Knowledge and Growth in the Very Long-Run

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Abstract. This paper proposes a theory for the evolution of knowledge diffusion and growth over the very long run. A feedback mechanism between capital accumulation and knowledge spillovers creates a unified growth theory that explains a long epoch of (quasi-) stasis and an epoch of high growth linked by gradual economic take-off. It is shown how the feedback mechanism can explain the Great Divergence, the failure of less developed countries to attract capital from abroad, the productivity slowdown in fully developed countries, and why R&D effort, TFP growth, and income growth are jointly rising along the transition towards modern growth. Finally, it is explained how a First Industrial Revolution, brought forth by increasing propositional knowledge, triggered a Second Industrial Revolution from which onwards technological progress was increasingly produced by market R&D activities.

Keywords: Endogenous Growth, Knowledge Spillovers, R&D, Globalization, Unified Growth Theory, Productivity Slowdown, Roaring Twenties.
JEL: O10, O30, O40, E22.

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1. INTRODUCTION

All standard models of endogenous growth rely on knowledge spillovers of some form in order to generate perpetually rising productivity. The strength (i.e. the production elasticity) of knowledge spillovers is usually assumed to be a constant with some debate about whether the constant is exactly one or smaller than one (Jones, 1995, Ha and Howitt, 2007).

The present paper instead allows for a variable strength of knowledge spillovers. More specifically, it is assumed that capital-embodied technological change improves the transmission of knowledge by reducing the effective distance between firms. This way capital accumulation (more horses, trains, cars, airplanes etc.) improves the appropriation of knowledge. More knowledge spillovers lead to higher factor productivity which in turn triggers even more investment and higher growth.

In lack of a better word the improving contact of economic agents that manifests itself in a higher degree of knowledge spillovers will be called globalization, acknowledging that a more widespread diffusion of technology is only one of the many facets of globalization. Globalization is, like economic growth, understood as a process. Yet unlike growth, globalization has a well defined end, namely when the economy is fully globalized. We can thus measure how far globalization has already progressed in an interval running from 0 to 1. This measure will be called the degree of globalization g.

In retrospect any economy has started out at or close to a state where $g = 0$. In the initial state economic agents (firms) were at most connected with their nearest neighbor(s) and the economy could be conceptualized as a regular network. As time proceeds and the economy develops, g converges towards an upper bound, at most towards 1, i.e. towards a state where all agents (firms) are connected with each other.

Standard models of endogenous growth (built on Romer 1986, 1990) usually fail to capture the notion of globalization as *a process*, i.e. the fact of *increasing* diffusion of technology starting out at no or little globalization at a time before the era of modern growth. Instead it is assumed that once discovered knowledge spills over completely (or at a time-invariant degree) across the whole economy. In other words, standard models are built to explain knowledge diffusion in a fully globalized economy. It is thus no wonder that they fail to get the historical evolution of income and factor productivity right. They either produce no adjustment dynamics or they predict that growth of income and productivity are at their highest rate initially, i.e. when the level of income is low, and that growth rates are subsequently decreasing.

In fact, however, high income growth and globalization are quite recent phenomena, in particular when one is concerned with a very long-run perspective. One or two thousand years ago the world was certainly much less globalized and income per capita was not visibly growing. Between the epoch of stasis and the epoch of high growth lies a phase of take off: the Industrial Revolution and the onset of globalization, which interestingly occurred roughly at the same time between the mid-18th and the mid-19th century (Mokyr, 2005; O'Rourke and Williamson, 2002, 2005).

While there is still a debate about how large income and productivity growth exactly were during the Industrial Revolution, economic historians unanimously emphasize that the take off was *gradual* (e.g. Crafts, 1995, 2004, Temin, 1997, Antras and Voth, 2003, Clark, 2007). Such a gradual take off can be explained by the feedback mechanism between knowledge spillovers and capital accumulation proposed in the present paper. The feedback mechanism produces a unified growth theory in the sense that a unique model explains the evolution of income over the very long-run, linking the epoch of (quasi-) stasis with the epoch of balanced growth through a phase of gradual industrialization.

With contrast to the established unified growth theory (Galor, 2005), the present paper does not rely on an interaction between economic and demographic variables. The disregard of population dynamics should of course not be misread as a downplaying of the demographic channel. It just helps to keep the model simple, to disentangle effects, and to present the globalization-feedback as a theoretically stand-alone mechanism, which, of course, in practice interacts with demographic forces. For the same reason the paper considers a very simple behavior of households resulting in a constant savings rate. This closes a savings rate channel, which could – at least partly – produce reasonable transitional dynamics as well, and helps to establish the globalization-feedback as a genuinely new channel in the theory of economic growth over the very long-run.[1]

The notion of a gradual evolution of knowledge spillovers, both over time and over space,

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[1] An interaction between poverty and the savings rate can produce increasing income growth along the transition but it, of course, fails to explain the secular rise of TFP growth. See Steeger (2000), Carroll et al. (2000), Strulik (2008). The focus on a constant population closes as well a third channel operating through the interaction of population dynamics and market size (Kremer, 1993, Jones, 2001, Strulik and Weisdorf, 2008). An incomplete list of important contributions to the standard channel operating through the interaction between fertility and education comprises Boucekkine et al. (2002), Doepke (2004), Galor and Weil (2002), Galor and Moav (2002), Galor and Mountford (2008), Kögel and Prskawetz (2001), Lucas (2002), Strulik (2004), and Tamura (2002). See Galor (2005) for a comprehensive review.

is supported by empirical evidence. Keller (2002) and Bottazzi and Peri (2003) show that knowledge spillovers are spatially localized and decay strongly with geographic distance. Keller's study also demonstrates that the degree of localization has shrunk substantially over time. Similarly, Jaffe et al. (1993) find localization effects for the links between patent creation and patent citation on the level of country, state, and metropolitan area, and that localization fades gradually over time. Sokoloff (1988) provides evidence that inventive activity during early industrialization was strongly related to the proximity to navigable inland waterways and that an expansion of the canal network was followed by a rise of patenting in the newly connected areas.

On a more aggregate level Dreher (2006) has constructed an index of globalization and provides evidence in favor of a causal effect of globalization on economic growth. Subdividing the index into different categories he finds a significant correlation with growth not only for economic integration (confirming earlier results like, for example Frankel and Romer, 1999) but also for social integration (personal contacts and information flows) albeit with some hints on possible reverse causality.

For *given* capital stock, some economies are certainly better equipped than others to appropriate knowledge. This could be so because economies differ in institutions that foster or hinder the appropriation of knowledge created elsewhere. Keller and Shiue (2008) provide evidence that early European globalization was to a larger degree affected by capital accumulation (the expansion of the railway network) than by institutional change (customs liberalization and currency agreements). However, they also document an indirect effect of institutions on economic performance in that better institutions improved the rate of adoption of steam trains. For the modern world Coe et al. (2008) provide evidence that institutional differences are an important determinant for the national appropriation of international R&D spillovers. The present model takes the efficiency of knowledge appropriation for given capital stock as a parameter. Comparative statics show that low institutional efficiency can prevent industrialization, globalization, and convergence towards perpetual growth. The model thus displays multiple equilibria and generates the Great Divergence (Pomeranz, 2000).

Starting with Grossman and Helpman (1991) the diffusion of knowledge through international trade has been investigated by quite a large literature using multi-country endogenous growth models. This literature has by now reached a high degree of technical sophistication (see Eaton and Kortum, 1999, as a prime example). Because of their high complexity, however, these models are solved only for the balanced growth path. Adjustment dynamics, i.e. gradual globalization and economic take-off, are not analyzed.

In order to allow for a theoretical exploration of adjustment dynamics the present paper proposes a technically much less involved framework. The feedback effect between globalization and growth is integrated into two popular models of endogenous growth, the Arrow (1962)–Romer (1986) learning-by-doing model and the Romer (1990) R&D-driven model. The learning-by-doing – or, strictly speaking, learning-by-investing – approach is certainly the appropriate framework to discuss economic development for most of human history since before the mid 19-th century technological advance was not (much) brought forth as a market activity of formally trained scientists (Mokyr, 2005).

Since then, however, production of knowledge has increasingly become a market activity, rendering the Romer (1986) model increasingly less appropriate. In order to accommodate for this development the globalization-feedback is also integrated into a Romer (1990) framework where technical change is driven by costly R&D. It is shown that most of the results of the simpler model continue to hold. Additionally, the globalization-feedback allows to explain why R&D effort and TFP growth are jointly increasing, an empirical fact which has been left unexplained by standard R&D models.[2]

The remainder of the paper is organized as follows. The next section integrates the globalization feedback into the Romer (1986) framework. A formal discussion provides an intuition for the gradual evolution of productivity, the existence of multiple equilibria, and the productivity slowdown. The formal reasoning is supplemented by a calibration of the model and a numerical solution of adjustment dynamics from the year 1 AD to 2100. Section 3 integrates the globalization feedback into the Romer (1990) model, demonstrates that all main results continue to hold and rationalizes the joint increase of TFP and R&D effort. Again a quantitative solution of a numerically specified model completes the discussion. Section 4 integrates both models and explains why (slow) growth was produced through learning for most of our history and how the amplification of this process during the first Industrial Revolution endogenously initiated a second Industrial Revolution, from which on growth became increasingly driven by market

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[2] From a technical viewpoint a similar interaction between accumulation and productivity has recently been proposed by Zuleta (2008) and Peretto and Seater (2008). With contrast to the present paper, however, these authors investigate how private R&D manages to improve the production elasticity of the *privately* supplied factors. They are thus not dealing with the long-run evolution of knowledge spillovers and growth.

