Longevity and Growth

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Abstract

We develop a continuous time overlapping generations model to investigate the effect of longevity on economic growth. In our model, life expectancy is determined by individuals' choices of investments in a healthcare technology. We find that an improvement in the healthcare technology always increases the steady state growth rate, although the magnitude of this increase is small even for large improvements in life expectancy. During transition, per capita growth rates can experience a period of decline and may even become negative. Our findings fit well with recent empirical studies on the relationship of longevity and growth.

Keywords: economic growth, endogenous longevity, healthcare expenditures, population dynamics, transitional dynamics

JEL: O40, I10, J10

1 Introduction

There is no doubt that human longevity has increased substantially over the last decades. However, there is an ongoing debate how this affects short- and long-run economic growth. By now, there exists a number of growth models that examine the effect of exogenous variations in expected lifetime on economic development.[1] Expected lifetime, however, – and this is well recognized by these authors – is not given per se but can be influenced by investments in healthcare, such as improving sanitation, buying medication and inoculation, consulting a physician, etc. Hall and Jones (2007) report that the share of GDP spent for healthcare in the United States has risen from 5.2 percent in 1950 to 15.4 percent in 2000. Over the same period, average life expectancy at the date of birth increased from 68.2 years in 1950 to 76.9 years in 2000. They argue that healthcare investments are positively correlated with life expectancy. This hypothesis is strongly supported by Lichtenberg's (2004) empirical analysis of U.S. data from 1960–2001.

We develop an endogenous growth model with overlapping generations in continuous time that takes into account the positive relationship between healthcare investments and average lifetime. Individuals can influence their death probability at each instant of time by investments in a healthcare technology. Thus, instead of varying longevity exogenously, we focus on how economic growth reacts to improvements in the healthcare technology. In fact, we consider two different types of healthcare improvements. The first type increases the average expected lifetime when no individual investments in healthcare are undertaken. One could think of improvements in the sanitary infrastructure or infant mortality reduction. The second type increases the reduction of the instantaneous death probability for given healthcare expenditures. Examples include breakthroughs in medication, such as penicillin or streptomycin, or therapeutic breakthroughs, such as new diagnostic tools or surgeries.

Our main results are that the new steady state growth rate that the economy approaches after a positive shock in the healthcare technology is always higher than the steady state growth rate before the shock. However, using empirically reasonable parameter values, the increase in the steady state growth rate is almost negligible even for substantial improvements in the healthcare technology. The intuition is that although

[1]See e.g. De la Croix and Licandro (1999), Kalemli-Ozcan et al. (2000), Zhang et al. (2001) Kalemli-Ozcan (2002), Boucekkine et al. (2007)

individuals increase their savings in assets when they expect to live longer, they over-proportionally spend resources for health services to increase their lifetimes. Hence, in the long run, welfare substantially increases but rather through an increase in average lifetime than due to economic growth.

In order to examine short run effects, we apply a small open economy setting which allows us to analytically determine the transition dynamics of the economy after a shock in the healthcare technology. We find that a positive shock on the healthcare technology may be followed by a substantial decline of the per capita growth rates of capital and consumption. The growth rate of capital per capita may temporarily even become negative if the shock and/or the exogenously given growth rate of new-borns are sufficiently large. Intuitively, a positive shock to the healthcare technology induces individuals to increase their healthcare expenditures to further increase their average lifetime. In addition, the increase in average lifetime temporarily increases the population growth rate. Both effects reduce the growth rate of capital per capita. On the other hand, a higher expected lifetime gives higher incentives to save in assets, which increases the growth rate of capital per capita. For large positive shocks in the healthcare technology the first two (negative) effects outweigh the (positive) third one.

The empirical literature usually reports positive effects of longevity on per capita growth.[2] However a recent study by Acemoglu and Johnson (2007) finds that GDP per working age population shows relative declines in countries experiencing large increases in life expectancy. The interpretation that our theoretical model offers is that the negative growth effects are of temporary nature. Our model can well explain negative growth rates per capita over the time horizons under consideration in the Acemoglu and Johnson (2007) study if the shock in the healthcare technology is sufficiently large. Our theory predicts these negative effects to vanish in the long run.

There are other attempts to endogenize the individual's life expectancy such as Blackburn and Cipriani (2002), Aisa and Pueyo (2004), Chakraborty (2004), Chakraborty and Das (2005), Bhattacharya and Qiao (2006), and Finlay (2006). However, to the best of our knowledge, only Chakraborty and Das (2005), Bhattacharya and Qiao (2006), and Finlay (2006) fully endogenize expected lifetime in the sense that healthcare investments are directly chosen either by the government or the individual household. For example, Aisa and Pueyo (2004) consider a government decision on public expenditures for healthcare that increase life expectancy. However, the government only chooses tax

[2] See e.g. Bloom et al. (2004) for an overview.

rates, while the allocation of the tax income between public infrastructure for firms and healthcare is exogenously fixed. In an opposite way, Chakraborty (2004) considers a government's endogenous choice of investments in healthcare for an exogenously given tax rate. Blackburn and Cipriani (2002) assume that an individual's life expectancy is a by-product of her human capital, which is determined by her parents' investments.

All three papers with fully endogenous choices of investments in longevity, consider a two period overlapping generations model, where households have a certain probability to live in the second period. Growth is driven by human capital, in which households invest during the first period of life. Chakraborty and Das (2005) focus on the effects of endogenous longevity choices on inequality. They show that initial differences in economic and health status may perpetuate across generations when annuities markets are imperfect. Bhattacharya and Qiao (2006) argue that the presence of public input in private longevity, where the latter is endogenously chosen, may lead to chaotic dynamics in a neoclassical growth model. Finally, Finlay (2006) investigates individual's incentives to invest in human capital and higher expected lifetime dependent on the agents' degree of risk aversion. With respect to transitional dynamics, her simulation results indicate that higher risk aversion lowers investments in both human capital and healthcare.

In contrast to these papers, we develop a continuous time overlapping generations framework and focus on investments in physical capital rather than human capital. More importantly, however, we investigate the growth effects of changes in the healthcare technology rather than assuming a static one.

From a methodological perspective, we develop a model that combines the household side of Blanchard (1985) with the production side of Romer (1986). In this respect our model shares some similarities with Boucekkine et al. (2007). However, they do not consider endogenous investments in longevity. Instead, they study the effect of age-dependent mortality rates on economic growth. We deliberately abstract from age-dependent mortality in order to study, in its simplest form, the effects of longevity on economic growth resulting from the household's central trade-off between investments in assets and healthcare. In a next step it would be interesting to consider more realistic assumptions such as age-dependent mortality, retirement decisions or endogenous fertility to learn how this affects our results.

The paper is organized as follows. The next section introduces the model, determines the static market equilibria and derives the dynamics of the aggregate economy. In Section 3 we investigate the economy's steady state dynamics. Transitional dynamics are studied in Section 4 followed by a discussion of the theoretical results and how they relate to empirical evidence in Section 5. Section 6 concludes.

2 The Model

The model comprises a continuum of households. Households born at time s face a probability of dying p at all times $t \geq s$. Like in Blanchard (1985), the probability p is constant throughout the lifetime of each household, but in our model it may vary among different households. In fact, the probability of dying is determined by the level of medical treatment the household gets throughout its lifetime. At time of birth, a household chooses a level h of medical treatment which is fixed over its entire lifetime and determines the probability of dying via some healthcare technology H. For the remainder of the paper we assume the following healthcare technology:

$$H(h(s)) = p_0(1 - \psi h(s)). \tag{1}$$

Without medical treatment ($h(s) = 0$), households face a maximal probability of death equal to p^0. We further assume that the probability of dying $p(s)$ decreases with constant returns ψ in the level of medical treatment $h(s)$ (i.e., $H' < 0$, $H'' = 0$). The parameter ψ reflects the productivity of healthcare investments and is interpreted as the quality level of the health system. While p_0 reflects, for example, the sanitary infrastructure of the economy, ψ increases with the human capital of physicians, the efficiency of hospitals and the like.

