STRATEGIC DELEGATION OF PRICE SETTING AND THE BERTRAND PARADOX

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Abstract

We show that firm owners may delegate price setting to managers in order to circumvent the Bertrand outcome. In particular, the equilibrium price is above the expected marginal costs when managers' reservation utility is low enough, so that owners hire managers. The key features of our model are that marginal costs are uncertain, managers are risk-averse, and managers can be equipped with performance dependent wage schemes.

1. INTRODUCTION

Wambach (1999) explores the price setting of risk-averse firms when marginal costs are uncertain. He shows that in equilibrium firms set prices above expected marginal costs and make positive expected profits. But it is questionable whether firms are risk-averse.[1] Moreover, he neglects the separation between ownership and control which is present in most firms.[2]

We consider a model with multiple firms and uncertain marginal costs where each owner (she) is risk-neutral and can delegate the pricing setting to a risk-averse manager (he). His risk aversion plays, however, only a role for his price setting when his wage scheme depends on the risky profit of the firm. She can specify the scheme such that he faces no risk at all. Then she need not pay a risk premium and his price setting gets very aggressive. This may, however, not be in her interest since this would result in a low equilibrium price. We show that when managers' reservation utilities are low enough, in equilibrium, owners hire managers and equip them with wage schemes that highly depend on the firm's profit. Then managers set prices above expected marginal costs. This results in positive expected profits.

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[1]Indeed, it is standard in the literature to treat firms as risk-neutral.

[2]See Jensen and Meckling (1976).

Our model uses Schelling's (1956) insight that a principal can improve the outcome of a bargaining game if she delegates the bargaining power to an agent with a different objective function. We apply this general idea to a specific context in which several parties interact. Each manager's objective function is endogenous due to the endogeneity of the wage scheme. We restrict our attention to linear wage schemes.

The Bertrand Paradox is as follows. When two firms, with equal and constant marginal costs, compete with prices, firms set prices equal to marginal costs. It is paradox that two firms are sufficient to restore perfect competition. However, the literature shows that this is no longer true when (i) firms are capacity constrained (Allen and Hellwig 1986, 1989; Kreps and Scheinkman 1983), (ii) marginal costs are increasing (Dixon 1990, 1992; Dastidar 1995), (iii) consumers have positive search costs (Diamond 1971; Stahl 1989), (iv) there is repeated interaction (Friedman 1971; Fudenberg and Maskin 1986), (v) products are differentiated (Hotelling 1929), (vi) firms collude, or (vii) marginal costs are stochastic and firms are risk-averse (Wambach 1999). We offer a new explanation which is related to (vii):[3] Given that managers have sufficiently low reservation utilities, each firm owner hires a risk-averse manager to whom price setting is delegated and equips him with a wage scheme that depends highly on the firm's profit. Each manager will then set a price above expected marginal costs.

The rest of the paper proceeds as follows. Section 2 describes the model. Then we analyze it in Section 3. By applying the Pareto criterion as a selection device we can get rid of the multiplicity of equilibria. We reduce the set of potential equilibria to two. In the first, no managers are hired and owners set prices equal to expected marginal costs. In the second, each owner hires an manager and equips him with the same contract that exposes him to risk and managers set prices above expected marginal costs. When the reservation utility of managers is sufficiently low, the Pareto criterion selects the second equilibrium. Then the Bertrand Paradox is circumvented. When the reservation utility is high, it is too costly to hire managers, and we get the first equilibrium in which the Bertrand Paradox arises. In Section 4 we discuss the model. Interestingly, when we allow for secret or public renegotiations of wage schemes and simultaneously for renegotiations of the market price renegotiations will not occur. Moreover, anticipating renegotiations does not change the equilibrium. That is, despite complete information contracts can be used for strategic commitment even when they can be secretly or publicly renegotiated. Section 5 offers three extensions. First, we consider the case of more than

[3]In a sense, we give a micro foundation for why firms are risk-averse.

two firms. Second, we allow for heterogeneity of managers or cost shocks. We show that with both extensions the results yielded before stay qualitatively valid. Third, we relax the assumption that firms need to meet demand. We show that again, managers equipped with performance depended wage schemes set prices above expected marginal costs. When owners determine the wage scheme, there is, however, no equilibrium in which owners hire managers. In contrast, in a more general model that recognizes the positive effects of higher powered incentives schemes on problems due to the separation of ownership and control it can again be an equilibrium that managers are hired.

