The Contest Winner: Gifted or Venturesome?

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Abstract

We examine the chance of winning a contest when participants differ in both their talent and their attitude towards risk. For the case of CARA preferences, we show that the agent's probability of winning is increasing (decreasing) in the own (opponent's) skill level but decreasing (increasing) in the own (opponent's) degree of risk aversion. We conclude that the chance of winning may be higher for a low-skilled agent with a low degree of risk aversion than for a high-skilled agent with a high degree of risk-aversion. In many circumstances, especially in selection contests, such an outcome is undesirable.

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1 Introduction

May the best man win! Following this dictum, in many areas of life contests are used with the objective of selecting the most able. In business, entrepreneurs conduct job interviews or assessment centers in order to hire or promote the agent with the highest skills. In politics, elections are run where the candidates try to convince the voters of their abilities for holding office. In sports, athletes compete against each other in order to single out the strongest one.

But is it really the most gifted one who has the best chance of winning such a selection contest? Is the winner of the famous Tour de France really the most talented cyclist or simply the guy who fears least the consequences of doping? Is the president-elect indeed best suited for holding office or just the one who had the guts to invest more money into the campaign? Seemingly, an agent's success during a contest does not depend on her skills only but also on her attitude towards risk.

Despite this observation, the economic literature on asymmetric contests has mainly focussed on models where agents differ in just one dimension. In particular, the influence of skill heterogeneity and asymmetric risk aversion on the probability of winning a contest have not yet been analyzed simultaneously but only apart. On the one hand, Baik (1994) shows that, with respect to skills, the more talented agent has, ceteris paribus, the higher probability of winning the contest. On the other hand, authors like Skaperdas and Gan (1995) or Cornes and Hartley (2003) demonstrate for the case of constant absolute risk aversion that the less risk averse agent invests, ceteris paribus, more and has a higher probability of winning.

Given these results, now consider a contest between agent A, who is high skilled but highly risk averse, and agent B, who is low skilled but barely risk averse. Two natural questions arise: i) Who has the higher probability of winning the contest? ii) Who spends more effort?

To study this topic more closely, we engage a simple model of a two-person contest where agents differ in both their skill levels and their degree of constant absolute risk aversion.[1] We show that the agent's probability of winning is increasing (decreasing) in the own (opponent's) skill level but decreasing (increasing) in the own (opponent's) degree of risk aversion. Accordingly, the above questions cannot be answered generally without ambiguity. In particular, there will be cases in which the less able/risk-averse agent spends more effort and has a higher probability of winning the contest than the more

[1] Note the difference to the model of Hvide (2002), where both players have the same attitude towards risk ex-ante but where risk taking is a strategic variable that is endogenously determined in equilibrium of the contest.

able/risk-averse one.

Usually, in selection contests, such an outcome is undesirable.[2] The designer of the contest should therefore take measures in order to reduce the riskiness of the contest if she does not want to discriminate against risk averse agents. However, this is not a trivial task. As we will show, for example, it is a priori not clear whether the contest prize should be increased or decreased in order to improve the situation.

The remainder of this paper is organized as follows: The formal model is set up in Section 2 and used to derive the main results in Section 3. Section 4 illustrates the results and the tradeoff between talent and attitude towards risk by means of some numerical examples. Finally, possible extensions and scope for further research are discussed in Section 5.

2 The Model

In this section, we first introduce the basic assumptions we use in our analysis of the contest game. We then consider the individual maximization problems and derive a general condition characterizing the equilibrium of this game.

2.1 Basic assumptions

There are N agents participating in a winner-take-all contest competing for some rent $R > 0$. Each agent $i \in \{1, \dots, N\}$ has an initial wealth endowment I_i and can spend some resources $x_i \in [0, I_i]$ in order to improve her probability of winning p_i. This probability is determined by the following contest success function (CSF):

$$p_i := \frac{f_i(x_i)}{\sum_{j=1}^{N} f_j(x_j)}, \tag{1}$$

where $f_i : [0, I_i] \to \mathbb{R}_0^+$ is an increasing concave function of x_i satisfying $f_i(0) = 0$. For the sake of concreteness and ease of calculation we assume $N = 2$ and

$$f_i(x_i) := \theta_i x_i, \tag{2}$$

where $\theta_i > 0$ is a parameter expressing agent i's skill level. We also refer to θ_i as agent i's talent for the task required within the contest. Note that equation (2) states, reasonably enough, a complementarity between talent and effort, which is standard in the related literature (e.g. Baik, 1994).