As the probability of dying is constant over the whole lifetime, at any time $t \geq s$ all households that are born at time s and still alive face an expected remaining lifetime $T(s)$ of

$$T(s) = \int_t^\infty (t' - t)p(s) \exp[-p(s)(t' - t)]dt' = \frac{1}{p(s)} \ . \tag{2}$$

Although the lifetime of each household is stochastic, we assume that the size of each cohort is large enough for cohort sizes to decline deterministically over time.[3] At any point in time a new cohort is born. We abstract from fertility choices of households

[3]Formally, this is reflected in our assumption of a continuum of households.

and assume that the cohort size grows at a constant and exogenously given rate ν. Normalizing the cohort size at time $t = 0$ to unity, we obtain for the size of the population at time t

$$N(t) = \int_{-\infty}^{t} \exp[\nu s] \exp[-p(s)(t-s)] ds \ . \tag{3}$$

Independent of their time of birth $s \in (-\infty, \infty)$, households are supposed to maximize the expected discounted lifetime utility derived from consumption U

$$U(s) = \int_{s}^{\infty} V\big(c(t,s)\big) \exp\left[-\big(\rho + p(s)\big)(t-s)\right] dt \ , \tag{4}$$

where V is the instantaneous utility derived from consumption $c(t,s)$ at time t of the household born at time s, and ρ is the constant rate of time preference. We assume that V exhibits constant elasticity of intertemporal substitution σ[4]

$$V\big(c(t,s)\big) = \frac{c(t,s)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \ , \quad \sigma > 1 \ . \tag{5}$$

At any time alive, each household is endowed with one unit of labor, which is supplied inelastically to the labor market at wage $w(t)$ over the whole lifetime. In addition, households can save and borrow assets $b(t,s)$ at the interest rate $r(t)$. Households are born with no assets.

Although households are non-altruistic, the stochastic nature of life may leave unanticipated bequests. To avoid this, we assume a perfectly competitive life insurance market at which households can contract against their risk of unanticipated bequests. For each unit of assets the household can buy a life annuity which pays the return a as long as the household is alive. We assume that the insurance company can learn the probability of dying $p(s)$ of each cohort at no costs. Thus, insurance companies can offer annuities risklessly (due to the large cohort sizes). Perfect competition among insurance companies leads to fair annuity payments

$$a(t,s) = r(t) + p(s) \ . \tag{6}$$

If negative bequests are prohibited, households will hold their whole wealth in fair annuities. Denoting the costs of healthcare by $M\big(h(s)\big)$ the households' budget constraint reads

$$\dot{b}(t,s) = \big(r(t) + p(s)\big)b(t,s) + w(t) - c(t,s) - M\big(h(s)\big) \ , \qquad t \geq s \ , \tag{7}$$

[4]We restrict the parameter σ to $(1, \infty)$. This is important, because we need a utility representation with $V\big(c(t,s)\big) > 0$, as the trade off between investments in healthcare and average lifetime involves not only marginal but also absolute welfare comparisons. This will become obvious in Section 2.3.

with

$$b(s,s) = 0 \ . \tag{8}$$

2.1 Final Good Production

The economy comprises two sectors, the final good sector and the healthcare sector. We assume that both sectors operate at perfectly competitive conditions, in which all firms have access to the same constant returns to scale production technologies. As a consequence we restrict the analysis to a representative firm in each sector.

The representative firm in the final good sector produces a homogeneous consumption good via a Cobb-Douglas production technology

$$Y(t) = K(t)^{\alpha}\big(A(t)L^F(t)\big)^{1-\alpha} \ , \quad \alpha \in (0,1) \ . \tag{9}$$

where $K(t)$ and $L^F(t)$ denote the aggregate amount of capital and labor employed in production and $A(t)$ characterizes the technological level of the economy which is exogenous to the individual firm. In essence, we assume a "learning-by-investing" externality a la Romer (1986), but we do not allow for scale effects. That is, the technological level of the economy is given by the average capital stock per worker[5]

$$A(t) = \frac{K(t)}{L^F(t)} \ . \tag{10}$$

Capital depreciates at the constant rate δ. Profit maximization of the representative firm implies factor prices equal to their marginal productivity

$$r(t) = \alpha - \delta \ , \tag{11a}$$

$$w(t) = (1-\alpha)\frac{K(t)}{L^F(t)} \ . \tag{11b}$$

As the interest rate is constant due to the "learning-by-investing" externality, we use the abbreviation $r = r(t) = \alpha - \delta$.

2.2 Healthcare Sector

The representative firm in the healthcare sector provides medical treatment by employing labor in the healthcare technology H. Without loss of generality we assume

[5] In Romer (1986) it is assumed that $A(t) = K(t)$. By assuming the learning externality to be proportional to capital per worker, we correct for scale effects.

that one unit of labor produces one unit of medical treatment. In equilibrium, labor employed in the healthcare sector has to earn the same wage as labor employed in the final good sector. Consequently, given the levels $h(s)$ of medical treatment of each cohort alive, the total amount of labor employed in the healthcare sector equals

$$L^H(t) = \int_{-\infty}^{t} h(s) \exp[\nu s] \exp[-p(s)(t-s)] ds \ . \tag{12}$$

Due to the assumption that the healthcare sector is perfectly competitive, it offers medical treatment at marginal costs $w(t)$. Thus, we obtain for the costs of medical treatment

$$M\big(h(s)\big) = h(s) w(t) \ . \tag{13}$$

Inserting equations (11a) and (13) into the households' budget constraint (7) yields

$$\dot{b}(t,s) = \big(r + p(s)\big) b(t,s) + \big(1 - h(s)\big) w(t) - c(t,s) \ , \qquad t \geq s \ . \tag{14}$$

2.3 The Individual Households' Problem

Households maximize intertemporal utility (4) subject to conditions (7) and (8) we adopt a two step procedure. First, we assume that the level of medical treatment $h(s)$ and thus the probability of dying $p(s)$ are given, and seek for the optimal paths of $c(t,s)$ and $b(t,s)$. This yields the well known Euler equation

$$\frac{\dot{c}(t,s)}{c(t,s)} = \sigma\big(r - \rho\big) \ , \qquad t \geq s \ . \tag{15}$$

For given $h(s)$ the behavior of a household born at time s is characterized by the system of differential equations (7) and (15), the initial condition (8) and the transversality condition for the stock of assets

$$\lim_{t \to \infty} b(t,s) \exp\left[-\big(r + p(s)\big)(t-s)\right] = 0 \ . \tag{16}$$

Introducing the abbreviation

$$x\big(p(s)\big) = (1-\sigma) r + \sigma\rho + p(s) \ , \tag{17}$$

we obtain for the optimal paths of consumption $c(t,s)$ and assets $b(t,s)$

$$c(t,s) = c(s,s) \exp\left[\sigma(r-\rho)(t-s)\right] \ , \tag{18a}$$

$$b(t,s) = \frac{c(t,s)}{x\big(p(s)\big)} - \big(1 - h(s)\big) \int_t^{\infty} w(t') \exp\left[-\big(r + p(s)\big)(t'-t)\right] dt' \ , \tag{18b}$$

$$c(s,s) = x\big(p(s)\big)\big(1 - h(s)\big) \int_s^{\infty} w(t') \exp\left[-\big(r + p(s)\big)(t'-s)\right] dt' \ . \tag{18c}$$

