When are Multiple Tasks a Problem?

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Abstract

Production requires that workers are assigned to tasks and given incentives. Sometimes it is impossible to provide incentives such that production is carried out in a desired way, e.g. because workers focus effort on the wrong task (*multi-task problem*). While it is well-known that such problems can arise, the conditions under which this happens have –to my knowledge– not been systematically studied. Here, we examine when and how information structure and assignment of workers to tasks restrict what can be achieved by incentives. We show that at the heart of the impossibility to implement activities is an *identification problem*. We provide conditions under which this problem is caused by the assignment, i.e. when it is a multi-task problem, and when it can hence be overcome by assigning workers differently. The analytical framework encompasses well-known models on multitasking and advocacy.

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1 Introduction

Arguably one of the most important roles of the management of a company, organization or government agency is to ensure that production occurs in a desired manner. To this end, the management assigns workers to tasks and provides incentives to these workers. Typically, the information how exactly tasks have been accomplished is limited: the activities chosen by workers have to be deduced from signals. Since the provision of incentives can only be based on these signals, the power of the management to implement desired activities may be limited, too. As an example take the well-known advocates model by Dewatripont and Tirole (1999). In this model, it is impossible to use the observable decision by a decision maker in order to provide incentives for a single investigator to find arguments in favor and against the decision. Increasing the number of investigators and reducing the number of tasks assigned to each of them, however, solves this problem. While this example confirms that multiple tasks may be the reason why certain activities cannot be implemented, it has not yet been systematically studied when this is the case. This paper provides necessary and sufficient conditions when assigning multiple tasks to a worker hampers the implementation of activities. Put differently, it answers the question when multiple tasks pose a problem?

The agenda of most contract theoretical models is to determine which activity should be implemented given a specific contractual environment and to devise a mechanism that leads to this activity. In short, these models seek to characterize and implement the second-best. The present paper deals with an issue that is more fundamental. It identifies activities that can be implemented. The analysis thus abstracts from the reasons to implement a certain activity and asks the following basic question: If one wanted to implement a specific activity would it be possible? There are various models that deal with this question implicitly—some of which will be discussed later. Following the contract-theoretical tradition, they are dealing with specific frameworks, which differ considerably. The number of signals and agents varies; how activities affect signals changes; signals may or may not be correlated; and different tasks may interact with respect to opportunity costs or be separable. These differing assumptions render it difficult to single out the crucial features on which implementation hinges.

This paper provides a general framework to examine when multiple tasks limit implementation of activities and offers three main insights. First, it identifies the crucial feature that renders the implementation of activities impossible. At the heart of the impossibility is an *identification problem*: the agent who carries out multiple tasks can often generate the same signals with different activities. Whatever signal the incentive scheme rewards, an opportunistic agent will produce this signal using the activity most convenient to him; activities other than this convenient one, cannot be implemented. While correlation between signals is important for optimal contracts and the effect of tasks on costs affects the second-best choice, both are irrelevant for the question which activities can be implemented. Implementation only depends on how tasks affect signals.

Second, the article explains why a new assignment sometimes helps: The re-assigned agents have no longer a choice how to produce a signal. Given the behavior of their co-workers, any change in activity affects at least one signal, the identification problem is alleviated, and implementation becomes possible. Whether this *assignment solution* to the identification problem works depends on the available signals.

Third, the article provides simple necessary and sufficient conditions on the signal structure such that any activity can be implemented by an appropriate assignment of agents to tasks.

Section 2 introduces a general framework to model how the behavior of m agents at n tasks affects k signals. We formally define what we mean by implementable activities and multi-tasking problem. In addition, we borrow the notion of an identification problem from econometrics to describe the situation, where different activities lead to the same distribution of signals.

Section 3 examines different examples from the literature in which a desired activity cannot be implemented. These examples include various models from the multi-tasking article by Holmström and Milgrom (1991), the model to explain advocacy by Dewatripont and Tirole (1999), the model proposed by Ratto and Schnedler (2003, 2008) to provide an incentive-based rationale for specialization, and the model of accountability by Corts (2007). Each of these models is re-examined in the general framework and it is shown that they all feature an identification problem. For those models who feature assignment solutions, we observe that the assignment solves the identification problem This already indicates that only the effect of tasks on activities matters and neither the correlation between signals nor how activities interact in costs. It provides, however, no proof that it is only the identification problem which limits implementation. Also, the examples fail to answer the general question when the identification problem occurs and when it can be overcome by assignment. The section suggests variations on the discussed models and asks how they would affect whether activities can be implemented. The following two sections then answer these questions. They also offer a general analysis when the identification problem occurs, how it relates to limited implementation, and when multiple tasks are behind the identification problem.

Section 4 provides the formal basis for the claim that any restriction on implementable activities is due to an identification problem. Before establishing this result, we need to characterize activities that can be implemented when an agent is assigned a given set of tasks. We do so by a linear equation system (Proposition 1) and a rank condition (Corollary 1). Based on these results, it becomes possible to show that some activities cannot be implemented if and only if signals are not sufficiently rich (Proposition 2), where richness means the number of signals that are independently affected by tasks (rank of the Jacobi-matrix of the parameter vector). An immediate consequence of this result is that an identification problem is at the heart of any problem to implement an activity (Theorem 1).