[2] In the context of sales contests, Bono (2008) characterizes a situation where it might be desirable to promote less risk averse managers, since they exert, ceteris paribus, more effort.

Introducing risk aversion into the analysis of contests we follow the approach proposed by Skaperdas and Gan (1995) and Cornes and Hartley (2003), respectively and assume that the preferences of agent i can be expressed by the following utility function which exhibits constant absolute risk aversion (CARA):

$$u_i(W_i) := -e^{-\alpha_i W_i}, \tag{3}$$

where α_i is agent i's constant degree of absolute risk aversion. We include the limit case of a risk-neutral player i with $u_i(W_i) = W_i$ into the analysis, and refer to this situation as one in which $\alpha_i = 0$.

2.2 Individual objectives and equilibrium conditions

The contest is organized as a Cournot-Game. The players simultaneously choose their effort levels x_i in order to maximize their expected utility Eu_i from consumption W_i, which equals $I_i - x_i + R$ if agent i wins the contest and $I_i - x_i$ otherwise. Hence, for $i, j \in \{1, 2\}, i \neq j$,

$$\begin{aligned} Eu_i &= p_i u_i(I_i - x_i + R) + (1 - p_i)u_i(I_i - x_i) \\ &= -\left[\frac{\theta_i x_i}{\theta_1 x_1 + \theta_2 x_2} e^{-\alpha_1(I_i - x_i + R)} + \frac{\theta_j x_j}{\theta_1 x_1 + \theta_2 x_2} e^{-\alpha_1(I_i - x_i)}\right]. \end{aligned}$$

For ease of notation, we define

$$p_i' := \frac{\partial p_i}{\partial x_i} = \theta_i \frac{\theta_j x_j}{(\theta_1 x_1 + \theta_2 x_2)^2} \geq 0, \tag{4}$$

$$p_i'' := \frac{\partial^2 p_i}{\partial x_i^2} = -\frac{2\theta_i^2}{\theta_1 x_1 + \theta_2 x_2} \frac{\theta_j x_j}{(\theta_1 x_1 + \theta_2 x_2)^2} \leq 0, \tag{5}$$

$$\beta(\alpha) := \frac{\alpha}{1 - e^{-\alpha R}}, \tag{6}$$

$$\delta(\alpha) := \frac{\alpha e^{-\alpha R}}{1 - e^{-\alpha R}} = e^{-\alpha R}\beta\alpha. \tag{7}$$

It is easily verified that β is an increasing function of α and δ is a decreasing function of α (Skaperdas and Gan, 1995, supplementary appendix to Proposition 2). The first order condition (FOC) for an interior solution of agent i's maximization problem yields

$$p_i' = \beta(\alpha_i)(p_i e^{-\alpha_i R} + 1 - p_i). \tag{8}$$

Equation (8) implicitly defines the reaction function of agent i, i.e. her optimal effort x_i as a function of the opponent's effort x_j. Under the assumptions made, the rent seeking game has a unique Nash equilibrium in pure strategies, as shown by Cornes and Hartley (2003, Propostioin 3.3) and Yamazaki (2008), respectively. In order to compute this equilibrium under different parameter constellations, we divide the FOC of agent 1 by the FOC of agent 2, and note that $\frac{p_1'}{p_2'} = \frac{x_2}{x_1}$. Denoting by $q := \frac{p_2}{p_1} = \frac{\theta_2 x_2}{\theta_1 x_1}$ the competitive balance of the contest,[3] this yields

$$\frac{\theta_1}{\theta_2} q = \frac{\beta(\alpha_1)}{\beta(\alpha_2)} \cdot \frac{e^{-\alpha_1 R} + q}{q e^{-\alpha_2 R} + 1}. \tag{9}$$

Equation (9) can be transformed into a quadratic equation for the competitive balance in equilibrium; as $\frac{\theta_2 \delta(\alpha_1)}{\theta_1 \delta(\alpha_2)} > 0$, only the positive root yields a feasible solution $q > 0$, i.e.

$$q = \sqrt{\frac{\theta_2 \delta(\alpha_1)}{\theta_1 \delta(\alpha_2)} + \left(\frac{\theta_1 \beta(\alpha_2) - \theta_2 \beta(\alpha_1)}{2\theta_1 \delta(\alpha_2)} \right)^2} - \frac{\theta_1 \beta(\alpha_2) - \theta_2 \beta(\alpha_1)}{2\theta_1 \delta(\alpha_2)}. \tag{10}$$

A value $q < 1$ indicates that agent 1's probability of winning exceeds the one of agent 2, i.e. $p_1 > p_2$, and vice versa for $q > 1$.