Second, given the optimal consumption and assets paths (18) for given $h(s)$, we now seek the level of healthcare $h(s)$ which maximizes lifetime utility $U(s)$ given that $p(s) = H\big(h(s)\big)$. The necessary condition for the optimal $h(s)$ reads

$$\begin{aligned} &\int_s^\infty \frac{\partial c(t,s)}{\partial h(s)} V'\big(c(t,s)\big) \exp\left[-\big(\rho + p(s)\big)(t-s)\right] dt \\ &= \int_s^\infty (t-s) H'\big(h(s)\big) V\big(c(t,s)\big) \exp\left[-\big(\rho + p(s)\big)(t-s)\right] dt \ . \end{aligned} \tag{19}$$

The intuition for this condition is straightforward. In the optimum the decrease in lifetime utility from a marginal increase of healthcare expenditures due to decreasing lifetime consumption has to equal the increase in lifetime utility due to an increasing expected lifetime. Inserting equations (5) and (18a) yields

$$\begin{aligned} &\frac{\partial c(s,s)/\partial h(s)}{c(s,s)} \int_s^\infty \exp\left[-x\big(p(s)\big)(t-s)\right] dt \\ &= H'(h(s)) \frac{\sigma}{\sigma - 1} \int_s^\infty (t-s) \exp\left[-x\big(p(s)\big)(t-s)\right] dt \ . \end{aligned} \tag{20}$$

Utilizing (18c), this can be simplified to

$$\begin{aligned} &H'\big(h(s)\big) \left[\frac{\int_s^\infty w(t')(t'-s) \exp\left[-\big(r+p(s)\big)(t'-s)\right] dt'}{\int_s^\infty w(t') \exp\left[-\big(r+p(s)\big)(t'-s)\right] dt'} - \frac{1}{(1-\sigma)x\big(p(s)\big)} \right] \\ &= \frac{1}{1-h(s)} \ . \end{aligned} \tag{21}$$

From the particular functional form of our healthcare technology (1), we obtain that the marginal decrease of the death probability from spending an additional marginal unit of income in healthcare equals $p_0\psi$. The first-order condition can thus be written as

$$\begin{aligned} F\big(h(s)\big) \ := \ & p_0\psi \left[\frac{\int_s^\infty w(t')(t'-s) \exp\left[-\big(r+p(s)\big)(t'-s)\right] dt'}{\int_s^\infty w(t') \exp\left[-\big(r+p(s)\big)(t'-s)\right] dt'} - \frac{1}{(1-\sigma)x\big(p(s)\big)} \right] \\ & - \frac{1}{1-h(s)} = 0 \ . \end{aligned} \tag{22}$$

A closer look at the function $F\big(h(s)\big)$ reveals two important insights: First, as $p(s)$ cannot become smaller than 0, the term in brackets converges to a finite value when h increases to 1.[6] Second, the last summand of $F\big(h(s)\big)$ approaches $-\infty$ for $h \to 1$.

[6] Of course, this is only true if the integrals are finite and distinct from zero, which we assume.

Taken together, we infer that the limit of the left hand side of (22) with h approaching 1 is minus infinity. As a consequence, the marginal costs of healthcare investments are higher than the marginal benefits, whenever $F(h(s))$ is negative. With h approaching 1, almost the entire income is spent for medical treatment leaving very little for consumption. As the household's utility function satisfies the Inada-conditions, the opportunity costs of healthcare expenditures in utility of forgone consumption tends to infinity if h is close to 1. This fact immediately implies the following lemma.

Lemma 1 (Existence)
If $F\big(h(s)\big) > 0$ for some $h(s) \in [0, 1)$, then the household's utility maximization problem possesses a solution with positive healthcare expenditures.

Proof. A solution to the household's maximization problem, $h^\star(s)$, must satisfy equation (22) and $\partial F\big(h^\star(s)\big)/\partial h^\star(s) < 0$. Both requirements are ensured by $F\big(h(s)\big) > 0$ for some $h(s) \in [0, 1)$ and $\lim_{h(s)\to 1} F\big(h(s)\big) = -\infty$. □

If the wages are increasing at a constant rate, which will be the case in steady state, we have a unique solution to the household's problem.

Proposition 1 (Uniqueness)
If $w(t)$ grows at a constant rate, the optimal level of healthcare $h^\star(s)$ is unique. If $F\big(h(s)\big) > 0$ for some $h(s) \in (0, 1)$, it is positive. Otherwise the household chooses the corner solution $h(s)^\star = 0$.

The proof is given in Appendix A.1.

The intuition of the proof can be summarized as follows: If $w(t)$ grows at a constant rate, say g, the first order condition yields

$$F\big(h(s)\big) = -\frac{1}{1-h(s)} + p_0\psi\left[\frac{1}{r+p(s)-g} - \frac{1}{(1-\sigma)x\big(p(s)\big)}\right] = 0\ . \tag{23}$$

As shown in the proof, $F\big(h(s)\big)$ possesses at most two roots for $h(s) \in (0, 1)$. If there is only one root for $h(s) \in (0, 1)$, then it must be a maximum to the household's utility maximization problem as $\lim_{h(s)\to 1} F(h(s)) = -\infty$. If $F\big(h(s)\big)$ has two roots, the continuity of $F\big(h(s)\big)$ in the interval $(0, 1)$ and $\lim_{h(s)\to 1} F\big(h(s)\big) = -\infty$ require that the root with the lower value of healthcare spending is a minimum and the root with the higher $h(s)$ is the utility maximum. The case that the first order condition cannot be satisfied implies that $F\big(h(s)\big) < 0$ over the entire interval $h(s) \in (0, 1)$.

Hence, the marginal costs of healthcare services are larger than the marginal gains and the household reaches maximal utility by choosing $h^\star(s) = 0$.

Remark. Another aspect is important. In case the wage rate grows at a constant rate, the first order condition (23) is identical for all households irrespective of their date of birth. Consequently, all households spend the same fraction of income for medical treatment, and thus exhibit the same death probability p.

2.4 Aggregate Economy and Market Equilibrium

Investigating the aggregate economy, we first introduce aggregate household variables per capita, which are derived by integrating over all living individuals and dividing by the population size of the economy according to the scheme

$$z(t) = \frac{\int_{-\infty}^{t} x(t,s) \exp[\nu s] \exp[-p(s)(t-s)] \, ds}{N(t)} \ , \tag{24}$$

where $z(t)$ and $z(t,s)$ denote aggregate per capita respectively individual household variables.[7] The economy consists of five markets: the labor market, the capital market, the consumption good market, the market for annuities and the market for healthcare. We assume the economy to be in market equilibrium at all times t. In particular, this implies that labor demand equals the population size, i.e. $L^F(t) + L^H(t) = N(t)$, and capital per capita equals aggregate assets per capita, i.e. $k(t) = b(t)$.

Then, the dynamics of the aggregate economy is characterized by

$$\dot{c}(t) = \left[\sigma(r-\rho) - \frac{\dot{N}(t)}{N(t)}\right] c(t) + \Delta c(t) \ , \tag{26a}$$

$$\dot{k}(t) = \left(r - \frac{\dot{N}(t)}{N(t)}\right) k(t) + \left(1 - \frac{L^H(t)}{N(t)}\right) w(t) - c(t) \ , \tag{26b}$$

$$\dot{N}(t) = 1 - \int_{-\infty}^{t} p(s) \exp[-p(s)(t-s)] ds \ , \tag{26c}$$

[7] Differentiation with respect to time yields

$$\begin{aligned} \dot{z}(t) = {} & \frac{\int_{-\infty}^{t} \dot{z}(t,s) \exp[\nu s] \exp[-p(s)(t-s)] \, ds}{N(t)} + \frac{z(t,t) \exp[\nu t]}{N(t)} \\ & - \frac{\int_{-\infty}^{t} z(t,s) p(s) \exp[\nu s] \exp[-p(s)(t-s)] \, ds}{N(t)} - \frac{\dot{N}(t)}{N(t)} z(t) \ . \end{aligned} \tag{25}$$

$$\Delta c(t) = \frac{c(t,t)\exp[\nu t] - \int_{-\infty}^{t} c(t,s)p(s)\exp[\nu s]\exp[-p(s)(t-s)]ds}{N(t)} , \qquad (26d)$$

where $\Delta c(t)$ is difference in consumption per capita between people being born and people dying at time t.