Section 5 deals with the question when the identification problem can be overcome by an adequate assignment of agents to tasks. The main result draws on the observations from the previous section; it states that the identification problem can be eliminated with an assignment if and only if the signals faced by each agent are sufficiently rich (Theorem 2). While this theorem offers a general characterization, determining the richness of signals may be difficult. The rest of the section then provides necessary and sufficient conditions that are easier to verify.

First, it is shown that identification problems cannot be avoided by any assignment of agents to tasks if some tasks affect no signal (Corollary 2). Re-assignment is no solution in this case. The result has an interesting implication for Holmström and Milgrom's home contractor model (1991, Section 3.2). Since no assignment can solve the identification problem, it persists although each agent only carries out one task. Put differently, the identification problem is not caused by the multi-task nature; the difficulty to implement the desired activity in the home contractor model has nothing to do with how many tasks the agent exerts; in this sense, the home contractor model features no multi-tasking problem although it is often regarded to be an archetypical example for such a problem.

Second, we provide a condition on the relationship between richness of the signal structure (rank of the Jacobi matrix), number of agents and number of tasks such that all assignments lead to an identification problem (Proposition 3). This occurs when there are few agents, many tasks, and the signal space is poor. The result provides a simple counting rule to determine in many cases whether implementation is limited, which include some of the above examples and their extensions. In addition, the result extends an observation by Corts (2007) in many dimensions: it allows for risk-averse agents, deals with implementation of arbitrary activities (not only the first-best), and does not require contracts and signals to be linear.

Third and on a more positive note, we also identify a similar condition such that there is some assignment which allows to implement any activity (Proposition 4). In other words, we provide a sufficient condition for solving the identification problem by an appropriate assignment. The proposition presents an easy way to determine when re-assigning agents helps under many circumstances, which encompass, for example, the advocates and the specialization model. A special case of this proposition is that one signal per task suffices to implement any activity with a single agent. Corts (2007) has already observed that one signal per task is enough to implement the first-best with a risk-neutral agent given linear signals and contracts. The proposition thus generalizes his observation. Various models in Holmström and Milgrom (1991) evoke the one-signal-per-task assumption. From his observation, Corts (2007) concludes that these models do not exhibit multi-tasking problem in the limit as agents get risk-neutral. Proposition 4 shows that Corts' claim is not only valid in the limit but also holds for risk-averse agents.

We end with a discussion of the limitations of the model and some concluding remarks (Section 6).

2 The Model

Consider a principal who delegates an activity $a = (a_1, \ldots, a_n)$ to m agents, where $i = 1, \ldots, n$ are n tasks required for the activity and a_i are real numbers. Let N be the set of all tasks. Denote the set of tasks assigned to worker l by N_l. An assignment $\mathcal{A}$ is a disjoint decomposition of N assignment.

Opportunity costs. Activity a leads to (opportunity) costs $c^l_{\mathcal{A}}(a)$ for agent l given assignment $\mathcal{A}$, where costs are strictly convex in a. Note that costs may depend on activities of other agents at different tasks.

In addition, agent l has a benefit $b^l(t)$ from transfer t, where the benefit is increasing in the transfer ($b' > 0$), twice continuously differentiable, and weakly concave ($b'' \leq 0$). The utility of agent l amounts to $u(a,t) = b^l(t) -$

$c^l_{\mathcal{A}}(a)$.

Signals. The activity a affects k signals $S = (S_1, \ldots, S_k)$ in the following way. Each signal S_j depends on a real-valued parameter θ_j. This parameter is twice continuously differentiable and weakly concave in the activity a: $\theta'' \leq 0$. In addition, the signal S_j depends on influences beyond the control of the agents ϵ_j. These influences may have an arbitrary joint distribution.

Marginal effects of activity on costs and signals. Let a^l be the vector of activities carried out by agent l given assignment N^l. Denote the derivative with respect to this vector by ∇_{a^l}. Accordingly, the marginal effect on costs is represented by the $|N^l|$ dimensional vector $\nabla_{a^l} c^l_{\mathcal{A}}(a)$. The marginal effect on the parameters θ is the matrix $\nabla_{a^l}\theta(a)$, which has $|N^l|$ rows and k columns.

Let us now be more precise about the meaning of implementable.

Definition 1. *Activity a is implementable if there are transfers such that the activity is a Nash equilibrium.*

In case of a single agent, the activity simply has to be a maximizer of the agent's utility. Since the notion of multi-tasking problem is central to this article, a definition is warranted. Interestingly, the literature does not seem to offer a formal definition. However, in most cases the term "multitasking problem" is used to refer to a situation in which it is difficult to implement an allocation of effort or more generally an activity *because* an agent carries out more than one task. The following definition tries to capture this notion.

Definition 2 (Multi-tasking problem)**.** *A multi-tasking problem is present if activities cannot be implemented because an agent carries out multiple tasks.*

The impossibility to implement an activity is not necessarily due to a multi-tasking problem. If, for example, each agent carries out only one task but there are still activities that cannot be implemented, the reason cannot be a multi-tasking problem. Conversely, if it becomes possible to implement activities by reducing the number of tasks assigned to an agent, then a multi-tasking problem caused the initial limitation.

Terms from econometrics have been found useful to describe concepts in the principal-agent model at least since the sufficient statistic result. A key question later will be whether it is possible to recover the activity from the distribution of signals (or alternatively when observing infinitely many signal realizations). In order to describe situations, where this is not possible, we borrow the term "identification problem" from econometrics.

