Optimal Fiscal Policy and Human Capital Risk*

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Abstract

This paper studies optimal fiscal policy when households face uninsurable idiosyncratic human capital risk and the government provides public services to households regardless of their economic status. We consider a tractable macroeconomic model with risk-less physical capital and risky human capital in which the government chooses optimally a system of flat-rate taxes/subsidies as well as the level of spending on public services. We show that it is always socially optimal to subsidize investment in human capital as long as i) there is some human capital risk and ii) public services are valued by individual households. A quantitative study based on a calibrated version of the model reveals that the optimal human capital subsidy is substantial, but the corresponding welfare gains are small due to strong general equilibrium effects. However, for the small open economy version of the model with constant interest rate and wages, the welfare gains from implementing the optimal human capital subsidy are large.

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I. Introduction

There is by now an extensive empirical literature documenting that individual households face a substantial amount of labor income uncertainty.[1] Thus, if labor income is the return to human capital investment, a view that has been prominent in economics for some time, then human capital investment is highly risky. There is also strong evidence that governments of many developed countries spend a significant amount of total output on public services that are in principle available to all households regardless of their economic status. In other words, the government provides risk-free consumption services. In this paper, we show that these two facts taken together imply that the optimal fiscal policy requires the government to subsidize investment in (risky) human capital. Moreover, a quantitative study based on a calibrated version of the model reveals that the optimal human capital subsidy is substantial, but the corresponding welfare gains are small because of strong general equilibrium effects. However, for the small open economy version of the model with constant interest rate and wages (partial equilibrium), the welfare gain from implementing the optimal human capital subsidy are substantial.

There is a simple economic intuition for the optimality of human capital subsidies. We imagine a world in which the government may use non-distortionary taxation to finance the (optimal) provision of risk-free public services and individual households allocate their total investment between risk-free physical capital and risky human capital. Clearly, as long as households are risk-averse, in equilibrium average human capital returns must exceed average physical capital returns, that is, human capital is the high-return, high-risk investment

[1] Using individual data on labor income dynamics, estimates for the standard deviation of the permanent component of labor income shocks range from 0.15 in Carroll and Samwick (1997) over 0.19 in Meghir and Pistaferri (2004) up to 0.25 in Storesletten, Telmer and Yaron (2004). Jacobson, LaLonde and Sullivan concentrate on the specific issue of labor income dynamics after job displacement and find that long run earnings are on average 25 percent below the pre-displacement rate. For a review of the job displacement literature, see Kletzer (1998).

opportunity. In such a world, the portfolio choice between risk-free physical capital and risky human capital made by individual households is not socially optimal. To see this, consider a household who contemplates reducing physical capital investment by one dollar and using this extra dollar to invest in human capital. The benefit of this re-allocation of investment funds is an increase in total output (human capital returns are high) and the cost is an increase in consumption risk (human capital returns are risky). As long as the government does not waste resources, the private and social benefits of such a re-allocation of investment funds are the same. In contrast, private cost in general exceed social cost, and it therefore becomes optimal to subsidize human capital investment. To understand that private cost exceed social cost, we only have to note that the government will optimally choose to transform some of the additional output into risk-free public services, and that this effect is not taken into account by individual households who take government policy as given when making their investment choices.

There is also as simple intuition for the result that openness to international capital flows enhances the welfare gains from implementing the optimal human capital subsidy. The introduction of the subsidy increases human capital investment, which is good for economic growth since human capital is the high-return investment opportunity. However, in a closed economy without international capital flows, more human capital investment means that the "capital-to-labor-ratio", and therefore the wage rate, will be reduced. Thus, the increase in economic growth is dampened, though never overturned, by a decrease in average human capital returns. Our quantitative study shows that this dampening general equilibrium effect is quite large so that the implied welfare gains from a human capital subsidy are very small. In contrast, when the economy is small and fully integrated into the world economy, the increase in human capital investment is fully matched by an equal increase in physical capital investment coming from abroad so that the "capital-to-labor-ratio" and the wage rate remain unchanged. Our quantitative study shows that for this version of the model, large welfare gains can be reaped from implementing the optimal human capital subsidy.

In this paper, our theoretical and quantitative analysis is conducted using an extended version of the incomplete-market model developed in Krebs (2003,2006), which in turn is based on the human capital models of endogenous growth popularized by Lucas (1988), Manuelli and Jones (1990), and many others. The model of Krebs (2003,2006) assumes that production displays constant-returns-to-scale with respect to physical and human capital, and that there are many ex-ante identical, infinitely-lived households with log-utility preferences. These households make a consumption/saving decision and a decision regarding the allocation of total saving (investment) between physical capital and human capital. Investment in physical capital is risk-free, but investment in human capital is risky due to idiosyncratic shocks to the stock of human capital. Markets are incomplete in the sense that there are no explicit insurance markets for human capital risk. In this paper, we add to the basic framework discussed in Krebs (2003,2006) a government that chooses optimally a system of flat-rate taxes/subsidies as well as the level of spending on public services.

