Asymmetric cost pass-through in supply function equilibria, coordinating bids in power spot markets

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Abstract

In January 2007, first evidence of an asymmetric pass-through of CO2 emission allowance prices was reported for the German electricity spot market. Upward changes of carbon prices had a larger effect on power prices than downward changes. This paper explores the theoretical basis of such an asymmetry in the context of a supply function bidding duopoly. It interprets fluctuating carbon prices as a coordination mechanism for tacitly colluding firms and studies incentive compatibility in the repeated game. It is new in its attempt to model asymmetric behaviour in a spot market without relevant frictions, and provides a rationale for why the asymmetry shows up specifically for emission allowances. In conclusion, the paper provides the formal proof of a theorem which states that asymmetric price transmission is sustained up to a certain maximum level and that this mechanism is always preferred to non-cooperation.

1 Introduction

Following its deregulation in 1998, the German market for electricity generation has seen a wave of mergers leading to a four-firm oligopoly with two dominant firms [Bundeskartellamt, 2006]. In 2001, a wholesale spot market for electricity was established at the European Energy Exchange (EEX) in Leipzig. The market operates as a daily one-shot auction, in which power generators have to submit stepwise linear and non-decreasing supply functions, specifying the quantities they are willing to supply at each price, while buyers have to submit stepwise linear and non-increasing demand functions which specify the quantities they are willing to buy at each price. Aggregating the submitted demand and supply functions, the auctioneer chooses a market clearing equilibrium price at which trades are executed according to the submitted bids. The German setting is thus, with respect to market concentration and market design, quite similar to other deregulated power markets around the world.

In 2005, the European Union introduced the EU Emission Trading Scheme (EU ETS) which caps the total amount of CO_2 emissions for several industries but allows to trade the (freely distributed) emission allowances (EUA) among regulated firms. Since then, the prices of EUA reflect an opportunity cost for the generation of electricity with fossil fuel fired plants. The price for electricity in the spot market is determined by the marginal plant, and these plants are mostly gas or coal-fired. Hence, according to economic theory one would expect that an increase in the price of EUA would lead to an increase in the wholesale prices of electricity, while a decrease in the price of EUA would lead to a decrease in the wholesale prices of electricity. Indeed, following the introduction of the EU ETS, both the spot and future wholesale prices for power followed quite closely the increase in EUA prices, which peaked in April 2006. In May 2006, the carbon market crashed and the EUA price dropped from almost €30 per ton of CO_2 to less than €10 per ton. It continued to decrease and reached €0.80 per ton in February 2007. However, this steep price decline has not yet led to a corresponding drop in the wholesale price of electricity at EEX. Zachman and Hirschhausen [Zachmann and von Hirschhausen, 2008] find strong empirical evidence of an asymmetric effect of EUA prices on power prices at EEX, but they do not provide a theoretical explanation. That is where the following article attempts to shed light on.

Similar phenomena have been observed in various markets. The most prominent example is the US gasoline market in the early 1990s. Increasing crude oil prices caused sky-rocketing gasoline prices, but the inverse relation did not hold[1]. Peltzman [Peltzman, 2000] reports asymmetric pass-through of input prices for a large number of products mainly in the food sector, and states that this "points to a serious gap in a fundamental area of economic theory". In search of a theoretical explanation of asymmetric price adjustments, four prominent reasons are advanced in the literature: consumer search, menu costs, inventories and collusion.

The pattern of US gasoline prices can very well be explained by search costs, preventing consumers to search further when they observe declining prices [Lewis, 2001]. Suppliers then temporarily profit from less elastic demand and keep prices comparatively high. A second approach refers to menu costs. Here,

[1] The literature refers to 'rockets and feathers' because the downward adjustment reminded more of the falling of a dwindling feather [Borenstein et al., 1997].

it is the supplier who bears a cost for adjusting output prices. When the cost of 'selling at a wrong price' does not behave symmetrically around the current price, it may be profitable to adjust prices upward but not downward for some shocks [Kuran, 1983]. Thirdly, Reagan and Weitzman [Reagan and Weitzman, 1982] model asymmetric price adjustments without referring to search or menu costs. Instead, they consider inventories. With finite stocks and lags in production, upward shocks in demand might not be addressed by suppliers as quickly as negative demand shocks. Price adjustments will compensate the lack of flexibility in quantity and show an asymmetry. Finally, tacit collusion is probably the most popular and sometimes trivial explanation for asymmetric price adjustments in oligopolistic markets. A common prejudice says that firms raise extra profits by increasing prices faster in response to rising costs than they decrease prices in the opposite case. From a theory-based point of view, one may argue that asymmetric price adjustments are the result of a coordination game. When an oligopoly faces a decrease in costs, the level of the new equilibrium price will be lower than before. By omitting the downward adjustment, firms may signal their willingness to collude to competitors. Collusion can be sustained using the old price as a focal or trigger price. A caveat of this argument is that if signalling through prices is possible, the oligopoly might tacitly coordinate on higher prices without any need for an exogenous focal point.

