Reputation and Competition in a Market for Differentiated Products

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Abstract

This paper shows how the presence of uninformed consumers in a market for differentiated products induces firms to exert costly effort. Product quality observed (privately) by a consumer depends both on firms' effort levels and her type, which is a number in the unit interval. If all consumers are informed about their type, high effort provision is an equilibrium outcome only if a positive mass of consumers has *exactly* the same type. This assumption is usually not fulfilled in models of product differentiation. Uninformed consumers, who only know a distribution over their possible types, may be willing to switch firms after observing bad quality. Given that some consumers are uninformed, firms provide high effort in each period of an equilibrium even if the distribution over types is continuous. Thus, uninformed consumers exert a positive externality on informed ones. Furthermore, we show that the same result may obtain if consumers behave *non-strategically*, i.e. if they only compare prices and expected payoffs without paying attention to the incentive problem.

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1 Introduction

In standard economic theory, market participants are perfectly informed about all relevant details of a transaction. While this assumption has been criticized as unrealistic for long

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time (especially by those outside economic science), just in recent years economists have started to take this criticism more serious. Now settings have been analyzed in which some consumers may be inattentive to prices and/or quality of products as, for example, in Albrecht et. al. (2002), Armstong and Chen (2007) and Armstrong et. al. (2008). Some consumers may not be able to detect hidden add-on prices as in Ellison (2005) or Gabaix and Laibson (2006). Some consumers may be uninformed about their utility function in the future, see Heidhues and Kőszegi (2008). Sometimes consumers do not understand that only worthless products are sold in the market, see Spiegler (2006). Finally, some consumers do not know their demand as in Courty and Li (2000), Matthews and Persico (2007) or Inderst and Peitz (2008).

In most of these models, firms exploit uninformed consumers. The effect of uncertain consumers on informed ones is unclear: at best informed consumers become subsidized as in Ellison (2005) and Gabaix and Laibson (2006). However, often they suffer from a negative externality as, for example, in Armstong and Chen (2007). In contrast to this literature, we demonstrate that the presence of uncertain consumers might be necessary to ensure efficient market outcomes, in particular if we consider a market for differentiated goods.

We will concentrate on the simplest model of product differentiation: two firms are located on the extremes of the unit interval (the "Hotelling street"). In each period, they can choose whether to exert high or low effort. Each consumer of a continuum can trade with one of these firms. She then privately observes the outcome (or quality). If the chosen firm has exerted high effort, the probability of observing a good outcome is determined by her "type" (her location on the "Hotelling street"), which is a number in the unit interval: the higher her type, the lower (higher) is the probability of observing a good outcome if she has purchased the product from the first (second) firm. If the chosen firm has exerted low effort, she observes a bad outcome for sure. The type of a consumer represents her preferences: the firm might have exerted high effort, but the consumer probably dislikes some of the good's attributes and therefore observes a bad outcome. In each period, a new cohort of consumers enters the market. They live for two periods. Thus, consumers can condition their action in the second period of their life on the quality of the good purchased in the previous period. Two examples may illustrate for which markets this construction is applicable:

1. A consumer has to choose whether to dine in an Italian or a Greek restaurant. She knows that she enjoys both types of food with positive probability (i.e. she does not like all Italian or all Greek dishes), but prefers the Italian cuisine and therefore chooses the Italian restaurant. Even if she knows her preferences, she may

be disappointed by the food served in this restaurant, either because she does not like the menu served at that particular day or because the chef exerted low effort.

2. The owner of a company has to choose between two consulting firms. The first one is relatively good in finding new markets and distribution channels, while the second one is relatively good in decreasing production costs. The owner knows the current needs of her company (which involve both activities) and chooses the consulting firm, which offers the highest expected increase in profits. If the realized increase in profits is low, this might be due to bad luck and the fact that the competence of the consultant does not perfectly fit all needs of that company, or due to low effort provision by the consulting firm.

One can easily construct more examples. In this framework, imperfect private monitoring creates a problem that is similar to the one in Holmström (1999): given that firms exert high effort in each period, it may not be rational for a consumer to purchase from another firm after observing a bad outcome: if products are differentiated, most consumers may strictly prefer one product to the other one. In equilibrium, firms know how consumers will react to bad outcomes. Therefore, if they expect that most consumers will purchase their product regardless of what their experience was in the first period of their life, they have no incentive to exert costly effort.

Consumers who *exactly* know their type are ready to switch firms if and only if they are *exactly* indifferent between the two goods. Therefore, it depends on the distribution of consumers over types, whether fully cooperative equilibria (in which firms exert high effort in each period) exist or not: in the first step, we show that if there is a positive mass of consumers with *exactly* the same type, then such an equilibrium exists given that the costs of high effort are sufficiently small. However, if the type is a unique number in the unit interval, it seems to be unlikely that two consumers have *exactly* the same type. Note that in all models of product differentiation, the distribution over types is continuous. We show that with a continuous distribution over types, firms never exert high effort in equilibrium, even if the costs of high effort are arbitrary small.

In the second step, we demonstrate how to resolve this problem. We will assume that there are two fractions of consumers: "informed" consumers who *exactly* know their type, and "uncertain" consumers who only know a distribution over their possible types. In the two examples from above, an uncertain consumer can be interpreted as follows: an inexperienced consumer who knows little about whether she prefers Italian or Greek food, or a owner of a firm who does not know *exactly* the firm's problems and needs (she only has a rough intuition).