2. MODEL

There are two firms, 1 and 2. The firms compete à la Bertrand. Firm $i \in \{1, 2\}$ sets price p_i and sells x_i units. Consumers' demand is normalized to 2. In case of $p_1 = p_2$ each firm sells one unit. Firms have to meet demand, i.e., consumers can buy from firm i as much as they want for the preselected price p_i. Firm i's per unit production costs are z_i. Production costs are uncertain: z_i is drawn, *after* prices are set, from a normal distribution function with mean zero and variance $\tilde{\sigma}^2$.[4] There are no fix costs.

Firm i is owned by O_i. Each owner is risk-neutral and seeks to maximize the expected net profit of the firm.[5] Initially O_i has no manager. Before prices are set, O_i (she) can hire a manager M_i (he). He is risk-averse and has CARA utility. She can hire a manager by convincing a candidate to get her manager. This is done via a take-it-or-leave-it offer to the candidate. The offer specifies (i) who determines the firm's price

[4]Production costs can be uncertain, for example, due to uncertain input prices. Another reason is that R&D that may fail. Then a process innovation may not be achieved. Moreover, also selling a good may be risky, which has the same effect. Due too exchange rate risks, the home currency price of a good may be uncertain. Additionally, firms are often liable for defects of their products—and this is risky for firms. See Shavell (2003, Ch. 3).

Spulber (1995) considers a model where each firm knows its own costs, but not the costs of the other firms. He shows that firms set prices above marginal costs and therefore yield positive expected profits (see Section 2 of his paper). This is, however, also true when costs are common knowledge: the firm with the lowest marginal costs set its price equal to second-to-lowest marginal cost and yield positive profits (see Section 3 of his paper).

[5]One can think of the owner as a group of owners. Each group member holds a portfolio of assets and is fully diversified. It is hence sensible to assume that the members' common and single interest is to maximize the expected net profit of the firm.

and (ii) a wage scheme which is linear in the firm's profit π_i:

$$W_i = \alpha_i + \beta_i \pi_i.$$

When the candidate rejects, he receives the reservation wealth $\bar{u} > 0$. It will become clear below, that hiring a manager and not delegating the pricing setting is dominated by not hiring. So in the remainder we treat only the cases where (i) either a manager is hired and sets the price, or (ii) no manager is hired and the owner sets the price.

The timing is summarized in Figure 1.[6] There is perfect information. We only consider pure strategy, subgame perfect Nash equilibria, and use in case of multiplicity the Pareto criterion to narrow the equilibria set.[7] To avoid open set problems, we assume that for $\beta_i = 0$, M_i behaves as if $\beta_i \searrow 0$.

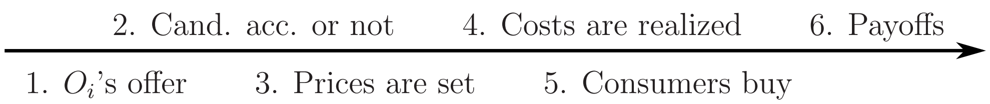

Figure 1: Timing

3. ANALYSIS

3.1. PARTIES PAYOOFS

Firm i's gross profit is

$$\pi_i = x_i(p_i - z_i).$$

When no manager is hired, O_i sets firm i's price p_i and her expected payoff is

$$\mathbb{E}[\pi_i].$$

When O_i hires a manager, M_i sets p_i and her expected payoff is

$$\mathbb{E}[\pi_i - W_i].$$

[6] The analysis and the results stay completely valid when consumers buy *before* the costs are realized.

[7] As we will see in the following analysis, the problem of multiplicity occurs at the price setting as well as at the hiring stage. Without a clear selection criterion, the analysis becomes completely arbitrary. We think the Pareto criterion is a clear, intuitive, and easy to manage selection device. The Pareto criterion is also used in many other papers, e.g., Wambach (1999) and a large part of the literature on infinitely repeated games.

Since the shock on his wealth is normally distributed, M_i's certainty equivalent of the wage is

$$C_i = \mathbb{E}[W_i] - \gamma \mathbb{V}[W_i] = \alpha_i + \beta_i \mathbb{E}[\pi_i] - \gamma \beta_i^2 \mathbb{V}[\pi_i], \tag{1}$$

where $\gamma > 0$ is the coefficient of absolute risk aversion. The expected profit and the variance of the profit are $\mathbb{E}[\pi_i] = x_i p_i$ and $\mathbb{V}[\pi_i] = x_i^2 \tilde{\sigma}^2$. We directly get

$$C_i|_{x_i=0} = \alpha_i, \tag{2}$$
$$C_i|_{x_i=1} = \alpha_i + \beta_i p_i - \beta_i^2 \sigma^2, \tag{3}$$
$$C_i|_{x_i=2} = \alpha_i + 2\beta_i p_i - 4\beta_i^2 \sigma^2, \tag{4}$$