When bids are made in functions, however, it is not straightforward to signal a high price agreement to one's competitors, especially when individual bids are not common knowledge. At EEX, for example, only the market clearing price and equilibrium quantity are reported after each auction. Moreover, the spot market design leaves no room for consumer search or menu costs. The instantaneous nature of electricity rules out inventories. Collusion thus remains the most likely candidate to explain an asymmetric cost pass-through.

The following article argues that input prices can be a well-functioning instrument to coordinate bids over a whole possible set of supply functions. The argument can intuitively explain why EUA prices are ideally suited to play this role, and a theoretical analysis shows that collusion is always preferred to non-cooperation. Section 2 will introduce a stylised static model of competition in supply functions and study joint profit maximisation in this setting. Section 3 explores the informational structure of the market and provides a raationale for why carbon prices qualify best as an instrument of collusion. Section 4 studies the incentive structure of the repeated game and section 5 concludes.

2 The Model

The theoretical analysis of competition in supply functions is of significant mathematical complexity and generally results in a multitude of possible equilibria as was shown by Klemperer and Meyer [Klemperer and Meyer, 1989] in their seminal work on the subject. Power spot market designs often impose supply function bidding to provide the necessary flexibility of supply. Most applications of the theory therefore focused on electricity markets.

To model competition in supply functions, the linear approach has proven to be quite useful in empirically motivated studies [Baldick et al., 2004]. Advancements have been made with numerical implementations, introducing asymmetric firms or allowing supply functions to be stepwise linear, e.g. by [Green, 1996] or [Baldick et al., 2004]. I will restrict my analysis to the symmetric linear duopoly case which has already been solved by Turnbull [Turnbull, 1983], and which provides a unique and analytically solvable equilibrium. In detail, linearity assumptions are made for demand, supply, and marginal costs. The following subsection introduces the simple model based on Turnbull. Subsequent parts will extend the analysis to cooperation and the repeated game.

A basic model of supply function equilibrium

Let i and j be two symmetric firms, producing quantities q_i, q_j at cost $C(q)$.

$$C(q) = \frac{c}{2}\,q^2 \text{ with } c > 0$$

The cost parameter c can be seen as an aggregate of carbon prices and other production costs: for instance, let p_e be the price for emission allowances needed for the first produced quantity of output q. Assume emission intensity to increase along the expansion path, such that the effect can be approximated by a quadratic relation. The costs of emissions are then given by $p_e \cdot q^2$.

Firms compete by submitting linear supply functions $q(p) = \beta\, p$. A strategy of firm i is a supply function $q_i(p)$ which is completely identified by the slope parameter $\beta_i > 0$. The auctioneer determines the equilibrium price $p^* > 0$ such that aggregate produced quantity $q_i(p^*) + q_j(p^*)$ equals demand $D(p^*)$. The functional forms of demand, the market clearing price p^*, and Π^i, the profit of firm i, are as follows:

$$\begin{aligned}
D(p) &= \tilde{N} - \gamma p && \text{with } \tilde{N}, \gamma \geq 0\\
p^* &= \frac{\tilde{N}}{\gamma + \beta_i + \beta_j}\\
\Pi^i &= p^* q_i - C(q_i)\\
\Pi^i(\beta_i, \beta_j) &= \left(\frac{\tilde{N}}{\gamma + \beta_j + \beta_i}\right)^2 (\beta_i - \frac{c}{2}\beta_i^2) && (1)
\end{aligned}$$

Demand fluctuations are covered by $\tilde{N}$, a positive random variable whose realisation is not known in advance[2]. Firm i then maximises its profit for each N over its residual demand $q_i = D(p) - q_j(p)$.

$$\max[\Pi^i] = p \cdot [D(p) - q_j(p)] \;-\; C([D(p) - q_j(p)])$$

The first order condition is

$$\begin{aligned}
\frac{\partial \Pi^i}{\partial p} &= [D(p) - q_j(p)] + [D'(p) - q_j'(p)] \cdot [p - C'] &&= 0\\
&= q_i - (\gamma + \beta_j)(p - c q_i) &&= 0\\
\Rightarrow\; \beta_i &= (\gamma + \beta_j) - c\beta_i(\gamma + \beta_j)
\end{aligned}$$

[2]This model can fully represent the more general model with affine supply functions $s(p) = \alpha + \beta p$ and costs $K(q) = c_0 + c_1 p + c_2 p^2$ as has been shown, for instance, by Baldick et al. [Baldick et al., 2004]: firms will always choose the intercept of the inverse supply function to equal the intercept of marginal cost. When firms are symmetric, arguing in prices net of the marginal cost intercept therefore allows, without loss of generality, to reduce the model to the linear one as it is presented here.

The last equation imposes a general condition on the strategic parameters β_i and β_j, which does not depend on the price or the random variable $\tilde{N}$. This illustrates a central feature of SFE models; firms choose an optimal supply schedule for *every* possible realisation of demand [Klemperer and Meyer, 1989]. Solving for β_i yields the best response function for firm i.

$$\beta_i = BR(\beta_j) \equiv \frac{(\gamma + \beta_j)}{1 + c(\gamma + \beta_j)} \tag{2}$$

The second order condition for profit maximisation holds, as is shown in appendix A.1. Indeed, the necessary and sufficient conditions are fulfilled at one and only one point given by the best response (2). Thus the profit of firm i increases with β_i for all $\beta_i \in [0, BR(\beta_j)]$ and decreases for all $\beta_i > BR(\beta_j)$. This property is needed for further discussion.