Definition 3 (Identification problem)**.** *An identification problem is present if there are two different activities by agent l that lead to the same distribution of signals: $a^l \neq \tilde{a}^l$ but $\theta(a^l) = \theta(\tilde{a}^l)$.*

3 Examples

In this section, we re-examine various examples from the literature using the framework from the preceding section. Particular attention is paid to the issue of implementability and the presence of identification problems. For some of these examples, it is possible to achieve the desired activity by assigning agents differently to tasks. It will turn out that after the respective re-assignments, the identification problem is no longer present.

The article by Holmström and Milgrom (1991) offers several different multi-tasking models. Their first proposition relates to the home contractor model. In this model, the principal hires a single agent ($m = 1$) to build a house . The principal is interested in two tasks ($n = 2$): attention to detail a_1 as well as in speedy work a_2. However, only timely completion can be observed; the signal ($k = 1$) is: $S(\theta(a), \epsilon) = \theta(a) + \epsilon$, where $\theta(a) = a_2$. Since two different values of a_1 lead to the same parameter, there is an identification problem. The agent in the example has a personal preference over the amount of attention to detail and finds any additional attention cumbersome. The two tasks are perfect substitutes in the cost function: increasing the attention to detail and reducing speed leads to the same opportunity costs for the agent: $c(a) = c(a_1 + a_2)$. In this example, it is impossible to implement the desired attention to detail. A direct consequence is the famous finding by Holmström and Milgrom (1991) that it may be optimal to ignore freely available signals and renounce incentives.

The same signal and cost structure also features in the low-powered-incentives model by Holmström and Milgrom (1991, Section 3.3). Here, the agent can produce output (a_1) and enhance an asset (a_2). The signal only measures output such that $\theta(a) = a_1$ and it is hence impossible to identify the activity. At the same time, asset enhancing activities are impossible to implement—unless the asset is owned by the manager.

In both of these models, the agent carries out two tasks and one might suspect that the multi-task character is responsible for the fact that desired activities cannot be implemented. Later, we examine this possibility in a more general model. We find that the desired activity cannot be implemented even if each agent only works on one task. Implementation is thus not limited by the multi-task character.

In their other models, Holmström and Milgrom (1991) assume that there is a signal for every activity i relevant to the principal:[1] $S_j(\theta_j, \epsilon_j) = \theta_j + \epsilon_j$ with $\theta_j(a) = a_j$ for all $j = i$. Since any change in activity is reflected in a parameter change, there is no identification problem. On the other hand, Holmström and Milgrom (1991) maintain the assumption that tasks are perfect substitutes in costs. The second-best activity in these models differs from the first-best activity. This could be for two reasons: (i) it is impossible to implement the first-best activity or (ii) it is possible but another activity is Pareto-better under the restrictions imposed by the models. The general analysis later will determine which of the two reasons applies.

The issue of implementing a desired activity arises also in Dewatripont and Tirole's (1999) famous analysis of advocacy. In their model, there is initially a single agent ($m = 1$) who collects evidence for option 1 (a_1) or option 2 (a_2) on behalf of some decision maker. It is observable what this decision maker has decided and the decision depends on the collected evidence. In a somewhat reduced form, the decision can be described as $S(\theta(a), \epsilon) = \theta(a) + \epsilon$, where $\theta(a) = a_1 - a_2$. An identification problem results because exerting more effort to find evidence for option 1 (larger a_1) and exerting more effort to support option 2 (larger a_2) cancel each other. A well-informed choice requires high effort in both dimensions. This activity, however, cannot be implemented because the agent can always reduce effort on both tasks without affecting the distribution of the decision. However, by assigning one agent the role of advocating option 1 and the other the role of advocating 2, each agent only affects the signal in one way, the identification problem is alleviated and effort by both agents can be implemented.

While effort on different tasks in Dewatripont and Tirole's model has opposing effects on the signal, Ratto and Schnedler (2003, 2008) deal with the situation that effort on tasks affects the result in the same direction. Consider a (possibly stochastic) production function with expected output θ, which positively depends on two inputs (a_1 and a_2). Now, assume that only the result of production $S(\theta(a), \epsilon) = \theta(a_1, a_2) + \epsilon$ is observable. Again, there is an identification problem: the same $\theta(a)$ can be obtained by reducing a_1 and increasing a_2; there are many combinations of the two inputs that

[1] In the model in their Section 4, there seem to be activities that are not reflected by signals. These activities, however, are not affecting the benefit of the principal.

lead to the same output. Some of these combinations may be preferred by the principal, e.g. because they imply less tear and wear. If a single agent ($m = 1$) decides on the inputs, the choice how to produce θ will be guided by the agent's opportunity costs and generally not coincide with the principal's preferred combination. The crucial observation by Ratto and Schnedler is that specialization has an advantage which has so far been neglected: it enables the principal to implement the desired input combination. If a specialist is employed for each task, the specialist at task i can only affect output by choosing his input a_1. By adjusting the marginal gains to each specialist from the output signal, the desired input combination can be obtained.

The advocates idea by Dewatripont and Tirole (1999) as well as the specialization solution by Ratto and Schnedler (2003, 2008) require that it is possible to separate tasks and assign them to different agents. While this is plausible in many contexts, it may be impossible in others. Then, the question arises how much and which information is needed to implement the desired activity with one agent. This question will be addressed in the analysis section.