where $\sigma^2 := \tilde{\sigma}^2 \gamma$. Each manager seeks to maximize his expected utility, given his wage scheme. This is equivalent to the maximization of the certainty equivalent. So to determine the equilibrium of the pricing game, we compare $C_i|_{x_i=2}$, $C_i|_{x_i=1}$, and $C_i|_{x_i=0}$:

$$C_i|_{x_i=2} \gtreqless C_i|_{x_i=1} \iff p_i \gtreqless 3\beta_i \sigma^2, \tag{5}$$

$$C_i|_{x_i=2} \gtreqless C_i|_{x_i=0} \iff p_i \gtreqless 2\beta_i \sigma^2, \tag{6}$$

$$C_i|_{x_i=1} \gtreqless C_i|_{x_i=0} \iff p_i \gtreqless \beta_i \sigma^2. \tag{7}$$

In equilibrium the participation constraint holds with equality. So O_i's expected payoff when she hires a manager is

$$\mathbb{E}\left[\pi_i - W_i\right]|_{x_i=0} = -\bar{u}, \tag{8}$$
$$\mathbb{E}\left[\pi_i - W_i\right]|_{x_i=1} = p_i - \beta_i^2 \sigma^2 - \bar{u}, \tag{9}$$
$$\mathbb{E}\left[\pi_i - W_i\right]|_{x_i=2} = 2p_i - 4\beta_i^2 \sigma^2 - \bar{u}. \tag{10}$$

3.2. THE CASE WHITH TWO MANAGERS

We start with the subcase of $\beta_2 \geq \beta_1 > 0$ and later argue that the other subcases are irrelevant. From the inequalities (5)-(7) we get that the set of sharing equilibria, i.e. the set where both firms sell one unit, is

$$\left\{p_1, p_2 | p_1 = p_2; p_1 \in [\beta_1 \sigma^2, 3\beta_1 \sigma^2] \cap [\beta_2 \sigma^2, 3\beta_2 \sigma^2]\right\}.$$

Next consider the non-sharing equilibria, where some firm i serves the whole market and firm $-i$ sells nothing. From (7) it follows that $p_i \leq \beta_{-i} \sigma^2$ must be true, otherwise M_{-i} would want to lower p_{-i}. From (4) it follows that in equilibrium $p_i \nearrow p_{-i}$, or if there is a price-grid with mesh size ϵ, $p_i = p_{-i} - \epsilon$. We will ignore the ϵ since it is assumed to be small. So the set of potential non-sharing equilibria, ignoring M_i's incentives for a moment, is

$$\left\{p_i, p_{-i} | p_i \nearrow p_{-i}; p_i \leq \beta_{-i}\sigma^2\right\}.$$

But does M_i not want to deviate? From (5) it follows that $p_i \geq 3\beta_i\sigma^2$, otherwise M_i can improve his expected payoff by increasing p_i to p_{-i}. Hence, the set of non-sharing equilibria is

$$\left\{p_1, p_2 | p_1 \nearrow p_2; p_1 \in [3\beta_1\sigma^2, \beta_2\sigma^2]\right\}.$$

Note, for firm 2 the set of non-sharing equilibria is always empty through $\beta_1 \leq \beta_2$.

To get rid of the multiplicity problem we apply the Pareto criterion. By applying the criterion the next lemma follows directly.

LEMMA 1: When $\beta_1 \in [\beta_2/3, \beta_2]$ we have a sharing equilibrium with $p_1 = p_2 = 3\sigma^2 \min\{\beta_1, \beta_2\}$. For $\beta_1 \leq \beta_2/3$ we have a non-sharing equilibrium with $p_1 \nearrow p_2 = \beta_2\sigma^2$.

Note, this equilibria are only equilibria of the subgame with two managers and $\beta_2 \geq \beta_1 > 0$. They need not be equilibria of the total game.

The next lemma restricts the equilibrium set of the subgame further.

LEMMA 2: $\beta_2 > \beta_1$ cannot be an equilibrium.

PROOF: When $\beta_1 \leq \beta_2 < 3\beta_1$ there is a sharing equilibrium. Then by lowering β_2 to β_1 the equilibrium price does not change, see Lemma 1. Nonetheless, $O_1 2$'s expected payoff increases since M_2's risk premium decreases, see (9).

When $\beta_2 > 3\beta_1$ there is a non-sharing equilibrium. O_2 can improve her payoff by setting $\beta_2 = 0$. Then his expected payoff is $2\beta_1\sigma^2 - \bar{u}$ instead of $-\bar{u}$.