If firms exert high effort in each period, uninformed consumers learn about their type by observing the outcome: if it has been good (bad), then the probability that she prefers goods of the previously chosen firm is also relatively high (low). Unlike informed consumers, uninformed ones may be ready to switch firms after observing a bad outcome (even if they would not do so if they knew their true type). Our first main result is that as soon as a positive mass of consumers is uninformed about their true type, fully cooperative equilibria again exist given that the costs of high effort are sufficiently small. We obtain equilibria where informed consumers rely on the fact that firms compete for uninformed consumers: prices must be set such that a substantial fraction of uninformed consumers buys from the same firm after observing high quality, and switches to the other firm after observing low quality. Otherwise, firms would not have an incentive to exert high effort and it would not be rational for consumers to trade with firms at positive prices.

For this result we need that all consumers behave strategically: they do not only compare prices and buy the product which offers the highest expected payoff to them, but they also stop buying at all if the price combination of the two firms no longer guarantees that firms exert high effort (as too few consumers would switch to another firm after observing a bad outcome). If consumers behave *non-strategically*, they act like in the one-shot Hotelling-game without moral hazard. Firms might have an incentive to cut prices in order to increase their market share. However, they will refrain from doing so if this triggers a price war, which wipes out all additional gains. Whenever prices remain stable over time and the distribution of uninformed consumers over expected types is continuous, low effort implies a loss of market shares in the next period. Therefore, our second main result is the following: given that the costs of high effort are sufficiently small and the distribution of uninformed consumers over expected types is continuous, fully cooperative equilibria may exist even if consumers behave *non-strategically*.

In this paper, we combine two structures: imperfect competition through product differentiation and imperfect private monitoring. Starting with Hotelling (1929), a large literature in economics spans around the question on how imperfect competition works and under what circumstances equilibria exist. The most prominent papers in this area are Salop (1979), Perloff and Salop (1985) and Caplin and Nalebuff (1991). Imperfect private monitoring has been the most important topic in the literature on infinitely repeated games, see Kondori (2002) for a introduction to this subject. At a later stage, we will discuss the relation of our findings to some important results in this literature.

To closest work to ours is Hörner (2002). He shows how to sustain cooperative equilibria through competition. If consumers become dissatisfied with their supplier of a product, they trade with another company. Thus, every firm has an incentive to produce high-quality products. In equilibrium, the only difference between firms is the level of their reputation: firms with little reputation charge low prices and firms with high reputation charge high prices. Hörner therefore solves three problems: (i.) how to provide incentives for firms to exert costly effort under imperfect private monitoring in (ii.) a perfectly competitive environment with free market entry, such that (iii.) firms with high reputation cannot profitably "milk" their reputation by exerting low effort. However, unlike in our approach, all consumers are *exactly* indifferent between firms. We therefore extend Hörner's analysis to a market for differentiated products.

The rest of the paper is organized as follows: in the next Section, we outline the framework of the model. In Section 3, we discuss the existence and non-existence of equilibria with high effort in each period and derive the main result. Section 4 demonstrates how to establish the same result if consumers behave *non-strategically*. Finally, the last Section concludes.

2 Framework of the model

Time is discrete and indexed by $t \in \{0, 1, ...\}$. In each period, two firms, A and B, may trade with a continuum of consumers. Consumers have unit demand. If a consumer traded with firm $j \in \{A, B\}$ in period t, she pays up front the price p_j^t and observes (privately) either a good, $j1$, or a bad outcome, $j0$, depending on the effort level exerted by her chosen firm. Denote by e_j^t the effort level of firm j in period t, which can be either high, $e_j^t = 1$, or low, $e_j^t = 0$, and is observed only by the firm itself. The costs of effort are given by c in case of high effort, and by 0 in case of low effort. The payoffs for a consumer associated with a good and a bad outcome are 1 and 0, respectively. Firms maximize the expected discounted sum of profits. The discount rate, $\delta \in (0, 1)$, is common to both firms and consumers.

2.1 Consumers

In each period, a new cohort of consumers of mass 1 is born. All cohorts are identical. Consumers live for two periods, i.e. there are "young" and "old" consumers in each period $t \geq 1$. Each consumer from a given cohort has an identity $i \in [0, 1]$ and a type (or preference) $r^i \in [0, 1]$. If consumer i trades with firm A in period t and $e_A^t = 1$, the probability of a good outcome for her is $(1 - r^i)$. If she trades with firm B and $e_B^t = 1$, the probability of a good outcome for her is r^i. In case of $e_j^t = 0$, the corresponding probabilities are 0. Let the distribution of consumers over types of each cohort be given by $F(r)$. We assume that no positive mass of consumers has type 0 or 1.[1]

There are two kinds of consumers, informed and uninformed ones. When born, consumer i only knows the distribution $G^i(r)$ over her possible types. If i is informed and has type $r^i = \hat{r}$, then

$$G^i(r) = \begin{cases} 1 & \text{if } r \geq \hat{r} \\ 0 & \text{if } r < \hat{r} \end{cases}. \tag{1}$$

If i is uninformed, then $G^i(r)$ is continuous and strictly increasing.[2] Denote

$$\tilde{r}^i = \mathbf{E}_{G^i}\left[r\right], \tag{2}$$

i.e. the expected type for a given distribution G^i. Consumers are Bayesian and update beliefs about their true type: if i purchases from firm A and she observes a good outcome, her expected type becomes

$$\tilde{r}^i_{A1} = \mathbf{E}_{G^i}\left[r \mid A1\right]. \tag{3}$$

If she observes a bad outcome and believes that A has exerted high effort, then her expected type is

$$\tilde{r}^i_{A0} = \mathbf{E}_{G^i}\left[r \mid A0\right]. \tag{4}$$

The same can be defined for purchases from firm B. One then can verify that

$$\tilde{r}^i_{A1} = \tilde{r}^i_{B0} \text{ and } \tilde{r}^i_{A0} = \tilde{r}^i_{B1}. \tag{5}$$

Thus, we characterize each consumer i by the collection $\{r^i, \tilde{r}^i, \tilde{r}^i_{A1}, \tilde{r}^i_{A0}\}$. For an informed consumer, these numbers are identical. For an uninformed consumer i, we have

$$0 < \tilde{r}^i_{A1} < \tilde{r}^i < \tilde{r}^i_{A0} < 1. \tag{6}$$

2.2 Strategies and equilibrium

The sequence of events in each period is as follows:

1. Young consumers are born.
2. Firms set prices.
3. Each consumer decides whether to purchase a good or not. If yes, she decides with which firm she wants to trade and pays the price.
4. Firms exert high or low effort.