The last example is from Corts (2007). Take a sales force consisting of two agents ($m = 2$) who cater to two customers. Catering to customer $j = 1, 2$ involves two tasks: selling ($a_{1+2(j-1)}$) and customer care ($a_{2+2(j-1)}$). There is hence a total of four tasks: $a = (a_1, a_2, a_3, a_4)$, where the first two are related to customer 1 and the second two to customer 2. The only observable signals are the sales to customer j: $S_j(\theta_j(a), \epsilon) = \theta_j(a) + \epsilon_j$, where $\theta_j(a) = a_{1+2(j-1)} + a_{2+2(j-1)}$. In case of individual accountability, where each agent is assigned to one customer, an identification problem arises which is very similar to that of Ratto and Schnedler (2003, 2008). The same output can be produced by convincing customer j to buy more (larger $a_{1+2(j-1)}$) or by better customer care (larger $a_{2+2(j-1)}$). Joint accountability, where one agent is responsible for selling while the other cares for the customers, alleviates this problem: the only way for the agent who is responsible for customer care to increase sales is by better customer care.

In the sales force example, each task affects only one customer. On the other hand, some tasks such as marketing could also influence more than one customer. How would this affect implementability of activities? The discussion of this question will be postponed to the analysis section when we have developed the necessary tools.

The examples indicate that the identification problem seems to be a general feature that prevents implementation. The last three examples suggest that re-assigning tasks is a rather general method to overcome the identification problem and achieve implementation. Of course, the examples are based on models that feature many specific assumptions concerning, for example, the risk-attitudes and opportunity costs of agents, linearity of signals, etc.

The next section uses the general model from Section 2 in order to show that the identification problem is indeed the reason for *any* failure to implement a desired activity. Section 5 then deals with the question when the identification problem can be overcome.

4 Implementation and identification problem

The main result in this section will be that any failure to implement a desired activity is due to an identification problem. In order to state this result, it is necessary to characterize the set of all implementable activities . This section offers two such characterizations (Proposition 1 and Corollary 1). Based on these results, it is possible to determine when all activities can be implemented (Proposition 2. Finally, we use this result to show that the set of implementable activities is limited if and only if there is an identification problem.

Without loss of generality, we focus on a single agent l who is assigned tasks N^l and chooses activity a^l in this section. For the moment, we regard the assignment $\mathcal{A}$ and the activity chosen by other agents, a^{-l}, as fixed. To facilitate notation let $c^l(a^l)$ be the costs of agent l and $\theta^l(a^l)$ the signal parameter given assignment $\mathcal{A}$ and activity a^{-l}. Since we are dealing only with agent l in this section, it simplifies the exposition to slightly abuse notation and suppress the index l.

The following proposition characterizes all implementable activities using a simple equation system.

Proposition 1 (Proportionality characterization for implementable activities)**.** *The set of activities that can be implemented with agent l given assignment $\mathcal{A}$ and signal structure S is:*

$$A^S := \{\tilde{a} | \nabla_a \theta|_{a=\tilde{a}} \lambda = \nabla_a c(a)|_{a=\tilde{a}}, \lambda \in I\!R^k\}.$$

Proof. Notice that a is from an open interval. We are hence dealing with an inner solution and the first-order conditions are necessary conditions. Under the assumptions about S, the expected utility of the unit is differentiable and the first-order conditions take the form:

$$\sum_{j=1}^{k} \frac{dE}{d\theta_j} \cdot \frac{d\theta_j}{da_i} = \frac{\partial c}{\partial a_i} \text{ for } i = 1, \ldots, n.$$

Using these conditions, we first show that $\tilde{a}$ has to lie in the set in order to be implementable. Then, we suppose that $\tilde{a}$ lies in the set and propose a scheme to implement $\tilde{a}$

First, we show that $\tilde{a}$ has to lie in the set in order to be implementable. Suppose $\tilde{a} \notin \{\tilde{a} | \nabla_a \theta|_{a=\tilde{a}} \lambda = \nabla_a c(a)|_{a=\tilde{a}}, \lambda \in IR^k\}$. Then, no λ exists and for some i, the first-order condition must be violated. Thus, the respective activity cannot be implemented.

Second, we show that $\tilde{a}$ can be implemented if it lies in the set. Consider a linear scheme that pays $t_0 + t_1 S_1 + \ldots t_k S_k$, where $t = (t_0, \ldots, t_k)$ is from some compact interval. Given this scheme, the expected value is weakly concave in a (see Lemma 1), the maximization problem by the unit is concave, and the first-order condition is necessary and sufficient for a maximizer. It now remains to be shown that there is a t such that $\tilde{a}$ fulfills the first order condition. This would follow if $\frac{dE}{d\theta_j} = \lambda_j$ for all j and some t^*. Note that $\frac{dE}{d\theta_j} = \mathrm{E}\left[u'(t_0 + \sum_{j'} t_{j'} S_{j'}) t_j \cdot \frac{\partial S}{\partial \theta_j}\right]$. Hence,

$$\frac{dE}{d\theta_j} = \lambda_j \Leftrightarrow t_j = \frac{\lambda_i}{\mathrm{E}\left[u'(t_0 + \sum_{j'} t_{j'} S_{j'}) \cdot \frac{\partial S}{\partial \theta_j}\right]} \text{ for } j = 1, \ldots, k \tag{1}$$