R&D. In other words, it explains the transition from propositional knowledge towards prescriptive knowledge as the main driver of technological progress (Mokyr, 2005). Section 5 abandons constant savings rates and shows how the amplification of knowledge diffusion and increasing savings or investment generate overshooting behavior of growth of income and TFP during the second phase of industrialization (the Roaring Twenties). The final section concludes.

2. Globalization of an Arrow-Romer Economy

We consider a simple overlapping generations version of the Arrow (1962) – Romer (1986) learning-by-doing model. The concept of overlapping generations is useful in order to investigate adjustment dynamics analytically but it is not driving the results. The main results can also be derived by imposing a constant savings rate in a non-overlapping generations model in continuous time (see Appendix). Recall that the original version of the simple learning-by-doing model, also known as the Ak growth model, displays no adjustment dynamics and that ad hoc repair of this shortcoming (Jones and Manuelli, 1992) produces the wrong adjustment dynamics with respect to the historical facts by predicting that growth of income per capita is falling along the transition towards the balanced growth path (see also the detailed discussion of these models in Barro and Sala-i-Martin, 2004, Ch. 4).

2.1. **Households.**

Consider an economy populated by two overlapping generations. Members of the young generation supply one unit of labor, earn wages w_t, and divide their labor income on current consumption c_t^1 and on savings for the second period of life. Members of the old generation do not work and live off the returns on their savings. More specifically, we assume that the young individuals of period t maximize utility $u_t = \log(c_t^1) + \beta \log(c_{t+1}^2)$ where β is the discount factor. They face the current period's budget constraint $c_t^1 = w_t - s_t$ and the next period's budget constraint $c_{t+1}^2 = R_{t+1} s_t$ where R_{t+1} is the expected gross interest rate and s_t are savings. This standard OLG setup provides the well-known solution for savings (1).

$$s_t = \frac{\beta}{1+\beta} \cdot w_t. \tag{1}$$

There is no population growth. The size (mass) of a generation is normalized to one.

2.2. **Firms.**

There exists a continuum of size one of competitive firms. Firms produce a homogenous output using a Cobb-Douglas production function and employing capital and labor. In period t a firm i employs capital $k_t(i)$ and labor $\ell_t(i)$ and produces output $y_t(i) =$

$k_t(i)^\alpha[A_t(i)\ell_t(i)]^{1-\alpha}$ where total factor productivity $A_t(i)$ is exogenous to the single firm. For simplicity and without loss of generality we assume that capital depreciates fully within one generation. Profit maximization implies that production factors are demanded such that factor prices equal the (private) marginal product, i.e. $w_t = (1-\alpha)k_t(i)^\alpha A_t(i)^{1-\alpha}$, $r_t = \alpha k_t(i)^{\alpha-1}A_t(i)^{1-\alpha}$ where w_t denotes wages and r_t denotes the interest rate. Aggregate (i.e. average) employment is denoted by $k_t = \int_0^1 k_t(i)\mathrm{d}i$ and $\ell_t = \int_0^1 \ell_t(i)\mathrm{d}i = 1$.

2.3. **Knowledge Spillovers.** As proposed by Arrow (1962) and also suggested by Romer (1986) we think of knowledge embodied in capital goods such that knowledge and capital are used in fixed proportions in production. This allows to conceptualize knowledge spillovers as a positive function of aggregate capital stock. The existing literature assumes that knowledge spills over completely by imposing a linear association between k_t and $A_t(i)$. Here, we allow the degree of knowledge spillovers to be a positive function of the connectivity of the economy, measured by the degree of globalization g_t. In other words, the more globalized an economy is the more important becomes knowledge created elsewhere for own production, implying an increasing production elasticity of knowledge. Equation (2) is the simplest way to formalize this fact.

$$A_t(i) = \bar{A}{k_t}^{g_t}. \tag{2}$$

In a completely localized economy knowledge spillovers between firms are at their minimum, $g_t = 0$, and the model is isomorph to the neoclassical growth model. In a completely globalized world $g_t = 1$, and firms are capable to appropriate all the knowledge created elsewhere. For $g_t = 1$ the model is isomorph to (an overlapping generations version of) the original Arrow-Romer setup and knowledge spillovers generate perpetual economic growth at a constant rate.

In between these extrema we have a developing economy at an intermediate degree of knowledge spillovers. Knowlege spillovers are jointly rising with capital accumulation and the economy is possibly converging towards perpetual growth. But convergence towards balanced growth is not self-evident. Because the degree of knowledge spillovers is itself endogenous, the economy may get stuck in midst of the process of globalization and growth. It will be shown that whether stagnation occurs and how fast the economy develops is influenced by the knowledge-independent productivity $\bar{A}$, a parameter which can be thought of comprising the effect of institutional quality on efficiency.

2.4. **Static equilibrium.** In equilibrium all firms make the same choices, $\ell_t(i) = \ell_t$, $k_t(i) = k_t$ for all i. Inserting this fact and (2) into wages we get (3).

$$w_t = (1-\alpha)\bar{A}^{1-\alpha} k_t^{\alpha+(1-\alpha)g_t}. \tag{3}$$

Wages are increasing in the capital endowment of the workplace and this effect is in turn increasing in g_t because a high degree of globalization implies that much knowledge is embodied in capital goods (machines), a fact that amplifies worker productivity for any given size of k_t.

2.5. **Globalization and Growth.** According to the OLG setup the capital stock with which the next period's young generation is working is determined by the savings decision of this period's young generation, $k_{t+1} = s_t$. Inserting (1) – (3) we get the equation of motion (4).

$$k_{t+1} = a k_t^{\alpha+(1-\alpha)\cdot g_t} \tag{4}$$

where $a \equiv (1-\alpha)\bar{A}^{1-\alpha}\beta/(1+\beta)$.

The final element is a positive feedback from the size of the capital stock on the degree of globalization because capital goods (e.g. horses, ships, trains, cars, airplanes, and computers) alleviate travel and information exchange.

$$g_t = g(k_t), \qquad g(0) = 0,\ \ g' > 0,\ \ g''(0) > 0,\ \ \lim_{k\to\infty} g(k) = 1. \tag{5}$$

In words, we assume that globalization is at its lowest level (of zero) when there is no capital and that it is everywhere an increasing function of the capital stock. When the capital stock goes to infinity the economy becomes fully globalized in the sense that everyone is connected with everyone through long-distance links and the effective distance between firms is at its minimum such that all knowledge spills over to all firms. We also assume that $g(k_t)$ is convex close to the origin, implying that the first units of capital have a comparatively minor impact on globalization. This assumption is not essential for the qualitative results but crucial to get the quantitative adjustment dynamics right. A popular function that fulfils all assumptions is the logistic function.

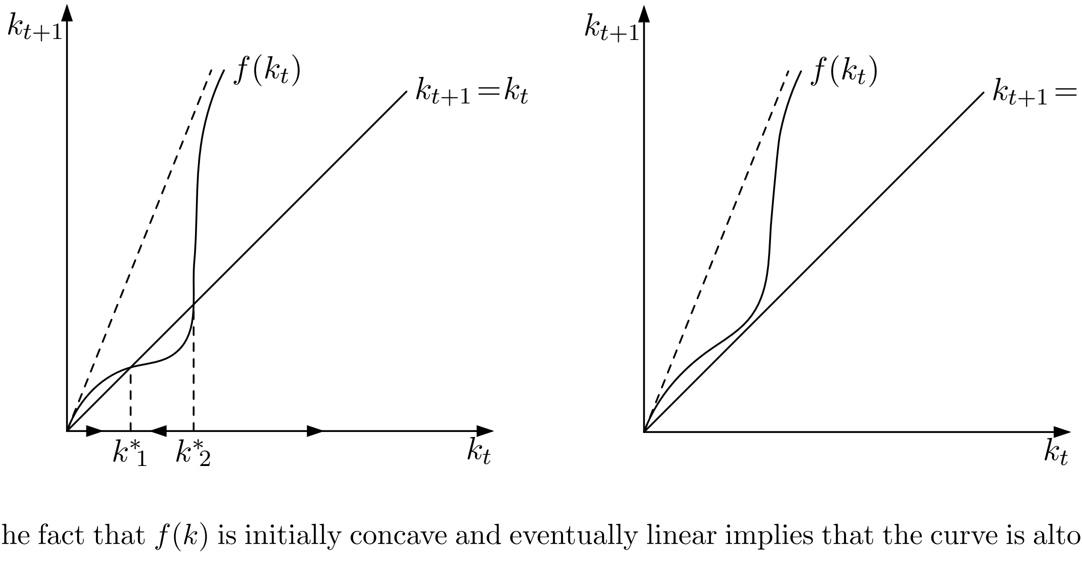
2.6. **Steady-State and Dynamics.** Inserting (5) in (4) we get the economy represented by a single difference equation of k.

$$k_{t+1} = f(k_t) = a k_t^{\alpha + (1-\alpha) \cdot g(k_t)}. \tag{6}$$

Inspection shows that $\lim_{k \to \infty} k_{t+1} = a k_t$, implying that the economy is capable of long-run growth if $a > 1$, a condition which we henceforth assume to hold in order to keep the problem interesting.