[1] This ensures that for almost all consumers this is a game of private monitoring.

[2] This ensures that there is a density function $g_i(r) > 0$ for all $r \in [0, 1]$, such that the true type receives positive support. We do not make any assumption on the relation between r^i and $G^i(r)$.

5. Outcomes occur and payoffs are realized.

6. Old consumers die.

A young consumer i can condition her action on prices p_A^t, p_B^t, the period number t and her expected type $\tilde{r}^i$. An old consumer i can condition her action additionally on the outcome she observed in the last period, i.e. $A1$, $A0$, $B1$ or $B0$, if she has traded with a firm. Firms A and B observe in each period $t \geq 1$ the full history of prices $\left\{(p_A^0, p_B^0), ..., (p_A^{t-1}, p_B^{t-1})\right\}$ and can condition price and effort level on this information. An equilibrium of this game refers to a perfect Bayesian equilibrium. It is called fully cooperative, if each firm j chooses $e_j^t = 1$ in each period t, and non-cooperative, if each firm j chooses $e_j^t = 0$ in each period t.

2.3 Aggregate Uninformedness

In order to characterize the distribution of uninformed consumers, we define for some $\varepsilon \in (0, 1)$:

Definition 1 *Consumers are ε-uninformed if there is a $y < 1$ and a permutation*

$$\pi_\varepsilon : [0,1] \to [0,1], i \to i^*,$$

such that for each $i \in [0,1]$ with *$\pi_\varepsilon(i) \geq y$ we have $|\tilde{r}_{Al}^i - \tilde{r}^i| \geq \varepsilon$ for $l \in \{0,1\}$.*

If consumers are ε-uninformed, then there is a positive mass of consumers who substantially change the expectation about their type in a fully cooperative equilibrium, regardless of what outcome they observe. We say that all consumers are informed if r^i, $\tilde{r}^i$, $\tilde{r}_{A1}^i$ and $\tilde{r}_{A0}^i$ are identical numbers of all $i \in [0, 1]$.

For some our second main result we will need a stronger notion of aggregate uninformedness. Let $F(r, \tilde{r}, \tilde{r}_{A0}, \tilde{r}_{A1})$ be the joint distribution of consumers over types, expected types and expected types after outcomes, i.e. the share of consumers whose true type is smaller or equal to r, whose expected type (when young) is smaller or equal to $\tilde{r}$, whose expected type after observing $A0$ or $B1$ is smaller or equal to $\tilde{r}_{A0}$, and whose expected type after observing $A1$ or $B0$ is smaller or equal to $\tilde{r}_{A1}$. We now can define:

Definition 2 *Consumers are globally uninformed if from $F(r, \tilde{r}, \tilde{r}_{A0}, \tilde{r}_{A1})$ we can derive a bounded and continuous density function $f(r, \tilde{r}, \tilde{r}_{A0}, \tilde{r}_{A1})$ which is positive for all values $(r, \tilde{r}, \tilde{r}_{A0}, \tilde{r}_{A1})$ with $0 < \tilde{r}_{A1} \leq \tilde{r} \leq \tilde{r}_{A0} < 1$ and zero otherwise.*

If consumers are globally uninformed, then for each $\tilde{r} \in (0,1)$ there is a positive mass of consumers with $|\tilde{r}^i_{Al} - \tilde{r}| \geq \varepsilon$ for each consumer i of this mass and for some small $\varepsilon > 0$. Thus, if consumers are globally uninformed, they are also ε-uninformed for some positive ε.

3 Existence and non-existence of fully cooperative equilibria

3.1 The non-existence problem if all consumers are informed

We start by analyzing under what circumstances fully cooperative equilibria exist if all consumers are informed. Assume first that there is a $\bar{r} \in (0,1)$ and a positive mass of consumers $y_{\bar{r}}$ whose type is equal to $\bar{r}$. In this case, $F(r)$ is discontinuous at $\bar{r}$. In period t of a fully cooperative equilibrium, these consumers must trade with firm A if

$$(1-\bar{r}) - p^t_A > \bar{r} - p^t_B \geq 0, \tag{7}$$

and must trade with firm B if

$$\bar{r} - p^t_B > (1-\bar{r}) - p^t_A \geq 0. \tag{8}$$

They are indifferent between the two firms if and only if

$$(1-\bar{r}) - p^t_A = \bar{r} - p^t_B \geq 0. \tag{9}$$

Now pick two positive values p_A and p_B such that (9) holds if $p^t_A = p_A$ and $p^t_B = p_B$. Consider the following strategy of a consumer i:

- Given that $r^i \neq \bar{r}$: Trade in period t with firm A if $(1-r^i) - p_A > r^i - p_B$, $p^t_A = p_A$ and $p^t_B = p_B$. Trade in period t with firm B if $(1-r^i) - p_A < r^i - p_B$, $p^t_A = p_A$ and $p^t_B = p_B$. Otherwise, do not trade.
- Given that $r^i = \bar{r}$: When young, trade with firms A and B with equal probability if $p^t_A = p_A$ and $p^t_B = p_B$. Otherwise, do not trade. When old, choose the same firm as in the last period if the observed outcome has been good and $p^t_A = p_A$ and $p^t_B = p_B$. If it was bad and $p^t_A = p_A$ and $p^t_B = p_B$, choose the other firm. Otherwise, do not trade.