The human capital model with incomplete markets analyzed in this paper is highly tractable in the sense that we can provide a complete characterization of equilibrium consumption and welfare that does not require knowledge of the endogenous wealth distribution. This tractability allows us to conduct a fairly general theoretical analysis of the main economic issues. In contrast, problems with tractability have often forced previous work in the literature to confine attention to purely quantitative analysis. Clearly, the tractability of the current model is only possible because certain assumptions about the nature of human capital production and human capital risk have been made. Though we would not claim that all the assumptions and implications of the model are exactly satisfied, we do show in this paper that the model's most crucial assumptions and implications are roughly consistent with the empirical evidence (see also Krebs (2003)). In particular, our way of modeling human capital risk in terms of idiosyncratic shocks to human capital implies a labor income process that has been extensively used by the empirical literature on labor income risk. Moreover, empirical estimates of human capital returns are consistently higher than the return on less risky financial investments, supporting the basic idea that human capital is a highly risky investment opportunity with high average returns. Finally, the empirical literature on household-level consumption data points towards the presence of a large amount of consumption risk, another empirical fact that can be explained by the model. Indeed, in the quantitative part of this paper, the body of empirical evidence on labor income risk, human capital returns, and consumption risk is used to calibrate the model and cross-check its empirical plausibility.

Additional literature to be discussed:

i) old two-period with risky human capital: no government spending, risk reduction effect of labor income tax, different effect and opposite result (for example, Eaton and Rosen (1980) and Stiglitz (1969)).

ii) human capital externalities as reason for human capital subsidy

iii) borrowing constraints as reason for human capital subsidy

iv) optimal capital income taxation (for example, Aiyagari (1995), Chamley (1986), Golosov, Kocherlakota, and Tsyvinski (2003), and Judd (1987)).

II. Model

This section develops the model that underlying the theoretical and quantitative analysis conducted in the next sections. The model considers households who invest in physical capital and human capital. Physical capital investment is risk-free, but human capital investment is risky due to labor market risk. Production takes place under constant-returns-to-scale with respect to physical and human capital. There is a government that uses incentive-neutral consumption taxes to finance the public provision of ”risk-free” consumption services. In addition, the government can use capital and labor income taxes/subsidies to change the investment choice made by individual households. We will discuss to versions of the model: a closed-economy version with endogenous wage and interest rate and a small-open economy version in which the domestic interest rate, and therefore the wage rate, is pinned down by the exogenous world interest rate.

II.1. Economy

Consider a discrete-time, infinite-horizon economy with one non-perishable good that can be consumed or invested. There is one domestic firm that produces the “all-purpose” good. The firm combines physical capital, K, with human capital, H, to produce output, $Y = F(K, H)$, where all variables are expressed in per household terms. The function F is a standard neoclassical production function. In particular, it displays constant-returns-to-scale. The firm rents the two input factors (physical and human capital) in competitive markets. We denote the rental rate of physical capital by r_k and the rental rate of human capital (the wage rate per efficiency unit of labor) by r_h. In each period, the firm hires capital and labor up to the point where current profit is maximized. Thus, the firm solves the following static maximization problem:

$$\max_{K_t, H_t} \{ F(K_t, H_t) - r_k K_t - r_h H_t \} . \tag{1}$$

Notice that Y is GDP since it is the output of the domestic firm. In contrast, GNP is determined by the decision of the domestic household, which will be discussed next. Of course, in the closed-economy version of the model the two concept coincide.

There are many ex-ante identical, infinitely-lived households with total mass of one. Households have identical preferences over private consumption plans $\{c_t\}$ and sequences of public services, $\{G_t\}$, where lower-case letters denote individual-specific variables and upper-case letters denote aggregate (per capita) variables. Preferences of individual households allow for a time-additive expected utility representation with one-period utility function of the logarithmic type:

$$U(\{c_t, G_t\}) = E\left[\sum_{t=0}^{\infty} \beta^t log c_t\right] + \nu \sum_{t=0}^{\infty} log G_t \ . \tag{2}$$

Let k_t and h_t stand for the stock of physical and human capital owned by an individual household, and let x_{kt} and x_{ht} stand for the corresponding investment in physical and human capital. We denote the rental rate of physical capital by r_k and the rental rate of human capital (the wage rate per efficiency unit of labor) by r_h. Capital and labor markets are perfectly competitive and the government taxes (or subsidizes) capital and labor income at the flat rate τ_{kt}, respectively τ_{ht}. In addition, the government can tax consumption at the rate τ_{ct}. Thus, the sequential budget constraint reads:

$$\begin{aligned} (1+\tau_{ct})c_t + x_{kt} + x_{ht} &= (1-\tau_{ht})r_k k_t + (1-\tau_{ht})r_h h_t \\ k_{t+1} &= (1-\delta_k)k_t + x_{kt} \ \ , \ \ k_t \geq 0 \\ h_{t+1} &= (1-\delta_h+\eta_t)h_t + x_{ht} \ \ , \ \ h_t \geq 0 \\ & (k_0, h_0, \eta_0) \ given \ . \end{aligned} \tag{3}$$

In (3) δ_k, respectively δ_h, denotes the (average) depreciation rate of physical capital, respectively human capital. The term η_t describes a household-specific shock to human capital. We assume that these idiosyncratic shocks are identically and independently distributed across households and across time (unpredictability of idiosyncratic shocks).[2]