With contrast to the standard model, the version with endogenous knowledge spillovers displays transitional dynamics. Figure 1 shows the phase diagram. For an intuition it may be helpful to recall that the $f(k)$ curve would exhibit a strictly concave slope in case of a neoclassical model. In case of an Ak growth model the $f(k)$ curve would exhibit the constant slope $a > 1$, reflected in the Figure by the dotted line. Here, for the model with globalization feedback, the concave slope is preserved near the origin because knowledge spillovers are very small for low k. As k gets larger and more knowledge is appropriated through capital accumulation, $f(k)$ becomes less concave and, eventually, linear. For high k the curve approaches asymptotically the dashed line of slope a.

Figure 1: Phase Diagram: Globalization in the Arrow-Romer Economy



The fact that $f(k)$ is initially concave and eventually linear implies that the curve is altogether concave-convex with a unique turning point at an intermediate value of k. This shape implies that the curve either cuts the identity line twice or never. These cases are displayed in Figure

1 on the left hand side and right hand side, respectively. If two intersections exist, then there exist two equilibria of stagnation (or three if we account for the origin). The first one at k_1^* is locally stable, while the second one at $k_2^* > k_1^*$ is unstable. If no intersection exists, the curve lies everywhere above the identity line implying that the economy's capital stock is perpetually growing. Observe also that – for k larger than at the turning point – the distance between $f(k)$ and k rises with rising k. Because the distance between $f(k)$ and k gives the (gross) rate of economic growth, this observation implies that the rate of economic growth is increasing as the economy converges towards balanced growth. The following proposition summarizes the results.

PROPOSITION 1. *For the Arrow-Romer model with endogenous knowledge spillovers there exists either two equilibria of stagnation, one unstable and one locally stable, or no equilibrium of stagnation. In the latter case the economy converges towards balanced growth. The growth rates of capital and income are increasing during the transition towards balanced growth.*

Intuitively, at an equilibrium of stagnation the negative force of decreasing returns with respect to the privately provided factors of production and the positive force of increasing knowledge spillovers are balancing each other. At the first equilibrium k_1^*, occurring along the concave branch of the $f(k)$ curve, the negative force of decreasing returns becomes dominating for k slightly larger than k_1^*, which renders the equilibrium locally stable. At the second equilibrium k_2^*, occurring at the convex branch of the $f(k)$ curve, the economy is already sufficiently globalized, and the knowledge spillover effect becomes dominating for k larger than k_2^*.

A higher value of the parameter a increases the slope of the dotted line and rotates it away from the identity line. As a consequence the balance growth rate (computed as $a - 1$) gets larger and stagnation becomes less likely. Inspecting the compound parameter a we find that it increases with increasing β, i.e. decreasing time preference (increasing life-expectancy) and with increasing $\bar{A}$, i.e. higher accumulation-independent factor productivity (higher general efficiency, higher quality of institutions). In other words, stagnation is more likely in low-β-low-$\bar{A}$ societies.

Note that the equilibrium of stagnation differs qualitatively from the usual poverty trap. Stagnation occurs irrespective of subsistence needs at an income level which – depending on parameter choice – may exceed by far the income level which is usually associated with subsistence. This way, the equilibrium of stagnation exhibits more potential to explain actually observed poor growth performance.[3] Furthermore, although this is strictly speaking outside the

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[3] See Kraay and Raddatz, 2007, for the difficulties incurred by calibrating actual economies to allow for stagnation

model, we usually associate subsistence production with a traditional society. With contrast, for the present model stagnation may occur in midst the process of globalization at an intermediate level of g, a fact that may help to explain the poor growth performance of countries which are less appropriately characterized as traditional societies.

With contrast to the subsistence argument for stagnation, which relies on national savings rates, the present model can provide also an explanation for why capital is not flowing from rich to poor countries. The argument is based on the *dilemma* originating from the fact that knowledge spillovers are external to the individual firm. In the neighborhood of the equilibrium of stagnation k_1^*, capital would not flow into the country because capital productivity is low. In turn, capital productivity is low because capital endowment per workplace is low such that learning-by-doing effects and knowledge spillovers are small. In order to escape from this situation a reform improving the general appropriability of knowledge $\bar{A}$ (or an reform improving the propensity save $\beta/(1+\beta)$) is needed such that $f(k_t)$ lies above k_t and productivity gains from learning are no longer egalized by privately decreasing returns on investment.

But the model does not rely on *literal* stagnation in order to produce poor growth performance over a very long stretch of time. Diagrammatically, very low growth occurs when the $f(k)$ curve is close to but still above the identity line. The fact that economies can spend millennia in this part of the diagram makes growth at glacier speed observationally equivalent to actual stagnation.

Qualitatively, however, it makes a big difference whether an economy stagnates at k_1^* in Figure 1 on the left hand side or whether it develops very slowly through the funnel on the right hand side of Figure 1. In the latter case the economy develops endogenously such that sooner or later growth at a positive rate becomes visible. Since it was invisible before, one may speak of an Industrial Revolution. From then on the growth rate of income is visibly increasing over time and approaches a high constant level. The model establishes a unified growth theory in the sense that no exogenous impulse is needed in order to connect the period of quasi-stasis with the period of balanced growth.

Capital and income per capita are growing at a monotonously increasing rate along the transition towards balanced growth, i.e. when the economy travels along the convex branch of $f(k)$-curve. Productivity growth, however, adjust non-monotonously in an inverted u-shaped way.

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at subsistence.

The economy experiences a *productivity slowdown* along the transition. To see this, obtain TFP growth from (2).

$$\gamma_{A_t} \equiv \frac{A_{t+1} - A_t}{A_t} = \frac{(k_{t+1})^{g_{t+1}}}{(k_t)^{g_t}} - 1.$$

Since g is small in comparison with k (which goes to infinity), we set $g_{t+1} \approx g_t$ such that $\gamma_{A_t} \approx (k_{t+1}/k_t)^{g_t} - 1$. Inserting (4) the expression simplifies to

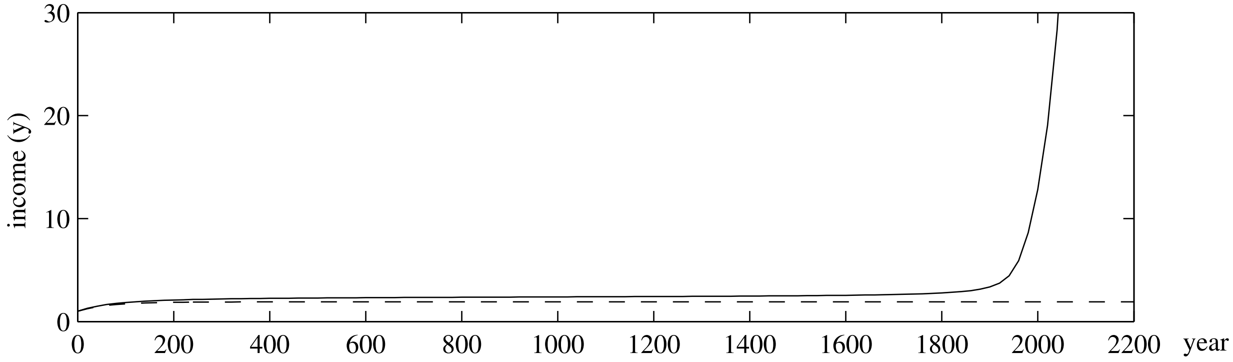
$$\gamma_{A_t} \approx a^{g_t} \cdot k_t^{g_t(1-\alpha)(g_t-1)} - 1.$$

We see that the rate of TFP growth approaches the growth rate of income, $a-1$ when the degree of globalization approaches one. Along the way, the first term rises monotonously with k and approaches a as g_t approaches one. The k_t term, however, grows non-monotonously. Taking the derivative with respect to k_t, noting that g_t is itself a function of k_t, we see that its sign is the same as the sign of $g_t(1-g_t) + k_t \log(k_t) g_t'(1-2g_t)$. While the sign of the first term is positive the sign of the second term is negative for $g_t > 1/2$ and $k_t > 1$, and the second term becomes dominating as k_t grows. In conclusion, γ_{A_t}' exhibits a root and TFP growth a maximum along the way to balanced growth. Note that the result obtains independently from the specification of the g function.

In order to check *quantitatively* whether the model is capable to explain the actually observed transition to modern growth we next consider a calibration of the model. For that purpose we have to specify a functional form of $g(k_t)$ such that g_t is increasing from zero to one for k_t increasing from zero to infinity. A simple function that does the trick is the logistic function, $g_t = \lambda/[\lambda + \exp(k_t - \kappa)]$. This gives us two parameters to influence adjustment dynamics. Roughly speaking, parameter λ determines the overall speed of globalization and the parameter κ specifies the capital stock (degree of economic development) at which globalization gets its highest momentum.