3 Steady State Dynamics

In the following, we will concentrate on the steady state dynamics of the economy. As the healthcare technology H is time-invariant, we know from the previous section that in steady state healthcare expenditures $h(s) = h$, mortality risk $p(s) = p$ and population size $N(t) = 1/(p + \nu)$ are constant. Then, the wage grows proportionally to the per capita stock of capital, i.e. $w(t) = (1 - \alpha)k(t)/(1 - h)$. As a consequence, the dynamics of the aggregate economy (26) reduces to a system of ordinary differential equations:

$$\dot{c}(t) = \sigma(r - \rho)c(t) - x(p)(p + \nu)k(t) , \qquad (27a)$$

$$\dot{k}(t) = (1 - \delta - \nu)k(t) - c(t) . \qquad (27b)$$

Note that this is a special case of Blanchard (1985), as we restrict attention to a special production technology (see Section 2.1). It is also a generalization, as we allow for arbitrary elasticities of intertemporal substitution σ and an exogenous growth rate ν of the cohort size.

Introducing the following abbreviations:

$$A = \sigma(r - \rho) , \qquad B = x(p)(p + \nu) , \qquad C = (1 - \delta - \nu) , \qquad (28)$$

we can write (27) as $\dot{c}(t) = Ac(t) - Bk(t)$ and $\dot{k}(t) = Ck(t) - c(t)$. We can now state the following proposition.

Proposition 2 (Steady State Dynamics)
In steady state, the economy possesses a unique balanced growth path, i.e. consumption and capital are growing at the same rate. The growth rate is positive if $B < AC$ and negative if $B > AC$. The economy is stationary if $B = AC$.

The proof is given in Appendix A.2. Note that we neglect trivial steady states where $c(t) = k(t) = 0 \ , \forall \, t$.

The proposition establishes that there is a unique balanced growth path for any given death probability p. The growth rate g can be calculated from the equations of motion. We will also refer to it as a function of p by $g^D(p)$. More precisely, we define

$$g^D(p) := \frac{1}{2}\left[A + C - \sqrt{(A-C)^2 + 4B}\right] \ . \tag{29}$$

In Section 2.3 we have shown that for every steady state growth rate g, there is a unique choice with respect to healthcare investments. As healthcare investments map one-to-one into death probabilities via the healthcare technology (1), we can perceive the household's choice of healthcare investments as a choice of its death probability. In this vein, we can rewrite the first order condition (23) as

$$F\big(p(s)\big) := p_0\psi\left[\frac{1}{p(s) - p_0(1-\psi)} + \frac{1}{r + p(s) - g} + \frac{1}{(1-\sigma)x\big(p(s)\big)}\right] = 0 \ . \tag{30}$$

In steady state, $F(p) = 0$ also defines a relationship between g and p. Solving for the steady state growth rate g, we refer to the first order condition (30) as $g^F(p)$.

The economy's steady state growth rate is then the solution to the system of these two equations, $g^D(p)$ and $g^F(p)$, or simply their intersection. The following proposition characterizes this intersection.

Proposition 3 (Steady state with positive healthcare investments)
In steady state, the economy grows on a balanced growth path with positive healthcare investments if $g^F(p_0) < g^D(p_0)$.

The proof is given in Appendix A.3.

In the proof we show that both functions, $g^D(p)$ and $g^F(p)$, are declining in the death probability p and that $g^D(p) < g^F(p)$ for the highest value of the death probability p. Thus, $g^F(p_0) < g^D(p_0)$ ensures that both functions intersect. Note that if the two functions intersect within the relevant interval, there exists one intersection at $p = \hat{p}$ for which

$$\left.\frac{\partial g^F(p)}{\partial p}\right|_{p=\hat{p}} < \left.\frac{\partial g^D(p)}{\partial p}\right|_{p=\hat{p}} , \tag{31}$$

holds. The following analysis will focus on this intersection.

We are particularly interested in how the steady state values of healthcare expenditures h, death probability p and growth rate g change with respect to an improvement in the healthcare technology. The healthcare technology (1) exhibits two parameters that influence the death probability p of the households. A decline in the parameter p_0 reduces the death probability that households face without any investments in healthcare. As an example think of hygienic standards in an economy, which surly affect average life expectancy without directly drawing on medical treatment. An increase in the parameter ψ increases the reduction in death probability that is purchased for any given healthcare investment h. This parameter directly reflects improvements in medical treatment.

The following proposition summarizes the comparative static results.

Proposition 4 (Comparative Statics)
In response to an increase in the productivity of healthcare investments (ψ), the economy will approach a steady state with a higher growth rate. The same is true for a decline of the death probability without medical treatment (p_0).
Formally, in steady state the following relationships hold:

$$\begin{aligned} &\frac{\partial p}{\partial p_0} > 0 \; , \qquad \frac{\partial g}{\partial p_0} < 0 \; , \\ &\frac{\partial p}{\partial \psi} < 0 \; , \qquad \frac{\partial g}{\partial \psi} > 0 \; . \end{aligned} \tag{32}$$

The proof is given in Appendix A.4.

Apart from the formal proof there is a kind of "Cobb-web" intuition to this result. Suppose the economy is in steady state with growth rate g and is suddenly hit by an improvement of ψ. Due to the first order condition this implies that for a given g the individual households would choose a higher expected lifetime, i.e. a lower value of p. According to $g^D(p)$ a lower value of p involves a higher growth rate. Given a higher growth rate the households would further reduce their death probability as $\frac{dg^F(p)}{dp} < 0$ and hence $\frac{dp}{dg} < 0$, and so on until the new intersection is reached. It follows from the properties of the functions $g^D(p)$ and $g^F(p)$, as given in the proof of Proposition 3, that this dynamic will converge. This illustrates that the economy approaches a steady state with a lower death probability and a higher growth rate in response to an increase in the productivity of healthcare investments.

Table 1 gives some intuition about the magnitude of the comparative static effects from an improvement in the healthcare technology. An important insight is that improvements in the healthcare technology hardly increase the growth rate, no matter whether

	$p_0 = 2.5\%$									
	$\nu = 0\%$					$\nu = 2\%$				
ψ	0.5	0.6	0.7	0.8	0.9	0.5	0.6	0.7	0.8	0.9
h	0	23.9%	42.2%	55.8%	66.4%	0	22.8%	41.3%	55.2%	65.9%
p	2.5%	2.14%	1.76%	1.38%	1.01%	2.5%	2.16%	1.78%	1.40%	1.02%
T	40	46.7	56.7	72.3	99.5	40	46.3	56.3	71.6	98.4
g	2.12%	2.15%	2.17%	2.20%	2.22%	2.00%	2.04%	2.08%	2.12%	2.15%

	$\psi = 0.7\%$									
	$\nu = 0\%$					$\nu = 2\%$				
p_0	2.5%	2.2%	1.9%	1.6%	1.3%	2.5%	2.2%	1.9%	1.6%	1.3%
h	42.2%	37.9%	30.9%	22.0%	9.1%	41.3%	36.4%	29.9%	20.9%	7.8%
p	1.76%	1.63%	1.49%	1.35%	1.22%	1.78%	1.64%	1.50%	1.37%	1.23%
T	56.7	61.5	67.1	73.9	82.17	56.3	61.0	66.5	73.2	81.4
g	2.17%	2.18%	2.19%	2.20%	2.21%	2.08%	2.10%	2.11%	2.12%	2.13%

Table 1: Steady state values for healthcare expenditures h, death probability p, average life expectancy T and growth rate g for different parameters of ψ (upper half) and p_0 (lower half), and for a population growth rate $\nu = 0$ (left half) and $\nu = 2\%$ (right half). The remaining parameters are set to $\alpha = 0.3$, $r = 3.5\%$, $\sigma = 1.5$ and $\rho = 2\%$.