When $\beta_2 = 3\beta_1$ there is is both, a sharing and a non-sharing equilibrium. When $\beta_1 < 1$, O_2 is in any case better-off by setting $\beta_2 = \beta_1$. For $\beta_1 \geq 1$, she is in any case better-off by setting $\beta_2 = 0$. □

Observe that when $\beta_i < 0$ manager M_i seeks to minimize profits by choosing p_i infinitely low. This cannot be optimal for O_i. Hence, there are no equilibria with $\beta_1 < 0$ or $\beta_2 < 0$. Moreover, for $\beta_i = 0$ the managers risk aversion plays no role and M_i sets the same prices as when O_i decides herself. Since hiring a manager is costly through $\bar{u} > 0$, it cannot be an equilibrium that $\beta_i = 0$. From Lemma 2 and these arguments, the next lemma follows directly.

LEMMA 3: When both owners hire managers it must hold that $\beta_1 = \beta_2 > 0$.

It remains to be explored whether or not owners indeed want to hire managers.

3.3. THE CASE WHITHOUT MANAGERS

O_i's expected payoff is

$$\mathbb{E}[\pi_i] = x_i p_i.$$

Due to the risk neutrality of owners, the standard Bertrand price setting arises: given some price of the other firm $p_{-i} > 0$, O_i seeks to grab the whole market by setting a price slightly below p_{-i}. Hence, in equilibrium $p_1 = p_2 = 0$. Therefore, both firms have zero expected profits and both owners yield zero expected payoffs. Put differently, the Bertrand Paradox arises.

3.4. THE CASE WHITH ONE MANAGER

Without loss of generality, suppose that O_1 hires no manager. Then O_1 itself sets the prices and acts like when she would have hired a manager and set $\beta_1 = 0$. From the case with two managers we know that when $\beta_2 > 0$ firm 2 sells nothing which yields an expected payoff $-\bar{u}$ for O_2. When $\beta_2 = 0$, O_2's expected payoff is again $-\bar{u}$. For $\beta_2 < 0$, her expected payoff approaches minus infinity. In all cases O_2 is better-off by not hiring a manager which yields an expected payoff of zero. Hence, it is never an equilibrium that only one owner hires a manager.

3.5. THE EQUILIBRIUM

We are left with two equilibrium candidates for the whole game: (i) no managers are hired and (ii) both owners hire managers and set $\beta := \beta_1 = \beta_2 > 0$. In the latter case we get from (9) and Lemma 1 that each owner's expected payoff is

$$\mathbb{E}[\pi_i - W_i]\big|_{x_i=1} = 3\sigma^2\beta - \beta^2\sigma^2 - \bar{u}. \tag{11}$$

Given that O_2 hires a manager and chooses $\beta_2 = \beta$, O_1 may choose not to hire. Then she yields an expected payoff of

$$2\beta\sigma^2, \tag{12}$$

since she sells two units, each for $\beta\sigma^2$, see Lemma 1. Comparing (11) and (12) yields that only for

$$\beta \in \left[\frac{1-\sqrt{1-4\bar{u}/\sigma^2}}{2}, \frac{1+\sqrt{1-4\bar{u}/\sigma^2}}{2}\right] \tag{13}$$

does O_1 not want to deviate by not hiring. By symmetry, this is also the condition for O_2. The intuitive interpretation is as follows. When β is low, the sharing profit is low, too. This profit is not high enough to compensate for the expected wage payment. That is, given that O_2 hires a manager, O_1 is better-off by not hiring. When β is high, the risk premium which managers require is huge. Since the risk premium must be ultimately born by the owner, the expected wage payment is huge, too. Deviating saves the huge wage payment and this overcompensates for the lower gross profit when an owner deviates.[8] So a hiring equilibrium is only possible when β is moderate.

Since $\beta \in \mathbb{R}$, we see from (13) that the existence of a hiring equilibrium requires

$$\bar{u} \leq \sigma^2/4. \tag{14}$$

The intuition is as follows. When an owner wants to grab the whole market she is best-off by not hiring a manager. The firm's gross profit from deviating through not hiring is only 2/3 of the sharing solution. But by deviating the owner saves the manager's wage. Hence, when it is very costly to hire a manager, i.e., if $\bar{u}$ is high, there cannot be an equilibrium in which managers are hired since deviating is too tempting. When $\bar{u}$ is sufficiently low the owners hire managers and no owner wants to deviate by not hiring.