In the following I set parameters such that the benchmark economy reflects roughly the historical evolution of England. The parameter β is set to 0.25 implying a savings rate of 20 percent. The parameter α is set to 0.6 in line with the empirical estimate of the capital-output elasticity (Mankiw et al., 1992). The parameters λ and κ are set such that a benchmark economy that starts in year 1 A.D. with a capital stock $k_0 = 1$ initiates an industrial revolution in the late 18th century and a productivity slowdown in the second half of the 20th century and gets the fivefold increase of income per capita observed for England from 1900 to 2000 about right.

Figure 3 Adjustment Dynamics – The Great Divergence



Parameters: $\alpha = 0.5$, $\beta = 0.25$, $\lambda = 50$, $\kappa = 8.72$, $k_0 = 1$ and $a = 1.48$ (basic run, solid line, convergence towards balanced growth at annual rate of 2 percent) and $a = 1.35$ (dotted line, convergence towards stagnation, potential growth rate 1.5 percent). Income has been normalized such that income $y_0 = 1$.

This leads to the estimates $\lambda = 50$ and $\kappa = 8.72$. For better comparison with the historical data, generations are converted to years after the simulation has finished. Generational growth rates are converted to annual ones assuming that a generation takes 20 years. The remaining parameter $\bar{A}$ is set such that the economy approaches a balanced growth rate of 2 percent annually, providing the estimate $a = 1.48$.

The solid line in Figure 2 shows the implied adjustment dynamics for income. Initially, at very low capital stock, the degree of globalization is very low. Diagrammatically the economy is in the neighborhood of the origin in the phase diagram of Figure 1. Consequently, it exhibits the usual adjustment dynamics of the neoclassical type, i.e. income is growing at a decreasing rate. After that period of early growth, the economy experiences the “dark Middle Ages”. Income appears to be constant for about a century and a half. It is, however, not *exactly* constantly. The imposed parameters do not support stagnation. Diagrammatically the $f(k)$ curve is close to yet above the identity line implying that the economy is actually growing, people are perpetually learning-by-doing, factor productivity is rising, and the economy becomes more globalized. From today’s perspective, however, all this happens at glacier speed providing the image of stagnation in a poverty trap. It takes up to the 18th century until (initially slight) improvement of income can be observed. In the 19th century income growth really gets momentum and in the 20th century the economy grows exponentially at an almost constant rate.

Dashed in lines in Figure 2 show the outcome of an alternative scenario. Everything else is kept from the benchmark run, “only” $\bar{A}$ is assumed to be somewhat lower such that $a = 1.35$.

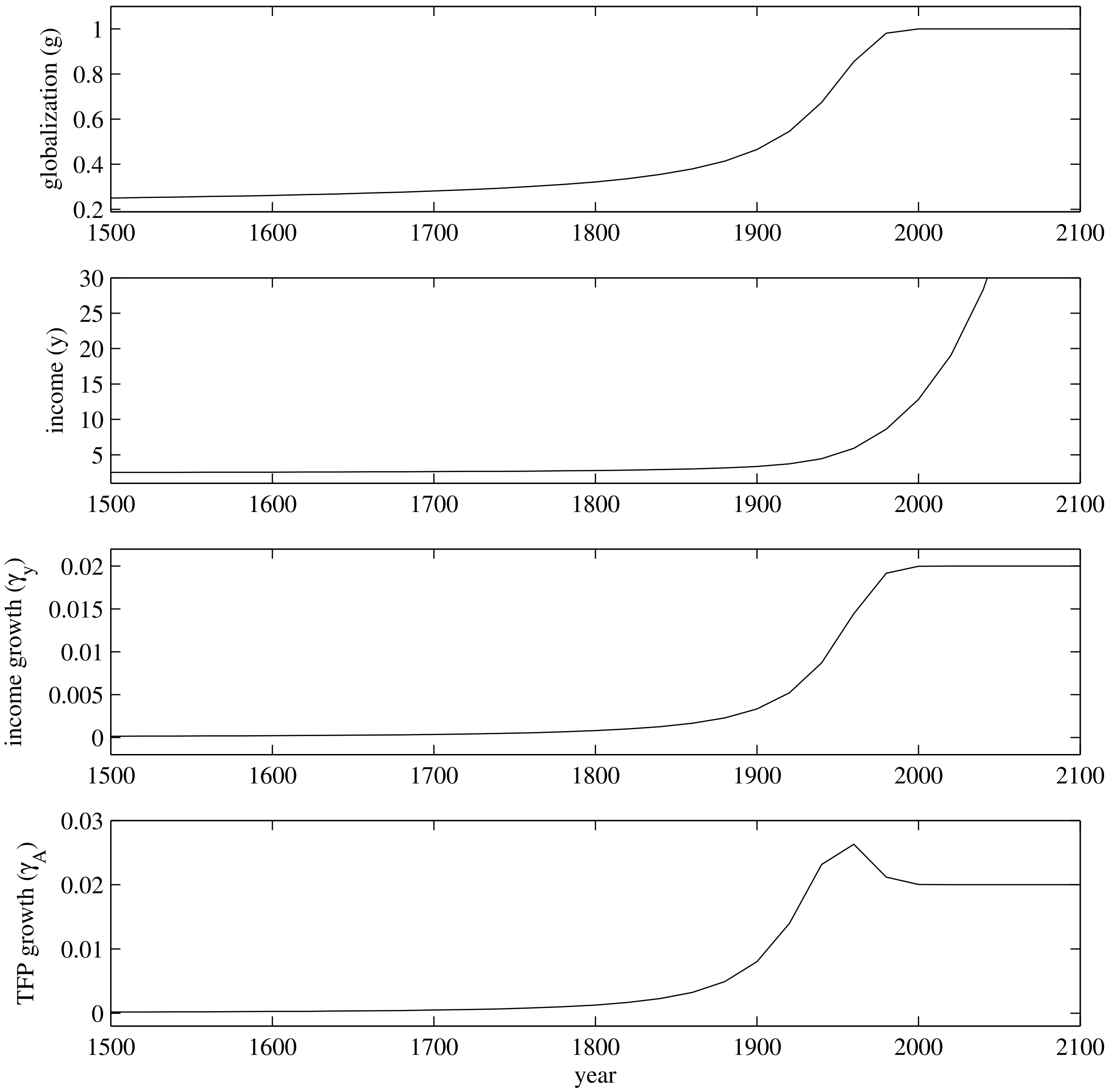
Intuitively the alternative economy is equipped with institutions that are less favorable for the adoption of knowledge created elsewhere. According to (2) factor productivity is lower than in the benchmark economy for any given level of capital stock and degree of globalization. If placed on its balanced growth path, the economy would generate a growth rate of 1.5 percent. Starting at initially low k, however, this path is not approached. Instead, the economy converges towards stagnation.

For most time of history the (non-) development of the alternative economy looks very similar to the benchmark economy. Only when the benchmark economy takes off to the Industrial Revolution the alternative economy is visibly left behind. We observe the Great Divergence (Pomeranz, 2000). With contrast to the benchmark economy, the alternative economy lacks the power to approach balanced growth endogenously. To break away from stagnation it needs an exogenous change, a reform improving the access to knowledge created elsewhere.

In Figure 3 we take a closer look at economic development of the benchmark economy. In order to make the take-off years more visible we focus on the period from year 1500 onwards. In the first panel we see that globalization begins to increase strongly in the early 19th century in line with the historical evidence (O'Rourke and Williamson, 2002, 2005, and Bordo, 2002). During the 19th and 20th century the speed of globalization increases until it reaches a turning point and slows down in late 20th century.

The third and fourth panel from the top show the rates of growth for income per capita and factor productivity. In line with the historical evidence both growth rates are close to zero during the Middle Ages and remain quite low – from today's perspective – during the early phase of industrialization and globalization in the 19th century (see e.g. Crafts, 2004). It takes until the 20th century that both growth rates are sharply on the rise and reach levels unseen so far in history. While income growth adjusts monotonously towards its balanced growth value, TFP growth exhibits a maximum. Observe that productivity begins to slow down when the speed of globalization (reflected by the slope of the g time series) reaches a maximum. Historically, this period was the onset of the IT age. According to the model mechanics, there is nothing alarming or frightening about the productivity slowdown. With increasing speed of globalization TFP growth rates were "just" overshooting and are subsequently returning to "normal", i.e. converging towards balanced growth level from above.

Figure 3: Adjustment Dynamics: Take-off to Modern Growth and Productivity Slowdown



Parameters as for Figure 2 (solid lines).

3. Globalization of a Romer-Jones Economy

In this section we give up the notion of knowledge creation through learning-by-doing. Instead we investigate the globalization feedback in the Romer (1990) framework where knowledge is produced through costly R&D as a market activity. Similarly to the intellectual history of the learning-by-doing model, the market R&D model originally displayed no adjustment dynamics. Jones (1995) proposed a modification of the model that produces adjustment dynamics. The modification was certainly less ad hoc than the "repair" of the learning-by-doing model although it followed a similar idea: Jones suggested to replace linear returns w.r.t. existing knowledge in the creation of new knowledge by decreasing returns.