Property 1.

$$\frac{\partial \Pi_i(\beta_i, \beta_j)}{\partial \beta_i} = \begin{cases} > 0 & \textit{if } \beta_i \in [0, BR(\beta_j)[\\ = 0 & \textit{if } \beta_i = BR(\beta_j) \\ < 0 & \textit{if } \beta_i > BR(\beta_j). \end{cases}$$

Note that the upper bound of the best response $\lim_{\beta_j \to \infty} BR(\beta_j) = \frac{1}{c}$ corresponds to the marginal cost bid. Any strategy $\beta < 1/c$ defines a *less* competitive bid. The lower bound $\lim_{\beta_j \to 0} BR(\beta_j) = \frac{\gamma}{1+\gamma c}$ is the trace through all Cournot bids in a market with the given demand and cost functions. From the derivative of the best response,

$$0 \; < \; BR'(\beta_j) = \frac{1}{(1 + c(\gamma + \beta_j))^2} \; < \; 1,$$

we can see that the slope parameters β_i and β_j are strategic complements and that the nested best response function constitutes a contraction map. In other words: an increase of β_j also increases i's best response, but with lower magnitude and vice versa. Thus, no matter what the initial level of β_j is, an iteration of mutual best responses necessarily converges to an equilibrium between Cournot and Bertrand strategies.

Solving for the symmetric best response yields only one positive solution, which is the unique supply function equilibrium in non-cooperative strategies: $\beta_i = \beta_j = \beta_{SFE}$).

$$\beta_{SFE} \equiv \frac{\sqrt{\gamma^2 + \frac{4\gamma}{c}} - \gamma}{2} \tag{3}$$

Joint profit maximisation

The Nash equilibrium established above is the solution to the non-cooperative one-shot game. Assuming that cooperative strategies are feasible, firms might do better by coordinating their bids and thus maximising joint profits. Coordination is of interest if joint profit maximisation outperforms the profits of the non-cooperative equilibrium. The cooperative programme for two symmetric firms is $\max_\beta [2\Pi(\beta,\beta)]$. FOC:

$$\frac{\partial 2\Pi(\beta,\beta)}{\partial\beta} = 2\left[\left(2\cdot\frac{-2N^2}{(\gamma+2\beta)^3}\right)\cdot\left(\beta-\frac{c}{2}\beta^2\right)+\left(\frac{N^2}{(\gamma+2\beta)^2}\right)\cdot(1-c\beta)\right] = 0$$

$$\Leftrightarrow -4\left(\beta-\frac{c}{2}\beta^2\right)+(2\beta+\gamma)(1-c\beta) = 0$$

which solves to

$$\beta = \beta_{col} \equiv \frac{\gamma}{2+c\gamma}. \tag{4}$$

Joint profit maximisation over the same set of strategies $(\beta_i,\beta_j)\in \mathbb{R}^2_+$ yields a different optimal strategy compared to the non-cooperative equilibrium: $\beta_{col} < \beta_{SFE}$. This implies that cooperation pays. The local second order condition holds as well, and is checked in appendix A.2. Since there is a unique maximum, profits increase for joint variation of β in $[0,\beta_{col}]$, and decrease beyond β_{col}:

Property 2.

$$\frac{\partial\Pi_i(\beta,\beta)}{\partial\beta} = \begin{cases} >0 & \textit{if } \beta\in[0,\beta_{col}[\\ =0 & \textit{if } \beta=\beta_{col} \\ <0 & \textit{if } \beta>\beta_{col}. \end{cases}$$

Note that the quantities supplied by firms bidding β_{col} are even below the Cournot quantities. At this stage, it might be worthwhile discussing the empirical relevance of such a quasi-monopoly solution. To my knowledge, there are only theoretical studies considering such joint profit maximisation beyond the Cournot-solution. An early paper by Bolle [Bolle, 1992] discusses the danger of tacit collusion in supply functions for deregulated electricity markets. The author argues that the monopoly outcome could be sustained by an arbitrary large number of firms when consumers are paying a fixed flat rate price for electricity. A more recent paper compares the possibility of collusion in quantity competition and in supply function competition [Ciarreta and Gutiérrez-Hita, 2006]. The authors conclude that under supply function competition collusive agreements are sometimes easier to sustain, depending on the number of competitors and the slope of demand. If and how those collusive agreements can be installed, however, is not addressed.

In contrast to some theoretical contributions, the empirical literature on power spot markets mainly considers equilibria within the range between Cournot and Bertrand strategies (see e.g. [Green and Newberry, 1992]). This is due to the highly inelastic demand for electricity. Even Cournot models have been accused of exaggerating the exertion of market power when realistic assumptions for the elasticity of demand are made [Baldick et al., 2004]. This was, among others, an important reason to motivate the use of SFE models in applied studies. Perfectly collusive or monopolistic strategies would result in prices far above the Cournot outcome and are unlikely to be observed.