ψ rises or p_0 declines. In the numerical example, the maximal change in the growth rate is observed for an increase in ψ and a population growth rate of $\nu = 2\%$ (upper right quadrant in Table 1). Although an increase from $\psi = 0.5$ to $\psi = 0.9$ results in drastic changes in the economy, as the healthcare expenditures rise from 0 to 65.9% of the wage level and the average life expectancy rises from 40 to 98.4 years, the steady state growth rate only rises from 2.0% to 2.15%, an increase of only 0.15%. The effect on the growth rate is even smaller for the other three scenarios. Our model thus indicates that improvements in healthcare technology may have a large impact on overall welfare, but this impact is hardly driven by economic growth but rather by improvements in the average life expectancy.

4 Transitional Dynamics

In the last section we have analyzed the effects of improvements in the healthcare technology on the steady state values of the economy. Our main results were that improvements in the healthcare technology have a significant impact on the households' healthcare expenditures, and thus on the death probability and average life expectancy.

However, the effect on the steady state values of the growth rate was almost negligible. In the following, we investigate the transition from an original steady state towards a new steady state when the healthcare technology exhibits a one shot improvement at time $t = 0$.

The transition dynamics of the model introduced in Section 2 is beyond analytical tractability. The main reason is that given a set of exogenous parameters, the household's choice of healthcare expenditures h depends on the net present value of the future wage income, which in turn depends on the aggregate stock of capital per capita in the economy. As the aggregate stock of capital does, in general, not grow at a constant rate, each cohort along the transition would choose a different level of healthcare expenditures h. As healthcare expenditures determine the death probability p, this in turn affects the level of household savings, which determine the aggregate stock of capital per capita, and thus the net present value of the future wage income. Formally, this can be seen from the system dynamics of the aggregate economy (26). Inserting equations (18c), (26c) and (26d) into the equations of motion for aggregate consumption per capita (26a) and aggregate capital per capita (26b) yields a system of integro-differential equations, with $p(s)$ given implicitly by equation (30). To circumvent these difficulties, we consider a *small open economy* setting.

4.1 Small Open Economy

We assume that the one shot improvement in the healthcare technology only affects a small economy, leaving the rest of the world unchanged. The economy's production factors capital and labor are fully mobile, but the country is too small to affect world prices. Consequently, both factors capital and labor earn the constant world interest rate r and the world market wage $w(t)$, which we assume to grow at the constant rate g, i.e. $w(t) = w_0 \exp[gt]$. From the remark of Proposition 1 we know that under these conditions all households choose the same unique level of healthcare expenditures h, which corresponds to a certain death probability p, irrespective of the time of birth s.

Moreover, for a fixed interest rate r, a wage level growing at the constant rate g and given healthcare expenditures h, we can directly solve for the optimal consumption and asset paths of the households born at time s. Introducing the abbreviation

$$y(p) = r + p - g \ , \tag{33}$$

we obtain

$$
\begin{aligned}
c(t,s) &= w_0(1-h)\frac{x(p)}{y(p)}\exp[gs]\exp[\sigma(r-\rho)(t-s)] \ , && (34a)\\
b(t,s) &= w_0\frac{1-h}{y(p)}\big(\exp[gs]\exp[\sigma(r-\rho)(t-s)]-\exp[gt]\big) \ . && (34b)
\end{aligned}
$$

Applying the aggregation rule (24) yields for the aggregate system dynamics:

$$
\begin{aligned}
c(t) &= (1-h)\frac{x(p)}{y(p)}\frac{p+\nu}{p+\nu+g-\sigma(r-\rho)}w(t) \ , && (35a)\\
b(t) &= \frac{1-h}{y(p)}\frac{\sigma(r-\rho)-g}{p+\nu+g-\sigma(r-\rho)}w(t) \ . && (35b)
\end{aligned}
$$

As $x(p) > 0$, $y(p) > 0$ and $1-h > 0$, both aggregate consumption and capital per capita are increasing over time if

$$
p+\nu > y(p)-x(p) = \sigma(r-\rho)-g > 0 \ . \tag{36}
$$

In the following, we assume this condition to hold. Then, for any given and constant death probability p, the aggregate per capita values of consumption and capital grow at the same constant rate g as the wage level.

4.2 Exogenous Shock in Healthcare Technology

Now, we assume that at time $t = 0$ there is an exogenous improvement in the healthcare technology. This implies that all households born at time $s < 0$ have chosen the same healthcare expenditures h^{old}, which corresponds to a death probability p^{old}, but all households born at time $s \geq 0$ choose a different level of healthcare expenditures h^{new}, which corresponds to the death probability p^{new}. For all improvements in the healthcare technology $p^{old} > p^{new}$ holds. However, if the healthcare technology has improved by a decline in p_0, then $h^{old} > h^{new}$, while $h^{old} < h^{new}$ for an increase in ψ. This implies the following relationships for individual households' consumption and asset holdings:

$$
\begin{aligned}
&\frac{\partial c(t,s)}{\partial \psi} < 0 \ , \qquad \frac{\partial b(t,s)}{\partial \psi} \lesseqqgtr 0 \ ,\\
&\frac{\partial c(t,s)}{\partial p_0} \lesseqqgtr 0 \ , \qquad \frac{\partial b(t,s)}{\partial p_0} < 0 \ .
\end{aligned}
\tag{37}
$$

Equally important as the individual changes in consumption and asset holdings are the demographic changes in the economy due to a decrease in the death probability, and thus an increase in expected lifetime. In fact, while the population before the shock was given by $\exp[\nu t]/(p^{old}+\nu)$ and was growing at the constant rate ν, after the shock we derive for the population

$$
\begin{aligned}
N(t) &= \int_{-\infty}^{0} \exp[\nu s] \exp[-p^{old}(t-s)] ds + \int_{0}^{t} \exp[\nu s] \exp[-p^{new}(t-s)] ds \\
&= \frac{\exp[-p^{old} t]}{\nu + p^{old}} + \frac{\exp[\nu t] - \exp[-p^{new} t]}{\nu + p^{new}} \ . \qquad (38)
\end{aligned}
$$

The first term relates to the population born before the shock at time $t = 0$ and the second term relates to the population born after the shock. As people born after the shock exhibit a lower death probability p^{new}, the economy experiences a period of increased population growth in the short run and returns to the rate ν in the long run. This can be seen from the population growth rate $g_N(t) = \dot{N}(t)/N(t)$ given by

$$
g_N(t) = \frac{p^{new}(\nu + p^{old}) \exp[-p^{new} t] - p^{old}(\nu + p^{new}) \exp[-p^{old} t] + \nu(\nu + p^{old}) \exp[\nu t]}{(\nu + p^{old})(\exp[\nu t] - \exp[-p^{new} t]) + (\nu + p^{new}) \exp[-p^{old} t]} \ . \qquad (39)
$$

The change in population dynamics needs to be considered when deriving aggregate per capita values. Taking account of the fact, that death probabilities change for households born before and after time $s = 0$, we obtain the following aggregation rule:

$$
\begin{aligned}
z(t) &= \frac{1}{N(t)} \Bigg[\int_{-\infty}^{0} z(t,s) \exp[\nu s] \exp[-p^{old}(t-s)] ds \\
&\quad + \int_{0}^{t} z(t,s) \exp[\nu s] \exp[-p^{new}(t-s)] ds \Bigg] \ . \qquad (40)
\end{aligned}
$$