At first sight the Jones-modification produced the right adjustment dynamics because now the share of the workforce occupied with R&D was predicted to rise over time in accordance with the historical evidence. Yet Jones failed to get the historical adjustment dynamics completely right because the model also counterfactually predicts that TFP growth decreases along the transition. The generated *negative* correlation between R&D effort and TFP growth is a simple consequence of decreasing returns in R&D. The fact that it becomes subsequently harder to come up with new ideas is partly compensated by pulling more workers into R&D and partly (because research gets more expensive) by slower growth in the production of new ideas. Subsequently it will be shown that the globalization feedback produces the "historically correct" adjustment dynamics in the sense that market R&D effort and TFP growth are *jointly* rising when the economy converges towards the balanced growth path.[4]

With contrast to the original Romer (1990) structure we consider again an overlapping generations economy and re-use equation (1) from the last section. In order to discuss adjustment dynamics qualitatively (in a phase diagram) we make the simplifying assumption that patents last for one generation. We keep from the last section the association between capital accumulation and degree of globalization (5). Otherwise we take the Romer (1990) setup, which is discussed in great detail in many textbooks, so that here its description can be brief.

3.1. **Final Goods Sector.** In any period t final goods y_t are produced by a large number of firms (of measure one) with constant returns to scale (i.e. competitively) using a Cobb-Douglas technology and the inputs labor (L_t^Y) and a range of intermediate goods. At time t there are A_t different varieties of intermediate goods available.

$$y_t = B(L_t^Y)^{1-\alpha} \sum_{i=1}^{A_t} x_{i,t}^{\alpha}. \tag{7}$$

Here, $x_{i,t}$ denotes the quantity of good i and B is a constant.

Taking wages w_t and prices $p_{i,t}$ as given, firms maximize profits, which renders the indirect

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[4] The Jones-modification pays a high price in order to get the R&D-effort dynamics right: the model predicts that there will be no long-run growth without population growth and that long-run growth is semi-endogenous, i.e. invariant with respect to (standard) economy policy. With contrast, the present model explains rising R&D effort as a phenomenon of transitional dynamics in the original Romer-setup implying that perpetual growth does not rely on population growth and growth is fully endogenous.

demand functions for labor and intermediate goods.

$$w_t = (1-\alpha)y_t/L_t^Y, \qquad p_{i,t} = \alpha(L_t^Y)^{1-\alpha}(x_{i,t})^{\alpha-1} \tag{8}$$

for all i since all intermediates enter production symmetrically.

3.2. **Intermediate Goods Sector.** Each intermediate good is produced by a single firm under monopolistic competition. A firm that has the right to produce good i can transform x_i units of raw capital into x_i units of specialized capital. Facing demand functions $p_{i,t}(x_{i,t})$ according to (10) and given the price of raw capital r_t, producers maximize profits.

$$\pi_{i,t} = p_{i,t}(x_{i,t}) \cdot x_{i,t} - r_t x_{i,t}. \tag{9}$$

The first order condition provides the price as a markup on factor costs.

$$p_{i,t} = r_t/\alpha \tag{10}$$

for all i. Thus, all capital goods sell at the same price r_t/α and all capital goods are demanded in equal quantities $x_{i,t} = x_t$. We thus drop the good index i henceforth.

Since $x_{i,t} = x_t$ for all i, the aggregate capital stock is computed as $k_t = \sum_{i=1}^{A_t} x_t$ and aggregate output can be written as

$$y_t = Bk_t^\alpha (A_t L_t^Y)^{1-\alpha}. \tag{11}$$

3.3. **The R&D Sector.** With contrast to the last section, production of new knowledge is conceptualized as a market activity. The R&D sector consists of a large number (of measure one) of competitive firms producing new ideas in the sense of blueprints for new intermediate goods. Employment of L_t^A allows the production of $\bar{\delta}L_t^A$ new ideas. Productivity in research $\bar{\delta}$ is taken as given by the single firm but is itself endogenously determined by the magnitude of accessible knowledge. The degree of knowledge spillovers in turn depends on the degree of globalization g_t so that $\bar{\delta} = \delta {A_t}^{g_t}$. Summarizing,

$$\Delta A_t \equiv A_{t+1} - A_t = \delta {A_t}^{g_t} L_t^A. \tag{12}$$

For a completely globalized economy in which all knowledge is accessible by everyone ($g_t = 1$) the R&D production function coincides with the one of the original Romer model.

Newly invented products get patented. In order to allow for a theoretical investigation of adjustment dynamics we follow Aghion and Howitt (2009, Chapter 4) and make the simplifying assumption that a patent holds for one period (i.e. one generation) and that afterwards the monopoly right to produce a good passes to someone chosen at random from the next generation. Through this simplification we get rid of intertemporal (dynastic) problems of patent holding and patent pricing while keeping the basic incentive to create new knowledge intact. Specifically, free entry into intermediate goods production implies that producers acquire blueprints (patents) at price π_t. Profit maximization in the R&D sector then leads to

$$w_t = \pi_t \delta {A_t}^{g_t}. \tag{13}$$

3.4. **Equilibrium and Dynamics.** Using (8) and (13), a labor market equilibrium requires

$$w_t = \pi \delta {A_t}^{g_t} = (1-\alpha) Y_t / L_t^Y. \tag{14}$$

Next, insert (8) and (10) into profits (9) to obtain $\pi_t = (1-\alpha)\alpha Y_t / A_t$. Insert this expression into (14) and use labor market clearing, i.e. the fact that $1 = L_t = L_t^Y + L_t^A$, to get sectoral employment as a function of the current state of technology.

$$L_t^Y = \min\left\{1, \frac{1}{\alpha\delta} \cdot {A_t}^{1-g_t}\right\} \qquad \Leftrightarrow \qquad L_t^A = \max\left\{0,\ 1 - \frac{1}{\alpha\delta} \cdot {A_t}^{1-g_t}\right\}. \tag{15}$$

Insert (15) into (12) to obtain the evolution of knowledge.

$$\Delta A_t = \max\left\{0,\ \delta {A_t}^{g_t}\left(1 - \frac{1}{\alpha\delta} \cdot {A_t}^{1-g_t}\right)\right\}. \tag{16}$$

Finally insert (15), (11), (8), and (1) into $k_{t+1} = s_t$ to get the evolution of capital.

$$\Delta k_t = a {A_t}^{1-\alpha-\alpha(1-g_t)} k_t^{\alpha} - k_t \tag{17}$$

where $a \equiv (1-\alpha)\beta/(1+\beta)/(\alpha\delta)^{\alpha} B$. Dynamics of the economy are described by the two-dimensional system of difference equations (16) and (17), taking the feedback on globalization (5) into account.

Inspect (16) and (17) to see that for the fully globalized economy (where $g = 1$) there exists a balanced growth path along which productivity and capital grow at the common constant rate $\delta - 1/\alpha$, which is the solution from the original Romer setup. Here we focus on adjustment dynamics and on the questions if, how, and when the balanced growth is approached. For now we assume that parameters (and initial values) are such $L_t^A > 0$ in (15) in order to keep the problem interesting. Otherwise, without any research, the model would boil down to the neoclassical growth model.[5]

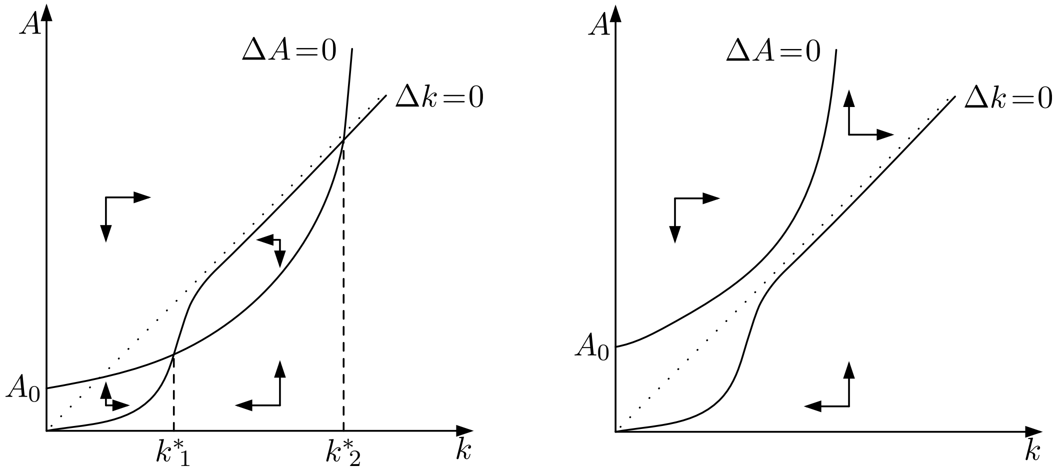
For a phase diagram analysis in the $k - A$ space the $\Delta k_t = 0$-locus is computed as

$$A_t = \left(k_t^{1-\alpha}/a\right)^{1/(1-\alpha-\alpha(1-g(k_t))}.$$

Observe that for $k_t \to \infty$ and thus $g(k_t) \to 1$ we have $A_t = (1/a)^{1/(1-\alpha)} \cdot k_t$ implying that the isocline converges from below towards a ray originating from the origin with slope $(1/a)^{1/(1-\alpha)}$. For $g_t \to 0$ we have $A_t = k_t^{(1-\alpha)/(1-2\alpha)}/a^{1/(1-2\alpha}$. In order to avoid uninteresting case differentiation we henceforth assume $\alpha < 1/2$ such that the isocline has convex shape for small k_t as shown in Figure 4. Observe also that we have $\Delta k_t < 0$ above the curve and $\Delta k_t > 0$ below since $k_t^\alpha < k_t$.