3 The Results

In this section, we compare the outcome of the contest for different parameter constellations and derive some insightful comparative static results. We start with the case of symmetric, i.e. identical, players as a benchmark. We then analyze successively how the outcome changes if we introduce heterogeneity between the agents with respect to their skills only, their risk attitude only, and both skill and risk aversion.

3.1 The symmetric benchmark

In this subsection, we derive the equilibrium of the contest as well as its comparative statics properties for the case of identical players.

Proposition 1 *Suppose* $\theta_1 = \theta_2 = \theta > 0$ *and* $\alpha_1 = \alpha_2 = \alpha \geq 0$.

(a) *The equilibrium is symmetric with equal winning probabilities* $p_1^* = p_2^* = \frac{1}{2}$ *(i.e.* $q = 1$*) and effort levels, which do not depend on the skill level*

[3] Some authors use the difference in winning probabilities as an alternative measure of 'competitive balance' or 'closeness' of the contest (see e.g. Runkel, 2006a,b).

θ:

$$x_1^* = x_2^* = x_{sym}^* := \begin{cases} \frac{R}{4} & if \quad \alpha = 0 \\ \frac{e^{\alpha R}-1}{2\alpha(e^{\alpha R}+1)} & if \quad \alpha > 0 \end{cases}$$

(b) The effort levels are increasing in the prize: $\frac{\partial x_{sym}^*}{\partial R} > 0$.

(c) The effort levels are decreasing in the degree of risk aversion: $\frac{\partial x_{sym}^*}{\partial \alpha} < 0$.

The proof can be found in Appendix A. Parts (a) and (b) of Proposition 1 are straightforward generalizations of the respective results in the case of risk-neutral players (Baik, 1994) and, as such, very intuitive: The intensity of competition among equal competitors does not depend on the (skill) level the contest takes place at, but is positively related to the rent offered.

Part (c) of Proposition 1 resolves the general ambiguity result of Konrad and Schlesinger (1997) for the case of preferences with CARA. Note that a higher degree of risk aversion has two opposing effects on the individual investment decision. On the one hand, there is the so called *gambling effect*: Since participation in the contest comes along with an uncertain payment, it may be regarded as a lottery, which the agents invest the less into the more risk averse they are. On the other hand, there is a so called effect of *self-protection*: By spending more effort, the players can reduce their probability of loosing the contest. Therefore, the more risk averse they are the more they invest. Under our assumptions, the gambling effect outweighs the effect of self protection.

Traditionally, the literature conducts the comparative statics with respect to the *dissipation rate*, which is defined as the fraction ρ of the rent that is 'wasted' in form of aggregate effort, i.e. $\rho := \frac{\sum_{j=1}^{N} x_j}{R}$.

Corollary 1 *For* $\theta_1 = \theta_2 = \theta > 0$ *and* $\alpha_1 = \alpha_2 = \alpha > 0$*, the equilibrium rent dissipation rate equals*

$$\rho_{sym} = \frac{e^{\alpha R} - 1}{\alpha R(e^{\alpha R} + 1)} < 1$$

and is decreasing in both the prize R *and the degree of risk aversion* α*.*

The proof can be found in Appendix A. As shown by Hillman and Samet (1987) in a more general framework, less than the full rent will be dissipated if agents are risk averse. The comparative statics of Corollary 1 are in line with the simulations run by Hillman and Katz (1984) for the case of logarithmic utilities and with the results in Long and Vousden (1987) for contests with rents that are divisible among agents.[4] Put differently, for the case of preferences with CARA, the intuition holds that less of the rent will be wasted if the agents are more risk averse or if the stakes are higher.

3.2 Asymmetric skills

In this subsection, we derive the equilibrium of the contest as well as its comparative statics properties for the case of players with the same degree of risk aversion but different skill levels.

Proposition 2 *Suppose, without loss of generality,* $\theta_1 > \theta_2 > 0$ *and* $\alpha_1 = \alpha_2 = \alpha \geq 0$.