The random variable η_t represents uninsurable idiosyncratic labor income risk. A negative human capital shock, $\eta_t < 0$, can occur when a worker loses firm- or sector-specific human capital subsequent to job termination (worker displacement). In order to preserve the tractability of the model, the budget constraint (3) rules out extended periods of un-

[2]The budget constraint (3) makes two implicit assumptions about the accumulation of human capital. First, it lumps together general human capital (education, health) and specific human capital (on-the-job training). Second, (3) does not impose a non-negativity constraint on human capital investment ($x_{ht} \geq 0$).

employment because it assumes that the wage payment is received in each period. Thus, the emphasis is on earnings uncertainty, not employment uncertainty. A decline in health (disability) provides a second example for a negative human capital shock. In this case, both general and specific human capital might be lost. Internal promotions and upward movement in the labor market provide two examples of positive human capital shocks ($\eta_t > 0$).

The budget constraint (3) permits households to save ($x_{kt} > 0$) and dissave ($x_{kt} < 0$) at the going interest rate, but rules out the possibility of negative financial wealth (no credit market). Thus, one might conjecture that the equilibrium will change once households are allowed to accumulate debt. However, this is not the case for the model analyzed here since the non-negativity constraints in (3) will never bind in equilibrium. More precisely, the introduction of a risk-free bond does not change the equilibrium allocation as long as the bond interest rate, r_b, is given by $r_b = r_k - \delta_k$.

For given initial state (k_0, h_0, η_0) and given fiscal policy, $\{\tau_t, G_t\}$, where $\tau_t = (\tau_{ct}, \tau_{kt}, \tau_{ht})$, an individual household chooses a plan, $\{c_t, k_t, h_t\}$, that maximizes expected lifetime utility (1) subject to the budget constraint (2), where each (c_t, k_{t+1}, h_{t+1}) is a function of the history of idiosyncratic shocks, $\eta^t = (\eta_1, \ldots, \eta_t)$. Note that the tax variables, τ_t, may depend on t, but not on idiosyncratic shocks η_t. In this sense, the tax system does not provide insurance against idiosyncratic human capitals shocks. Indeed, if there is no government spending on public services, $G_t = 0$, then optimal taxes are zero: $\tau_t = 0$ (see below). In contrast, when the government provides public service to everyone independent of their economics status (independent of η^t), then $\tau_t = 0$ will not be optimal anymore.

The budget constraint (2) can be rewritten in a way that shows how the households's optimization problem is basically a standard intertemporal portfolio choice problem. To see this, define total wealth of an individual household as $w_t \doteq k_t + h_t$ and the fraction of total wealth invested in physical capital, respectively human capital, as $\theta_t = k_t/w_t$, respectively

$1-\theta_t = h_t/w_t$. Using this notation, the budget constraint reads:

$$\begin{aligned} w_{t+1} &= (1+r_t)w_t \; - \; c_t \qquad (4) \\ & w_t \geq 0 \; , \; 0 \leq \theta_t \leq 1 \; , \\ & (w_0, \theta_0, \eta_0) \; given \; . \end{aligned}$$

with the total investment return defined as

$$r_t \doteq \theta_t \left((1-\tau_{kt}) r_k - \delta_k \right) + (1-\theta_t) \left((1-\tau_{ht}) r_h - \delta_h + \eta_t \right) \; .$$

We assume that the government runs a balance budget in each period (no government debt). Thus, the government budget constraint reads:

$$\tau_{kt} r_{kt} E[\theta_t w_t] + \tau_{ht} r_{ht} E[(1-\theta_t) w_t] + \tau_{ct} c_t = G_t \qquad (5)$$

II.2. Equilibrium

Introduce the aggregate capital-to-labor ratio $\tilde{K} \doteq K/H$ and the production function $f = f(\tilde{k})$ with $f(\tilde{K}) \doteq F(\tilde{K}, 1)$. Using this notation, the first-order conditions associated with the domestic firm's static maximization problem read

$$\begin{aligned} r_{kt} &= f'(\tilde{K}_t) \qquad (6) \\ r_{ht} &= f(\tilde{K}_t) - \tilde{K}_t f'(\tilde{K}_t) \; . \end{aligned}$$

In the small open-economy version of the model, the interest rate is equal to the exogenous world interest rate, which we assume to be constant: $r_k - \delta_k = r_{world} - \delta_{world}$.[3] In this case, the first equation in (6) pins down the domestic capital-to-labor ratio, $\tilde{K}$. The second equation in (6) then determines the wage rate per unit of human capital, r_h. In the closed-economy model, the interest rate is endogenous, and (6) only determines the return functions

[3]Equating the before-tax interest rates amounts to the assumption that domestic and foreign residents are taxed at rates determined by their respective home country.

$r_k(\tilde{K})$ and $r_h(\tilde{K})$. The equilibrium value of $\tilde{K}$ can only be determined by considering the investment choices of domestic households and imposing market clearing which in our case reads:

$$\tilde{K}_t = \frac{E[\theta_t w_t]}{E[(1-\theta_t)w_t]} \ . \tag{7}$$

More formally, we have the following definitions:

Definition 1. Suppose the domestic interest rate is pinned down by the exogenous world interest rate, $r_k - \delta_k = r_{world} - \delta_{world}$, and the wage rate (per unit of human capital) is determined through (6). For given initial distribution over individual states, π_0, an equilibrium of the small-open-economy model is a fiscal policy, $\{\tau_t, G_t\}$, and an allocation, $\{c_t, w_t, \theta_t\}_{alloc}$, so that i) for each initial state, (w_0, θ_0, η_0), the corresponding plan, $\{c_t, w_t, \theta_t\}$, maximizes expected lifetime utility (2) subject to the sequential budget constraint (4) and ii) the government budget constraint (5) is satisfied.