Applying this rule to individual household's consumption and asset holdings yields for aggregate per capita consumption and capital for $t \geq 0$

$$
\begin{aligned}
c(t) &= \frac{w_0}{N(t)} \left[\frac{x(p^{old})}{y(p^{old})} \frac{1 - h^{old}}{p^{old} + \nu + g - \sigma(r - \rho)} \exp\left[-\big(p^{old} - \sigma(r - \rho)\big) t \right] \right. \qquad (41a) \\
&\quad \left. + \frac{x(p^{new})}{y(p^{new})} \frac{1 - h^{new}}{p^{new} + \nu + g - \sigma(r - \rho)} \Big(\exp[(\nu + g)t] - \exp\left[-\big(p^{new} - \sigma(r - \rho)\big) \right] \Big) \right] , \\
b(t) &= \frac{w_0}{N(t)} \left[\frac{1 - h^{old}}{y(p^{old})} \left(\frac{\exp\left[-(p^{old} - \sigma(r - \rho)) t \right]}{p^{old} + \nu + g - \sigma(r - \rho)} - \frac{\exp[-(p^{old} - g)t]}{p^{old} + \nu} \right) \right. \qquad (41b) \\
&\quad + \frac{1 - h^{new}}{y(p^{new})} \left(\frac{\exp[(\nu + g)t] - \exp\left[-(p^{new} - \sigma(r - \rho)) t \right]}{p^{new} + \nu + g - \sigma(r - \rho)} \right. \\
&\quad \left. \left. - \frac{\exp[(\nu + g)t] - \exp[-(p^{new} - g)t]}{p^{new} + \nu} \right) \right] .
\end{aligned}
$$

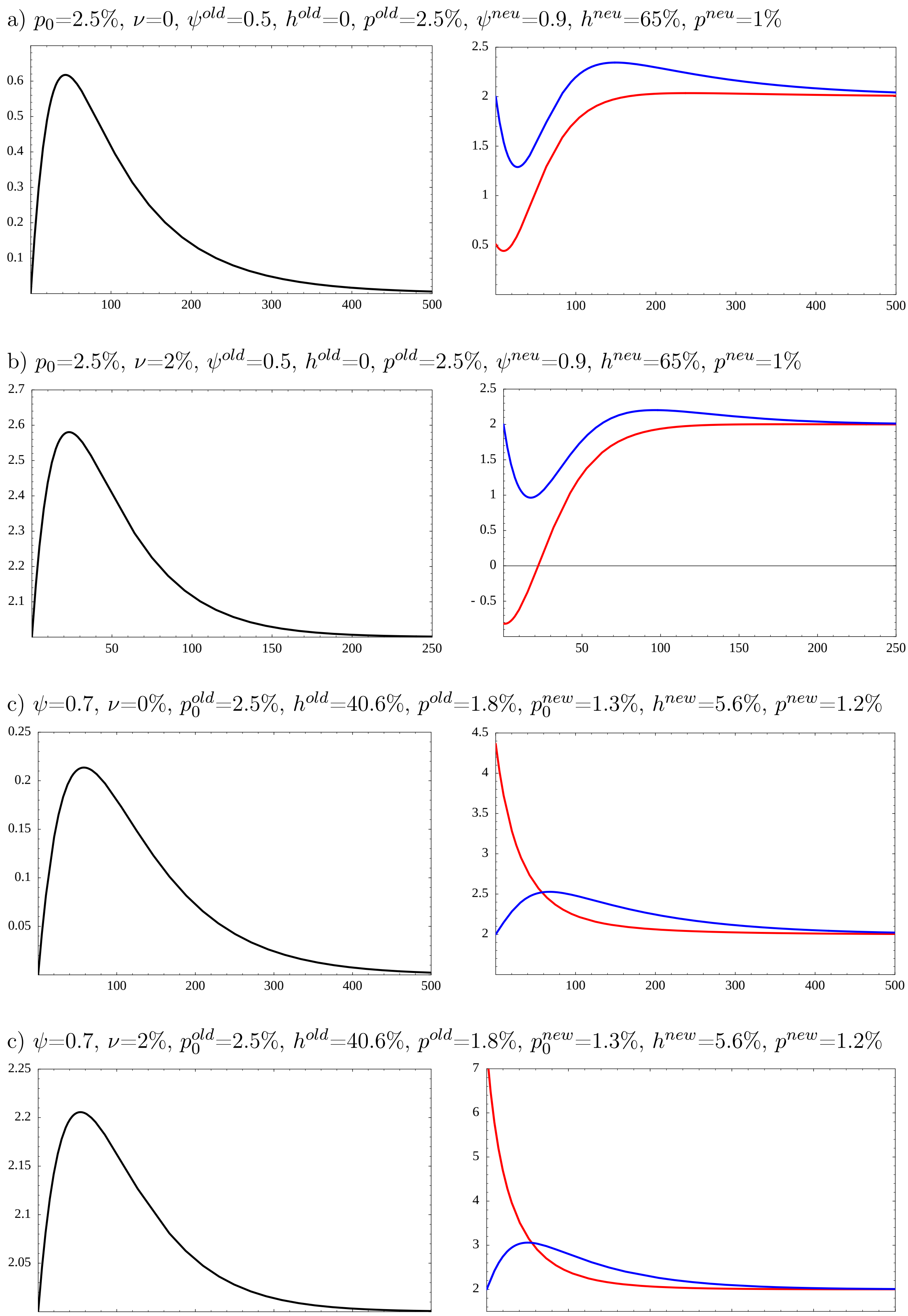

Figure 1: Population growth rates (black) and aggregate per capita growth rates of consumption (red) and capital (blue) during transition for four scenarios. The remaining parameters are set to $g = 3\%$, $r = 3.5\%$, $\sigma = 1.5$ and $\rho = 2\%$.

Figure 1 shows the population growth rates and the aggregate per capita growth rates of consumption and capital for an exogenous shock in the healthcare technology. Part a) and b) show the effect of an increase in ψ, while part c) and d) refer to a drop in p_0. In all cases $p^{new} > p^{old}$. This implies a positive level effect on population, as people born at time $t = 0$ and later exhibit a longer average lifetime compared to people born at $s < 0$. We also observe that the transition to the new population level is faster if the exogenously given growth rate ν of the cohort size is higher. This is because the percentage of people born after $t = 0$ grows faster the higher is the growth rate ν.

As aggregate capital is a stock variable, the transition occurs in a smooth way, i.e. the growth rate of aggregate capital per capita is continuous, starting at g, the level before the exogenous shock. This does not hold for aggregate consumption per capita which can freely adjust at the time of the exogenous shock $t = 0$. Whether aggregate per capita growth rates of consumption and capital go down or up during the transition ultimately depends on the individual household's behavior. From conditions (37) we know that household's consumption decreases for an increase in ψ and household's asset holdings increase for a drop in p_0. This is also reflected in the aggregate per capita growth rates. In part a) and b) the aggregate consumption per capita growth rate starts below its steady state value of $g = 2\%$ and converges back to it over time. In a similar vein the aggregate per capita growth rate of capital in part c) and d) shows an inverted U shape: starting from its steady state level it increases during the transition, peaks and converges back to the steady state growth rate of 2%. The transitional dynamics of the growth rate of aggregate capital per capita in part a) and b), and of the growth rate of aggregate consumption per capita in part c) and d) are ambiguous. The reason is that the signs of $\partial \left(\frac{1-h}{y(p)} \right) / \partial \psi$ and $\partial \left((1-h) \frac{x(p)}{y(p)} \right) / \partial p_0$ can be positive, zero and negative, depending on the whole set of exogenous parameters. We also observe that a positive exogenous growth rate of the cohort size ν pronounces the transitional deviations from the steady state growth rate. The reason is that for positive ν the share of people born after $s = 0$ grows faster in the economy giving them a stronger impact on the aggregate per capita values.

5 Discussion

In this section we discuss two issues. First, we clarify the difference between the small open economy and the closed economy. Second, how do our results relates to empirical evidence.