(a) In equilibrium,

$$1 > q = \begin{cases} \frac{\theta_2}{\theta_1} & \text{if} \quad \alpha = 0 \\ \sqrt{\frac{\theta_2}{\theta_1} + \left(\frac{e^{\alpha R}}{2}\frac{\theta_1 - \theta_2}{\theta_1}\right)^2} - \frac{e^{\alpha R}}{2}\frac{\theta_1 - \theta_2}{\theta_1} & \text{if} \quad \alpha > 0, \end{cases}$$

i.e. the player with the higher skill level has the better probability of winning, $p_1^* > p_2^*$.

(b) The higher the agent's skill level, the better her probability of winning, i.e. $\frac{\partial q}{\partial \theta_1} < 0 < \frac{\partial q}{\partial \theta_2}$.

(c) (i) Under risk neutrality ($\alpha = 0$)*, competitive balance is independent of the prize, i.e.* $\frac{\partial q}{\partial R} = 0$.

(ii) In the case of risk aversion ($\alpha > 0$)*, the higher the prize, the better the winning probability of the more talented agent, i.e.* $\frac{\partial q}{\partial R} < 0$.

(d) The higher the degree of risk aversion, the better the winning probability of the more talented agent, i.e. $\frac{\partial q}{\partial \alpha} < 0$.

The proof can be found in Appendix A. Parts (a) and (b) of Proposition 2 are straightforward generalizations of the respective results in the case of risk-neutral players (Baik, 1994) and, as such, very intuitive: The more talented agent has the better chance of winning and this probability is the higher the larger the gap in skills is. Part (c) of Proposition 2 shows that the

[4] They contrast, though, to the diametric result in Fabella (1992). However, Konrad and Schlesinger (1997, footnote 11) report that his "result is not correct as the paper contains several serious errors".

neutrality result for risk neutral agents (Runkel, 2006a, Proposition 1 (b)) does not hold for risk averse players: A higher prize increases the 'riskiness' of the contest which is worse for the less skilled agent being more likely to loose. A similar intuition also drives the result in Part (d) of Proposition 2.

Can there also be said something about the equilibrium effort levels? If the agents are risk neutral, they will exert the same effort level $x_1^* = x_2^* = \frac{R\theta_1\theta_2}{(\theta_1+\theta_2)^2}$ in equilibrium, which will be maximal for equal skills $\theta_1 = \theta_2$ (Baik, 1994). Under risk aversion, however, the equilibrium effort levels will differ if and only if agents differ in skills. To see this, consider the relative effort $\xi := \frac{x_2^*}{x_1^*} = \frac{\theta_1}{\theta_2} q$ in equilibrium; then the following statements hold.

Corollary 2 *Suppose, without loss of generality, $\theta_1 > \theta_2 > 0$ and $\alpha_1 = \alpha_2 = \alpha > 0$.*

(a) The higher the skill level of agent i the higher her relative equilibrium effort, i.e. $\frac{\partial \xi}{\partial \theta_1} < 0 < \frac{\partial \xi}{\partial \theta_2}$.

(b) In equilibrium, the agent with the higher skill level exerts more effort, i.e. $x_1^* > x_2^*$.

The proof can be found in Appendix A. The results confirm the complementary character of effort and skill and back up the much cited anecdotal evidence for talent to come along with diligence.

3.3 Asymmetric risk aversion

In this subsection, we derive the equilibrium of the contest as well as its comparative statics properties for the case of players with the same skill level but different degrees of risk aversion.

Proposition 3 *Suppose, without loss of generality, $\theta_1 = \theta_2 = \theta > 0$ and $\alpha_1 > \alpha_2 \geq 0$.*

(a) In equilibrium,

$$q = \sqrt{\frac{\delta(\alpha_1)}{\delta(\alpha_2)} + \left(\frac{1}{2}\frac{\beta(\alpha_2) - \beta(\alpha_1)}{\delta(\alpha_2)}\right)^2} - \frac{1}{2}\frac{\beta(\alpha_2) - \beta(\alpha_1)}{\delta(\alpha_2)}.$$

(b) For small differences in risk aversion, the player with the higher degree of risk aversion exerts less effort and has the smaller probability of winning, i.e. $\alpha_1 = \alpha_2 + \varepsilon$, with $\varepsilon > 0$ sufficiently small, implies $x_1^* < x_2^*$ and $p_1^* < p_2^*$.

