on a comparison of firm performance to the performance of other, similar firms. Hence, managers will focus their actions on improving the performance of their firm relative to a set of competitor firms. Therefore, many firms appear to maximize their market share at the expense of profits, a strategy which corresponds to relative behavior. Individuals may also care about their own payoffs relative to those of others and not just about the absolute level of their payoffs. These individuals have relative rather than absolute preferences. Empirical evidence on relative preferences is given in Bewley (1995), Clark and Oswald (1996), Sobel (2005), Kapteyn et al. (1997), Solnik and Hemenway (1998) and Levine (1998). Their results indicate that there are many situations in which individuals exhibit relative preferences.

Introducing agents with negatively interdependent preferences in a two-stage rent-seeking contest translates into adding another strategic component to the theoretical decision-making process. The relative maximizer's aim is to beat the average payoff and this is not only achieved by increasing one's own payoff, but also by reducing the payoff of the other players. Behavior that lowers the own payoff and the rival's payoff even more has been termed "spiteful" (Hamilton (1970)). In the context of two-stage contests, it can occur via lowering the opponents (group) probability of winning the contest.

To date, there is hardly any theoretical work on the characteristics and the potential impact of negatively interdependent preferences on contest outcomes, even though Leininger (2008) has shown that evolutionary stable preferences in contests are negatively interdependent. One exception is Guse and Hehenkamp (2006) who analyze rent-seeking contests with a heterogeneous population in which part of the players are absolute payoff maximizers while others are also concerned about their relative position. Their paper shows that players with negatively interdependent preferences experience a strategic advantage in general two-players contests and in n-players-contests with non-increasing marginal efficiency. In contrast to this contribution, this chapter examines homogeneous populations in which the players have either absolute preferences or negatively interdependent preferences, to find the influence of preference types on the aggregate rent-seeking and therewith on the contest-structure choice.

The remainder of the paper is structured as follows: In Chapter 2 we present the Between-Group model and analyze it separately for players with absolute payoff preferences following Katz and Tokatlidu (1996) and players with negatively interdependent preferences. Afterwards, in Chapter 3 we analyze the Semi-Finals model under both assumptions. In Chapter4 we compare the rent dissipations of the two contests to investigate the influence of the assumed preferences on the choice of the contest-structure. The theoretical results are applied to the case of USA and German elections in Chapter 5 and to the case of awarding prestige sport events in Chapter 6. Last in Chapter 7 we conclude.

2 The Between-Group model

Assume that a contest-designer offers a fixed rent V for example in the form of a political mandate to a member of group X or of group Y. Group sizes are exogenously given and identical. Groups X and Y consist of n homogeneous risk neutral members. Each player has the same valuation for the rent. A member of a group is given the chance to compete directly for the rent V in the second stage of the contest only if her group has won the rent in the first stage. In the first round members of each group attempt to obtain the rent for their own group. The two groups play an inter-group Tullock contest, which means that a group of players can make an investment in order to participate in a lottery. The higher the group's investment, the higher is its chance of winning the prize. Hence, its initial rent-seeking is group-oriented. Denote the first round rent-seeking pursued by member i of group X and member j of group Y by x_{1i} and y_{1j} respectively, and denote the probability that group X (Y) is awarded the rent by p_X (p_Y). We assume the probability that a given group will win the rent depends on the value of the group's investment relative to the investment of both groups:[2] Following Tullock (1980), the probability that group X wins the rent is given by the ratio

$$p_X = \frac{\sum_{i=1}^{n} x_{1i}}{\sum_{i=1}^{n} x_{1i} + \sum_{j=1}^{n} y_{1j}} \tag{1}$$

Eq.(1) is referred to as the group success function. We assume constant returns to scale technology. The probability that group Y wins the rent, p_Y, is equal to $1 - p_X$.

After the completion of the first round, each member of the winning group engages in rent-seeking activities in order to win the rent V for herself. The members of the winning group play an intra-group Tullock contest. Denote the second round rent-seeking investment pursued by member i of group X and member j of group Y by x_{2i} and y_{2j} respectively. If group X won the first round, then its ith member will win in the second round with a probability

$$p_{xi}(x_{21}, \ldots, x_{2n}) = \frac{x_{2i}}{\sum_{l=1}^{n} x_{2l}} \tag{2}$$

[2] For an axiomatic foundation of group contest success functions see Münster (2008).

The first round of the contest does not offer individuals an ultimate payoff; it only determines the probability that a contestant will participate in the second round. The marginal benefit of an individual's first round investment is the increased probability to win the ultimate rent. Hence, the rent-seeking activities of individuals in the first round are determined by optimizing their expected payoff, given that all individuals act rationally and optimally in the second round.

First, we analyze the model under the assumption that players are absolute payoff maximizers following Katz and Tokatlidu (1996) and Stein and Rapoport (2004). In a second step, we assume rent-seekers to be relative payoff maximizers. This extends the analysis of a two-stage group rent-seeking contest to an additional strategic component. Analogous to Kockesen et al. (2000), the focus of this chapter lies on intra-group symmetric equilibria, in which all players choose the same effort level in each group. We solve for a subgame perfect equilibrium outcome of this game via backward induction. Therefore, we start the analysis with the second stage. All previous rent-seeking expenditures are sunk at this stage.

2.1 BG contest with absolute payoff maximizers

In this section we present the main results, adopted to our particular setting, of the models presented in Katz and Tokatlidu (1996) and Stein and Rapoport (2004). We use the special case of two equal sized groups and equal valuations of the rent of all players.

Assume without loss of generality that group X wins the contest in the first stage. Therefore, we analyze the second stage of the contest from the perspective of the players of group X. For group Y the analysis applies equivalently. In the second round the ith member of group X maximizes her utility F_{2i}^{abs}, which is equal to her payoff Π_{2i}^{abs} in the second stage

$$F_{2i}^{abs} = \Pi_{2i}^{abs}(x_{21}, \ldots, x_{2n}) = p_{xi}V - x_{2i} \tag{3}$$

where p_{xi} is the success probability of player i chosen as in eq.(2).