Given that the firms A and B exert high effort and charge the prices p_A and p_B in each period, this strategy is optimal for all consumers. Given that all consumers play according to this strategy, firm A faces the following trade-off: If $e^t_A = 0$, then in period

t it gains c, but its continuation value reduces by $\frac{1}{2}\delta y_{\bar{r}} p_A(1-\bar{r})$.For firm B, this value is given by $\frac{1}{2}\delta y_{\bar{r}} p_B \bar{r}$. Both terms are positive. Therefore, if

$$c < \frac{1}{2}\delta y_{\bar{r}} \min\left\{p_A(1-\bar{r}), p_B\bar{r}\right\}, \tag{10}$$

then no firm j can gain by exerting low effort or by choosing $p_j^t \neq p_j$. We have shown:

Proposition 1 *Suppose that all consumers are informed. If there is a positive mass of consumers with the same type* $\bar{r} \in (0,1)$ *and* c *is sufficiently small, then there is a fully cooperative equilibrium.*

Note that in any fully cooperative equilibrium there must be a mass of consumers who carry out a "punishment" after observing bad outcomes. This punishment is only rational for them if they are indifferent between the two firms. Thus, we always need a positive mass of consumers with *exactly* the same type to establish a fully cooperative equilibrium. From (10) we see that the critical value of c increases in δ, $y_{\bar{r}}$ and the prices p_A, p_B: the more consumers are of type $\bar{r}$ and the closer $\bar{r}$ is to 0.5, the more looses a firm which exerts low effort.

It is important to mention the relationship of Proposition 1 with some results in the literature on imperfect private monitoring: in Ely and Välimäki (2002) and Piccione (2002), cooperation in the infinitely repeated prisoner's dilemma with private monitoring is established with a strategy combination that keeps each player indifferent between cooperation and defection in each period. The play of each agent is a best response to each possible history of signals her opponent might have received. As a player's belief about her opponent's history is no longer needed to compute her optimal continuation strategy, equilibria with this property are called "belief-free", see Ely et. al. (2005). In our framework of differentiated goods, it is impossible to keep each player indifferent between her options. However, we use in Proposition 1 that at least a fraction of consumers is indifferent between the two firms. In equilibrium, a consumer does not have to worry about what all other consumers might have observed. Thus, we have derived belief-free equilibria.

As we argued in the introduction, a sizeable fraction of consumers with *exactly* the same type might not exist. In this case, $F(r)$ is continuous on the interval $[0,1]$. Informed consumers cannot credibly commit to carry out a punishment as long as they are not indifferent between both firms. We therefore can show:

Proposition 2 *Suppose that all consumers are informed. If there is no positive mass of consumers with the same type* $\bar{r} \in (0,1)$ *and* $c > 0$*, then any equilibrium is non-cooperative.*

Proof. Assume this is not the case and there is an equilibrium such that there is at least one period in which at least one firm exerts effort with positive probability. As consumers know the period number t, they can derive from the firms' strategies the probability μ_j^t with which a firm j exerts high effort in period t. There is no positive mass of consumers with the same type, therefore almost all consumers have a unique best response in each period: A consumer i purchases from A in period t if

$$\mu_A^t(1 - r^i) - p_A^t > \mu_B^t r^i - p_B^t \geq 0, \tag{11}$$

and purchases from B if

$$\mu_B^t r^i - p_B^t > \mu_A^t(1 - r^i) - p_A^t \geq 0, \tag{12}$$

and does not trade at all if

$$\max\left\{\mu_A^t(1 - r^i) - p_A^t, \mu_B^t r^i - p_B^t\right\} < 0. \tag{13}$$

For given consumers' behavior, a firm's continuation value is not affected by its effort-decision. With $c > 0$, each firm can gain by always exerting low effort. ■

Whenever $F(r)$ is continuous and all consumers are informed, firms exert low effort in each period of an equilibrium. We could hope for equilibria with high effort provision in some periods, if consumers were long-lived and we relax the assumption of conditional independence of outcomes: whenever a consumer observes a bad outcome and she knows that most likely many other consumers (with very similar type) also have observed a bad outcome, it could be rational for her to switch firms. Hence, a punishment-phase of low effort and no trade between a firm and (most) consumers could be started.[3] We will not pursue this approach any further. Instead, we concentrate on the case where not all consumers know *exactly* their type.

3.2 Existence of fully cooperative equilibria if some consumers are uninformed

We now show that the problem of non-existence of a fully cooperative equilibrium can be solved if some consumers are uninformed. In a fully cooperative equilibrium, uninformed consumers revise the value of their expected type after having purchased the good for the first time. For example, a consumer i who traded with firm A in period t updates the expectation about her type from $\tilde{r}^i$ to $\tilde{r}^i_{Al}$ if she observed Al, $l \in \{0, 1\}$. It follows from (6) that if prices p_A^t, p_B^t, p_A^{t+1}, p_B^{t+1} are such that

$$(1 - \tilde{r}^i) - p_A^t > \tilde{r}^i - p_B^t \geq 0, \tag{14}$$

[3] For more on private strategies in games of imperfect private monitoring, see Bhaskar and Obara (2002), Mailath and Morris (2002), Matsushima (2004) and Kandori and Obara (2006).

and

$$\tilde{r}^i_{A0} - p_B^{t+1} > (1 - \tilde{r}^i_{A0}) - p_A^{t+1} \geq 0, \tag{15}$$

and

$$(1 - \tilde{r}^i_{A1}) - p_A^{t+1} > \tilde{r}^i_{A1} - p_B^{t+1} \geq 0, \tag{16}$$

then in a fully cooperative equilibrium consumer i trades with firm A in period t and with firm B (A) in period $t+1$ given that $A0$ ($A1$) has been observed. Firm A will loose this consumer for sure if it exerts low effort in period t, but will keep her with positive probability if it exerts high effort. We now can establish the existence of fully cooperative equilibria:

Proposition 3 *Suppose that for some $\varepsilon > 0$ consumers are ε-uninformed. If c is sufficiently small, then there is a fully cooperative equilibrium.*

Proof. As consumers are ε-uninformed there exist $y < 1$ and a permutation π_ε with the property mentioned in Definition 1. From the well-ordering theorem it follows that there also exists a permutation $\bar{\pi}_\varepsilon$, such that (i.) for each $i \in [0,1]$ with $\bar{\pi}_\varepsilon(i) \geq y$ we have $|\tilde{r}^i_{Al} - \tilde{r}^i| \geq \varepsilon$ for $l \in \{0,1\}$ and (ii.) for $i, i' \in [0,1]$ with $\bar{\pi}_\varepsilon(i') > \bar{\pi}_\varepsilon(i) \geq y$ it holds that $\tilde{r}^i \leq \tilde{r}^{i'}$. Let $\bar{\pi}_\varepsilon^{-1}$ be the inverse of $\bar{\pi}_\varepsilon$.

Step 1. There exist values $y_1, y_2 \in (y, 1)$ with $y_1 < y_2$, such that for two consumers $i, i' \in [0,1]$ with $\bar{\pi}_\varepsilon(i), \bar{\pi}_\varepsilon(i') \in [y_1, y_2]$ it holds that $\left|\tilde{r}^i - \tilde{r}^{i'}\right| < \varepsilon$. To see this, assume that this claim does not hold. Define

$$\bar{k} = \min\{k \in \mathbb{N} : k\varepsilon > 1\} \tag{17}$$

and choose $n \in \mathbb{N}$ large enough such that

$$y + \frac{\bar{k}}{n}\varepsilon < 1. \tag{18}$$

Define further

$$y^{(0)} = y, \tag{19}$$

$$y^{(k)} = y + \frac{k}{n}\varepsilon. \tag{20}$$

Because of the second property of $\bar{\pi}_\varepsilon$, we must have

$$\tilde{r}^{\bar{\pi}_\varepsilon^{-1}(y^{(1)})} \geq \varepsilon. \tag{21}$$

Otherwise, we would have

$$\tilde{r}^{\bar{\pi}_\varepsilon^{-1}(y_2)} - \tilde{r}^{\bar{\pi}_\varepsilon^{-1}(y_1)} < \varepsilon \tag{22}$$

for $y_1 = y^{(0)}$ and $y_2 = y^{(1)}$ and the claim would hold. From induction it follows that

$$\tilde{r}^{\bar{\pi}_\varepsilon^{-1}(y^{(k)})} \geq k\varepsilon \tag{23}$$

for all $k \in \{2, ..., \bar{k}\}$ and therefore

$$\tilde{r}^{\bar{\pi}_\varepsilon^{-1}(y^{(\bar{k})})} > 1, \tag{24}$$

which of course is not possible. Thus, the claim must be true.

Step 2. Pick two values $y_1, y_2 \in (y, 1)$ with the property mentioned above. Define

$$y^* = \frac{y_1 + y_2}{2}. \tag{25}$$

Choose positive prices p_A and p_B, such that

$$\left(1 - \tilde{r}^{\bar{\pi}_\varepsilon^{-1}(y^*)}\right) - p_A = \tilde{r}^{\bar{\pi}_\varepsilon^{-1}(y^*)} - p_B > 0. \tag{26}$$

This is possible as $\tilde{r}^{\bar{\pi}_\varepsilon^{-1}(y^*)} \in (0, 1)$. Consider the following strategy combination:

- Each consumer i purchases in period t the good which maximizes her expected period payoff (if both offer the same expected payoff, she chooses each good with equal probability) for given belief about her type if and only if $p_A^t = p_A$ and $p_B^t = p_B$. Otherwise, she does not trade.

- Each firm j exerts high effort an charges the price $p_j^t = p_j$ in each period t.

We show that these strategies support an equilibrium: clearly, consumers cannot gain by deviating and firms cannot gain by charging prices different from p_A or p_B, respectively. A young consumer i with $\bar{\pi}_\varepsilon(i) \in [y_1, y^*]$ trades with firm A at least with probability $\frac{1}{2}$, and a young consumer i with $\bar{\pi}_\varepsilon(i) \in [y^*, y_2]$ trades with firm B at least with probability $\frac{1}{2}$. An old consumer i with $\bar{\pi}_\varepsilon(i) \in [y_1, y_2]$ will purchase product A if she observed $A1$ or $B0$, and will purchase product B if she observed $A0$ or $B1$. Let r_A be defined by

$$F(r_A) = 1 - \frac{y_2 - y_1}{2}. \tag{27}$$

As no positive mass of consumers has type $r = 1$, we have that $r_A < 1$. At least half of those consumers i with $\bar{\pi}_\varepsilon(i) \in [y_1, y^*]$ have a type $r^i \leq r_A$. Similarly, let r_B be defined by

$$F(r_B) = \frac{y_2 - y_1}{2}. \tag{28}$$

As no positive mass of consumers has type $r = 0$, we have that $r_B > 0$. At least half of those consumers i with $\bar{\pi}_\varepsilon(i) \in [y^*, y_2]$ have a type $r^i \geq r_B$. If a firm A exerts low effort in period t her continuation value decreases by at least

$$\frac{1}{2}\delta\left(\frac{y_2 - y_1}{2}\right) p_A \left(1 - \frac{1}{2} r_A\right), \tag{29}$$

regardless of what happens thereafter. For B the corresponding value is given by

$$\frac{1}{2}\delta\left(\frac{y_2 - y_1}{2}\right) p_B \left(\frac{1}{2} r_B\right). \tag{30}$$