Definition 2. For given initial distribution over individual states, π_0, an equilibrium of the closed-economy model is a fiscal policy, $\{\tau_t, G_t\}$, an allocation, $\{c_t, w_t, \theta_t\}_{alloc}$, and a system of rental rates (investment returns), $\{r_{kt}, r_{ht}\}$, so that i) for each initial state, (w_0, θ_0, η_0), the corresponding plan, $\{c_t, w_t, \theta_t\}$, maximizes expected lifetime utility (2) subject to the sequential budget constraint (4), ii) the profit maximization conditions (6) are satisfied, iii) the market clearing condition (7) and iv) the government budget constraint (5) hold.

Finally, we adopt the standard definition of (constrained) Pareto optimality:

Definition 3. An equilibrium allocation is Pareto optimal if there is no other equilibrium allocation that yields higher expected lifetime utility for (almost) all initial types of households, (w_0, θ_0, η_0).

III. Theoretical Results

III.a. Equilibrium for Given Fiscal Policy

The first-order conditions (Euler equations) associated with the household's utility maximization problem read

$$\begin{aligned} c_t^{-1} &= \frac{1+\tau_{c,t+1}}{1+\tau_{ct}} \beta E\left[\left(1 + r(\theta_{t+1}, \tilde{K}_{t+1}, \tau_{k,t+1}, \tau_{h,t+1}; \eta_{t+1})\right) c_{t+1}^{-1}\right] \\ 0 &= E\left[\left((1-\tau_{h,t+1}) r_h(\tilde{K}_{t+1}) - \delta_h + \eta_{t+1} - (1-\tau_{k,t+1}) r_k(\tilde{K}_{t+1}) + \delta_k\right) c_{t+1}^{-1}\right] . \end{aligned} \tag{8}$$

with total return on investment, $r_{t+1} = r(.)$, defined as in (4). The first equation in (8) says that the utility cost of investing (saving) one more unit of the good must be equal to the expected discounted utility gain of doing so, and the second equation states the equality of expected (marginal utility weighted) returns on the two investment opportunities. Notice that we take the unconditional expectations over η_{t+1} in (8) because of the assumption that idiosyncratic shocks are independently distributed over time.

In the appendix we show that any plan satisfying the budget constraint (4) yields finite expected lifetime utility. Hence, any solution to (4) and (8) that satisfies a corresponding transversality condition is also a solution to the utility maximization problem. Direct calculation shows that the plan

$$\begin{aligned} c_t &= \frac{1-\beta}{1+\tau_{ct}} \left(1 + r(\theta_t, \tilde{K}_t, \tau_{kt}, \tau_{ht}; \eta_t)\right) w_t \\ w_{t+1} &= \beta \left(1 + r(\theta_t, \tilde{K}_t, \tau_{kt}, \tau_{ht}; \eta_t)\right) w_t \end{aligned} \tag{9}$$

solves (4), (8), and a corresponding transversality condition (see appendix) if θ is the solution to

$$E\left[\frac{(1-\tau_{ht}) r_h(\tilde{K}_t) - \delta_h + \eta_t - (1-\tau_{kt}) r_k(\tilde{K}) + \delta_k}{1 + r(\theta_t, \tilde{K}_t, \tau_{kt}, \tau_{ht}; \eta_t)}\right] = 0 . \tag{10}$$

Notice that the optimal portfolio choice, θ, is the same for all households regardless of the wealth, w, and their current idiosyncratic shock, η_t. The intuition for this result runs as follows. Each household's decision problem is a standard intertemporal portfolio choice problem with two assets (linear investment opportunities), one risk-free asset (physical capital) with return $(1-\tau_k)r_k - \delta_k$ and one risky asset (human capital) with return $(1-\tau_h)r_h - \delta_h + \eta$. Because households have CRRA-preferences, the optimal portfolio share θ is independent of the wealth level w. Because the random variable η is serially uncorrelated and identically distributed across households, the optimal portfolio is independent of the realization of η.

In the small open-economy version of the model, the interest rate is constant and given, and through (5) this pins down the constant capital-to-labor ratio of domestic firms, $\tilde{K}$, as well s the constant wage rate, r_h. Thus, (9) and (10) (in conjunction with the initial values for physical and human capital) provide a complete solution of the model. Notice also that the capital-to-labor ratio chosen by domestic households is $\theta/(1-\theta)$, which in general is not equal to the capital-to-labor ratio, $\tilde{K}$, chosen by domestic firms. In contrast, when the economy is closed, these two have to be equal in order to ensure market clearing (condition (7)), and we therefore have:

$$E\left[\frac{(1-\tau_{ht})r_h(\theta_t/(1-\theta_t)) - \delta_h + \eta_t - (1-\tau_{kt})r_k(\theta_t/(1-\theta_t)) + \delta_k}{1 + r\left(\theta_t, \theta_t/(1-\theta_t), \tau_{kt}, \tau_{ht}; \eta_t\right)}\right] = 0 \;, \tag{11}$$

where we used the fact that the optimal portfolio choice, θ, is independent of wealth, w, so that the market clearing condition (7) simply becomes:

$$\tilde{K}_t = \frac{\theta_t}{1-\theta_t} \;. \tag{12}$$

To sum up, we have the following proposition:

Proposition 1. The solution to the individual household maximization problem is given by (9) and (10). For given fiscal policy, this solution also determines the equilibrium for the small-open-economy model. For the closed-economy model, the equilibrium rental rates are determined jointly with the equilibrium capital-to-labor ratio through (11).