The $\Delta A_t = 0$ isocline is given by $A_t = (\alpha\delta)^{1/(1-g(k_t))}$. The curve originates from $A_0 \equiv \alpha\delta$ and is hyper-exponentially growing in k_t. We have $\Delta A_t < 0$ above and $\Delta A_t > 0$ below the curve.

Figure 4: Phase Diagram: Globalization and R&D-based Growth



The shape of the isoclines renders two qualitatively different possibilities, which are displayed in Figure 4. If the isoclines do not intersect, as shown at the right hand side, there exists no equilibrium of stagnation and the economy converges towards the balanced growth path along which it grows at rate $\delta - 1/\alpha$. If the curves intersect they do so exactly twice because the $\delta A_t = 0$–curve originates from positive A_0 and has (hyper-) exponential shape while the

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[5] The next section integrates the R&D-model into the learning-by-doing framework, which allows to start out in a situation where $L_0^A = 0$ without implying stagnation.

$\Delta k_t = 0$–curve originates from zero and converges towards a straight line with finite slope. From the arrows of motion developed above follows that the equilibrium at k_1^* is locally stable while the one at k_2^* is unstable.

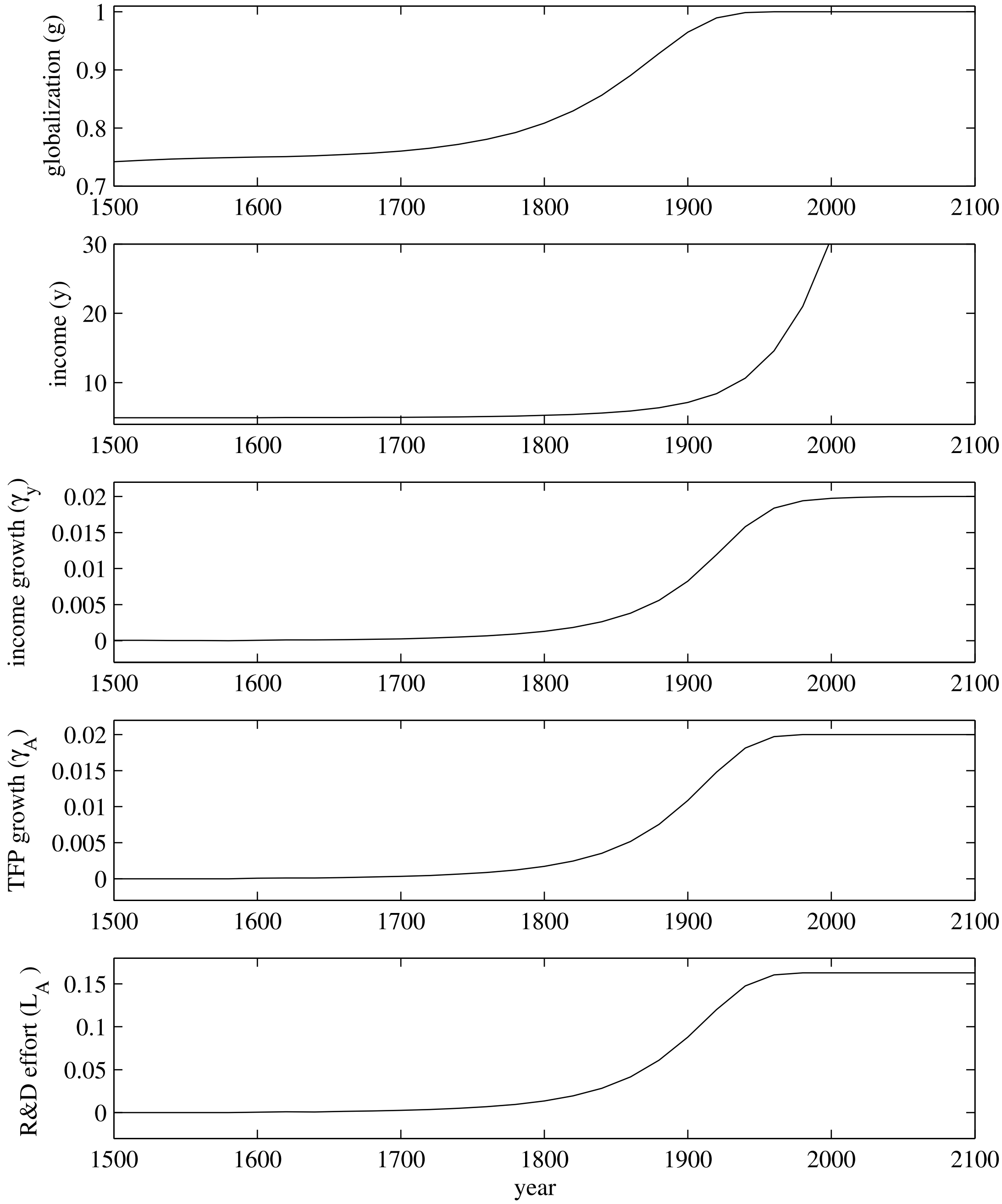
In conclusion the R&D-based model produces adjustment dynamics that are qualitatively similar to those obtained within the simpler learning-by-doing framework. In particular, it generates also a Great Divergence. Starting at low k_0 an economy in which A_0 is sufficiently small (reflecting institutions unfavorable to appropriate knowledge such that $\bar{\delta}$ is small) get stuck at an equilibrium of stagnation k_1^* while an otherwise identical economy equipped with larger A_0 travels towards balanced growth. Again, stagnation may occur above subsistence level at an intermediate stage of modernization (globalization). And again, successful convergence towards balanced growth goes hand in hand with increasing globalization.

The phase diagram is less informative about the speed of convergence and the timing of the take off. In order to investigate this behavior in detail we consider again a calibration of the model. For that purpose I set again $\beta = 0.25$ such that $s = 0.2$. Focussing on a narrow definition of capital I set $\alpha = 0.4$ and calibrate δ such that the economy grows at a rate of 2 percent annually along the balanced path. Assuming again a length of a generation of 20 years implies $\delta = 2.98$. I take the same parameters for the g function as for the learning-by-doing model, $\lambda = 50$ and $k = 8.72$. Again the model generates an Industrial Revolution around 1800.

Figure 5 shows the adjustment dynamics from year 1500 onwards. The take off of income growth is again accompanied by a take off of the degree of globalization. The take off is again gradual and, again, the model gets the 5 fold increase of income per capita in England during the 20th century about right. More importantly, the model continues to predict that TFP growth is gradually increasing during the Industrial Revolution, a historical fact that cannot be displayed within the original Romer (1990) or Jones (1995) framework. Nevertheless the model predicts also a secular increase of R&D effort measured by the share of the workforce engaged in this sector. It thus predicts that TFP growth and market R&D effort are *jointly* increasing during industrialization, a prediction in line with the historical facts (Mokyr, 2005).

Increasing R&D effort causes knowledge to spill over more easily because more recently developed goods (e.g. cars vs. trains) allow faster travel of people and ideas. Higher knowledge diffusion increases the productivity of R&D further, a fact that drags even more people into R&D which in turn further improves the diffusion of knowledge etc. Yet unlike economic growth

Figure 5: Adjustment Dynamics: Globalization and R&D-based Growth



Parameters: $\alpha = 0.4$, $\beta = 0.25$, $\delta = 2.98$, $\lambda = 50$, $\kappa = 8.72$, $k_0 = 1$.

the improvement of knowledge diffusion has a well-defined end, namely when knowledge spills over completely. This is why the economy does not explode but converges towards balanced growth instead.

The feedback mechanism between R&D and knowledge spillovers rationalizes the so called "scale effect" and possibly helps to reconcile the different position that growth researchers take with this respect. It also suggests that expressions like “growth on the knife edge” associated with spillovers of degree one could be misleading. Spillovers of degree one are the natural end of a process running from zero to full globalization.

4. The Endogenous Rise of Intellectual Property Rights and Market R&D

From a very long-run perspective the R&D-based model is subject to a similar criticism as the learning-by-doing model. It is the appropriate tool to investigate technological advances in fully developed economies today and (to some lesser degree) a hundred years ago, but it is certainly less suited to investigate technological advance in the Middle Ages. Ideally one would like to combine both approaches such that the learning by-doing model explains technological advances for most of human history but gets increasingly replaced by the market R&D model from the onset of the Industrial Revolution onwards.

In order to built such a framework we distinguish, following Mokyr (2005), between propositional knowledge and prescriptive knowledge. Propositional knowledge comprises the artisanal, informal, and empirical part of knowledge, knowledge that is appropriated through learning-by-doing. Prescriptive knowledge comprises the written down instructions of how a particular invention works, knowledge that can (potentially) be patented and that is brought forth by market R&D activities.