Assuming a symmetric Nash equilibrium within each group $x_{2i} = x_{2l} = x_2$ and a regular interior solution, the first-order condition can be solved and gives the optimal equilibrium effort

$$x_2^{abs} = \frac{n-1}{n^2}V \tag{4}$$

Substituting the optimal effort given in eq.(4) of each member of group X in the payoff function defined in eq.(3), we find that the individual's valuation of entering round two, $\tilde{V}_{BG}^{abs}$, is $\frac{1}{n}V - \frac{n-1}{n^2}V$. If a player enters the second round of the contests she can expect to earn this amount of the rent. The amount is the difference between the expected share of the rent she gets minus her effort chosen in this stage. Every player will bear this optimal payoff in mind when finding an optimal effort level for the first stage. Hence, in the first round each group-member solves the rent-seeking maximization for a reduced rent: $\tilde{V}_{BG}^{abs} = \frac{1}{n^2}V$.

In the first stage of the contest, assuming the group success function of eq.(1), an absolute payoff maximizing individuum i of group X tries to maximize her own utility F_{1i}^{abs}, which is equal to her payoff

$$F_{1i}^{abs} = \Pi_{1i}^{abs} = \frac{\sum x_{1k}}{\sum x_{1k} + \sum y_{1j}} \tilde{V}_{BG}^{abs} - x_{1i} \tag{5}$$

Solving the first order condition under the assumption that all members of each group are identical, we get the equilibrium solution

$$x_1^{abs} = y_1^{abs} = \frac{1}{4n^3}V \tag{6}$$

This is the optimal effort of a player of group X as well as of group Y in the first stage if all players have absolute preferences.

From a welfare perspective, we can now determine the degree of social waste. The degree of the social waste, the rent dissipation RD_{BG}^{abs}, is defined as the aggregate expenditures made in both stages of the contest divided by the amount of rent.

$$RD_{BG}^{abs} = (2nx_1^{abs} + nx_2^{abs})/V = (\frac{1}{2n^2} + \frac{n-1}{n}) \tag{7}$$

2.2 BG contest with relative payoff maximizers

We now repeat the analysis of the Between-Group contest under the assumption of relative payoff maximizers. The material payoff function of the second stage is again given as in eq.(3) and in eq.(5). The players maximize their expected utility given relative preferences: The utility function is of the form

$$F_{2i}^{rel} = \Pi_i^{rel}(\mathbf{x}) + \alpha \frac{1}{n} \sum_{j=1}^{n} \Pi_j^{rel}(\mathbf{x})$$

where $\alpha \in (-1, 0)$. The individual i maximizes a sum of her own material payoff and a weighted average payoff of all active players. The restriction that α can only take values between -1 and 0 ensures that the utility function of each player depends more on her own payoff than on the averaged payoff. The value $\alpha = 0$ conforms to the above analyzed case of absolute payoff maximizers.

We assume again without loss of generality that group X with n players wins the contest in the first stage. Therefore, we concentrate the analysis of the second contest stage on the players of group X. In the second round the ith member of group X maximizes

$$\begin{aligned} F_{2i}^{rel} &= \Pi_{2i}^{rel}(\mathbf{x}) + \alpha \frac{1}{n} \sum_{l=1}^{n} \Pi_{2l}^{rel}(\mathbf{x}) \\ &= \frac{x_{2i}}{\sum_{l=1}^{n} x_{2l}} V - x_{2i} + \alpha \frac{1}{n} \Big(V - \sum_{l=1}^{n} x_{2l} \Big) \end{aligned} \tag{8}$$

In this stage the individuum i is concerned with her payoff relative to the payoff of her own group members. Members of the other group have lost the first stage competition and therefore dropped out of the contest. The individuum i does not include the other group members in her utility function.

Assuming a symmetric Nash equilibrium within the group and a regular interior solution, the first-order condition for member i of group X is

$$\frac{\partial F_{2i}^{rel}}{\partial x_{2i}} = \frac{\sum x_{2l} - x_{2i}}{(\sum x_{2l})^2} V - 1 - \alpha \frac{1}{n} \stackrel{!}{=} 0. \tag{9}$$

Since we assume identical agents and search for symmetric equilibria, we set $x_{2i} = x_{2l} = x_2$. This gives the optimal effort of the second stage

$$x_2^{rel} = \frac{n-1}{n(n+\alpha)} V. \tag{10}$$

Eq.(10) is our interior equilibrium solution of the second stage. To find the equilibrium solution of the first round we calculate the benefit of entering the second stage and use this value as a reduced rent which the individuals can imaginary win in the first stage. Regarding all players and the optimal effort of the second stage as defined in eq.(10), we calculate the utility of entering the second stage.y win in the first stage. Regarding all players and the optimal effort of the second stage as defined in eq.(10), we calculate the utility of entering the second stage.

$$\tilde{V}_{BG}^{rel} = \frac{1+\alpha}{n(n+\alpha)}V + \alpha\frac{n\frac{1+\alpha}{n(n+\alpha)}}{2n}V = (1+\frac{\alpha}{2})\frac{1+\alpha}{n(n+\alpha)}V \tag{11}$$

Hence, in the first round each player solves the rent-seeking maximization for a reduced rent. Therefore, the utility of a relative payoff maximizing individuum i of group X in the first stage is $F_{1i}^{rel} = \Pi_i^{rel} + \alpha\bar{\Pi}^{rel}$, where $\bar{\Pi}^{rel}$ is the population mean payoff:

$$\begin{aligned} \bar{\Pi}^{rel} &= \frac{\sum_{l=1}^{n}\Pi_l^{rel} + \sum_{j=1}^{n}\Pi_j^{rel}}{2n} \\ &= \frac{p_X n\tilde{V}_{BG}^{rel} - \sum_{l=1}^{n} x_{1l} + p_Y n\tilde{V}_{BG}^{rel} - \sum_{j=1}^{n} y_{1j}}{2n} \end{aligned} \tag{12}$$

and the success function of the groups p_X respective p_Y chosen as in (1).