III.b. Optimal Fiscal Policy

For given fiscal policy, proposition 1 specifies the equilibrium plan of consumption and wealth chosen by individual households. The particular representation of the equilibrium plan allows us to characterize the Pareto optimal equilibrium in simple and transparent manner. In order to deal with the two cases of a small-open economy and a closed economy simultaneously, let us introduce the following notation for the equilibrium rate of return on total investment:

$$r_e(\Theta, \tau_k, \tau_h; \eta) \doteq \begin{cases} r(\Theta, \Theta/(1-\Theta), \tau_k, \tau_h; \eta) \; if \textit{ closed economy} \\ r(\Theta, \tilde{K}, \tau_k, \tau_h; \eta) \; if \textit{ open economy} \end{cases} ,$$

where for the small open economy the dependence on the exogenous $\tilde{K}$ is suppressed. Further, it turns out to be convenient to express government spending on public services as a fraction of private consumption:

$$C_{gt} \doteq \frac{G_t}{E[c_t]} .$$

Using the above notation and the representation of equilibrium plans given by proposition 1, the government budget constraint (5) can be written as

$$\begin{aligned} \tau_{kt} r_{kt} \Theta_t + \tau_{ht} r_{ht} (1 - \Theta_t) + \frac{\tau_{ct}}{1 - \tau_{ct}} (1 - \beta)(1 + r_e(\Theta_t, \tau_{kt}, \tau_{ht}; E[\eta_t])) &= \\ C_{gt} \frac{1 - \beta}{1 + \tau_{ct}} (1 + r_e(\Theta_t, \tau_{kt}, \tau_{ht}; E[\eta_t])) & , \end{aligned} \quad (13)$$

where r_k and r_h are exogenous for the open economy model and endogenous for the closed economy model. More compactly, we have

$$GC(\Theta_t, C_{gt}, \tau_{ct}, \tau_{kt}, \tau_{ht}) = 0 \quad (14)$$

where the function $GC(.)$ is defined by the government budget constraint (13) in the canonical way. Similarly, we can represent the implementability constraint (10,11), which expresses the fact that the portfolio choice Θ is individually optimal and, for the closed economy model, satisfies market clearing, as

$$IC(\Theta_t, C_{gt}, \tau_{kt} \tau_{ht}) = 0 , \quad (15)$$

where the function $IC(.)$ is defined through (10), respectively (11), in the obvious way. In short, we can think of an equilibrium as a joint sequence $\{\Theta_t, C_{gt}, \tau_t\}$ that satisfies the government budget constraint (14) and the implementation constraint (15).

For any equilibrium, $\{\Theta_t, C_{gt}, \tau_t\}$, expected lifetime utility from private consumption can be calculated as

$$\begin{aligned} E\left[\sum_{t=0}^{\infty}\beta^t log c_t\right] &= E\left[\sum_{t=0}^{\infty}\beta^t log\left(\frac{1-\beta}{1+\tau_{ct}}\beta^t\Pi_{n=0}^{t}(1+r_n)w_0\right)\right\} \\ &= h(w_0) - \sum_{t=0}^{\infty}\beta^t log(1+\tau_{ct}) + \sum_{t=0}^{\infty}\beta^t\sum_{n=0}^{t}E\left[log\left(1+r_e\left(\Theta_n,\tau_{kn},\tau_{hn};\eta_t\right)\right)\right] \ , \end{aligned} \tag{16}$$

where $h(.)$ is some function that also includes a constant. Note that the effect of any tax policy, $\{\tau_t\}$, on expected lifetime utility of private consumption (16) for given initial tax rates, τ_0, is independent of the initial type, (w_0, θ_0, η_0). The same result holds for equilibrium portfolio choices, $\{\Theta_t\}$. Thus, any Pareto optimal equilibrium is preferred by all types of households, (w_0, θ_0, η_0), over all alternative equilibria. In other words, the set of Pareto optimal equilibria is independent of the initial distribution over household types. More precisely, any Pareto optimal equilibrium is the solution to the following constrained social planner problem:

$$\begin{aligned} &\max_{\{\Theta_t, C_{gt}, \tau_{ct}, \tau_{kt}, \tau_{ht}\}} V\left(\{\Theta_t, C_{gt}, \tau_{ct}, \tau_{kt}, \tau_{ht}\}\right) \\ &subject\ to\ :\ \{\Theta_t, C_{gt}, \tau_{ct}, \tau_{kt}, \tau_{ht}\} \in \mathbf{F} \end{aligned} \tag{17}$$