Utilizing the globalization feedback again, knowledge in firm j acquired through learning is given by $A_t^L(j) = \bar{A}k_t^{g_t}$. This means that acquiring propositional knowledge is easier if there is a lot of investment to learn from and if knowledge diffuses more easily through the economy, i.e. at a high degree of globalization. Continuing to assume that there is a measure one of competitive firms producing final goods, aggregate production can be written as

$$y_t = (A_t^L L_t^Y)^{1-\alpha} \sum_{i=1}^{A_R} x_i^{\alpha}. \tag{18}$$

Intermediate goods are again provided by local monopolists. There exists the possibility to invest resources into targeted research to create new prescriptive knowledge ΔA_t^R. We continue to assume that existing prescriptive knowledge can be more easily accessed and thus used in the creation of new knowledge when the degree of globalization is large, i.e. when knowledge diffuses easily.

$$\Delta A_t^R = \delta (A_t^R)^{g_t} L_t^A. \tag{19}$$

Performing all the calculation from the last section we end up with the following sectoral employment.

$$L_t^Y = \min\left\{1,\ \frac{1}{\alpha\delta} \cdot \left(A_t^R\right)^{1-g_t}\right\} \qquad \Leftrightarrow \qquad L_t^A = \max\left\{0,\ 1 - \frac{1}{\alpha\delta} \cdot \left(A_t^R\right)^{1-g_t}\right\}. \tag{20}$$

This is exactly the same as (15) besides the fact that employment is determined "only" by the available *prescriptive* knowledge A_t^R because the generation of new propositional knowledge does not utilize market resources. Inserting (20) into (19) we obtain the evolution of prescriptive knowledge.

$$\Delta A_t^R = \max\left\{0,\ \delta (A_t^R)^{g_t}\left(1 - \frac{1}{\alpha\delta} \cdot \left(A_t^R\right)^{1-g_t}\right)\right\}, \tag{21}$$

which replaces (16).

Finally, utilizing the fact that all producers of intermediate goods are alike we arrive at the equation of motion for capital.

$$\Delta k_t = a\left[A_t^L \cdot A_t^R\right]^{1-\alpha} k_t^\alpha - k_t. \tag{22}$$

Note that propositional knowledge and prescriptive knowledge enter production and thus capital accumulation *multiplicatively.* They are thus indirectly, via capital accumulation and globalization, re-enforcing each other.

The main difference to the last section is that we let the economy start in an environment where $L_t^A = 0$ and thus $\Delta A_t^R = 0$. Although there exists in principle the possibility to create new prescriptive knowledge, i.e. to invest resources in targeted R&D in order to obtain intellectual property rights, nobody is willing to engage in this endeavour. Intuitively, the existing knowledge is too hard to access initially, g_t is too low, such that productivity in research is too low $\left(A_t^R\right)^{1-g_t} > \alpha\delta$ and there is no targeted research. There are no new blueprints developed and the number of products A_R stays constant.[6]

Figure 6 shows the adjustment dynamics. Although there is no market R&D activity initially, knowledge is increasing in the economy. Propositional knowledge grows (for example, new design of ships are developed) and with it grows the productivity of capital, leading to more capital accumulation (more ships) and more widespread diffusion of knowledge (increasing long-distance travel). The self-enforcing mechanism between the evolution of propositional knowledge and

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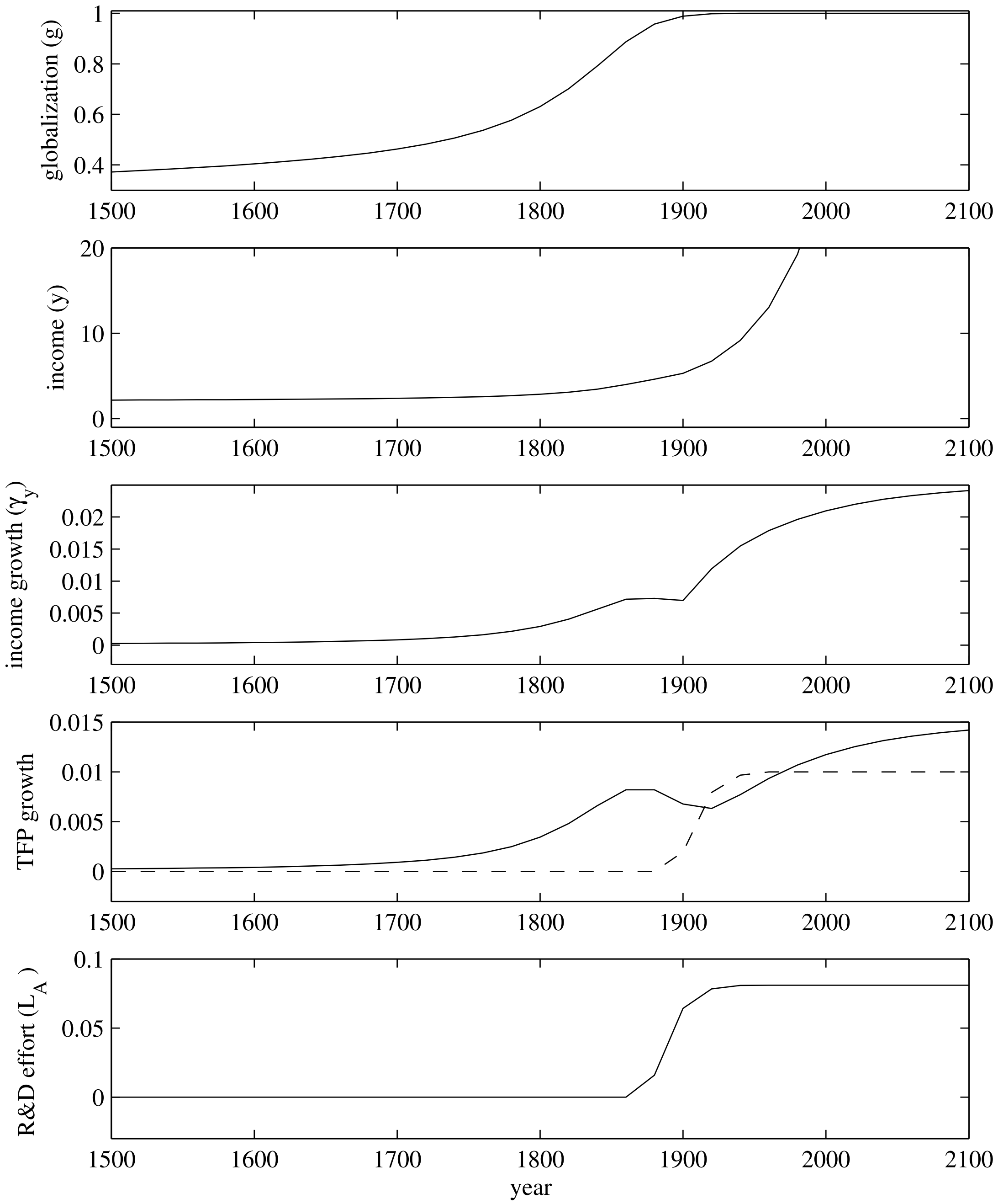
[6] In England a patent system was available for a long time in history but only sparingly used until the Industrial Revolution (Mokyr, 2005).

Figure 6: The First and the Second Industrial Revolution



Parameters: $\alpha = 0.4$, $\beta = 0.25$, $\delta = 2.72$, $\lambda = 50$, $\kappa = 8.72$, $k_0 = 1$. TFP time series for γ_{A_L} (solid line) and γ_{A_R} (dashed line).

diffusion of knowledge eventually causes a visible (yet gradual) take off of productivity growth and income growth in the late 18th century, the first phase of Industrial Revolution.

During the first Industrial Revolution propositional knowledge was not only increasing in size but becoming also increasingly accessible. As globalization gets momentum during the 19th century knowledge spillovers eventually become large enough for market R&D to be worthwhile. Formally, the exponent of $(A_t^R)^{1-g_t}$ becomes small enough such that the goods sector sets free labor for R&D. The second phase of the Industrial Revolution sets in.

At the beginning of the second Industrial Revolution, increasing R&D activities imply mostly that less labor is allocated to goods production. The new innovations thus provide initially small advances in income per capita growth. TFP growth through learning-by-doing even falls slightly during this period. Eventually, however the productivity gains and the improvement of knowledge diffusion become visible in income growth rates, which converge towards a new plateau. During this process market R&D is not replacing learning-by-doing. On the contrary, both processes are re-enforcing each other. Eventually, however market R&D becomes the main driver of productivity growth. At the new steady-state, income grows at an annual rate of 2.5 percent and market R&D grows at a rate of 1.5 percent.

5. R&D, Capital Accumulation, and the Roaring Twenties

It has been argued in the Introduction that secular increasing income growth (but not increasing TFP) can also be induced by a positive association between income and the savings rate and that we have neglected this mechanism in order to properly establish the knowledge feedback mechanism. We are now, finally, abandon this simplification and investigate both mechanisms together.