Suppose that (14) holds. Since we apply the Pareto criterion as a selection devise, we now select among the hiring equilibria. Maximization of (11) with respect to β, subject to the constraint (13), yields

$$\beta^* = \beta_1 = \beta_2 = \frac{1+\sqrt{1-4\bar{u}/\sigma^2}}{2}, \tag{15}$$

[8]The firm's gross profit from deviating is only 2/3 of the sharing solution: the firm sells two units instead of one, but the price is only on-third of the price which is yielded in the sharing solution, see Lemma 1.

which is $\in [.5, 1)$ and

$$\mathbb{E}[\pi_1 - W_1] = \mathbb{E}[\pi_2 - W_2] = \frac{1+\sqrt{1-4\bar{u}/\sigma^2}}{2} 3\sigma^2 - \left(\frac{1+\sqrt{1-4\bar{u}/\sigma^2}}{2}\right)^2 \sigma^2 - \bar{u},$$

which is positive. Our analysis is summarized as follows.

PROPOSITION 1: When $\bar{u} \leq \sigma^2/4$, both owners hire managers. Then the equilibrium prices are above expected marginal costs: $p_1 = p_2 = 3\sigma^2\beta^*$. Both firms have positive gross and net profits. When $\bar{u} > \sigma^2/4$, no managers are hired and equilibrium prices equal the expected marginal costs: $p_1 = p_2 = 0$. Both firms have zero gross and net profits.

Note, when marginal costs are certain, or managers are not risk-averse, or both, then no hiring equilibrium exists. (Formally, then $\sigma^2 = 0$ and (14) is violated.) To sum up, the equilibrium price is above the expected marginal costs when (i) marginal costs are uncertain, (ii) managers are risk-averse, and (iii) managers' reservation utility $\bar{u}$ is low enough.

4. DISCUSSION

4.1. MORE GENERAL CONTRACTS

With nonlinear contracts, for example, the owner can specify a base wage and a small bonus, where the bonus is only paid for certain gross profits. Hence, the manager has very strong incentives to set a certain price. Wage schemes may also not only depend on a firm's gross profit, but also on other measures. For example, each owner may set a contract where the manager receives a bonus only in case that the firm has a market share of one-half and sells to the monopoly price. Then in equilibrium both managers will set indeed monopoly prices. Another example with the same equilibrium is a wage scheme where each manager's wage depends on the sum of both firms' profits. However, as our previous analysis showed, prices above marginal costs may also result when each manager's wage scheme is just linear in a firm's profit.

4.2. OTHER PROBLEMS OF THE SEPERATION OF OWNERSHIP AND CONTROL

In a more general model, a firm's profit also depend on the effort chosen by its manager. Normally it is assumed that there is a moral hazard problem, that is a manager's effort is not observable (see Holmström, 1979). Then, for $\beta_i = 1$, M_i will choose the first best effort. Hence, setting $\beta > \beta^*$ and closer to 1 gets more attractive for each owner. Intuitively, a higher powered incentive scheme aligns the manager's interests with those of the owner.[9] Additionally, not hiring a manager gets less attractive, since then nobody can spend effort.

In our model a higher powered incentive scheme does not align the interests of the manager and the owner: With $\beta_i = 0$, firm i's manager sets prices like the owner; for $\beta_i > 0$ his objective function differs from the owner's and his price-setting gets less aggressive. Ultimately, however, this is beneficial for the owner: we know from before that, in equilibrium, managers set prices above expected marginal costs.

4.3. ORDER OF MOVES AND RENEGOTIATION

With sequential moves in the pricing subgame and $\beta_1 = \beta_2$, the unique subgame perfect equilibrium is $p = 3\beta_1\sigma^2$. This is also the equilibrium of the simultaneous move game selected by the Pareto criterion. More generally, it can easily be verified that every equilibrium we have selected in the simultaneous move pricing subgame is also an equilibrium in the sequential move pricing subgame. Similarly, every equilibrium the Pareto criterion selects in the simultaneous move hiring subgame is also an equilibrium in the sequential move hiring subgame. With sequential moves there is no need to apply the Pareto criterion. This has three implications. First, the use of the Pareto criterion as a selection devise is supported. Second, given a wage scheme selected by O_i for M_i, $O{-i}$ is not interested in renegotiating publicly the wage scheme of M_{-i} at a Stage 2.5.[10] Third, the price setters are not interested in renegotiating publicly the prices at a Stage 3.5.

Another theme are secret renegotiations, that is renegotiations which

[9] Jensen and Meckling (1976) show that a separation of ownership and control leads managers to undertake projects with large private benefits which are nonetheless inefficient. As with the moral hazard problem a higher powered incentive scheme mitigates this problem. There are also other problems caused by the separation of ownership and control (for a survey, see, for example, Shleifer and Vishny, 1997). Most of this problems are mitigated by higher powered incentives.