With these we get

$$F_{1i}^{rel} = \frac{\sum x_{1l}}{\sum x_{1l} + \sum y_{1j}}\tilde{V}_{BG}^{rel} - x_{1i} + \alpha\left(\frac{n\tilde{V}_{BG}^{rel}}{2n} - \frac{\sum x_{1l} + \sum y_{1j}}{2n}\right) \tag{13}$$

The first order condition is given by

$$\frac{\partial F_{1i}^{rel}}{\partial x_{1i}} = \frac{\sum y_{1j}}{(\sum x_{1l} + \sum y_{1j})^2}\tilde{V}_{BG}^{rel} - 1 - \frac{\alpha}{2n} \stackrel{!}{=} 0 \tag{14}$$

Following our assumption that players are identical, we have $\sum_{l=1}^{n} x_{1l} = nx_1 = \sum_{j=1}^{n} y_{1j}$. Substituting this in the FOC, we yield

$$x_1^{rel} = \frac{1}{(1+\frac{\alpha}{2n})4n}\tilde{V}_{BG}^{rel} = \frac{(1+\frac{\alpha}{2})(1+\alpha)}{2n(2n+\alpha)(n+\alpha)}V. \tag{15}$$

If we take a look at the contest with status seekers from a welfare perspective, we focus on the rent dissipation RD_{BG}^{rel} in the whole contest to determine the

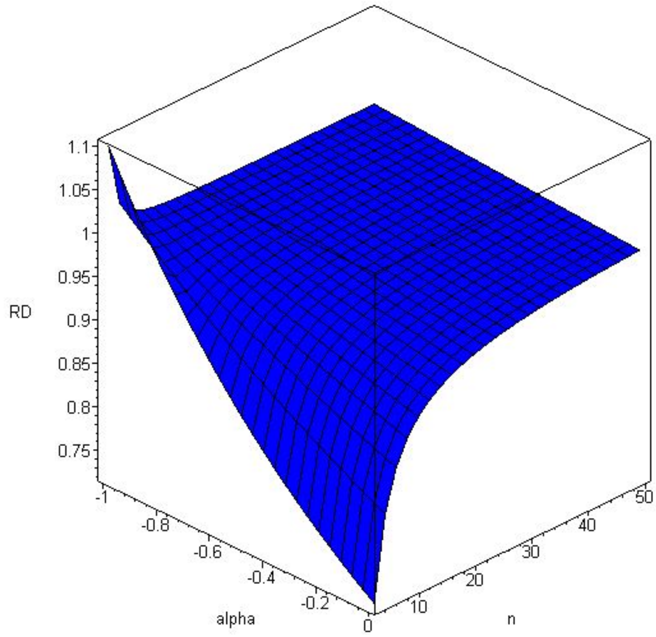

Figure 1: Rent dissipation of the Between-Group contest.

social waste. Therefore, we aggregate the expenditures of the single stages and add them up multiplied by their absolute frequency. The aggregate effort of the contest with relative payoff maximizers in proportion to the rent (figure 1) is given as

$$RD_{BG}^{rel} = (2nx_1^{rel} + nx_2^{rel})/V = \left(\frac{1 + \frac{\alpha}{2}(1+\alpha)}{(2n+\alpha)(n+\alpha)} + \frac{n-1}{n+\alpha} \right). \qquad (16)$$

2.3 Relationship between preference type and rent-seeking in the BG contest

We have seen, the second stage Nash equilibrium with relative payoff maximizers is given by $x_2^{rel} = \frac{n-1}{n(n+\alpha)}V$. In comparison, the unique pure strategy Nash equilibrium of this stage when players have absolute payoff maximizing preferences is given by $x_2^{abs} = \frac{n-1}{n^2}V$. When comparing the effort levels of the two Nash equilibria of the second stage, we find that the players interested in relative payoffs exert more effort gaining lower payoffs than the players maximizing absolute payoffs.

Hence, the incentive of the relative payoff maximizers to reach the second round is smaller. In a private good contest for V, as in the second stage of our model, the contest with relative payoff maximizers always leads to a higher expenditure than the one with absolute payoff maximizers.[3] The explanation therefor is that a marginal increase of efforts beyond the optimal absolute Nash level increases relative payoffs. It has second order negative effects on a player's own payoff (first derivative of his payoff function is zero at the absolute Nash level) but has a first order negative effect on the other players' payoff (cross-derivatives in the absolute Nash equilibrium are always negative). As a consequence, the difference between one's own and others' payoffs increases.

Focusing on the first stage of the two-stage contest there is the aggregate expenditure in the equilibrium with relative preferences always lower than those of the Nash equilibrium of players with absolute independent preferences. Nevertheless the explanation given above is still valid. How can it be? This result is driven by the different premises. Players valuations to enter the second round of the contest are unequal. If we compare the benefit of entering the second stage of absolute payoff maximizers with those of relative payoff maximizers, we find that absolute maximizers have a higher valuation of the rent in the first stage. This creates a much stronger effect than the effect arisen from the different preferences and explains the observed asymmetry in the player's behavior.

The effects of both stages counteract each other, but since the effect of the second stage is stronger, it dominates the first-stage effect. We can conclude

Result 1 *The total expenditure and with it the rent dissipation in equilibrium of the overall Between-Group contest with negatively interdependent preferences is higher than the overall expenditure and rent dissipation in the contest with absolute payoff maximizers.*

Proof. The idea of this proof is to show that the minimum of the rent dissipation with respect to α under the assumption of relative preferences lies at $\alpha = 0$ and this coincides with the rent dissipation under the assumption of absolute preferences. Partial derivative of the rent dissipation of the Between-Group contest

[3]See Leininger (2002) and Hehenkamp et al. (2004).

with respect to the preference parameter is given as

$$\frac{\partial RD_{BG}^{rel}}{\partial \alpha} = \frac{\frac{1}{2}+\alpha}{(2n+\alpha)(n+\alpha)} - \frac{1+\frac{\alpha(1+\alpha)}{2}}{(2n+\alpha)^2(n+\alpha)} - \frac{1+\frac{\alpha(1+\alpha)}{2}}{(2n+\alpha)(n+\alpha)^2} - \frac{n-1}{(n+\alpha)^2}$$