where the objective function in (17) is defined as

$$\begin{aligned} V\left(\{\Theta_t, C_{gt}, \tau_{ct}, \tau_{kt}, \tau_{ht}\}\right) &\doteq (1+\nu)h(w_0) - \sum_{t=0}^{\infty}\beta^t log(1+\tau_{ct}) + \sum_{t=0}^{\infty}\beta^t\sum_{n=0}^{t}E\left[log\left(1+r_e\left(\Theta_n,\tau_{kn},\tau_{hn};\eta_n\right)\right)\right] \\ &+\ \nu\sum_{t=0}^{\infty}\beta^t log\left(C_{gt}\right)\ +\ \nu\sum_{t=0}^{\infty}\beta^t\sum_{n=0}^{t}log\left(1+r_e\left(\Theta_n,\tau_{kn},\tau_{hn};E[\eta_n]\right)\right) \end{aligned}$$

and the feasibility set is

$$\mathbf{F} \doteq \{\{\Theta_t, C_{gt}, \tau_{ct}, \tau_{kt}, \tau_{ht}\} | \{\Theta_t, C_{gt}, \tau_{ct}, \tau_{kt}, \tau_{ht}\}\ solves\ GC\ and\ IC\}\ .$$

The constrained social planner problem (17) can be transformed into an unconstrained social planner problem as follows. Define the variable

$$T_t \doteq \Theta_t \tau_{kt} r_{kt} + (1-\Theta_t)\tau_{ht} r_{ht} \ ,$$

which measures to what extent total investment is taxed ($T > 0$), respectively subsidized ($T < 0$). Using the new notation, the government budget constraint (13) can be written as

$$\tau_{ct} = \frac{C_{Gt}(1-\beta)(1+\tilde{r}_e(\Theta_t)+T_t) - T_t}{(1-\beta)(1+\tilde{r}_e(\Theta_t)+T_t)+T_t} \ , \tag{18}$$

where

$$\tilde{r}_e(\Theta) \doteq \begin{cases} \Theta_t\left(r_k\left(\frac{\Theta_t}{1-\Theta_t}\right)-\delta_k\right)+(1-\Theta_t)\left(r_h\left(\frac{\Theta_t}{1-\Theta_t}\right)-\delta_h\right) \ if\ closed\ economy \\ \Theta_t\left(r_k-\delta_k\right)+(1-\Theta_t)\left(r_h-\delta_h\right) \ if\ open\ economy \end{cases} ,$$

Clearly, (18) defines a function $\tau_{ct} = \tau_c(\Theta_t, C_{gt}, T_t)$ with

$$\begin{aligned} \frac{\partial \tau_c}{\partial \Theta} &> 0 \\ \frac{\partial \tau_c}{\partial T} &< 0 \end{aligned} \tag{19}$$

where the first inequality uses $\tilde{r}_e'(\Theta) < 0$. Put differently, for any choice of (Θ, C_g, T), the government budget constraint can be made to hold by choosing τ_c according to the function $\tau_c(.)$ defined in (18). Similarly, direct calculation shows that for any choice of (Θ, C_g, T), the implementation constraint (10), respectively (11) will hold if capital and labor income taxes are

$$\begin{aligned} \tau_h &= \left(E\left[\frac{\frac{1-\Theta}{\Theta}(r_k-\delta_k)-(r_h-\delta_h)}{1+\tilde{r}_e(\Theta)+T+(1-\Theta)\eta}\right]\right)^{-1} E\left[\frac{r_h-\delta_h+\eta-(r_k-\delta_k)+\frac{T}{\Theta}}{1+\tilde{r}_e(\Theta)+T+(1-\Theta)\eta}\right] \\ \tau_k &= \frac{T-(1-\Theta)r_k}{\Theta r_k}\tau_h \ . \end{aligned} \tag{20}$$

Clearly, (20) defines functions $\tau_h = \tau_h(\Theta, C_g, T)$ and $\tau_k = \tau_k(\Theta, C_g, T)$, which ensure that the implementation constraint is satisfied. In short, we have shown that the constrained social planner problem (17) is equivalent to the unconstrained social planner problem

$$\max_{\{\Theta_t, C_{gt}, T_t\}} \tilde{V}\left(\{\Theta_t, C_{gt}, T_t\}\right) \tag{21}$$

where the objective function in (21) is defined as

$$\begin{aligned}\tilde{V}\left(\{\Theta_t, C_{gt}, T_t\}\right) &\doteq h(w_0) - \sum_{t=0}^{\infty}\beta^t log(1+\tau_c(\Theta_t, C_{gt}, T_t)) + \\ &\sum_{t=0}^{\infty}\beta^t \sum_{n=0}^{t} E\left[log\left(1+\tilde{r}_e\left(\Theta_n\right)+T_n+(1-\Theta_n)\eta_n\right)\right)] \\ &+ \nu\sum_{t=0}^{\infty}\beta^t log\left(C_{gt}\right) + \nu\sum_{t=0}^{\infty}\beta^t\sum_{n=0}^{t} log\left(1+\tilde{r}_e\left(\Theta_n\right)+T_n+(1-\Theta)E[\eta_n]\right))\end{aligned}$$

To sum up, the previous discussion has shown that the following equivalence result holds:

Proposition 2. Any Pareto optimal equilibrium allocation can be found by solving either the constrained social planner problem (17) or the unconstrained social planner problem (21).