5.1 Small Open Economy vs. Closed Economy

In our model, the households' main trade-off is on how to allocate income between consumption, healthcare expenditures and capital investments. This trade-off is present in both settings, the closed economy and the small open economy. They only differ in that the wage rate in the small open economy does not react to a change in the lifetimes of the households, whereas in the closed economy it does.

In general, an increase in the productivity of the healthcare technology possesses the following effects on aggregate capital per capita. First, the expenditures of healthcare either increase or decrease in response to the shock, and thus either reduce or increase the disposable income for savings and consumption. Trivially, the direct effect of a reduction of healthcare expenditures on savings is positive (and vice versa). Second, the analysis shows that an increase in the productivity of the healthcare technology always leads to a longer lifetime of the individuals. This increases their incentive to save, and hence possesses a positive effect on aggregate capital per capita. Third, due to the longer lifetime of the households, population growth increases during the transition to the new steady state. This has a negative effect on aggregate per capita values. Fourth, a larger population implies that wages are decreasing leading to lower income, which negatively affects the decision on both healthcare expenditures and savings. Hence, the effect on aggregate capital per capita is ambiguous. On the one hand, lower healthcare expenditures dampen population growth leading to an increase in capital per capita. On the other hand, lower savings of the households decrease capital per capita.

By not affecting the growth rate of the wages, the small open economy excludes the last (second order) effect but includes all the direct effects (the first three effects of the previous list) of a shock to the healthcare technology on the growth rate of capital per capita. Hence, whether the small economy, in comparison to the closed economy, over- or underestimates the short run effects due to a change in healthcare technology is not clear and depends on the specific parameter values of the utility function and the production function. From a more technical perspective, as discussed in Section 4, the main difference between the closed economy and the small open economy is that in the latter the net present value of labor income over the whole lifetime can be calculated due to a fixed growth rate of the wage. As the steady state growth rates before and after the shock usually differ very little in magnitude, we are confident that the small open economy does not yield values that are totally off those in the closed economy. With this line of thinking, the difference should decline the lower is the households' rate of time preference.

Certainly, the small open economy setting imposes strong assumptions that do not match reality. However, for the overwhelming majorities of countries, the closed economy setting (which is used in much of the literature) does so, too. It is not obvious which of the two better approximates reality. Of course, this also depends on the particular country under consideration. With this in mind, we now discuss how the results in our paper relate to empirical evidence on the link between longevity and growth.

5.2 Empirical Evidence

In most of the empirical literature there seems to be a consensus that increases in life expectancy are associated with higher rates of per capita growth.[8] However, there is some suspicion that at least some of the studies suffer from endogeneity problems or an omitted variable bias. For this reason Acemoglu and Johnson (2007) used unprecedented improvements in life expectancy, i.e. shocks to mortality by disease, as an instrument for changes in life expectancy and find no or slightly positive effects on total GDP and even show negative effects on GDP per working age population if the shock in expected lifetime is large.

Comparing these findings with the predictions of our theory, we interpret the exogenous shock on expected lifetime as a positive shock to the healthcare technology (i.e. to ψ). For sufficiently large shocks, our model can well explain negative growth rates per capita over the time horizons under consideration in Acemoglu and Johnson (2007). Our theory predicts these negative effects to vanish in the long run.

During the transition, our model also reflects other observations of the Acemoglu and Johnson (2007) study. For example, that increased life expectancy has a composition effect on the population, as the fraction of the population under age 20 increases in their short run estimation. In line with our theory, this effect dies out in the long run sample. Further, we see Acemoglu and Johnson (2007)'s empirical result that countries

[8] See, for example, Gallup and Jeffrey (2001), Bhargava et al. (2001), Barro (1996), Boucekkine et al. (2007), Lorentzen et al. (2008). Only very few studies find small negative or no effects of longevity of growth such as Caselli et al. (1996). An overview can be found in Bloom et al. (2004).

with a larger decline in predicted mortality experienced higher population growth as a justification for neglecting fertility choices in our theoretical analysis. However, it would nevertheless be interesting to include endogenous fertility in future research.

Finally, our theory also accounts for the rise in healthcare expenditures over the last decades as reported by e.g. Hall and Jones (2007) and Lichtenberg (2004).

6 Conclusion

We introduced an overlapping generations endogenous growth model in continuous time to investigate the link between life expectancy, which is the result of endogenous investments in healthcare, and economic growth. We find that positive shocks in the healthcare technology increase both expected life expectancy and steady state growth rates of the economy. However, simulation results suggest that the magnitude of the latter effect is negligibly small, even for substantial improvements in the healthcare technology. Thus, welfare improvements due to improvements in the healthcare technology largely stem from an increase in average lifetime. In addition, we find that a positive shock on the healthcare technology may be followed by a substantial decline of the per capita growth rates of capital and consumption, which may even be negative in the short run.

Although our model is relatively simple, our results fit very well with the bulk of empirical evidence on the link between healthcare, life expectancy and economic growth. Nevertheless, it is a fruitful avenue for further research to incorporate more realistic assumptions such as age-dependent mortality, retirement decisions or endogenous fertility. In our model, improvements in the healthcare technology are modelled as exogenous shocks. Another challenge for future research is to endogenize these improvements.

Appendix

A.1 Proof of Proposition 1

The proof proceeds as follows: With $w(t)$ growing at a constant rate, we are able to solve all integrals in the first order condition. Then we show that in the interval $h(s) \in (0,1)$, $F\big(h(s)\big)$ will have at most two roots. As $\lim_{h(s)\to 1} F\big(h(s)\big) = -\infty$, we know that one of them must satisfy the sufficient condition for a maximum, i.e. $\frac{\partial F\big(h(s)\big)}{\partial h(s)} < 0$. This establishes existence. Uniqueness follows from the fact that $F\big(h(s)\big)$ is continuous in the interval $h(s) \in (0, 1)$. Thus, if there are two roots, the one with the smaller value of healthcare spending $h(s)$ must have $\frac{\partial F(h(s))}{\partial h(s)} > 0$, and hence is a utility-minimum to the household's problem.

Solving all integrals under the assumption that $w(t)$ grows at the constant rate g, the first order condition reads

$$F(h(s)) = \frac{-1}{1-h(s)} + p_0\psi\left[\frac{1}{r+p(s)-g} - \frac{1}{(1-\sigma)x\big(p(s)\big)}\right] = 0. \tag{A.1}$$

It is convenient to rewrite the first order condition in terms of $p(s)$. As our production function for longevity is a bijection of $p(s)$ and $h(s)$, solving for the optimal death probability $p(s)$ is equivalent to determining the optimal level of healthcare expenditures. Further, we use the abbreviation $p_{min} = p_0(1-\psi)$, which is the smallest death probability that can be achieved by spending the entire income for medical treatment.

$$F(p(s)) = p_0\psi\left[\frac{1}{p(s)-p_{min}} + \frac{1}{r+p(s)-g} + \frac{1}{(1-\sigma)x\big(p(s)\big)}\right] = 0. \tag{A.2}$$

After some minor manipulations, the first order condition writes

$$\begin{aligned} &\frac{\big[p(s)-p_{min}\big](1-\sigma)x\big(p(s)\big) - \big[p(s)-p_{min}\big]\big[r+p(s)-g\big]}{\big[p(s)-p_{min}\big]\big[r+p(s)-g\big](1-\sigma)x\big(p(s)\big)} \\ &\quad + \frac{\big[r+p(s)-g\big](1-\sigma)x\big(p(s)\big)}{\big[p(s)-p_{min}\big]\big[r+p(s)-g\big](1-\sigma)x\big(p(s)\big)} \;=\; 0\;. \end{aligned} \tag{A.3}$$

From the transversality condition we obtain $r + p(s) - g > 0$. $x\big(p(s)\big)$ is also assumed to be strictly positive, otherwise the household's lifetime utility would not be finite. We know further that $p(s) - p_{min} > 0$ for $h(s) < 1$. This implies that in the interval $p(s) \in (p_0, p_{min})$ and with $\sigma \neq 1$, the denominator of the first order condition is different from zero. Hence the optimal level of healthcare spending requires the numerator to become zero.