First, we show

$$\frac{\partial RD_{BG}^{rel}}{\partial \alpha} < 0.$$

This is obvious for $\alpha \in [-1, -\frac{1}{2}]$ since all four terms in the partial derivative are negative. For $\alpha \in [-\frac{1}{2}, 0]$ some more considerations are necessary since the first term is positive. We show that the negative fourth term overtops the positive first term:

$$\begin{aligned} & \frac{\frac{1}{2}+\alpha}{(2n+\alpha)(n+\alpha)} < \frac{n-1}{(n+\alpha)^2} \\ \Leftrightarrow \quad & (\frac{1}{2}+\alpha)(n+\alpha) < (n-1)(2n+\alpha) \end{aligned}$$

Now, using the allowed range $[-\frac{1}{2}, 0]$ for α we can displace $(\frac{1}{2}+\alpha)(n+\alpha) \leq \frac{n}{2}$ and $(n-1)(2n+\alpha) \geq (n-1)(2n-\frac{1}{2})$. Therewith, it is sufficient to show that

$$\begin{aligned} & \frac{n}{2} < (n-1)(2n-\frac{1}{2}) \\ \Leftrightarrow \quad & 4n^2 - 6n + 1 > 0 \end{aligned}$$

With the condition $n \geq 2$ this results in $4n^2 - 6n + 1 \geq 8n - 6n + 1 = 2n + 1$ which is obviously bigger than zero.

We have profen that the partial derivative of RD_{BG}^{rel} with respect to α is always smaller than zero ($\frac{\partial RD_{BG}^{rel}}{\partial \alpha} < 0$) and approximate zero for $n \to \infty$. The rent dissipation function is strictly monotonic decreasing in α. A decrease of the preference parameter α strictly increases the rent dissipation in the Between-Group contest. The minimum of the rent-dissipation function is situated at $\alpha = 0$, because α is restricted to the interval $[-1, 0]$. If $\alpha = 0$ the rent-dissipations are equal $RD_{BG}^{rel} = RD_{BG}^{abs}$. Knowing that a decrease in α results in an increase of the rent-dissipation we can state that for $\alpha \in [-1, 0)$ the strict inequality $RD_{BG}^{rel} > RD_{BG}^{abs}$ holds. ■

3 Semi-Finals model

In contrast to the Between-Group model, in the Semi-Finals model the idea is that group members of the same group have to compete for a mandate in the first stage. Once, one individual wins the mandate, she must compete in a second stage against the winner of the first stage of the rival group for the rent V. This is appropriate when the competition consists of a semi-finals round and final round with the semi-final winners selected from a partition of the players. A classic example are sport events in which first stage events are used for qualifying for the final round. Another example is an election process consisting of primaries and final elections.

The Semi-Finals model assumes that $2n$ risk neutral and homogeneous players compete in a two-stage contest for a single indivisible rent V. Each player has the same valuation for the rent. The $2n$ players are arranged exogenously in two groups of same size, group X and group Y. Each player competes within her group. Member i of group X invests effort x_{1i} in the first stage. One winner is selected from each group using Tullock's contest success function $\frac{x_{1i}}{\sum_{l=1}^{n} x_{1l}}$ as the probability that player i of this group advances to the second stage. On this stage the two winners of the semi-finals stage compete against each other with the winner again chosen according to the Tullock contest rule $\frac{x_2}{x_2+y_2}$. We assume again constant returns to scale technology. The technology will not be changed during the whole contest and is the same for all players.

The first round of the contest does not offer individuals an ultimate payoff; it only determines the probability that a contestant will participate in the second round. The marginal benefit of an individual's first round investment is the increased probability to win the ultimate rent. Hence, the rent-seeking activities of individuals in the first round are determined by optimizing their expected payoff, given that all individuals act rationally and optimally in the second round.

First we analyze the model under the assumption that players are absolute payoff maximizers following Stein and Rapoport (2004). In a second step, we assume rent-seekers to be relative payoff maximizers. This extends the analysis of a Semi-Finals two-stage rent-seeking contest to an additional strategic component. Like in the last section the focus lies on intra-group symmetric equilibria, where all players choose the same effort level. We solve for a subgame perfect equilibrium outcome of this game via backward induction. Therefore we start the analysis with the second stage. All previous rent-seeking expenditures are sunk at this stage.

3.1 SF contest with absolute payoff maximizers

In this subsection we summarize the main results presented in Stein and Rapoport (2004) for the case of absolute payoff maximizing preferences, two groups and identical rent valuations. In the second round the winning member i of group X maximizes her utility, which is equal to the payoff.

$$F_{2i}^{abs} = \Pi_{2i}^{abs} = \frac{x_2}{x_2 + y_2} V - x_2 \tag{17}$$

Assuming a symmetric Nash equilibrium $x_2 = y_2$ and a regular interior solution we can solve the first-order condition. This gives the equilibrium effort

$$x_2^{abs} = \frac{1}{4} V. \tag{18}$$

Substituting the optimal effort defined in eq.(18) in the utility function eq.(17), we find that the individual's valuation of entering round two $\tilde{V}_{SF}^{abs}$ is $\frac{1}{4}V$. If a player enters the second round of the contests she earns this expected amount of the rent. Every player will bear this optimal payoff in mind when finding an optimal effort level for the first stage. Hence, in the first round each player solves the rent-seeking maximization for a reduced rent: $\tilde{V}_{SF}^{abs} = \frac{1}{4}V$.

In the first stage of the contest, an absolute payoff maximizing individuum i of group X tries to maximize her expected payoff Π_{1i}, which is at the same time the utility F_{1i}^{abs} and is given as

$$F_{1i}^{abs} = \Pi_{1i}^{abs} = \frac{x_{1i}}{\sum x_{1l}} \tilde{V}_{SF}^{abs} - x_{1i} \tag{19}$$

Solving the first order condition under the assumption that all players have identical payoff maximizing preferences, we get the solution

$$x_1^{abs} = \frac{1}{4} \frac{n-1}{n^2} V \tag{20}$$

This is the optimal effort of a player of group X as well as of group Y in the first stage if all players have absolute preferences.