Therefore, each firm exerts high effort in each period if

$$c < \frac{1}{2}\delta \left(\frac{y_2 - y_1}{2}\right) \min \left\{ p_A \left(1 - \frac{1}{2} r_A\right), p_B \left(\frac{1}{2} r_B\right) \right\}. \tag{31}$$

By construction, the term on the right-hand side is positive. Thus, there is a fully cooperative equilibrium if c is sufficiently small. ■

Most importantly, this result shows that there must not be a positive mass of consumers with *exactly* the same type (as in Proposition 1) and/or the same information: each two consumers i', i'' may differ in their types $r^{i'}$, $r^{i''}$ and in their information $\{\tilde{r}^{i'}, \tilde{r}^{i'}_{A0}, \tilde{r}^{i'}_{A1}\}$, $\{\tilde{r}^{i''}, \tilde{r}^{i''}_{A0}, \tilde{r}^{i''}_{A1}\}$. We therefore obtain belief-free equilibria, which are robust to small changes in consumers' types and information. Note that informed consumers rely on the fact that firms compete for those uninformed consumers who would switch to another firm after observing a bad outcome. Thus, uninformed consumers exert a positive externality on informed ones.

In the equilibria of Proposition 3, firms are restricted in their ability to influence market shares by price-setting in a fully cooperative equilibrium: prices must be set such that (14), (15) and (16) hold for a positive mass of consumers. Otherwise, they probably do not have an incentive to exert high effort. As consumers know that, they must stop purchasing products if the proportion of prices does no longer guarantee the provision of high effort. In the next section, we will deal with consumers who are not capable to exercise this punishment.

Clearly, the higher is δ, the higher can be c. Also the extent of aggregate uninformedness influences the critical value of c: the more consumers with similar beliefs about their type are ready to switch to another company (in case of a bad outcome), the more looses the firm if it exerts low effort. Furthermore, the critical value of c increases in $\min\{p_A, p_B\}$: c can be relatively high if for those young consumers, who would switch to another firm after a bad outcome, the expected type is around 0.5. Finally, the relation between the true type r^i and the distribution over types G^i of uninformed consumers is important: if consumer i is young and supposed to switch to firm B (A) after trading with firm A (B) after observing a bad outcome, she will do so with very high probability if her true type is close to 1 (0). The loss to firm A (B) from exerting low effort is small if most uninformed consumers have a type close to 1 (0). Thus, the critical value of c is relatively high if the true type of most uninformed consumers is around 0.5.

If consumers were long-lived and could purchase goods in more than two periods, they could have more opportunities to learn about their true type. They may also switch firms in later periods after bad outcomes. However, with more observations arriving, the change in the expected type decreases after each observation: the probability that a relatively old uninformed consumer i switches to another firm after observing a bad outcome may be smaller than the probability that a young consumer i switches to another firm, in particular if the true type r^i is close to 0 or 1. Therefore, it is important for the robustness for our result that a substantial amount of young, uninformed consumers enters the market in each period.

4 Less sophisticated consumers

To establish fully cooperative equilibria, we implicitly made an assumption that deserves some discussion: consumers stop trading in a period t if the proportion of prices p_A^t and p_B^t no longer guarantees that firms exert high effort. One may argue that most consumers are less sophisticated: they only compare prices and expected payoffs without paying attention to the incentive problem. However, if consumers always act as in standard models of product differentiation, a firm might have an incentive to lower its price in order to increase market shares. Prices then might be such that it does not pay off for firms to exert high effort: probably, too few consumers would switch firms after observing bad outcomes. We define:

Definition 3 *Consumers are "non-strategic" if they purchase the good, which offers the highest expected payoff for them (assuming that both firms exert high effort).*

In this section, we deal with *non-strategic* consumers and suggest a solution for this problem. Throughout the section, we will assume that consumers are globally uninformed. Using this property, we first derive the distribution of consumers over expected types, which is the joint distribution of young and old consumers over expected types in all periods $t \geq 1$ if all young consumers trade in each period and firms always exert high effort. We then find an equilibrium strategy for firms, such that in each period (i.) all consumers trade with firms and (ii.) it pays off for firms to exert high effort.

4.1 The distribution of consumers over expected types

If consumers are globally uninformed, we can define

$$f^o(r^*) = \int_0^1 f(r, \tilde{r}_{A0} = r^*) r dr + \int_0^1 f(r, \tilde{r}_{A1} = r^*)(1-r) dr, \quad (32)$$

where

$$f(r,\tilde{r}_{A0}) = \int_0^1\int_0^1 f(r,\tilde{r},\tilde{r}_{A0},\tilde{r}_{A1})d\tilde{r}d\tilde{r}_{A1}, \tag{33}$$

$$f(r,\tilde{r}_{A1}) = \int_0^1\int_0^1 f(r,\tilde{r},\tilde{r}_{A0},\tilde{r}_{A1})d\tilde{r}d\tilde{r}_{A0}. \tag{34}$$

Thus, $f^o(r^*)$ is the density of old consumers over expected types if all of them traded with firms and firms exerted high effort in the previous period. Furthermore, define

$$f^y(r^*) = \int_0^1\int_0^1\int_0^1 f(r,\tilde{r}=r^*,\tilde{r}_{A0},\tilde{r}_{A1})drd\tilde{r}_{A0}d\tilde{r}_{A1}, \tag{35}$$

i.e. the density of young consumers over expected types. The distribution of young consumers over expected types is

$$\hat{F}^y(r^*) = \int_0^{r^*} f^y(r)dr, \tag{36}$$

and the distribution of consumers over expected types then is given by

$$\hat{F}(r^*) = \frac{1}{2}\int_0^{r^*} \left(f^y(r) + f^o(r)\right)dr. \tag{37}$$