Thus, the desired t^* exists if the right-hand side of equation (1) has a fixed point. Since $\mathrm{E}\left[u'(t_0 + \sum_{j'} t_{j'} S_{j'}) \cdot \frac{\partial S}{\partial \theta_j}\right]$ is continuous in t and t is from a compact interval, Brouwer's fixed point theorem ensures the existence of t^*. Accordingly, we can implement $\tilde{a}$ using this t^*. □

This result is noteworthy because it highlights which aspects of the problem matter for implementation. The central role is played by the effect of the agent on the signals and his marginal costs. Conversely, substitutability of effort in the cost function is irrelevant for implementation; the cost function may take any shape as long as it remains strictly convex. Also, it does not matter for implementability whether the agent is risk-averse or risk-neutral. From the proof of the result, one can see that even a restriction to linear contracts does not hamper implementability.

From a technical point of view the proposition is helpful because it reduces the question of implementability to the question of whether there is a solution to a linear equation system. This means that we can use standard results from linear algebra to check for implementability. For example, the result that a linear equation system has a solution if the rank of the extended coefficient matrix is equal to the rank of the coefficient matrix allows us to derive the following corollary.

Corollary 1 (Rank characterization for implementable activities)**.** *The set of activities a from the open set A that can be implemented with agent l given assignment $\mathcal{A}$, and signal structure S is:*

$$A^S := \{\tilde{a}|rank(\nabla_a\theta) = rank(\nabla_a\theta|\nabla_a c)\}.$$

This characterization of implementable activities is useful because it expresses implementability in terms of two real numbers: the ranks of two matrices. By exploiting the structure of these matrices, we can derive necessary and sufficient conditions for any activity to be implementable.

Proposition 2 (Implementation and richness of signals)**.** *Any activity can be implemented if and only if signals are sufficiently rich: $rank\nabla_a\theta|_{\tilde{a}} \geq n$ for all $\tilde{a} \in A$*

Proof. Let us first show that any activity can be implemented if $\text{rank}\nabla_a\theta|_{\tilde{a}} \geq n$ for all $\tilde{a}$. Take an arbitrary $\tilde{a}$. From $\text{rank}(\nabla_a\theta) \geq n$, it follows that $n \leq k$. Thus, $min(n, k+1) = n$ and $\text{rank}(\nabla_a\theta|\nabla_a c) \leq n$ (i). On the other hand, $\text{rank}(\nabla_a\theta|\nabla_a c) \geq \text{rank}(\nabla_a\theta)$ (ii). But (i) and (ii) together imply: $n \geq \text{rank}(\nabla_a\theta|\nabla_a c) \geq \text{rank}(\nabla_a\theta) \geq n$. Applying Corollary 1 yields that the activity can be implemented.

Next we show that the rank condition must hold if any activity can be implemented. We do so by contradiction. Suppose that for some $\tilde{a}$, which by assumption is implementable, it holds that $\text{rank}\nabla_a\theta|_{a=\tilde{a}} < n$. Denote the parameter value induced by $\tilde{a}$ as $\tilde{\theta} := \theta(\tilde{a})$. Since θ is concave, the set $A(\tilde{\theta}) := \{a|\theta(a) \leq \tilde{\theta}\}$ is convex. On the other hand, costs c are strictly convex in a and accordingly there is a unique $\tilde{a}$ on $A(\theta)$ that minimizes costs. Next consider the neighborhood of $\tilde{a}$. Due to $\text{rank}\nabla_a\theta|_{a=\tilde{a}} < n$, the set $\{a|a\nabla_a\theta|_{a=\tilde{a}} = (0, \ldots, 0)'\}$ is not empty, and there are values a in the neighborhood of $\tilde{a}$ that yield the same parameter value θ. There is thus an $a \neq \tilde{a}$ such that $\theta(a) = \tilde{\theta}$. This activity cannot be implemented because $\tilde{a}$ is the unique maximizer on $A(\tilde{\theta})$. Thus, we have a contradiction to the assumption that all a can be implemented. $\square$

The proposition embodies the idea that implementation of activities is limited because signals are not sufficiently rich to reflect all changes in the activity. A very simple situation, where this is the case, is when there are less signals than tasks ($k < n$). Then, $\text{rank}\nabla_a \theta(a)$ must be smaller than n, and some activities cannot be implemented.

From the idea that implementability requires that signals are sufficiently rich to reflect all changes in activities, it is only a small step to the identification problem. Indeed, Proposition 2 can be used to prove the crucial role that identification problems play for implementation.

Theorem 1 (Implementation and identification problem)**.** *The implementation of activities is limited if and only if there is an identification problem.*

Proof. First, we show that some activities cannot be implemented if there is an identification problem. By definition, an identification problem is present if there are activities a and a' such that $\theta(a) = \theta(a')$. Since θ is continuously differentiable, there is an activity $\tilde{a} \in A$ such that $0 = \theta(a) - \theta(a') = \nabla_a \theta(\tilde{a})(a - a')$. This, however, implies that $\text{rank}\nabla_a \theta(\tilde{a}) < n$. So, Proposition 2 yields that some activity is not implementable.