From a welfare perspective, we can now determine the degree of social waste. Social waste or rent dissipation RD_{SF}^{abs} is defined as the aggregated expenditure by all players in proportion to the rent V.

$$RD_{SF}^{abs} = (2nx_1^{abs} + 2x_2^{abs})/V = 1 - \frac{1}{2n} \tag{21}$$

3.2 SF contest with relative payoff maximizers

We now repeat the analysis of the Semi-Finals contest under the assumption of relative payoff maximizers. The material payoff function is again given as

$$\Pi_i^{rel} = \frac{x_i}{\sum_{l=1}^n x_l} V - x_i \tag{22}$$

The players maximize their expected utility. The utility function is of the form $F_i^{rel} = \Pi_i^{rel}(x) + \alpha \frac{1}{n} \sum_{l=1}^n \Pi_l^{rel}(x)$ where $\alpha \in (-1, 0)$. The individual i maximizes a sum of her own material payoff and a weighted average of payoffs of all other active players. The restriction that α can only take values between -1 and 0 ensures again that the utility function of each player depends more on her own payoff than on the averaged payoff.

We assume again without loss of generality that member i of group X and member j of group Y win the contest in the first stage. In the second round the ith member maximizes

$$\begin{aligned} F_{2i}^{rel} &= \Pi_{2i}^{rel} + \alpha \frac{\Pi_{2i}^{rel} + \Pi_{2j}^{rel}}{2} && (23) \\ &= \frac{x_{2i}}{x_{2i} + y_{2j}} V - x_{2i} + \alpha \frac{V - x_{2i} - y_{2j}}{2}. && (24) \end{aligned}$$

In this stage the individuum i is concerned with her payoff relative to the payoff of her active rival. Other members of the groups have lost the first stage competition and therefore dropped out of the contest. Individuum i does not include dropped out players in her utility function. Therefore, the utility function just includes two individuals.

Assuming a symmetric Nash equilibrium and a regular interior solution, the first-order condition for player i is

$$\frac{\partial F_{2i}^{rel}}{\partial x_{2i}} = \frac{y_{2j}}{(x_{2i}+y_{2j})^2}V - 1 - \alpha\frac{1}{2} \overset{!}{=} 0. \tag{25}$$

Since we assume identical agents and search for symmetric equilibria, we set $x_{2i} = y_{2j} = x_2$. This gives

$$x_2^{rel} = \frac{1}{2(2+\alpha)}V. \tag{26}$$

To calculate the benefit of entering the second round we have to take a broader view on the whole game. In the second stage the individuals with relative preferences want to maximize their payoff in comparison to those of the players, who are still active in the contest. Under this assumption the optimal effort is given as $\frac{1}{2(2+\alpha)}V$ in the second stage. Now, analyzing the first stage, individuals additionally compete with all $2n$ active players. Therefore, the players want to maximize their material payoff compared to those of all other players. The benefit of entering the second round is now the difference in the material payoff between individual i and the average over all other players. Regarding all players we calculate the utility of entering the second stage.

$$\begin{aligned}\tilde{V}_{SF}^{rel} &= \frac{1}{2}V - \frac{1}{2(2+\alpha)}V + \alpha\left(\frac{V - 2\frac{1}{2(2+\alpha)}V}{2n}\right) \\ &= \left(\frac{(n+\alpha)(1+\alpha)}{2n(2+\alpha)}\right)V \end{aligned} \tag{27}$$

Hence, in the first round each player solves the rent-seeking maximization for a reduced rent. Therefore, the utility of a relative payoff maximizing individuum i in group X in the first stage is

$$F_{1i}^{rel} = \frac{x_{1i}}{\sum x_{1l}}\tilde{V}_{SF}^{rel} - x_{1i} + \alpha\left(\frac{2\tilde{V}_{SF}^{rel} - \sum x_{1l} - \sum y_{1l}}{2n}\right). \tag{28}$$

The first order condition is given by

$$\frac{\partial F_{1i}^{rel}}{\partial x_{1i}} = \frac{\sum x_{1l} - x_{1i}}{(\sum x_{1l})^2}\tilde{V}_{SF}^{rel} - 1 - \frac{\alpha}{2n}. \tag{29}$$

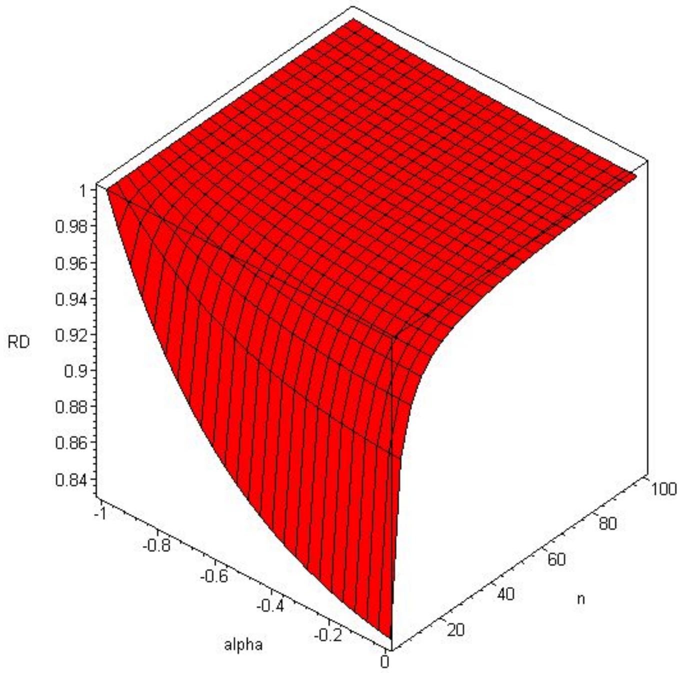

Figure 2: Rent dissipation of the Semi-Finals contest.