(c) *The higher the prize, the smaller the winning probability of the more risk averse agent, i.e.* $\frac{\partial q}{\partial R} > 0$.

The proof can be found in Appendix A. Part (b) of Proposition 3 reproduces the respective results in Skaperdas and Gan (1995, Proposition 2b), Cornes and Hartley (2003, Proposition 3.4), and Bono (2008, Proposition 1) for the specific framework at hand. The intuition here is similar to the corresponding result in the symmetric equilibrium. Since, under the assumptions made, the gambling effect outweighs the effect of self-protection, the less risk averse agent will spend more effort and thus have a higher chance of winning the contest. An analogous reasoning explains the result of Part (c). An increasing prize raises the 'riskiness' of the contest which is worse for the more risk averse agent.

3.4 Asymmetric skills and risk aversion

In this subsection, we argue that the agent with the higher skill level might nevertheless be very likely to lose the contest if, at the same time, she exhibits a higher degree of risk aversion. Such an outcome often is undesirable, especially in the case of a selection contest. A selection contest aims at finding out who is most productive in fulfilling a certain task and, hence, wants to rank the agents according to their skills rather than attitude towards risk. Examples range from job interviews to tournaments in sports with its dictum: *May the best man win!*

Part (a) of Proposition 2 shows that for a given equal degree of risk aversion agent 1, with the higher skill level, has a higher chance of winning. Part (b) of Proposition 3 states that, starting from the symmetric equilibrium, agent 1 has a smaller winning probability if his degree of risk aversion increases marginally. Therefore, by the continuity of the relevant functions, the implicit functions theorem implies the following

Corollary 3 *There exists a set of parameter values with* $\theta_1 > \theta_2$ *and* $\alpha_1 > \alpha_2$ *such that* $q > 1$, *i.e.* $p_1^* < p_2^*$.

Corollary 3 characterizes a situation where the presence of heterogeneity with respect to the agents' attitude towards risk induces some kind of failure of the contest: The probability of selecting the agent with the lower skills is higher than the probability of selecting the agent with the higher skills. Put differently, the venturesome has a good chance of beating the gifted.

Moreover, note that a higher prize on the one hand increases the chance of winning for the agent with the higher skill level (Proposition 2(c)), but on the other hand decreases the chance of winning for the agent with the higher degree of risk aversion (Proposition 3(c)). Consequently, from the viewpoint of contest design, it is a priori not clear how the prize of the contest should be chosen in order not to end up in such an undesirable situation characterized by Corollary 3. The numerical examples presented in the next section will clarify these remarks.

4 Numerical Examples

In this section we illustrate our findings by means of some numerical examples.

4.1 Venturesome beats gifted

First, we provide an example for a situation in which the chance of winning is higher for a low-skilled agent with a low degree of risk aversion (the venturesome) than for a high-skilled agent with a high degree of risk-aversion (the gifted).

Example 1 *Suppose* $\theta_1 = 10$, $\theta_2 = 9$, $\alpha_1 = 1$, $\alpha_2 = \frac{1}{2}$. *Accordingly, player 1 (the gifted) is more talented but also more risk averse than player 2 (the venturesome).*

(a) For $R = \ln(4)$ *we compute* $q = 1$, *i.e. the gifted and the venturesome have the same probability of winning the contest.*

(b) For $R = \ln(9)$ *we compute* $q = \frac{\sqrt{1161}+21}{40} > 1$, *i.e. the gifted has a lower probability of winning the contest than the venturesome.*

4.2 Adjusting the contest prize

Furthermore, the example in the previous subsection shows that an increasing contest prize R may increase the winning probability of the venturesome. In this subsection, however, we provide an example for a situation, in which the opposite is true.

Example 2 *Suppose* $\theta_1 = 2$, $\theta_2 = 1$, $\alpha_1 = 1$, $\alpha_2 = \frac{1}{2}$. *Again, player 1 (the gifted) is more talented but also more risk averse than player 2 (the venturesome).*

(a) For $R = \ln(4)$ *we compute* $q = \frac{1}{3}$, *i.e. the gifted has a higher probability of winning the contest than the venturesome.*

(b) For $R = \ln(9)$ *we compute* $q = \frac{1}{4} < \frac{1}{3}$, *i.e. the winning probability of the talented increases even further as the contest prize* R *increases.*

5 Concluding Remarks

We have examined the chance of winning a contest when participants differ in both their talent and their attitude towards risk. For the case of CARA preferences, we have shown that the agent's probability of winning is increasing (decreasing) in the own (opponent's) skill level but decreasing (increasing) in the own (opponent's) degree of risk aversion. Hence there are situations in which the chance of winning is higher for a low-skilled agent with a low degree of risk aversion than for a high-skilled agent with a high degree of risk-aversion.