Let consumers now behave *non-strategically*: a young consumer i trades with firm A in period t if and only if

$$(1-\tilde{r}^i) - p_A^t \geq \tilde{r}^i - p_B^t \geq 0, \tag{38}$$

and with firm B if and only if

$$\tilde{r}^i - p_B^t > (1-\tilde{r}^i) - p_A^t \geq 0. \tag{39}$$

An old consumer i does the same for given expected type $\tilde{r}_{A0}^i$ or $\tilde{r}_{A1}^i$. Given that $p_B^t \leq 1-p_A^t$ and both firms exert effort in each period, A's profit in period $t \geq 1$ is given by

$$2\hat{F}\left(\frac{1-p_A^t+p_B^t}{2}\right)p_A^t - c, \tag{40}$$

while B's profit in period $t \geq 1$ is

$$2\left(1-\hat{F}\left(\frac{1-p_A^t+p_B^t}{2}\right)\right)p_B^t - c. \tag{41}$$

Given that $p_B^t > 1-p_A^t$, these numbers are $2\hat{F}\left(1-p_A^t\right)p_A^t - c$ and $2\left(1-\hat{F}\left(p_B^t\right)\right)p_B^t - c$, respectively.

4.2 Fully cooperative equilibria with *non-strategic* consumers

We now suggest the following strategy of firms: they share the market equally by charging a price $p_A^t = p_B^t = 0.5$ in each period t, such that all consumers trade with one firm. Therefore, the distribution of consumers over expected types in period $t = 0$ is given by $\hat{F}^y(r^*)$, and for $t \geq 1$ by $\hat{F}(r^*)$. Whenever a firm changes its price to increase its market share, the other firm starts a price war for finite time, which wipes out all potential gains from this deviation. Nevertheless, prices remain positive during this price war: given that consumers are globally uninformed and c is sufficiently low, each firm then still has an incentive to exert high effort.

The assumption that a firm can carry out a punishment for "wrong" pricing decisions of its rival is more realistic than the assumption that consumers can do that. In order to establish these equilibria, we make an assumption on the distributions over expected types $\hat{F}^y(r^*)$ and $\hat{F}(r^*)$:

Assumption (**A**) *$\hat{F}^y(r^*)$ and $\hat{F}(r^*)$ are such that (i.) $\hat{F}^y(0.5) = \hat{F}(0.5) = 0.5$ and (ii.) the density of both distributions is strictly increasing in the interval $[0, 0.5)$ and strictly decreasing in the interval $(0.5, 1]$.*

In particular, (**A**) ensures that no firm can increase its profits by increasing its price: the expected type of most consumers is around 0.5. Those consumers would no longer trade with a firm if prices are larger than 0.5. (**A**) can be made even less restrictive: it is not necessary that $\hat{F}^y(0.5)$ or $\hat{F}(0.5)$ is exactly equal to 0.5. The density of $\hat{F}^y(r^*)$ or $\hat{F}(r^*)$ must not necessarily increase (decrease) everywhere in the interval $[0, 0.5)$ ($(0.5, 1]$). However, in its recent form (**A**) simplifies substantially the proof of our second main result:

Proposition 4 *Suppose that consumers are globally uninformed. If* (**A**) *holds, δ is sufficiently high and c is sufficiently low, then there is a fully cooperative equilibrium in which consumers behave non-strategically.*

Proof. see Appendix. ■

In these collusive equilibria, the critical value of c again increases in δ and depends on the distribution of consumers over types, expected types and expected types after outcomes: the more consumers there are whose expected type $\tilde{r}$ is below (above) 0.5, whose expected type after outcomes is above (below) 0.5 and whose real type is close to 0.5, the higher is the loss of firm A (B) after exerting low effort.

5 Conclusion

This paper explains why it may be important to have uninformed consumers in a market for differentiated products. If the distribution over types is continuous, all consumers *exactly* know their type and privately observe the quality of the purchased good there is no way to provide incentives for firms to exert high effort in each period. The market outcome would always be inefficient. If there are uninformed consumers who do not know *exactly* their type, but a distribution over possible types, firms can compete for these consumers. Unlike perfectly informed consumers, uninformed ones may trade with another firm after observing bad quality. Given that the costs of high effort are sufficiently small, competition for uninformed consumers is a sufficient commitment to provide high quality. An efficient market outcome then is restored.

We then discussed what is necessary to make this mechanism work. It is probably not a good idea to trust on the sophistication of consumers: they probably do not punish firms by not buying their products whenever prices are such that it no longer pays off for firms to produce high-quality goods. We showed how collusion between firms could solve this problem: if a price war is started after "wrong" pricing decisions, firms refrain from changing their prices. Given that prices are constant over time, the threat that uniformed consumers switch firms after observing bad quality again exists and induces firms to exert high effort.

In order to keep our analysis simple, we left out some important issues. For example, we did not discuss the location choice of firms. In standard models of product differentiation, the optimal location of a firm depends on the distribution of consumers over types as well as on the location of its rival. Recall that in our model the distribution over types was four-dimensional, as there were young and old consumers, and the old ones had potentially different expectations about their type. Thus, including location choice in our framework is not straightforward and can be a topic of future research.[4]

Our paper is supposed to be a contribution to the question of how competition supports the provision of high quality products and efficiency. We emphasized that in the complex situation of private monitoring, there are efficient equilibria even if agents' information and sophistication is severely limited. This is in stark contrast to Hörner (2002) who says on the same issue that "customer competence [...] is an important ingredient". Future research may identify more of these market structures.

[4] Papers that discussed the existence of equilibria in Hotelling-games with location choice and price competition are Neven (1986), Tabuchi and Thisse (1995) and Anderson et. al. (1997). These results are potentially helpful for extending the present model by location choice.