In order to generate a rising savings rate we assume that the elasticity of marginal utility from current consumption is decreasing in the level of consumption. The utility function is given by $u_t = \log(c_t^1 - \bar{c}) + \beta \log(c_{t+1}^2)$. The utility maximizing choice of savings is thus given by

$$s_t = \frac{\beta}{1+\beta}(w_t - \bar{c}),$$

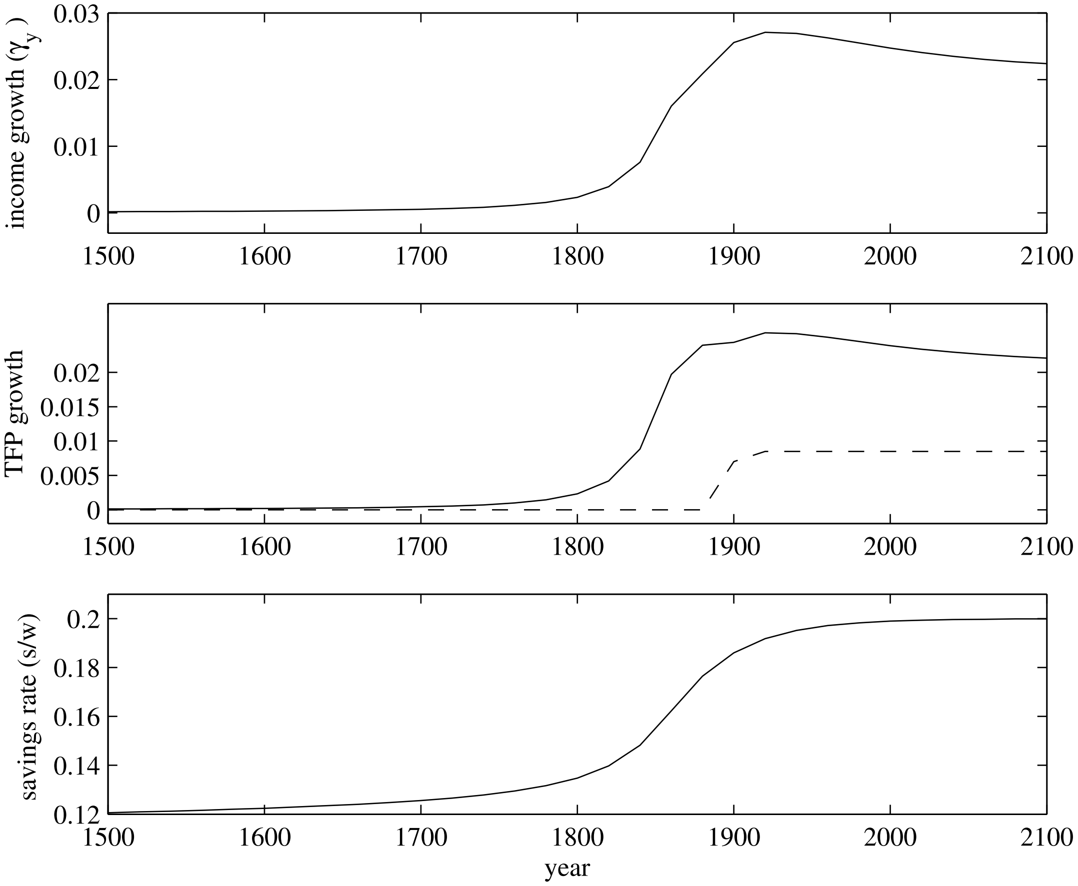
which replaces (1). Consequently, the equation of motion for capital (22) is replaced by

$$\Delta k_t = a \left[A_t^L \cdot A_t^R\right]^{1-\alpha} k_t^{\alpha} - k_t - \frac{\beta}{1+\beta}\bar{c}.$$

The remainder of the model and the numerical specification is kept from the previous section. Figure 7 shows the implied adjustment dynamics for $\bar{c} = 12$.

The phase of industrialization is now additionally characterized by secularly rising savings rates. Increasing capital accumulation and increasing knowledge growth are re-enforcing each other such that TFP growth and income per capita growth are overshooting. In line with the empirical evidence (Field, 2003; Harrison and Weder, 2009) the model predicts the highest growth, above steady-state, during the years 1920-30.

Figure 7: Capital Accumulation and the Roaring Twenties



Parameters as for Figure 6 and $\bar{c} = 12$.

While growth of income and TFP overshoots, the savings rate adjusts monotonously. This behavior generates a positive association between the (lagged) savings rate and income growth in the first phase of industrial revolution, i.e. when countries are relatively poor, and a negative association at later stages, i.e. when countries are relatively rich. A similar inverted-u shape is predicted for the association between savings rates and TFP growth. Finally, when the country becomes very rich the savings rate balances at a constant level while productivity and income continue to grow. Thus the model predict no association between savings and growth for very rich countries. Using cross-countries regressions similar results have recently been found by Aghion et al. (2009). If we assume that the countries of their sample are at the time of observation at different states of industrialization the finding supports the proposed theory.[7]

6. Conclusion

This paper has proposed a feedback mechanism between knowledge diffusion and capital accumulation. Capital embodied technological progress (incorporated, for example, in ships, trains, and planes) alleviates the travel of people and ideas. More capital accumulation leads to better diffusion of knowledge, which raises factor productivity, which in turn leads to even more accumulation and better diffusion of knowledge etc. Unlike growth, the process of improving knowledge diffusion has a certain end, the fully globalized economy, a fact that generates convergence towards balanced growth.

The diffusion-accumulation feedback has been incorporated in standard models of endogenous growth, which were, after the amendment, capable to produce adjustment dynamics in line with the historical record of England and the Western world. In particular, a long epoch of (quasi-) stasis is connected with an epoch of high growth by gradual economic take-off and globalization. Economic take off, however, is not self evident. If institutions are not sufficiently supporting the appropriation of knowledge (if knowledge spillovers are sufficiently low for given capital stock) an economy may get stuck in an equilibrium of stagnation. Actual stagnation of one economy may be visibly indistinguishable from quasi-stagnation of another for a long stretch of time but eventually, with the onset of industrialization, the Great Divergence of the two economies becomes increasingly visible.

For the model version with market R&D it has further been shown why R&D effort, TFP growth, and income growth are jointly rising during the Industrial Revolution. An integration of the learning-by-doing setup and the market R&D setup has demonstrated how the creation of propositional and prescriptive knowledge interact and, in particular, how a long phase of growth of exclusively propositional knowledge eventually triggers a transition towards market R&D activities. In such a modern society growth of propositional and prescriptive knowledge are re-enforcing each other and the latter becomes eventually the dominant driver of economic

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[7] Aghion et al. (2009) propose an alternative explanation for their finding based on the catch-up process of backward countries. By assuming an exogenously given and constant productivity growth rate for the technological leader, their model focusses on today's LDCs and cannot be applied to the Industrialization of Europe and the Western world.

progress.

An extension of the model towards endogenous savings has demonstrated how income growth and globalization are further amplified through increasing savings- or investment rates during industrialization. The amplification is strong enough to create overshooting behavior of growth during the second phase of industrialization (the Roaring Twenties). Afterwards the economy adjust towards the balanced growth path from above, i.e. with decreasing rates of TFP- and income growth.

In order to establish the diffusion-accumulation feedback as a theoretically stand-alone mechanism of growth over the very long run, the standard mechanism of unified growth theory, the interaction of education and fertility has not been investigated. It remains a challenging task for future research to integrate both drivers of long run development in a model where the diffusion-accumulation mechanism interacts with demographic variables.

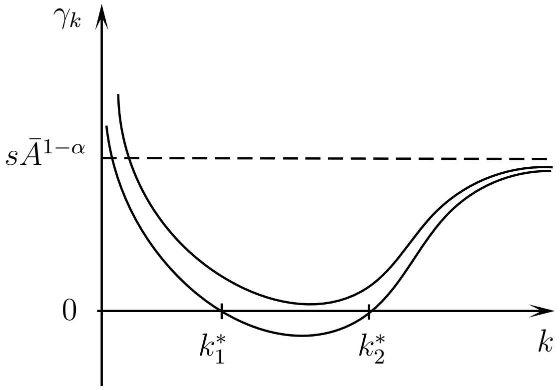
Appendix

In the following it is shown that the main results obtained in the main text continue to hold in a Solow-type model in continuous time. Starting point is the evolution of capital $\dot{k} = sy$ where s is the constant savings rate. Inserting the per-capita production function $y = k^{\alpha} A^{1-\alpha} \ell^{-\alpha}$, normalizing $\ell = 1$ and inserting the globalization feedback (2) and (5) we arrive at

$$\gamma_k \equiv s\bar{A}^{1-\alpha} k^{(1-\alpha)(g(k)-1)}.$$

For very small $k \to 0$ we have $\gamma_k = s\bar{A}^{1-\alpha} k^{-(1-\alpha)}$. The growth rate is high and falling in k (the neoclassical part of the model). For $k \to \infty$ we have $\gamma_k \equiv s\bar{A}^{1-\alpha}$. The growth rate approaches a positive constant (the Ak part of the model). Because $g'(k) > 0$ we are left with the two qualitatively different possibilities for model dynamics shown in Figure A.

Figure A: Globalization in the Continous Time



In a k–γ_k diagram the γ_k cuts the abscissa either twice or never. If it cuts twice, we have two equilibria, the first one at k_1^* locally stable and the other one at k_2^* unstable. If no intersection exists, a domain may exist where the γ_k curve is close to the abscissa, implying positive growth at slow (and possibly for centuries invisible) speed. If no equilibrium exists any economy eventually approaches the full globalization and balanced growth at rate $s\bar{A}^{1-\alpha}$.

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