[10] Technically speaking, M_{-i} and O_{-i} cannot yield a Pareto improvement for themselves through renegotiations. Hence, the contract will not be renegotiated. Throughout we assume that there cannot be contracts between parties of different firms (perhaps for antitrust reasons). This assumption is very sensible. Otherwise, the owners can make a contract which states that each firm has to charge the monopoly price. That is, by collusion the Bertrand Paradox can be circumvented and there is obviously no need for managers.

become not visible to the other party.[11] Suppose, for example, that initially both owners hire managers and set $\beta_1 = \beta_2 = \beta^*$. Then we have predicted that M_2 chooses $p_2 = 3\sigma^2\beta^*$. Given this, O_1 may like to compensate her manager secretly with another contract, for example with a fix wage contract. The argument may be as follows: Setting secretly a fix wage (i) saves O_1 the risk premium and (ii) leads to the price pair $p_2 = 3\sigma^2\beta^*$, $p_1 = 3\sigma^2\beta^* - \epsilon$ which results in a huge π_1. Hence, the hiring equilibrium seems to be not robust to secret renegotiations. This is false if one allows not only for renegotiations of the wage scheme, but also for renegotiations of prices. Returning to the example before, M_2 would lower p_2 to p_1. Then M_1 want to lower p_1 again which results finally in the equilibrium $p_1 \nearrow p_2 = \beta_2\sigma^2$. But this is just the equilibrium when M_1's wage scheme would have been publicly renegotiated. From before we know that no party has in equilibrium an incentive to renegotiate publicly since they cannot yield a Pareto improvement. This arguments should clarify that our results are robust to both, secret and public renegotiations.

That is, in our model, contracts can be used for strategic commitment even when they can be secretly or publicly renegotiated. This is in stark contrast to the findings of the literature which allows for renegotiations at only one stage, see Hart and Moore (1988) or Maskin and Tirole (1988); Dewatripont (1988, p. 378) summarizes this view: "Under complete information, it follows that contracts are useless for the purpose of strategic commitment since ... they cannot allow agents to sustain outcomes which are not allocatively efficient."

5. THREE EXTENSIONS

We first allow for more than two firms and then for heterogeneity of managers. The results yielded before stay qualitatively valid. We finally consider the case when firms do not have to meet the demand. We show that managers equipped with performance depended wage schemes set prices above expected marginal costs. When owners determine the wage scheme, there is, however, no equilibrium in which owners hire managers. The reason therefore is that when they hire managers, each owner wants to set lower incentives than the other. Hence, the Bertrand paradox arises. We argue that this is not true in a more general and plausible

[11] In Caillaud, Jullien, and Picard (1995) secret renegotiations are at the heart of their principal-agent model. They consider a setting where there is asymmetric information and parties are risk neutral. In their Bertrand example the goods are differentiated. Also Dewatripont (1988) considers a principal-agent model with secret renegotiations and asymmetric information.

model that recognizes the positive effects of higher powered incentives schemes on problems due to the separation of ownership and control.

5.1. MORE THAN TWO FIRMS

Let there be $N > 2$ firms. First we consider the case when the demand is 2, as in the case with only two firms. Later we will look at the case when the demand is N.

As in the case with two firms one can reduce the set of potential equilibria to two: (i) all owners hire managers and set the same wage scheme $\beta > 0$, (ii) no managers are hired. The certainty equivalent when each firm sells $2/N$ is

$$C_i|_{x_i=2/N} = \alpha_i + \frac{2}{N}\beta_i p_i - \frac{4}{N^2}\beta_i^2\sigma^2. \tag{16}$$

O_i's expected payoff is then

$$\mathbb{E}\left[\pi_i - W_i\right]|_{x_i=2/N} = \frac{2}{N}p_i - \frac{4}{N^2}\beta_i^2\sigma^2 - \bar{u}. \tag{17}$$

When all owners hire managers and set $\beta_i = \beta$, the maximal price for which $C_i|_{x_i=2/N} \geq C_i|_{x_i=2}$ is

$$p = 2\frac{N+1}{N}\beta\sigma^2. \tag{18}$$

There are also other prices for which market sharing is an equilibrium, but the Pareto criterion selects this price. Each owner's expected payoff in this case is

$$\mathbb{E}\left[\pi_i - W_i\right]|_{x_i=2/N} = 4\frac{N+1}{N^2}\beta\sigma^2 - \frac{4}{N^2}\beta^2\sigma^2 - \bar{u}. \tag{19}$$