6 Appendix: Proof of Proposition 4

If c is sufficiently small, we can choose a $p_{low} > 0$ such that

$$2p_{low} < 0.5 - c \tag{42}$$

If δ is sufficiently close to 1, we can choose a $T \in \mathbb{N}$ such that

$$\frac{1-\delta}{1-\delta^{T+1}} < \frac{0.5 - p_{low}}{0.5 + c} \tag{43}$$

and

$$\frac{1-\delta}{\delta^{T} - \delta^{2T}} < \frac{0.5 - c}{p_{low} + c}. \tag{44}$$

Consider the following strategy of firm $j \in \{A, B\}$:

- Exert high effort in each period. Charge $p_j^0 = 0.5$ and start Phase I in period 1.
- Phase I: Charge $p_j^t = 0.5$ if $p_j^{t-1} = p_{-j}^{t-1} = 0.5$. Otherwise, start Phase II in period t.
- Phase II: If Phase II starts in period t, charge $p_j^\tau = p_{low}$ in all periods $\tau \in \{t, ..., t+T-1\}$. Then, if $p_j^\tau = p_{-j}^\tau = p_{low}$ in all of these periods, charge $p_j^{t+T} = 0.5$ and start Phase I in period $t+T+1$. Otherwise, start Phase II again in period $t+T$.

By using the one-stage deviation principle, we show that this strategy can support a fully cooperative equilibrium in all periods $t \geq 1$. For period 0 (when there is only one cohort of consumers), (**A**) ensures that the same reasoning applies.

Assume first that there was no deviation from this strategy up to period t. If firm A complies to this strategy in period t, its normalized payoff is

$$(1-\delta)(0.5-c) + \delta(0.5-c). \tag{45}$$

If firm A charges $p_A^t \neq 0.5$ in period t, it follows from (**A**) that A's period payoff is less than 1. In this case, A's normalized payoff is at most

$$(1-\delta) + \left(\delta - \delta^{T+1}\right)(p_{low} - c) + \delta^{T+1}(0.5 - c). \tag{46}$$

If A charges $p_A^t = 0.5$ and exerts low effort, it loses a positive mass of consumers in period $t+1$: as consumers are globally uninformed, there is a positive mass of consumers $y_{\tilde{r}}^A$ with (i.) $\tilde{r}^i \in (0.5-\varepsilon, 0.5)$, (ii.) $r^i \geq \frac{1}{2}$ and (iii.) $|\tilde{r}_{Al}^i - \tilde{r}^i| \geq \varepsilon$ for $l \in \{0,1\}$, for each consumer i of this mass and for some $\varepsilon > 0$. By exerting low effort, firm A looses more than $\frac{1}{4}\delta y_{\tilde{r}}^A$. Therefore, (43) implies that A prefers to comply to the strategy if c is sufficiently low and δ is sufficiently high.

Assume now that Phase II has started in period t. Consider a period $\tau \in \{t, ..., t+T-1\}$. If A charges a price p_A^τ with $1 - p_A^\tau \geq p_{low}$, A's period payoff in period τ is at most

$$G^A(p_A^\tau, p_{low}) = 2\hat{F}\left(\frac{1 - p_A^\tau + p_{low}}{2}\right) p_A^\tau. \tag{47}$$

From (**A**) it follows that $G^A(p_{low}, p_{low}) = p_{low}$ and $G^A(p_A^\tau, p_{low}) < 2p_{low}$ for a p_A^τ with $1 - p_A^\tau \geq p_{low}$. (42) ensures that $G^A(p_A^*, p_{low}) < 0.5 - c$, where p_A^* is the price which maximizes (47). If A charges a price p_A^τ with $1 - p_A^\tau < p_{low}$, A's period payoff in τ is at most

$$G^A(p_A^\tau, p_{low}) = 2\hat{F}\left(1 - p_A^\tau\right) p_A^\tau. \tag{48}$$

From (**A**) we then get that

$$\begin{aligned} \frac{\partial G^A(p_A^\tau, p_{low})}{\partial p_A^\tau} &= 2\hat{F}\left(1 - p_A^\tau\right) - 2p_A^\tau \hat{f}\left(1 - p_A^\tau\right) \\ &= 2\int_0^{1-p_A^\tau} \hat{f}(x)dx - 2p_A^\tau \hat{f}\left(1 - p_A^\tau\right) \\ &< 2\int_0^{1-p_A^\tau} \hat{f}(1 - p_A^\tau)dx - 2p_A^\tau \hat{f}\left(1 - p_A^\tau\right) \\ &= 2\left(1 - p_A^\tau\right)\hat{f}(1 - p_A^\tau) - 2p_A^\tau \hat{f}\left(1 - p_A^\tau\right) < 0, \end{aligned} \tag{49}$$

where

$$\hat{f}(r) = f^y(r) + f^o(r). \tag{50}$$

Therefore, we have $G^A(p_A^\tau, p_{low}) < G^A(p_{low}, p_{low})$ for all p_A^τ with $1 - p_A^\tau < p_{low}$. Furthermore, it holds that $G^A(p_{low}, p_{low}) = p_{low} < 0.5 - c$. Thus, if A complies to the strategy in period τ, its normalized payoff is

$$\left(1 - \delta^{T-(\tau-t)}\right)(p_{low} - c) + \delta^{T-(\tau-t)}(0.5 - c). \tag{51}$$

If firm A charges $p_A^\tau \neq p_{low}$, A's normalized payoff is at most

$$(1 - \delta)2p_{low} + \left(\delta - \delta^{2T-(\tau-t)}\right)(p_{low} - c) + \delta^{2T-(\tau-t)}(0.5 - c). \tag{52}$$

If A charges $p_A^\tau = p_{low}$ and exerts low effort, it looses a positive mass of consumers in period $\tau + 1$ such that A's profits decrease. Therefore, (44) implies that A prefers to comply to the strategy if c is sufficiently low and δ is sufficiently high. As firm B faces the same trade-offs, it has no incentive to deviate from the proposed strategy.

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