Following our assumption that group members are identical within each group, we have $\sum_{l=1}^{n} x_{1l} = nx_1$. Substituting this in the FOC, we yield the equilibrium effort level

$$\begin{aligned} x_1^{rel} &= \frac{1}{1+\frac{\alpha}{2n}} \frac{n-1}{n^2} \tilde{V}_{SF}^{rel} \\ &= \frac{1}{1+\frac{\alpha}{2n}} \frac{n-1}{n^2} \left(\frac{(n+\alpha)(1+\alpha)}{2n(2+\alpha)} \right) V. \end{aligned} \tag{30}$$

If we take a look at the contest from a welfare perspective, we focus on the rent dissipation RD_{SF}^{rel} in the whole contest to determine the social waste. The aggregate effort of the contest with relative payoff maximizers in proportion to the rent V can be seen in figure 2 and is given as

$$RD_{SF}^{rel} = \frac{2nx_1^{rel} + 2x_2^{rel}}{V} = \frac{1}{(1+\frac{\alpha}{2n})} \frac{(n-1)}{n^2} \frac{(n+\alpha)(1+\alpha)}{(2+\alpha)} + \frac{1}{2+\alpha}. \tag{31}$$

3.3 Relationship between preference type and rent-seeking in the Semi-Finals contest

We have seen that the second stage equilibrium with relative payoff maximizers is given by $x_2^{rel} = \frac{1}{2(2+\alpha)}V$. In comparison the unique pure strategy Nash equilibrium with absolute payoff maximizers is given by $x_2^{abs} = \frac{1}{4}V$. When comparing the effort levels of the two Nash equilibria of the second stage, we find that the player interested in relative payoffs invests more effort but gaining lower payoffs than the agent maximizing absolute payoffs. The competition on the second stage between the relative payoff maximizers is stronger. Therefore, they waste more effort. The higher effort reduces the expected payoff. Hence, the incentive of the relative maximizers to reach the second round and therewith the valuation of the rent in the first stage is smaller. These different valuations of the rent in the first stage creates a asymmetry in the player's behavior. In the equilibrium of the first stage with relative preferences the aggregate expenditure is always lower than those of the Nash equilibrium of players with absolute independent preferences. The effects of both stages counteract each other, but since the effect of the second stage is stronger, it dominates the first-stage effect.

Result 2 *The total expenditure and therewith the rent dissipation in equilibrium of the overall Semi-Finals contest with negatively interdependent preferences is higher than the overall expenditure in the Semi-Finals contest with absolute payoff maximizers.*

Proof. The idea of this proof is to show that the minimum of the rent dissipation with respect to α under the assumption of relative preferences lies at $\alpha = 0$ and this coincides with the rent dissipation under the assumption of absolute preferences. The rent dissipation function is differentiable. The partial derivative of the rent dissipation of the Semi Finals contest with respect to the preference parameter is given as

$$\begin{aligned}\frac{\partial RD_{SF}^{rel}}{\partial \alpha} &= \frac{2(n-1)(1+\alpha)}{(1+\frac{\alpha}{2n})n^2(2+\alpha)} - \frac{(n-1)(n+\alpha)(1+\alpha)}{2n^3(1+\frac{\alpha}{2n})(2+\alpha)} \\ &\quad - \frac{(n-1)(n+\alpha)(1+\alpha)}{(1+\frac{\alpha}{2n})n^2(2+\alpha)^2} - \frac{1}{(2+\alpha)^2}\end{aligned}$$

First, we show $\frac{\partial RD_{SF}^{rel}}{\partial\alpha} < 0$. Leaving the negative fourth term out makes the derivative even bigger. Therefore, we can analyze the first three terms:

$$\frac{2(n-1)(1+\alpha)}{(1+\frac{\alpha}{2n})n^2(2+\alpha)} - \frac{(n-1)(n+\alpha)(1+\alpha)}{2n^3(1+\frac{\alpha}{2n})(2+\alpha)} - \frac{(n-1)(n+\alpha)(1+\alpha)}{(1+\frac{\alpha}{2n})n^2(2+\alpha)^2} < 0$$

$$\begin{aligned}
&\Leftrightarrow \quad 2 - \frac{n+\alpha}{2n+\alpha} - \frac{n+\alpha}{2+\alpha} < 0 \\
&\Leftrightarrow \quad -n^2 + 3n + \alpha < 0
\end{aligned}$$

This is obviously fulfilled for $n \geq 4$ for all values of α and for $n = 3$ with $\alpha \in [-1, 0)$. The case $n = 3$ with $\alpha = 0$ can be shown by simply plugging in the values into $\frac{\partial RD_{SF}^{rel}}{\partial\alpha}$. The derivative is still negative. Now, it is left to proof that $\frac{\partial RD_{SF}^{rel}}{\partial\alpha} < 0$ for $n = 2$:

$$\frac{2(1+\alpha)}{(1+\frac{\alpha}{4})4(2+\alpha)} - \frac{(2+\alpha)(1+\alpha)}{16(1+\frac{\alpha}{4})(2+\alpha)} - \frac{((2+\alpha)(1+\alpha)}{(1+\frac{\alpha}{4})4(2+\alpha)^2} - \frac{1}{(2+\alpha)^2} < 0$$

$$\begin{aligned}
&\Leftrightarrow \quad \frac{2(1+\alpha)}{(4+\alpha)(2+\alpha)} - \frac{(1+\alpha)}{(4+\alpha)^2} - \frac{(1+\alpha)}{(4+\alpha)(2+\alpha)} - \frac{1}{(2+\alpha)^2} < 0 \\
&\Leftrightarrow \quad \alpha^2 - 2\alpha - 12 < 0
\end{aligned}$$

The last inequality is always fulfilled since $\alpha \in [-1, 0]$.