A close look reveals that the numerator is a quadratic function in $p(s)$ and thus possesses at most two roots in the interval (p_0, p_{min}). Now we can apply the arguments stated at the beginning of the proof: Due to the fact that $F_p(p(s))$ is continuous and approaches minus infinity for $p(s)$ approaching p_{min}, the root must be a maximum to the household's problem if there exists only one root. If there are two roots in the relevant range, the one involving the higher death probability will be a utility minimum and the one with the lower death probability a utility maximum.

Suppose now the first order condition cannot be satisfied in the interval $h(s) \in (0,1)$. In this case $\lim_{h(s)\to 1} F(h(s)) = -\infty$ implies that $F(h(s)) < 0$ in the whole interval. This indicates that the marginal utility costs of healthcare spending are higher than the marginal gains, and hence the corner solution $h(s) = 0$ is chosen. □

A.2 Proof of Proposition 2

We proof the existence of the balanced growth path (BGP) by contradiction. Suppose there is no BGP, but capital would grow at a constant rate g_k and consumption at the rate g_c, with $g_k \neq g_c$.

(1) If $g_c > g_k$, $\lim_{t\to\infty} \frac{k(t)}{c(t)} = 0$. From the equations of motion (27), we then obtain long run growth rates of $g_c = \sigma(\alpha - \delta - \rho)$ and $g_k = -\infty$. This cannot be a solution, as $k(t) \geq 0$ for all t.

(2) If $g_c < g_k$, $\lim_{t\to\infty} \frac{k(t)}{c(t)} = \infty$. This implies that in the long run, $g_k = 1 - \delta$ and $g_c = -\infty$. As $c(t)$ must be positive, this cannot be a steady state of the economy.

The only possibility that remains is a BGP with $g_c = g_k = g$.

To proof uniqueness and the second part of the proposition we calculate the growth rate g and show that under the given conditions it possesses the respective sign.

First, we solve the equations of motion for $\frac{c(t)}{k(t)}$ under the assumption that $g_c = g_k = g$. We obtain

$$\left(\frac{c(t)}{k(t)}\right)_{1/2} = \frac{1}{2}\left[C - A \pm \sqrt{(A-C)^2 + 4B}\right] \,. \tag{A.4}$$

As $B > 0$ for all $p > 0$, there is only one economically sensible solution (where $\frac{c(t)}{k(t)} > 0$), which is

$$\frac{c(t)}{k(t)} = \frac{1}{2}\left[C - A + \sqrt{(A-C)^2 + 4B}\right] \,. \tag{A.5}$$

This proofs the uniqueness of the BGP.

We now calculate the growth rate by inserting into $g = C - \frac{c(t)}{k(t)}$ and obtain

$$g = \frac{1}{2}\left[A + C - \sqrt{(A-C)^2 + 4B}\right] \ . \tag{A.6}$$

The growth rate on the BGP is positive if and only if $g > 0$. After some trivial manipulations, this condition yields $B < AC$. This also implies $g < 0$ if $B > AC$ and $g = 0$ if $B = AC$. □

A.3 Proof of Proposition 3

First, we show that the functions $g^D(p)$ and $g^F(p)$ exhibit the following properties:

$$\frac{\partial g^D(p)}{\partial p} < 0 \ , \qquad \frac{\partial g^F(p)}{\partial p} < 0 \ , \qquad g^D(p_{min}) < \lim_{p \to p_{min}} g^F(p) \ , \tag{A.7}$$

with $p_{min} = p_0(1-\psi)$.

(1) $\frac{\partial g^D(p)}{\partial p} < 0$ follows directly from taking the derivative of the function (29) and the fact that $\frac{\partial B}{\partial p} > 0$.

(2) The slope of function $g^F(p)$ is given by

$$\frac{\partial g^F(p)}{\partial p} = -\frac{\frac{\partial F(p(s))}{\partial g}}{\frac{\partial F(p(s))}{\partial p}}. \tag{A.8}$$

The second order condition requires $\frac{\partial F_p(p(s))}{\partial p} > 0$. Taking the partial derivative of $F_p(p(s))$ with respect to g by using (30) reveals that $\frac{\partial F(p(s))}{\partial g} > 0$. Hence $g^F(p)$ declines in p.

(3) For $g^D(p_{min})$ note that the term $B = x(p)(p + \nu)$ is finite for all $p \in [0,1]$. As A and C do not depend on the death probability p, $g^D(p_{min})$ approaches a finite value, as p decreases to p_{min}:

$$g^D(p_{min}) = \frac{1}{2}\left[A + C - \sqrt{(A-C)^2 + 4x(p_{min})(p_{min} + \nu)}\right] \ . \tag{A.9}$$

Solving the first order condition (23) for g and taking the limit $p \to p_{min}$, we obtain

$$\lim_{p \to p_{min}} g^F(p) = r + p_{min} \ . \tag{A.10}$$

Hence, the claim is that $g^D(p_{min}) < r + p_{min}$. Note that this condition must hold for all equilibrium values of p and g, otherwise the transversality condition cannot be satisfied. Rewriting the condition to

$$A + C - \sqrt{(A - C)^2 + 4x(p_{min})(p_{min} + \nu)} < 2(r + p_{min}) \ , \tag{A.11}$$

yields after some trivial manipulations

$$\sigma(r - \rho) < r + p_{min} \ . \tag{A.12}$$

This condition is equivalent to $x(p_{min}) > 0$, which is necessary for the individual's household's lifetime consumption to be finite and has already been assumed.

Second, the conditions (A.7) indicate that both functions are declining in the death probability p and that they will only intersect if the slope of $g^F(p)$ is sufficiently smaller than that of $g^D(p)$. This implies the statement of the proposition. □

A.4 Proof of Proposition 4

From equation (29) we see that $g^D(p)$ is independent of ψ for given p. This is not true for $g^F(p)$. Using the first order condition, we obtain

$$\frac{\partial p}{\partial \psi} = -\frac{\frac{\partial F_p(p)}{\partial \psi}}{\frac{\partial F_p(p)}{\partial p}} < 0. \tag{A.13}$$

That is, for any given growth rate g, the individual household would choose a lower death probability if the productivity of the healthcare system increased.

This argument can also be made directly via $g_F(p)$. Independent of ψ, $g_F(p)$ approaches $\alpha - \delta + p_{min}$ in the limit $p \to p_{min}$. Hence the ordinate intercept of $g_F(p)$ does not change. However, the slope $\frac{\partial g_F(p)}{\partial p}$ changes according to

$$\frac{d^2 g^D(p)}{dp d\psi} = -\frac{\frac{\partial^2 F_p(p)}{\partial p \partial \psi}\frac{\partial F_p(p)}{\partial g} - \frac{\partial^2 F_p(p)}{\partial g \partial \psi}\frac{\partial F_p(p)}{\partial p}}{\left(\frac{\partial F_p(p)}{\partial g}\right)^2}. \tag{A.14}$$

Using (30), we obtain $\frac{\partial^2 F_p(p)}{\partial g \partial \psi} = 0$, $\frac{\partial F_p(p)}{\partial g} > 0$, $\frac{\partial^2 F_p(p)}{\partial p \partial \psi} > 0$, and consequently $\frac{d^2 g}{dp d\psi} < 0$. This means that the negative slope of $g^F(p)$ becomes steeper in response to an increase in ψ.

As $g^D(p)$ is unaffected by a change in ψ and is declining in p, the steeper slope of $g^F(p)$ implies that the functions intersect at a lower value of p, which involves a higher steady state growth rate.

The proof with respect to a decline in the death probability without healthcare investment moves along the same arguments. The only difference is that the ordinate intercept of $g^F(p)$ declines as well. $\square$

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