Second, we show that there is an identification problem if some activity cannot be implemented. If some activity cannot be implemented, $\text{rank}\nabla_a \theta(a)|_{a=\tilde{a}} < n$ for some $\tilde{a}$ by Proposition 2. This means that there is an a in the neighborhood of $\tilde{a}$ such that $\theta(a) = \theta(\tilde{a})$. There is hence an identification problem. □

The theorem establishes that the intuition from the initial examples is correct: the source of the impossibility to implement desired activities lies in the identification problem. Other aspects of the problem (substitutability of effort in costs, degree of risk-aversion, etc.) may affect which activity the principal wants to implement but they do not limit which activity she can implement. The theorem also highlights that the only way to overcome a multi-tasking problem is to eliminate the identification problem. The next section examines when the identification problem can be eliminated by a different assignment of agents to tasks.

5 Identification by assignment

Building on the results of the previous section, we characterize when the identification problem can be alleviated by an appropriate assignment. We also provide necessary and sufficient conditions that are simpler to verify.

Since we are dealing with multiple agents in this section, we will no longer suppress the index l pertaining to agents. The following result directly extends Proposition 2 to the multi-agent setting.

Theorem 2. *The identification problem can be alleviated with assignment $\mathcal{A}$ if and only if rank$\nabla_{a^l} \geq |N^l|$ for all l and a.*

Proof. If $\nabla_{a^l} \geq |N^l|$ for all l, then the implementation problem can be separated into m individual problems. In each of these problems all activities can be implemented by Proposition 2.

Conversely, if $\nabla_{a^l} < |N^l|$ for some l, Proposition 2 implies that some activities cannot be implemented for this agent. Accordingly, the identification problem persists by Theorem 1. □

This theorem gives a general characterization when an assignment solves the identification problem. Determining the richness of signals (rank $\nabla_a \theta|_{N_j}$), however, may be difficult. The following result provides a simpler necessary condition for assignments to solve the identification problem.

Corollary 2. *The identification problem can only be alleviated if each task affects some signal: for all tasks i there must be a signal k such that $\frac{\partial \theta_k}{\partial a_i} \neq 0$.*

Proof. Suppose there is a task i such that for all k $\frac{\partial \theta_k}{\partial a_i} = 0$. Take an arbitrary assignment $\mathcal{A}$ and l be the agent who carries out task i. Then, $\nabla_{a^l}\theta < |N^l|$ for l and the identification problem cannot be alleviated given assignment $\mathcal{A}$ by Theorem 2. Since the assignment was arbitrary, the identification problem cannot be alleviated by any assignment. □

Observe that in the home contractor model and the asset enhancement model by Holmström and Milgrom (1991), there is one activity, which is important to the principal, but affects no signal. Consequently, there is no assignment of agents to tasks that solves the identification problem. In particular, the desired activity cannot be implemented even if each agent only carries out one task. This shows that the problem in these two models is no multitasking problem in the sense that it is not caused by the fact that the agent carries out multiple tasks. The home contractor and asset enhancement model are hence examples for identification problems that are not multitasking problems.

It is not sufficient that each task affects some signal to eliminate the identification problem. The following proposition offers another necessary condition.

Proposition 3. *The identification problem can only be alleviated by some assignment if signals are sufficiently rich: $\tilde{k}m \geq n$, where $\tilde{k} := \min_a rank\nabla_a\theta$.*

Proof. If $\tilde{k}m < n$, where $\tilde{k} := \min_a \text{rank}\nabla_a\theta$, then there is an $\tilde{a}$ such that

$$\underbrace{\text{rank } \nabla_a\theta|_{a=\tilde{a}} + \ldots + \text{ rank}\nabla_a\theta}_{m}|_{a=\tilde{a}} < \sum_{l=1}^{m} |N^l|.$$

In particular,

$$\sum_{l=1}^{m} \text{ rank } \nabla_a^l\theta|_{a=\tilde{a}} < \sum_{l=1}^{m} |N^l|. \tag{2}$$

Now, take an arbitrary assignment $\mathcal{A}$ and suppose that any activity can be implemented. By Theorem 2, it must then hold that rank $\nabla_{a^l}\theta \geq |N^l|$ for all l. This, however, contradicts equation (2). Consequently not all activities can be implemented and the identification problem persists for any assignment. □

The simple counting rule in this proposition offers a quick way to show that implementation is limited with a single agent in Dewatripont and Tirole (1999) and Ratto and Schnedler (2003, 2008). In both cases, there is only one signal ($k = 1$) and the minimal number of independent signals is $\tilde{k} = \min_a \text{rank}\theta(a) = 1$. With two tasks ($n = 2$), there are hence at least $m = 2$ agents needed to implement any desired activity. If we are limited to one agent, then at least two signals are necessary to overcome the identification problem.