We have profen that the partial derivative of RD_{SF}^{rel} with respect to α is always smaller than zero ($\frac{\partial RD_{SF}^{rel}}{\partial\alpha} < 0$). The rent dissipation function is strictly monotonic decreasing in α. An decrease of the preference parameter α increases the rent dissipation in the Semi-Finals contest. The minimum of the rent-dissipation function is situated at $\alpha = 0$, because α is restricted to the interval $[-1, 0]$. Iff $\alpha = 0$ the rent-dissipations are equal $RD_{SF}^{rel} = RD_{SF}^{abs}$. ■

4 Comparison of BG contest and SF contest

An equilibrium x_i displays overdissipation (full dissipation, underdissipation) if and only if $\sum_{i=1}^{n} x_i > (=, <)V$, respectively. In equilibrium of the Between-Group contest as well as of the Semi-Finals contest we find for absolute payoff maximizer always underdissipation and for relative payoff maximizers underdissipation as long as $\alpha > -1$. Comparing the rent dissipations of the Between-Group contest with those of the Semi-Finals contest, we find the following:

Result 3 *i.) Total expenditure and rent dissipation under the assumption of absolute payoff maximizers in a Semi-Finals contest is always higher or equal than in a Between-Group contest.*

ii.) Total expenditure and rent dissipation of relative payoff maximizers in a Semi-Finals contest is for a sufficiently small α and $n \geq 3$ lower or equal than in a Between-Group contest.

Proof. ad i.) To show: $RD_{BG}^{abs} \leq RD_{SF}^{abs}$

$$\frac{1}{2n^2} + \frac{n-1}{n} \leq \frac{2n-1}{2n} \Leftrightarrow \frac{1}{n} \leq 1$$

The last inequality is always fulfilled because there are always at least two players in the contest, $n \geq 2$.

ad ii.) To show: $RD_{BG}^{rel} \geq RD_{SF}^{rel}$

$$\left(\frac{(1+\frac{\alpha}{2})(1+\alpha)}{(2n+\alpha)(n+\alpha)} + \frac{n-1}{n+\alpha} \right) \geq \frac{2(n-1)(n+\alpha)(1+\alpha)}{n(2n+\alpha)(2+\alpha)} + \frac{1}{2+\alpha}$$

The preference parameter $\alpha \in (-1, 0)$ has to fulfill the following inequalities to satisfy the above condition:

$$-1 \leq \alpha \leq -\frac{3n^2 - 4n - \sqrt{n(9n^3 - 36n^2 + 44n - 16)}}{3n-4}$$

This dependency is shown in the right panel of Figure 3. For a given number of players n the sufficient preference parameter α for the relation $RD_{BG}^{rel} \geq RD_{SF}^{rel}$ is calculated. ■

Let's now endogenize the contest structure. One might imagine a contest organizer who is interested in maximizing the efforts expended by the players.[4] The designer can choose between the Between-Group contest and the Semi-Finals contest. He picks the contest structure with the highest rent-dissipation. The preferred contest structure depends on the assumed type of players preferences. If the contest organizer believes that the players have absolute preferences, he should prefer a Semi-Finals model to elicit higher total aggregate effort. However, if he thinks they have strong negatively interdependent preferences, he should favor in the most cases the Between-Group model.

[4]See Gradstein and Konrad (1999) for endogenizing contest structures.

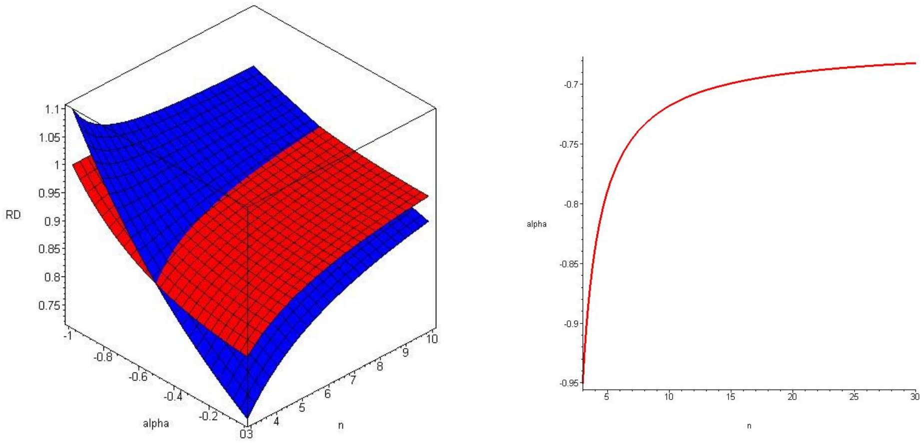

Figure 3: Left panel: Rent dissipation for the BG contest (blue) and SF contest (red). Right panel: Dependency of α to n. For preference parameter smaller than this α the rent dissipation of the Semi-Finals contest under the assumption of relative payoff maximizers is lower than the rent dissipation in the Between-Group contest.

5 Application: Electoral rules

An election is a decision making process where a population chooses an individual to hold official offices. This is the usual mechanism by which modern democracy fills offices in the legislature, sometimes for regional and local government. There is a high universal acceptance of elections as a tool for selecting representatives in modern democracies. In most democratic political systems, there is a range of different types of election. Here, we compare the United States presidential election with the election of a federal minister of the German cabinet concerning the arising rent dissipation. To give a benchmark for the aggregated spendings during a presidential election in the USA we refer to a table provided by Center for Responsive Politics on the CNN web page[5]: total spending by presidential candidates 240 million US-$ in 1996, 343 million US-$ in 2000, 718 million US-$ in 2004 and 586 million US-$ in 2008.

The election process of the President of the United States is split into two stages

[5]See CNN web page: http://edition.cnn.com/ELECTION/2008/money/index.html.