My reason to discuss a perfect cartel solution is twofold: first, the result gives a benchmark down to which point capacity withholding is theoretically profitable. Second, since the cooperative strategies exaggerate capacity withholding, it is evident that these strategies are unlikely to describe the current situation and that their implementation would arouse great public indignation. These conclusions are particularly relevant to the study of the repeated game in the next section.

3 Coordination in the Repeated Game

Power spot markets typically work on a daily basis with the shortest possible lag between market clearance and physical delivery. Demand might fluctuate, but the rules and participants of the market are rather stable, which corresponds to a classic setting of a repeated oligopoly game. For the repeated game of the duopoly described in section 2, the objective of firm i is to maximise the present value of all future profits. Let $\hat{\Pi}_t^i$ be the expected value of i's profit in period t with respect to the random demand parameter $\tilde{N}$. Time indices for the strategic variables are omitted for simplicity.

$$\hat{\Pi}_t^i(\beta_i, \beta_j) \equiv \mathbb{E}_{\tilde{N}_t}\left[\Pi^i\right] = \frac{\mathbb{E}[\tilde{N}^2]}{(\gamma + \beta_i + \beta_j)^2}\left(\beta_i - \frac{c_t}{2}\beta_i^2\right) \tag{5}$$

This definition is symmetric to the definition of profits in equation (1). Now, let r represent the day-to-day interest rate to discount future profits. The program for intertemporal profit maximisation of firm i in period $t = 0$ is

$$\max\left[\sum_{t=0}^{\infty}\left(\frac{1}{1+r}\right)^t \hat{\Pi}_t^i(\beta_i, \beta_j)\right] \tag{6}$$

Evidently, the one-shot Nash equilibrium $\beta_i = \beta_j = \beta_{SFE}$ from section (2) is also a solution to the repeated game. But it is a well known result that in repeated oligopoly games a continuum of collusive outcomes beyond the one-shot Nash strategies can be sustained as equilibria by a self-policing cartel because cooperative strategies are enforced when firms fear punishment in future periods (see e.g. [Tirole, 1988] for price competition, [Green and Porter, 1984] for Cournot competition or [Ciarreta and Gutiérrez-Hita, 2006] for competition in supply functions). An oligopoly engaged in collusion typically earns significantly higher profits, but has to agree on the equilibrium it wants to enforce. Thus, firms have to coordinate.

Coordination of strategies might work through overt communication, a tacit agreement or signalling. Overt communication is generally ruled out by law. A natural focal point for tacitly colluding firms is the monopoly solution which would be chosen by a perfect cartel. But the perfect cartel solution for power spot markets, as discussed in section 2, is likely to exaggerate price increases far beyond a feasible level and thus would immediately trigger governmental action. A tiny departure from the competitive equilibrium, however, might be less visible and already increase firms profits remarkably. To realise these profits, firms have to overcome the dilemma of multiple equilibria and uncertainty about the rivals' strategy. As mentioned before, individual bids at EEX are not reported. When the market has cleared, each firm learns the equilibrium price, the quantity it sold and the size of the whole market. It can therefore deduce ex-post from the number of its competitors if the rest of the market bid higher or lower quantities for the same price. It cannot deduce the slope or shape of other firms supply functions. The lack of visibility hinders coordination of individual bids by direct signalling. Suppose a firm tries to induce collusive bidding through direct signalling: it would risk large losses before rivals could perceive the collusive device. A coordinated simultaneous move of all firms, however, might increase profits for each of the players.

Asymmetric cost pass-through

Supposed that initially all firms bid their non-cooperative one-shot equilibrium strategies β_{SFE}: If the oligopoly aims at realising collusive profits, suppliers have to agree on '*when?*' and '*how much?*' they want to tighten their supply. They need a coordinating mechanism, which leads to a simultaneous departure from the one-shot equilibrium. Such a coordinating mechanism should (a) be tacit for legal reasons, should (b) lead to small but repeated joint variation towards the collusive strategies, should (c) be irregular enough not to be easily recognisable by an outsider but (d) has to be precise and easily interpretable by each firm.

An asymmetric pass-through of input prices can fulfil these requirements. The slope of the equilibrium supply function β_{SFE} decreases in costs. A not fully transmitted downward cost shock will result in an ex-post less competitive bid, incorporating 'historical' or say '*fictive*' costs. To describe this formally, let $c_t \in [\underline{c}, \bar{c}]$ be the realisation of the fluctuating cost parameter $\tilde{c}_t$ in period t. Now, define a measure Δ_t of non-transmitted downward cost shocks for each $t \in \mathbb{R}_+$.

$$\Delta_t \equiv \begin{cases} 0 & \text{if } t = 0 \\ \Delta_{t-1} & \text{if } t > 0,\ c_t \geq c_{t-1} \\ \Delta_{t-1} + x \cdot (c_{t-1} - c_t),\ \text{with } x \in [0,1] & \text{if } t > 0,\ c_t < c_{t-1} \end{cases} \tag{7}$$