Since such an outcome is undesirable in many circumstances, the contest should be designed in order to reduce the riskiness of the game. In our model, the only parameter available for contest design is the contest prize. We have shown, by the means of a numerical example, that the optimal prize depends, however, on the parameters of the model. To extend the analysis in this direction, one could think of a framework in which the contest designer has more instruments at hand. Imagine, for example, a situation where the designer can influence the agents' effort costs (which are normalized to 1 in our model).

Many scandals in business, politics, and sports lead to the impression that the agent's success during a selection contest is not always based on her superior skills but the result of her cheating. Sportsmen dope, politicians betray, managers bribe. In real life, besides plain effort, cheating is an illegal but often applied possibility for the agent to enhance the probability of winning. While it is intuitive that the availability of a cheating technology increases the riskiness of the contest, without a formal analysis it is not clear how the availability of such a technology influences the relative winning probabilities of the gifted and the venturesome.

A Appendix

Proof of Proposition 1.

Since the case of risk neutral agents is discussed extensively in the literature (see e.g. Baik, 1994), we concentrate on risk averse agents.

(a) For $\theta_1 = \theta_2 = \theta > 0$ and $\alpha_1 = \alpha_2 = \alpha > 0$ equation (10) immediately implies $q = 1$. Hence, the equilibrium is symmetric with equal winning probabilities $p_1^* = p_2^* = \frac{1}{2}$ and effort levels $x_1^* = x_2^* = x_{\text{sym}}^*$, which may be easily computed from the FOC (8).

(b) $\frac{\partial x_{\text{sym}}^*}{\partial R} = \frac{e^{\alpha R}}{(e^{\alpha R}-1)^2} > 0$.

(c) Using the identity $e^X = \sum_{k=0}^{\infty} \frac{X^k}{k!}$ for any real X, one verifies that

$$\frac{\partial x^*_{\text{sym}}}{\partial \alpha} = \frac{1 + 2\alpha R e^{\alpha R} - e^{2\alpha R}}{2\alpha^2 (e^{\alpha R} + 1)^2} < 0.$$

□

Proof of Corollary 1.

The equilibrium rent dissipation rate can be computed immediately from the equilibrium effort levels in Proposition 1. Denoting $A := \alpha R$ we calculate

$$\frac{\partial \rho_{\text{sym}}}{\partial A} = \frac{1 + 2Ae^A - e^{2A}}{A^2 (e^A + 1)^2} < 0,$$

where the last inequality is verified, again, using the identity $e^X = \sum_{k=0}^{\infty} \frac{X^k}{k!}$ for any real X. From this inequality it is also apparent that $\rho_{\text{sym}} < 1$.

□

Proof of Proposition 2.

Again, we concentrate on risk averse agents.

(a) For $\theta_1 > \theta_2 > 0$ and $\alpha_1 = \alpha_2 = \alpha > 0$, the value of q can be immediately computed from equation (10). Moreover, $\theta_1 - \theta_2 > 0$ and, hence,

$$q = \sqrt{\frac{\theta_2}{\theta_1} + \left(\frac{e^{\alpha R}}{2}\frac{\theta_1 - \theta_2}{\theta_1}\right)^2} - \frac{e^{\alpha R}}{2}\frac{\theta_1 - \theta_2}{\theta_1} < \sqrt{\frac{\theta_2}{\theta_1}} < 1.$$

(b) We have to show that $\frac{\partial q}{\partial \theta_1} < 0 < \frac{\partial q}{\partial \theta_2}$. One easily verifies that either inequality is equivalent to

$$1 + \sqrt{\frac{\theta_2}{\theta_1} e^{2\alpha R} + \left(\frac{e^{2\alpha R}}{2}\frac{\theta_1 - \theta_2}{\theta_1}\right)^2} - \frac{e^{2\alpha R}}{2}\frac{\theta_1 - \theta_2}{\theta_1} > 0,$$

which is obviously true as the root-term exceeds $\frac{e^{2\alpha R}}{2}\frac{\theta_1 - \theta_2}{\theta_1}$.