Straightforward but tedious calculation shows that the objective function in (20) is strictly concave. Since the choice set is obviously convex, the social planner problem (21) has at most one solution. In the Appendix we show that the maximization problem has indeed a solution. Thus, there is a unique solution to the social planner problem (21), and therefore a unique Pareto optimal equilibrium. Using the first-order condition approach, in the Appendix we also show the following properties of this equilibrium:

Proposition 3. Let $\{\Theta_t, C_{gt}, T_t\}$, respectively $\{\Theta_t, C_{gt}, \tau_{ct}, \tau_{kt}, \tau_{ht}\}$, be the solution to the social planner problem (17), respectively (21), and Θ' be the equilibrium portfolio choice when $\tau_{kt} = \tau_{ht} = 0$ has been chosen. Then:

i) Optimality of a stationary fiscal policy:

$$\begin{aligned}(\Theta_t, C_{gt}, T_t) &= (\Theta, C_g, T) \\ (\Theta_t, C_{gt}, \tau_{ct}, \tau_{kt}, \tau_{ht}) &= (\Theta, C_g, \tau_c, \tau_k, \tau_h)\end{aligned}$$

ii) Optimality of human capital subsidy:

$$\tau_h < 0$$
$$\Theta < \Theta'$$

iii) Optimality of subsidizing total investment:

$$T < 0$$

iv) Optimal level of government spending:

$$C_g = \nu$$

Proof. See Appendix

IV. Quantitative

IV.a. Calibration

The quantitative analysis is based on an economy with normally distributed human capital shocks, $\eta \sim N(0, \sigma_\eta^2)$, and a Cobb-Douglas production function: $f(\tilde{k}) = A\tilde{k}^\alpha$. We use $\alpha = .36$ to match capital's share in income and $\delta = .06$ (annually) as a compromise between the higher depreciation rate of physical capital used in the literature (but see also Cooley and Prescott [1995] for an argument that $\delta_k = .05$) and the probably lower depreciation rate of human capital. The values of the fundamental parameters A, σ_η^2, and β are chosen so that the model is roughly consistent with the US evidence along three dimensions: saving, growth, and labor income risk. More specifically, we require that per capita consumption growth satisfies $\mu_g = E[c_{i,t+1}/c_{it}] - 1 = .02$ and that the implied saving rate is $s_k = x_{kt}/y_t = .25$. For the annual US data on saving and growth, see Summers and Heston [1991]. Finally, we match observed labor income risk by requiring $\sigma_y = \sigma_\eta/(1+\tilde{k}) = .15$. This approach yields $A = .2674$, $\beta = .9465$, and $\sigma_\eta = .2633$.

The choice of $\sigma_y = \sigma_\eta/(1+\tilde{k}) = .15$ is made to ensure consistency with the empirical results of a number of micro studies on labor income risk. More specifically, in the model economy log-labor income of household i, y_{hit}, is given by $y_{hit} = (r_h + \delta)\, h_{it}$. Using the equilibrium condition (proposition 1) $h_{i,t+1} = \beta\,[1 + \theta r_k + (1-\theta)(r_h + \eta_{it}]\, h_{it}$, we find

$$\begin{aligned} log\, y_{hi,t+1} - log\, y_{hit} &= log h_{i,t+1} - log h_{it} \\ &= log\beta + log\,(1 + \theta r_k + (1-\theta)(r_h + \eta_{it})) \\ &\approx d + \tilde{\eta}_{it}\ , \end{aligned} \tag{22}$$

where $d = log\beta + \theta r_k + (1-\theta) r_h$ and $\{\tilde{\eta}_{it}\}$ is a sequence of i.i.d. random variables with $\tilde{\eta}_{it} = (1-\theta)\eta_{it}$. Hence, the logarithm of labor income follows (approximately) a random walk with drift d and error term $\tilde{\eta}_{it} \sim N(0, \sigma_y^2)$, $\sigma_y = (1-\theta)\sigma_\eta$.[4] The random walk specification is often used by the empirical literature to model the permanent component of labor income risk [Carroll and Samwick 1997, Meghir and Pistaferri 2001, and Storesletten et al. 2001]. Thus, their estimate of the standard deviation of the error term for the random walk component of annual labor income corresponds to the value of $\sigma_y = (1-\theta)\sigma_\eta$. In our baseline model we use $\sigma_y = .15$, which lies on the lower end of the spectrum of estimates found by the empirical literature. For example, Carroll and Samwick (1997) find .15, Meghir and Pistaferri (2004) estimate .19, and Storesletten et al. (2004) have .25 (averaged over age-groups and, if applicable, over business cycle conditions).