Firms bidding according to fictive costs $c_t + \Delta_t$ transmit costs asymmetrically, where the parameter x determines the degree of downward rigidity. $x = 0$ implies perfect pass-through of all costs shocks, while $x = 1$ represents total downward rigidity. In the linear case, such a downward rigid strategy is fully described by the following slope parameter:

$$\beta_{APT}(\Delta_t) \equiv \frac{\sqrt{\gamma^2 + \frac{4\gamma}{c_t + \Delta_t}} - \gamma}{2} \tag{8}$$

Given a starting date $t = 0$, the value of Δ_t measures precisely the distance from the non-cooperative equilibrium in each period. In contrast to a simple mark up, the cost shock is not shifting, but turning the supply functions towards higher prices. This makes a significant difference because the danger of zero supply at positive prices is eliminated. When more flexible forms of supply functions are allowed, a cost shock gives a sophisticated signal affecting the whole shape of a function. Nevertheless, each firm will be able to interpret this signal because all firms should be used to handle costs and supply. No further coordination apart from an initial tacit agreement on the starting date is needed to implement this mechanism. If all firms incorporate the same value of Δ_t, no firm has to bear the loss of a unilateral departure from its best response. Therefore, the quality of the external signal $\tilde{c}_t$ is essential for the efficiency of this mechanism. $\tilde{c}_t$ has to fluctuate continuously with small amplitude and an easily and precisely observable outcome. Emission allowances fulfil this function perfectly: they are auctioned daily, they show continuous price fluctuations, and they are an input to all fossil fuel fired plants, as opposed to solely gas or coal. Another important point is the question of initial timing. There should be an unambiguous date $t = 0$ when the game starts to obtain the symmetry of bids. The commencement of EU ETS provides a natural focal point for the start of the collusive game. No other input but EUA has this property, so the introduction of emissions trading in 2005 provided a well suited opportunity for the installed power oligopoly to collude tacitly by an asymmetric pass-through of EUA prices. If this is true, the discreetness of the mechanism has been witnessed by reality. It took two years and an immense crash in the carbon market 2006 to reveal the asymmetry. Of course the same mechanism would work with any other common cost shock, but the features of EUA seem to be specifically well suited to constitute a collusive device. This might explain why the asymmetries found by Zachmann and Hirschhausen [Zachmann and von Hirschhausen, 2008] are significant only for carbon prices.

The incentive constraint in the repeated game

Lowering offer curves only pays to all firms until the optimal collusive solution β_{col} is reached. From then on firms will have an interest in transmitting costs symmetrically ($x = 0$), such that the oligopoly acts like a perfect cartel. Define Δ_{max} to be the maximum value for Δ_t which implements the collusive bid.

$$\Delta_{max} \equiv \arg[\Delta \in \mathbb{R}_+, \, \beta_{APT}(\Delta) = \beta_{col}] = \frac{4 + c\gamma}{\gamma(3 + c\gamma)} \tag{9}$$

As mentioned above, it is questionable if this level of Δ_t is politically feasible. Here it only gives an upper bound on capacity withholding for the repeated game.

The greater concern is individual incentive feasibility. In short, expected future profits resulting from sustained collusion must exceed profits from cheating and playing the individual best response. Define the deviating strategy as

$$\beta_{dev}(\Delta_t) = BR(\beta_{APT}(\Delta_t)) \tag{10}$$

Deviation can be detected ex-post even if individual bids are not common knowledge. Dividing total quantity by the number of suppliers gives the average market supply at a given price. If a firm realises that this point lies significantly above its own bid, it can infer that there was deviation. Once cheated, a tricked firm will seek to play its individual best response as well, since the collusive agreement has been broken. By repeated anticipation of each other's best responses, both firms will go straight back to their one-shot equilibrium strategies β_{SFE}. Lacking a new starting point for coordination, firms cannot do better than the one-shot equilibrium for all following periods. So the structure of the repeated game is as follows: starting at $t = 0$ with $\Delta_0 = 0$, firms bid the asymmetrically adjusting strategies β_{APT}. When a firm deviates, it plays its best response according to (10). After collusion has broken, both firms are back at the non-cooperative equilibrium for all following periods. If the perfect cartel situation is reached without deviation, firms will jointly bid like a perfect cartel.

The resulting incentive constraint for sustained collusion in period τ is

$$\mathbb{E}_{\tilde{c}_t}\left[\sum_{t=\tau}^{\infty}\frac{\hat{\Pi}_{APT}(\Delta_t)}{(1+r)^{t-\tau}}\right] \geq \mathbb{E}_{\tilde{c}_t}\left[\hat{\Pi}_{dev}(\Delta_\tau)\right] + \mathbb{E}_{\tilde{c}_t}\left[\sum_{t=(\tau+1)}^{\infty}\frac{\hat{\Pi}_{SFE}}{(1+r)^{t-\tau}}\right] \quad (11)$$

with $\hat{\Pi}_{APT}$ representing profits from sustained collusion at the level Δ_τ, $\hat{\Pi}_{dev}$ being deviative profits and $\hat{\Pi}_{SFE}$ the profits at the non-cooperative equilibrium.