The condition in the proposition is necessary but not sufficient. Even if each task affects some signal and the condition is fulfilled, it may not be possible to alleviate the identification problem. As an example consider a variation on the sales force model by Corts (2007). Suppose that sales to the first customer cannot be individually targeted, so that a_1 is a general marketing effort that affects both customers. Then, sales to the first customer depend (as before) on marketing effort (a_1), customer care (a_2), and factors beyond the agents' control (ϵ_1): $S_1(\theta_1(a) = \theta_1(a) + \epsilon_1$, with $\theta_1(a) = a_1 + a_2$. Sales to the second customer, however, now depend on marketing (a_1), sales effort that is individually targeted to this customer (a_3), customer care (a_4), and factors beyond the agents' control (ϵ_2): $S_2(\theta_2(a), \epsilon_2) = \theta_2(a) + \epsilon_2$, with $\theta_2(a) = a_1 + a_3 + a_4$. Since signals are linear, the marginal effects of activities on the parameter are constant in a and the marginal effect matrix becomes

$$\nabla_a \theta = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \end{pmatrix}.$$

The rank of the matrix is two because the two rows are linearly independent; accordingly, the minimum rank is $\tilde{k} = 2$. Like in the original example, there are four tasks ($n = 4$). If we maintain the assumption that there are two agents ($m = 2$), this example meets the necessary condition of Proposition 3: $\tilde{k}m = 2 \cdot 2 \leq n = 4$. Still, the identification problem persists however we assign tasks to agents. Consider the set of tasks affecting the second customer $\{1, 3, 4\}$. In any assignment, at least two of these tasks are carried out by one agent. This agent then has more than one way to generate the same $\theta_2(a)$.

Consequently, the condition featured in Proposition 3 is not sufficient. The following result offers a sufficient condition.

Proposition 4. *The identification problem can be alleviated by some assignment of agents to tasks if each task affects some signal and signals are sufficiently rich:* $m > n - \tilde{k}$*, where* $\tilde{k} := \min_a \mathit{rank} \nabla_a \theta$.

Proof. Since $\tilde{k} := \min_a \operatorname{rank} \nabla_a \theta$, there are at least $\tilde{k}$ independent columns of $\nabla_a \theta$ for any activity. Assign the tasks belonging to these columns to the first agent. For this agent, it now holds that $\operatorname{rank} \nabla_{a^1} \theta \geq \tilde{k} = |N^1|$. Since $m > n - \tilde{k}$, there remain at least $n - \tilde{k}$ agents who are not yet assigned to tasks. At the same time, there are at exactly $n - \tilde{k}$ tasks, which are not yet assigned. We thus have sufficient agents such that each of the remaining agents only has to carry out one of the $n - \tilde{k}$ remaining tasks. For the agents $l \geq 2$, it thus holds that $\operatorname{rank} \nabla_{a^l} \theta = 1 \geq |N^l|$. Consequently, we have found an assignment such that $\operatorname{rank} \nabla_{a^l} \theta \geq \geq |N^l|$ for all $l = 1, \ldots, m$ and by Theorem 2, the identification problem is solved. $\square$

This proposition allows us to assess quickly whether the principal's choice set is limited or whether all activities can be implemented. Consider, for example, the situation in which there are two tasks, which both affect one signal as in the advocates model by Dewatripont and Tirole (1999) and the production specialization model by Ratto and Schnedler (2003, 2008). Since the signal is affected by both tasks, both entries in $\nabla_a \theta = (\frac{\partial \theta}{\partial a_1}, \frac{\partial \theta}{\partial a_2})$ are non-zero and hence $\tilde{k} = 1$. Then, two agents suffice ($m = 2$) to implement any desired activity. If the principal is for some reason restricted to one agent, we know from Proposition 3 that there must be at least two signals (otherwise $\tilde{k} = 1$). Proposition 4 informs us that this is not only a necessary but also a sufficient condition. Moreover, the second signal does not have to be dramatically different. Suppose the first signal is $S_1(\theta_1(a), \epsilon) = a_1 + a_2 + \epsilon_1$, then a second signal $S_2(\theta_2(a), \epsilon) = a_1 + (1 + \eta)a_2 + \epsilon_1$, with $\eta \neq 0$ suffices.

While the condition in Proposition 4 is sufficient to alleviate the identification problem, it is not necessary. The counter-example is the sales force model by Corts (2007). In this model, there are two independent rows ($\tilde{k} = 2$), two agents ($m = 2$), and four tasks ($n = 4$), so that the condition is violated ($m = 2 = n - \tilde{k} = 4 - 2$). Still, the identification problem is alleviated if one agent is assigned the sales related tasks while the other takes over customer care tasks.

This suggests that the inequality in the condition may be weakened to $m \geq n - \tilde{k}$. Then, however, our earlier variation on the sales force model, where marketing affects both customers provides a counter-example.

The proposition also enables us to examine the reason why the first-best is not chosen in various model variations suggested by Holmström and Milgrom (1991) in which $S_j(\theta_j, a) = \theta_j + \epsilon_j$ with $\theta_j(a) = a_j$ for all $j = i$. Recall that there are two candidate explanations: (i) it may not be possible to implement the first-best or (ii) it may not be desired to implement the first-best, e.g. because the insurance premiums attached to this implementation outweigh the benefits. Since each parameter θ_j is only affected by one task, namely a_j, the marginal effect matrix $\nabla_a \theta$ is the identity matrix and has minimum rank $\tilde{k} = n$. Proposition 4 then tells us that one agent suffices to implement all activities. In particular, it would be possible to implement the first-best. Consequently, the reason why the first-best is not implemented must lie in its high implementation costs in relation to the second-best.

6 Conclusion

Following the inspiring models in Holmström and Milgrom (1991), there has been a wealth of multiple task applications. When dealing with multiple task settings, however, it is often not distinguished between two different consequences of assigning multiple tasks to one person. First, it may prevent the implementation of desired activities. Second, it may affect the costs of those activities that can be implemented.