(c) (i) Trivial.

(d) and (c) (ii) Again denoting $A := \alpha R$, we have to show that $\frac{\partial q}{\partial A} < 0$. Using $\theta_1 - \theta_2 > 0$, one easily verifies that this is equivalent to

$$\frac{e^A}{2}\frac{\theta_1 - \theta_2}{\theta_1} < \sqrt{\frac{\theta_2}{\theta_1} + \left(\frac{e^A}{2}\frac{\theta_1 - \theta_2}{\theta_1}\right)^2},$$

which is obviously true.

□

Proof of Corollary 2.

Suppose $\theta_1 > \theta_2 > 0$ and $\alpha_1 = \alpha_2 = \alpha > 0$.

(a) We have to show that $\frac{\partial \xi}{\partial \theta_1} < 0 < \frac{\partial \xi}{\partial \theta_2}$. One easily verifies that either inequality is equivalent to

$$1 + \frac{e^{2\alpha R}}{2}\frac{\theta_1 - \theta_2}{\theta_2} < \sqrt{\frac{\theta_1}{\theta_2}e^{2\alpha R} + \left(\frac{e^{2\alpha R}}{2}\frac{\theta_1 - \theta_2}{\theta_2}\right)^2},$$

which is true as $1 < e^{2\alpha R}$.

(b) Noting that $\xi_{|\theta_1 = \theta_2} = 1$ and applying part (a) we conclude $\xi < 1$, i.e. $x_1^* > x_2^*$, for all $\theta_1 > \theta_2$.

□

Proof of Proposition 3.

Suppose $\theta_1 = \theta_2 = \theta > 0$ and $\alpha_1 \geq \alpha_2 > 0$.

(a) The value of q can be immediately computed from equation (10).

(b) Evaluating

$$\frac{\partial q}{\partial \alpha_1} = \frac{1}{2}\left(\sqrt{\frac{\delta(\alpha_1)}{\delta(\alpha_2)} + \left(\frac{1}{2}\frac{\beta(\alpha_2) - \beta(\alpha_1)}{\delta(\alpha_2)}\right)^2}\right)^{-1} \cdot \left[\frac{\delta'(\alpha_1)}{\delta(\alpha_2)} - \frac{1}{2}\frac{\beta'(\alpha_1)}{\delta(\alpha_2)}\frac{\beta(\alpha_2) - \beta(\alpha_1)}{\delta(\alpha_2)}\right] + \frac{1}{2}\frac{\beta'(\alpha_1)}{\delta(\alpha_2)}$$

at $\alpha_1 = \alpha_2 = \alpha$ yields

$$\frac{\partial q}{\partial \alpha_1}_{|\alpha_1 = \alpha_2 = \alpha} = \frac{1}{2}\frac{\delta'(\alpha) + \beta'(\alpha)}{\delta(\alpha)} > 0 \tag{11}$$

which is positive, since $\delta(\alpha) > 0$ and

$$\delta'(\alpha) + \beta'(\alpha) = \frac{e^{2\alpha R} - (1 + 2\alpha R e^{\alpha R})}{(e^{\alpha R} - 1)^2} > 0.$$

The last inequality can be verified using the identity $e^X = \sum_{k=0}^{\infty} \frac{X^k}{k!}$ for any real X.

Applying the result (11) and noting that $q_{|\alpha_1 = \alpha_2} = 1$ for symmetric agents, we find $\bar{\varepsilon} > 0$ such that $\alpha_1 = \alpha_2 + \varepsilon$ implies $q > 1$ and thus $p_1^* < p_2^*$ for all $0 < \varepsilon < \bar{\varepsilon}$. However, as $\theta_1 = \theta_2$, $p_1^* < p_2^*$ is possible if and only if $x_1^* < x_2^*$.

(c) The higher the prize, the smaller the winning probability of the more risk averse agent, i.e. $\frac{\partial q}{\partial R} > 0$: So far, we have only examples (e.g. for $\theta_1 = \theta_2$, $\alpha_1 = 1 > \frac{1}{2} = \alpha_2$, we compute $q_{|R=\ln(4)} < q_{|R=\ln(9)}$), but no general proof for this statement.

□

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