$$\left.\begin{array}{lcl} \hat{\Pi}_{APT}(\Delta_\tau) & = & \hat{\Pi}^i(\beta_{APT}(\Delta_\tau),\beta_{APT}(\Delta_\tau)) \\ \hat{\Pi}_{dev}(\Delta_\tau) & = & \hat{\Pi}^i(\beta_{dev}(\Delta_\tau),\beta_{APT}(\Delta_\tau)) \\ \hat{\Pi}_{SFE} & = & \hat{\Pi}^i(\beta_{SFE},\beta_{SFE}) \end{array}\right\} \quad (12)$$

I am interested in Δ_τ as the critical variable for sustained collusion and will treat r as a parameter[3]. Note that it is sufficient to analyse the constraint with respect to one critical value of Δ_τ for the current period τ and all future periods $t \geq \tau$. Let Δ_{IC} denote this critical value, such that the constraint (11) still holds. Only the left hand side of the inequality is affected by future increases of Δ_t. So when $\Delta_\tau > \Delta_{IC}$, (11) is not verified, and considering future increases of Δ_t is meaningless because collusion already broke. When (11) is slack, considering $\Delta_t > \Delta_\tau$ does not harm the constraint for all $\Delta_\tau \leq \Delta_{max}$, because this would only increase the left hand side, representing future profits from sustained collusion (see Property 2). To resume this argument: assuming no future changes in Δ_t is the most critical scenario for the incentive constraint which implies that the constraint holds for all other scenarios as well. It would be interesting to derive an analytical solution for this critical value Δ_{IC}, but unfortunately, it seems impossible to solve the constraint explicitly. Nevertheless, the following section provides analytical results from the overall behaviour of the inequality.

4 Solving the Repeated Game

The complexity of the incentive constraint lies not only in its functional form, but also in the presence of *two* random variables. So there are some assumptions to be made about the stochastic properties of cost and demand shocks.

Assumption 1. The anticipated probability distribution of $\tilde{N}_t$ is independent of cost shocks and of time.

Assumption 2. The anticipated probability distribution function $f(c_t)$ is the same for all t and $\mathbb{E}[\tilde{c}_t] = c_\tau$ for all $t \geq \tau$.

Assumption 1 is the less critical assumption: Independence of cost and demand shocks seems reasonable for the case of input prices and electricity consumption. The assumption of constant expected demand over time might as well be replaced by assuming a linear time trend, which would simply translate into a different discount factor. For the sake of simplicity, I did not consider a time trend here.

[3]This is a crucial difference to [Ciarreta and Gutiérrez-Hita, 2006]

Assumption 2 is quite restrictive but inevitable in order to obtain analytical results. Numerical implementations might overcome this drawback in the future, but this is clearly beyond the scope of this paper.

The assumptions permit to treat constraint (11) in a much more convenient form. Expected profits with respect to demand shocks can be expressed as in (5); expected profits with respect to cost shocks are obtained by integrating over all possible values of $c_t \in [\underline{c}, \overline{c}]$. Independence of time allows to sum up equivalent terms for future expected profits, which form a geometric sequence. The incentive constraint can now be specified as follows.

$$\hat{\Pi}_{APT}(\Delta_\tau) + \int_{\underline{c}}^{\overline{c}} f(c)\ \left(\tfrac{1}{r}\right) \hat{\Pi}_{APT}(\Delta_\tau)\ \mathrm{d}c \geq \hat{\Pi}_{dev}(\Delta_\tau) + \int_{\underline{c}}^{\overline{c}} f(c)\ \left(\tfrac{1}{r}\right) \hat{\Pi}_{SFE}\ \mathrm{d}c \tag{13}$$

The left hand side denotes profits from sustained collusion. The right hand side represents a one-period profit from deviation plus the discounted profits from non-cooperation in the future. Both sides of the inequality show integrals over c, which, however, do not include the whole term: for the current period τ the realisation of costs is known, but demand is not. Firms therefore calculate based on the actual cost parameter and expected profits with respect to demand. Future profits are uncertain with respect to cost and demand fluctuations. Expected profits with respect to both variables are discounted at the interest rate r. Note again that future changes in Δ_t would only increase profits from sustained collusion. Thus, using the current value Δ_τ for the evaluation of future profits does not affect the validity of the incentive constraint.

Still, the constraint is not solvable for a specific critical value[4], but insights can be gained, analysing the overall behaviour for variations in Δ_τ. The results are summed up in the following theorem, which is proved in three successive claims. Figure 1 might serve as an illustration.

Theorem. *For an arbitrary set of parameters $\{r, \gamma, c_\tau\}$, firms always prefer to transmit costs asymmetrically when they are at the one-shot equilibrium, for a continuous interval $\Delta_\tau \in [0, \Delta_{IC}]$ with $\Delta_{IC} > 0$, but never beyond Δ_{IC}.*

Proof of the Theorem

Claim 1. *For a given pair of parameters (r, γ), there exists a unique value $\Delta_{IC} \in \mathbb{R}_+$, such that the incentive constraint (13) is verified at Δ_{IC} and not verified for all $\Delta_\tau > \Delta_{IC}$.*

Proof. At $\Delta_\tau = 0$, $\beta_{APT} = \beta_{dev} = \beta_{SFE}$, thus the inequality (13) becomes an identity and the constraint is verified. I will now analyse how the inequality evolves when Δ_τ increases.