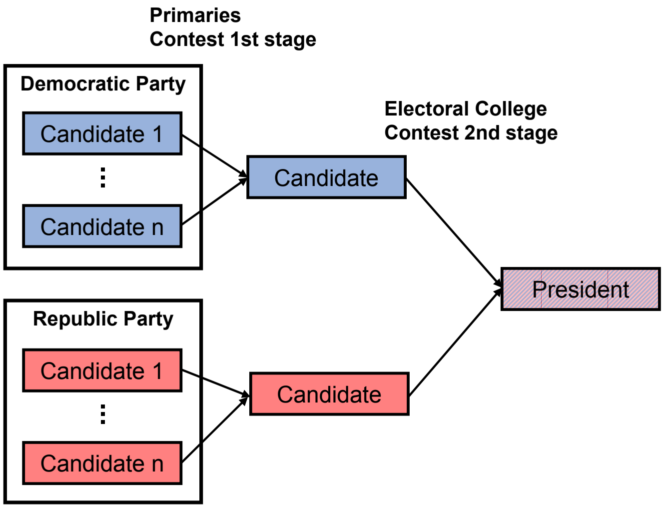

Figure 4: United States presidential election.

as shown in Figure (4). Primary elections, the first stage of the contest, serve to narrow down a field of candidates. Once a candidate has been elected for each party, the two candidates need to compete for office in the general elections. The presidential primary election is the first stage in the process of electing the President of the United States of America. A primary election (nominating primary) is an election in which voters in a jurisdiction select candidates for a subsequent election. In other words, primary elections are one means by which a political party nominates candidates for the following general election. A political party is a political organization that seeks to attain political power within a government. There are two major political parties in the United States, the Democratic Party and the Republican Party. The general election following the primaries is the second and final step. This election system coincides with the above explained Semi-Finals model. There are two groups, the two political parties, competing on the first stage in primaries trough expanding effort in election campaigns. The winning member is nominated for the second stage, the general election, where she again spends effort in campaigning against the winner of the other party.

By contrast the election of federal minister of the German cabinet coincides with the Between-Group model. The election occurs in two stages as shown in Figure (5). The first stage is the election of the Federal Diet. The Federal Diet is

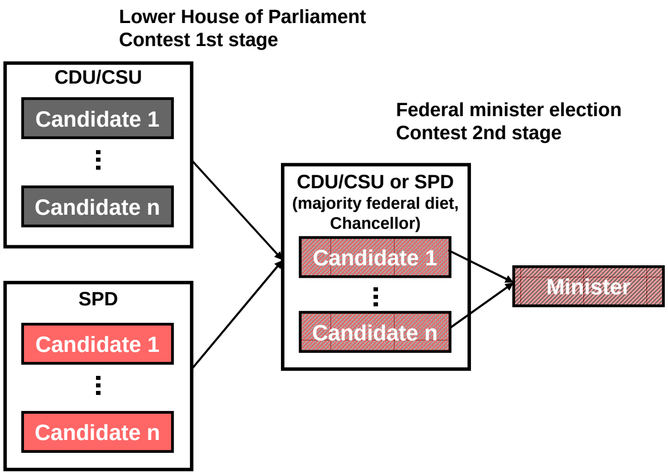

Figure 5: Elections of a federal minister of the German cabinet.

the lower house of the German Parliament. Germany has a multi-party system with two strong parties, the Christian Democratic Union (CDU) and the Social Democratic Party (SPD). According to experience one of these major parties supplies the Chancellor of Germany and therewith nominates the federal ministers. Therefore, we model two groups, the parties, which compete through expanding effort in a election campaign for the majority in the Federal Diet. Simplified seen, on a second stage the winning group, the party which holds the majority, has the right to elect the Chancellor and the federal minister out of their party. For it they expand again effort in the election process.

In view of the above analysis of the Between-Group model and the Semi-Finals model we can say that under the assumption of absolute players-preferences the rent dissipation of the Between-Group model is lower. This means that under this assumption the resources in the German election system are spent more efficiently than in the USA system. If we act on the assumption that politicians are status seeker, which means they have negatively interdependent preferences, the resources are spend more efficiently in the United States presidential election than in the German election. Summing up, it depends on the assumption of preferences whether resources are spent more efficiently in a USA election system with primaries or a German election structure.

6 Application: Awarding prestige sport events

This section compares the awarding systems of the Olympic Games and the World Cup. To host a match of the World Cup cities have to go through a two-stage contest. On a first stage the country where the city belongs to has to apply for the World Cup. Once the country is assigned to the World Cup the individual cities in the country can bid for being home for a match. This awarding system of the World Cup coincides with the above explained Between-Group model.

In a stylized way the awarding system of the Olympic games coincides with the Semi-Finals model. Countries around the world select cities within their national territory to put forward bids for hosting the Olympic Games. On the first stage cities in one country fight for the nomination as a candidate. On the second stage the elected cities of each country bidding to host the Summer Olympic Games or the Winter Olympic Games compete aggressively to have their bid accepted by the International Olympic Committee (IOC). The IOC members, representing most of the member countries, vote to decide where the Games will take place. Typically, the decision is made approximately seven years prior to the games. In recent years, the contest for the right to host the games has grown increasingly fierce.

Comparing the Between-Group model with the Semi-Finals model is a strongly simplified comparison of the awarding systems of the World Cup and the Olympic Games. Given the above found results we can state that under the assumption of absolute payoff maximizers the rent-dissipation of the Between-Group model is lower and therewith the World Cup awarding system is more efficient. If participating cities are spiteful and have negatively interdependent preferences, the resources are spent more efficiently in the awarding system of the Olympic Games.

7 Conclusion

This chapter examines the rent dissipation in two-stage rent-seeking contests. Two different types of contests in which rent-seeking activities take place in two stages are compared. Changing the common assumption of purely self-interested preferences in rent-seeking contests, it is shown that the choice of the contest structure is ambiguous. It depends on the type of preference which the players exhibit. We find that rent dissipation given absolute payoff maximizers in a Between-Group contest is lower than in a Semi-Finals contest and given strongly relative seekers it is the reverse.

On the one hand this result carries over to the case of elections: It depends on the assumption of preferences whether resources are spent more efficiently in a USA election system with primaries or a German election structure. On the other hand it carries over to the case of awarding prestige sport events: There the efficiency of the awarding system of the World Cup or the Olympic Games depends on the assumed preferences.

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