We have to check that no owner wants to deviate by not hiring. When all owners, except O_i, hire and choose β, O_i will snap the whole market for (equate $C_{-i}|_{x_{-i}=0}$ and $C_{-i}|_{x_{-i}=2/N}$)

$$p_i = \frac{2}{N}\beta\sigma^2.$$

This yields an expected payoff for O_i of

$$\mathbb{E}\left[\pi_i\right]|_{x_i=2} = \frac{4}{N}\beta\sigma^2.$$

Comparing this to (19) yields that the set of hiring equilibria is

$$\left\{\beta|\beta \in \left[\frac{1-\sqrt{1-N^2\bar{u}/\sigma^2}}{2}, \frac{1+\sqrt{1-N^2\bar{u}/\sigma^2}}{2}\right]\right\}.$$

The set is non-empty for $\bar{u} \leq \sigma^2/N^2$. The number of firms must be low enough so that a hiring equilibrium exists. But as we know from before, even with only two firms a hiring equilibrium may not exist. When the set is non-empty the Pareto criterion selects

$$\beta_N^* = \frac{1+\sqrt{1-N^2\bar{u}/\sigma^2}}{2}$$

which is $\in [.5, 1)$. As with two firms, when a hiring equilibrium exists, it Pareto dominates the not hiring equilibrium.

PROPOSITION 2: Suppose there are N firms and market demand is 2. When $\bar{u} \leq \sigma^2/N^2$, all owners hire managers. Then the equilibrium prices are above expected marginal costs: $p_1 = ... = p_N = 2\frac{N+1}{N}\beta_N^*\sigma^2$. Firms have positive gross and net profits. When $\bar{u} > \sigma^2/N^2$, no managers are hired and equilibrium prices equal the expected marginal costs: $p_1 = ... = p_N = 0$. Firms have zero gross and net profits.

Note, the two firm case is just the special case with $N = 2$. The comparative statics of the hiring equilibrium when $\bar{u} < \sigma^2/N^2$ are: $d\beta^*/dN < 0$ and $dp^*/dN < 0$. So both, the equilibrium incentives and prices are decreasing in the number of firms. It is straightforward to show that also the expected payoff of an owner is decreasing in N. As N becomes large, the condition for a hiring equilibrium, $\bar{u} \leq \sigma^2/N^2$, is violated. Therefore, the Bertrand paradox arises and there is perfect competition.

Finally consider the case when the market demand is N, i.e., proportional to the number of firms. Simple calculations yield that for $\bar{u} \leq \sigma^2/4$ the selected hiring equilibrium is

$$\beta^* = \frac{1+\sqrt{1-4\bar{u}/\sigma^2}}{2},$$

which is independent of N. For $\bar{u} > \sigma^2/4$ no managers are hired.

PROPOSITION 3: Suppose there are N firms and market demand is N. When $\bar{u} \leq \sigma^2/4$, all owners hire managers. Then the equilibrium prices are above expected marginal costs: $p_1 = ... = p_N = \beta^*\sigma^2(N+1)$. Each firm has positive gross and net profits. The latter are $\beta^*\sigma^2(N+1) - \beta^{*2}\sigma^2 - \bar{u}$. When $\bar{u} > \sigma^2/4$, no managers are hired and equilibrium prices equal the expected marginal costs: $p_1 = ... = p_N = 0$. Firms have zero gross and net profits.

Observe that for $\bar{u} \leq \sigma^2/4$ the equilibrium price, as well as the equilibrium expected payoff of each owner, are increasing in N. So even a large number of firms is not sufficient to approximate perfect competition.[12]

5.2. HETEROGENEOUS MANAGERS OR COST SHOCKS

When managers are heterogenous, or firms face different cost shocks, or both, one has to use the firm specific σ_1^2 and σ_2^2 instead of just σ^2. We consider the two firm case.

PROPOSITION 4: Suppose there are two firms, market demand is 2, and $\sigma_1^2 > \sigma_2^2$. When $\bar{u} \leq \sigma_2^2/4$, all owners hire managers. Then the equilibrium prices are above expected marginal costs: $p_1 = p_2 = 3\tilde{\beta}_2\sigma_2^2$, where $\tilde{\beta}_2 = \frac{1+\sqrt{1-4\bar{u}/\sigma_2^2}}{2}$. Moreover, $\beta_2 = \tilde{\beta}_2$ and $\beta_1 = \tilde{\beta}_2\frac{\sigma_2^2}{\sigma_1^2}$. Both firms have positive gross and net profits. The latter are $\mathbb{E}\left[\pi_1 - W_1\right] = 3\tilde{\beta}_2\sigma_2^2 - (\tilde{\beta}_2\frac{\sigma_2^2}{\sigma_1^2})^2\sigma_1^2 - \bar{u}$ and $\mathbb{E}\left[\pi_2 - W_2\right] = 3\beta_2\sigma_2^2 - (\tilde{\beta}_2)^2\sigma_2^2 - \bar{u}$. When $\bar{u} > \sigma_2^2/4$, no managers are hired and equilibrium prices equal the expected marginal costs: $p_1 = p_2 = 0$. Firms have zero gross and net profits.