First, consider the left hand side of (13), representing discounted profits of sustained collusion as a function of Δ_τ. We know from Property 2 that starting from the non-cooperative equilibrium β_{SFE}, a joint decrease of β increases profits until $\beta = \beta_{col}$. Further decreases of β then decrease joint profits. With $\beta \to 0$, there is no supply and therefore there are no profits. β_{APT} is decreasing in Δ_τ, thus Π_{APT} increases with Δ_τ until $\Delta_\tau = \Delta_{max}$ and then decreases with Δ_τ. When Δ_τ goes to infinity, $\Pi^i(\beta_{APT}(\Delta_\tau), \beta_{APT}(\Delta_\tau))$ approaches zero.

[4] For the interested reader: the explicit form of (11) can be obtained, using the definitions in (12), (1), (8), (10) and (3).

Considering the integral on the left hand side of the inequality (13), we know that this is merely a sum of expected profits with both firms playing β_{APT}, hence this side of the inequality first increases, and then decreases with Δ_τ, finally approaching zero.

Now consider the right hand side of the inequality (13). The only term depending on Δ_τ is the profit of deviation Π_{dev}. Its derivative with respect to Δ_τ is

$$\frac{\partial\hat{\Pi}^i(\beta_{dev},\beta_{APT})}{\partial\Delta_\tau} = \underbrace{\frac{\partial\hat{\Pi}^i_{dev}}{\partial\beta_j}}_{<0}\underbrace{\frac{\partial\beta_{APT}}{\partial\Delta_\tau}}_{<0} + \underbrace{\frac{\partial\hat{\Pi}^i_{dev}}{\partial\beta_i}}_{=0}\frac{\partial\beta_{dev}}{\partial\Delta_\tau}$$

The first summand is positive; profits always increase the more the rival withholds its capacity. The second summand is zero because the deviating firm plays its best response and due to Property 1, the partial derivative is zero at the best response. Therefore, deviative profits increase continuously with higher Δ_τ.

Finally, we know that (13) is verified at $\Delta = 0$; that the left hand side of the inequality first increases, then decreases in Δ and approaches zero. The right hand side increases continuously. Therefore, a maximum value $\Delta_{IC} \geq 0$ exists where both sides of the inequality coincide and never beyond. □

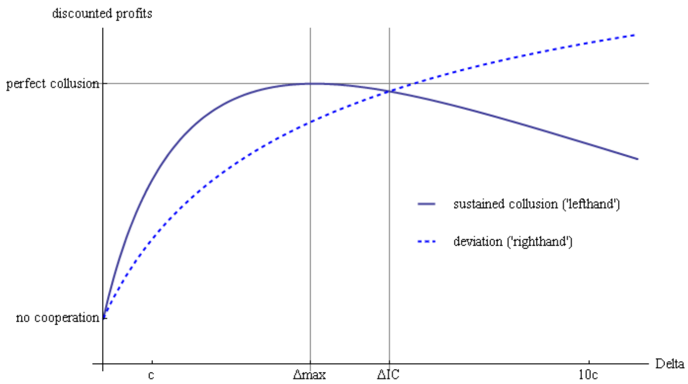

Figure 1: Both sides of the incentive constraint over Δ_τ, low interest rate

Claim 2. *All firms weakly prefer coordination by asymmetric cost pass-through to the non-cooperative equilibrium. Asymmetric cost pass-through is strictly preferred in the neighbourhood of the non-cooperative equilibrium.*

$$\exists\,\check{\Delta} > 0, \text{ such that (13) holds at } \Delta_\tau = \check{\Delta}, \text{ and } \left[\Delta_\tau = \check{\Delta}\right] \succ \left[\Delta_t = 0\right].$$

Note that Claim 2 implies $\Delta_{IC} > 0$.

Proof. We know that (13) holds as an identity at $\Delta_\tau = 0$. To hold for larger values of Δ_τ, the left hand side of the inequality has to follow a higher slope than the right hand side. This condition can be phrased as the 'Inequality of Derivatives':

$$\frac{\partial}{\partial \Delta_\tau}\left[\hat{\Pi}_{APT}(\Delta_\tau) + \int_{\underline{c}}^{\overline{c}} f(c)\ \left(\tfrac{1}{r}\right)\hat{\Pi}_{APT}(\Delta_\tau)\ dc\right] > \frac{\partial}{\partial \Delta_\tau}\hat{\Pi}_{dev}(\Delta_\tau)\ \Bigg|\ \Delta_\tau = 0. \tag{14}$$

A sufficient condition for (14) is

$$\frac{\partial}{\partial \Delta_\tau}\hat{\Pi}_{APT}(\Delta_\tau) = \frac{\partial}{\partial \Delta_\tau}\hat{\Pi}_{dev}(\Delta_\tau) \qquad |\Delta_\tau = 0 \tag{14.a}$$

and

$$\frac{\partial}{\partial \Delta_\tau}\int_{\underline{c}}^{\overline{c}} f(c)\ \left(\tfrac{1}{r}\right)\hat{\Pi}_{APT}(\Delta_\tau)\ dc > 0 \qquad |\Delta_\tau = 0 \tag{14.b}$$