The second consequence is not specific to multiple task settings. For example, in the traditional single-task moral-hazard problem, the first-best is implementable but rendered to costly by insurance aspects. Recent contributions to the principal-agent literature examine a different source for the costs of implementation, for example the fact that the worker has specific knowledge about production (Raith, 2008). The model without specific knowledge has been extensively researched (Feltham and Xie, 1994; Feltham and Wu, 2000; Baker, 2000, 2002; Datar, Cohen Kulp, and Lambert, 2001). But even in this situation, it is not yet well understood how multiple task and insurance issues interact. For example, Schnedler (2008) demonstrates that a performance measure that is not aligned with the principal's benefit may generate a higher surplus than an otherwise identical but aligned measure.

Here, we have studied the more fundamental question, whether multitasking renders it impossible to implement certain activities. The problem that multiple tasks hamper implementation has so far only been documented for risk-neutral workers, linear signals, and the first-best activity (see Proposition 1 and 2 in Ratto and Schnedler 2003, 2008 and Proposition 6 in Corts 2007). We have extended the framework to allow for risk-aversion and a wide class of signals. In this general framework, we have determined when activities in general (and not only first-best activities) cannot be implemented because an agent is assigned multiple tasks.

We found that limits on the set of implementable activities are always due to the problem that different activities lead to the same distribution of signals (Theorem 1). Apart from likelihood-ratio and sufficient statistic, there is hence a third concept from econometrics that matters for the principal-agent model: identification.

We have seen that some identification problems are caused by multiple tasks. Examples are the observation from Dewatripont and Tirole's model that a single investigator cannot be induced to exert effort to find exculpatory and incriminating evidence or the finding by Ratto and Schnedler (2003, 2008) that it is impossible to obtain the desired input combination with a single worker if only output is observable.

We have also encountered identification problems that are not due to multiple tasks. These include the home contractor model and the asset enhancement models by Holmström and Milgrom (1991). In both models, the impossibility to implement desired activities persist even if each worker carries out only one task.

Finally, we have observed situations in which agents carry out multiple tasks but no identification problem arises. These include all models by

Holmström and Milgrom (1991) in which there is a signal for every task—the so-called “one signal-per-task assumption” (a name introduced by Corts 2007). But also the sales force model by Corts (2007) in which there are only two signals for four tasks.

By offering a general characterization when the identification problem is caused by multiple-tasks and can thus be alleviated by a different assignment (Theorem 2) as well as simpler necessary (Corollary 2 and Proposition 3) and sufficient conditions (Proposition 4), the paper provides a first but necessary step toward answering the question when multiple tasks are a problem.

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Lemma 1 (Weakly concave expected value)**.** *$E\left[u(t_0 + t_1 S_1 + \ldots + t_k S_k)|\theta\right]$ is weakly concave in a.*

Proof. First, we compute the first and second derivative of the expected value in θ, and show that the latter is not positive definite. Then, use that θ itself is a weakly concave function in a.

$$\frac{d}{d\theta_i}\mathrm{E}\left[u(t_0 + t_1 S_1 + \ldots + t_k S_k)|\theta\right] = \mathrm{E}\left[u' \cdot t_i \frac{\partial}{\partial\theta_i} S_i\right] \tag{3}$$

$$\frac{d^2}{d\theta_i^2}\mathrm{E}\left[u(t_0 + t_1 S_1 + \ldots + t_k S_k)|\theta\right] = \mathrm{E}\left[u'' \cdot t_i \frac{\partial}{\partial\theta_i} S_i t_i \frac{\partial}{\partial\theta_i} S_i + u' t_i \frac{\partial^2}{\partial\theta_i^2} S_i\right] < 0 \tag{4}$$

$$\frac{d^2}{d\theta_i d\theta_j}\mathrm{E}\left[u(t_0 + t_1 S_1 + \ldots + t_k S_k)|\theta\right] = \mathrm{E}\left[u'' \cdot t_i \frac{\partial}{\partial\theta_i} S_i t_j \frac{\partial}{\partial\theta_j} S_i + u' t_i \frac{\partial^2}{\partial\theta_i \partial\theta_j} S_i\right], \tag{5}$$

$$= \mathrm{E}\left[u'' \cdot t_i \frac{\partial}{\partial\theta_i} S_i t_j \frac{\partial}{\partial\theta_j} S_i\right], \tag{6}$$

where the last equality follows from the fact that θ_j only affects S_j but not S_i. Define $\tilde{s} := (\frac{\partial}{\partial\theta_1} S_1 t_1, \ldots, \frac{\partial}{\partial\theta_k} S_k t_k)$. Then,

$$\nabla_{\theta\theta}\mathrm{E}\left[u(t_0 + t_1 S_1 + \ldots + t_k S_k)|\theta\right] = \mathrm{E}\left[u'' \cdot \tilde{s}'\tilde{s} + u' \cdot \mathrm{diag}(t_i \frac{\partial^2}{\partial\theta_i^2} S_i)\right] =: M,$$

where diag is the matrix with the respective entries on the diagonal. Observe that these entries as well as u'' are negative while u' is positive. Hence,

$x'Mx \leq 0$ for all x, the matrix is not positive definite, and the expected value is not convex in θ. Finally, observe that by assumption θ itself is a weakly concave function in a. Consequently, the expected value is a weakly concave function in a. $\square$