Interestingly, in the hiring equilibrium O_1 sets weaker incentives than O_2. Moreover, O_1's expected payoff is higher than O_2's. That is, an owner is better-off with more cost uncertainty or a more risk-averse manager.

5.3. FIRMS NEED NOT MEET DEMAND

Previously we have assumed that firms have to meet demand, that is, consumers can buy every amount they wish from a firm. Now we assume instead that firms can choose how much to sell. We assume that firms decide how much they want to sell.

For the same reasons as before we assume that when managers are hired $\beta_1, \beta_2 \geq 0$. From (1) we get that

$$C_i = \alpha_i + \beta_i x_i p_i - \beta_i^2 x_i^2 \sigma^2.$$

[12]This result is counter to most of the oligopoly literature. A notable exception is Wambach (1999), see his Proposition 2.

In a model similar to Wambach's, Cheng (2002) shows that (i) Bertrand competition can be less "competitive" than Cournot competition and (ii) the highest Bertrand equilibrium price converges to one higher than the competitive.

Hence, C_i is a concave function of x_i with the maximum for

$$x_i^* = \frac{p_i}{2\beta_i\sigma^2}.$$

Without specifying exactly specifying the game in which firms determine how much they sell, we suppose that in equilibrium each firm sells the amount x_i^*, demand equals total supply, and when both firms sell they both charge the same price p. From $x_1^* + x_2^* = 2$ we get that

$$\hat{p} := p_1 = p_2 = \frac{4\beta_1\beta_2\sigma^2}{\beta_1 + \beta_2}. \tag{20}$$

O_i's expected payoff is then

$$\mathbb{E}[\pi_i - W_i]|_{\hat{p}} = -\bar{u} + \frac{8\beta_1^2\beta_2^2\sigma^2}{(\beta_1+\beta_2)^2}\left(\frac{1}{\beta_i} - \frac{1}{2}\right).$$

Given that O_1 hires a manager, she maximizes over β_1.

$$\frac{d\,\mathbb{E}[\pi_1 - w_1]|_{\hat{p}}}{d\beta_1} = \frac{8\beta_2^2\sigma^2}{(\beta_1+\beta_2)^3}\left(\beta_2 - \beta_1 - \beta_1\beta_2\right). \tag{21}$$

Without loss of generality suppose that $\beta_1 \geq \beta_2 > 0$. Then the previous condition is negative which means that this cannot be an equilibrium. Intuitively, each owner wants to set a lower powered incentives than the other. To understand why this is the case, look at the revenue which a firms make:

$$x_1^* \times \hat{p} = \frac{8\beta_1\beta_2^2\sigma^2}{(\beta_1+\beta_2)^2}.$$

It is straightforward that for $\beta_1 \geq \beta_2$ the derivative with respect to β_1 is non-positive. That is, firm 1's revenue is non-increasing in β_1. Since additionally the risk premium which M_1 charges is increasing in β_1, O_1 is better off by setting β_1 below β_2.

So given that both owner hire managers, in equilibrium we have $\beta_1 = \beta_2 = 0$. Then both managers set prices equal to marginal costs. That is, the Bertrand equilibrium arises. But since it is costly to hire managers, it is straightforward that it cannot be an equilibrium that a manager is hired. Hence, in equilibrium, no managers are hired and the Bertrand paradox arises.

From Section 4.2 we know that in a more general and more realistic model other problems arise when a manager is hired and that this problems are mitigated through a higher powered incentive scheme. Using this insight we assume that firm i's profit has another additive component $h(\beta_i)$ which increasing in β_i (at least for small β_i's). Then (21) changes to

$$\frac{d\,\mathbb{E}[\pi_i - W_i]|_{\hat{p}}}{d\beta_1} = \frac{8\beta_2^2\sigma^2}{(\beta_1+\beta_2)^3}\,(\beta_2 - \beta_1 - \beta_1\beta_2) + h'(\beta_1). \qquad (22)$$

Assuming that $h'(\beta_i) > \epsilon$ for small β_i, where ϵ is some small and positive number, yields that there is a symmetric equilibrium with $\beta_1 = \beta_2 > 0$. Equipped with such an incentives schemes, managers will set prices above marginal costs, see (20). So our result from the model with the "meet demand clause" is also valid here: given that $\bar{u}$ is small enough, owners hire managers and managers set prices above marginal costs.

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