There are at least two reasons why the above approach might underestimate human capital risk. First, a constant $\sigma_y = .15$ represents less uncertainty than a σ_y that fluctuates with business cycle conditions and has a mean of .15. Second, the assumption of normally distributed innovations understates the amount of idiosyncratic risk households face if the

[4] We have $\tilde{\eta}_{it}$ instead of $\tilde{\eta}_{i,t+1}$ in equation (22), and the latter is the common specification for a random walk. However, this is not a problem if the econometrician observes the idiosyncratic depreciation shocks with a one-period lag. In this case, (22) is the correct equation from the household's point of view, but a modified version of (22) with $\tilde{\eta}_{i,t+1}$ replacing $\tilde{\eta}_{it}$ is the specification estimated by the econometrician.

actual distribution has a fat lower tail. For strong evidence for such a deviation from the normal-distribution framework, see Brav, Constantinides, and Greczy (2002) and Geweke and Keane (2000). There is, however, also an argument that the current approach might overestimate human capital risk because it assumes that all of labor income is return to human capital investment. If some component of labor income is independent of human capital investment, as argued in Mankiw, Romer, and Weil (1992), and if this component is random (random endowment of genetic skills), then some part of the variance of labor income is not human capital risk.

The above calibration procedure ensures that the model economy matches as many features of the US economy as there are free parameters. It is also interesting to investigate how the calibrated model performs in matching additional features of the U.S. economy. For example, the implied values for the average return on physical and human capital are $r_k = 5.52\%$ and $r_h = 9.47\%$, respectively. The return $r_k = 5.52\%$ is higher than the observed real interest rate on short-term U.S. government bonds (1%), but lower than the observed real return on US equity (8%). Given that there is no aggregate risk, and therefore no equity premium, in the model, it is not clear which one of the many financial return variables should be used as a basis for calibration, and we therefore conclude that the implied value is within the range of reasonable values.[5] The implied average return on investment in human capital, $r_h = 9.47\%$, is in line with the estimates of rate of returns to schooling.[6] Notice that the implied excess return on human capital investment is $r_h - r_k = 3.95\%$. Thus, the model generates a substantial "human capital premium". Finally, according to the model individual consumption growth is normally distributed, $g = c_{i,t+1}/c_{it} - 1 \sim N(\mu_g, \sigma_g^2)$, and has a standard deviation of $\sigma_g = \beta\sigma_y = .1420$. This amount of consumption volatility is somewhat

[5]The RBC literature usually strikes a compromise and chooses the parameter values so that the implied return on capital is 4%, which is somewhat lower than the value used here.

[6]The estimates vary considerably across households and studies, with an average of about 10% (Krueger and Lindhal 2001).

lower than what is found in the data. For example, using CEX data on consumption of non-durables and services Brav et al. (2002) find that the standard deviation of quarterly consumption growth ranges from .06 to .12 for different household groups with an average of about .09. If quarterly consumption growth is i.i.d., then this corresponds to a standard deviation of annual consumption growth of .18.

We choose income tax rates of $\tau_k = \tau_h = 0$ so that we can compare an economy with optimal fiscal policy to an economy in which the government does not change the private incentives to invest. Note that even though average marginal tax rates on capital and labor income are around 40 percent in the US (Judd 1987, Lucas 1990), human capital investment is also heavily subsidized. For example, subsidies for education expenditures in the US range between 40 and 80 percent (Trostel, 1993). Thus, the US might not be so far away from an effective marginal tax rate of zero for human capital. We pick the preference parameter ν so that the implied ratio C_g/Y is equal to the average ratio of government spending to output for the US in the post WWII period. This yields $\nu = 0.455$. Finally, we choose the consumption tax, τ_c, so that the government budget constraint holds.

IV.b. Quantitative Results

To compute economically meaningful welfare effects of implementing the optimal fiscal policy, we calculate corresponding equivalent consumption variations. More precisely, we compute the percentage of lifetime consumption, ΔU, that we have to give an individual household to compensate him for the welfare loss of not living in an economy with optimal fiscal policy:

$$\begin{aligned} E\left[\sum_{t=0}^{\infty}\beta^t log\left((1+\Delta U)c_t\right)\right] + \nu\sum_{t=0}^{\infty} log\left((1+\Delta U)C_g E[c_t]\right) &= \\ E\left[\sum_{t=0}^{\infty}\beta^t log c_t^*\right] + \nu\sum_{t=0}^{\infty} log\left(C_g^* E[c_t^*]\right) & \quad , \end{aligned} \tag{23}$$

where c is consumption in the economy with $\tau_k = \tau_h = 0$ and c^* is the economy with optimally chosen τ_k and τ_h. Using the formula (9) for equilibrium consumption, the number

ΔU can easily be calculated.

Table 1 shows our main quantitative results.

Table 1: Macroeconomic Implications of Optimal Fiscal Policy[7]

	GE	PE
$\tau_{\mathbf{k}} - \tau_{\mathbf{h}}$	7.39 %	10.36 %
Δr_k	+ 0.57 %	0 %
Δr_h	-0.31 %	0 %
Δs_k	-0.49 %	+1.21 %
Δs_h	-8.55 %	+14.99 %
Δg	0.14 %	1.00 %
$\Delta \sigma_c$	0.34 %	6.20 %
ΔU	0.11 %	1.82 %

[7] Δr_k and Δr_h are the changes in the return to physical capital and human capital investment. Δs_k and Δs_h are the changes in the investment (saving) rates in physical and human capital, expressed as a fraction of output. Δg is the change in the growth rate of household consumption. ΔU is the change in welfare (expected lifetime utility) expressed as equivalent variation in lifetime consumption (see equation 23).

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