These two conditions can be checked separately. Expressing (14.a) for firm i with respect to strategies of firm i and j yields

$$\underbrace{\frac{\partial \hat{\Pi}^i_{APT}}{\partial \beta_i}}_{=0}\frac{\partial \beta_{APT}}{\partial \Delta_\tau} + \frac{\partial \hat{\Pi}^i_{APT}}{\partial \beta_j}\frac{\partial \beta_{APT}}{\partial \Delta_\tau} \quad = \quad \underbrace{\frac{\partial \hat{\Pi}^i_{dev}}{\partial \beta_i}}_{=0}\frac{\partial \beta_{dev}}{\partial \Delta_\tau} + \frac{\partial \hat{\Pi}^i_{dev}}{\partial \beta_j}\frac{\partial \beta_{APT}}{\partial \Delta_\tau}\ \Bigg|\ \Delta_\tau = 0$$

At $\Delta_t = 0$, no fictive costs are priced in and every firm bids the equilibrium strategy: $\beta_{APT}(0) = \beta_{dev}(0) = \beta_{SFE}$. Because β_{SFE} is the best response, the derivative of firm i's profit with respect to its own strategy is zero, no matter if it is willing to cooperate or not. As indicated above, the corresponding derivatives cancel. Moreover, all firms earn equilibrium profits: $\Pi_{APT}(0) = \Pi_{dev}(0) = \Pi_{SFE}$. What remains is an identity of the derivatives of firm i's non-cooperative equilibrium profit with respect to firm j's strategy, which is evidently true.

Now consider (14.b). This condition requires discounted expected future profits to have a strictly positive slope in Δ_τ. However, the only aim of collusion by asymmetric cost pass-through is increasing profits by not transmitting downward cost shocks. Consider the profit function $\Pi^i(\beta_i, \beta_j)$ at an arbitrary realisation of c. We know that β_{APT} strictly decreases in Δ_τ and from Proposition 2 that profits strictly increase for a joint decrease of (β_i, β_j) as long as $(\beta_i, \beta_j) > (\beta_{col}, \beta_{col})$. Therefore Π_{APT} strictly increases with $\Delta_\tau \leq \Delta_{max}$ and the same applies to the expected value of Π_{APT}. Thus, (14.b) is true at $\Delta_\tau = 0$, which implies that the inequality of derivatives holds. □

Claim 3. *The incentive constraint (13) is verified for all* $\Delta_\tau \in [0, \Delta_{IC}]$.

Establishing the proof of Claim 3 requires a proof of concavity, which can be found in appendix B and is provided upon request. The following reasoning uses the result from this appendix.

Proof. Consider both sides of the inequality as functions of Δ_τ. One can establish Claim 3 by proving a single crossing property for profits of deviation and profits of collusion if $\Delta_\tau > 0$. Figure 1 might serve as an illustration.

We know from the proof of Claim 2 that both sides of the inequality coincide at $\Delta = 0$, and that the left hand side starts with higher slope. We know from Property 2, that the left hand side increases with Δ until Δ_{max} and then decreases, finally approaching zero. On the contrary, we know from the proof of Claim 1 that the right hand side is continuously increasing in Δ.

First, suppose that no intersection took place in the interval $\Delta \in [0, \Delta_{max}]$. Thus, the left hand side still exceeds the right hand side at Δ_{max}. Because the left hand side is monotonically decreasing for all $\Delta > \Delta_{max}$, and the right hand side is monotonically increasing, they will intersect exactly once: at Δ_{IC}.

Now suppose there is an intersection within the interval $[0, \Delta_{max}]$ at an arbitrary $\overline{\Delta} > 0$. Indeed, one can prove concavity of both functions over this interval (the proof of concavity is in appendix B and provided upon request), therefore they can intersect maximally twice: one intersection is at 0 and one is at $\overline{\Delta}$. However, since no further intersection is possible when both functions are concave, the right hand side exceeds the left hand side at Δ_{max}. From this point on, the left hand side is decreasing, and the right hand side is increasing. Hence, no other intersection is possible for all $\Delta > \overline{\Delta}$. Therefore, $\overline{\Delta}$ is unique, and corresponds exactly to the definition of Δ_{IC} in Claim 1. □

Claim 1, 2 and 3 together imply the Theorem. The proof of the Theorem therefore ends with the proof of Claim 3.

The range of collusive equilibria in numerical examples

From a regulatory point of view, it would be interesting to know more about the level of 'fictive costs' which can be priced in before collusion breaks. Numerical examples show that these levels vary significantly with the price sensitivity of demand given by γ and less remarkably with the level of the interest rate r. Higher values of each of the both parameters typically narrow the interval for collusion. The interest rate, however, mainly determines another important feature: whether optimal collusion will be reached or not. In terms of the model developed throughout this paper, this corresponds to the order of Δ_{IC} and Δ_{max}. If $\Delta_{IC} < \Delta_{max}$, collusion breaks before the perfect cartel solution is reached; if $\Delta_{IC} > \Delta_{max}$, continued fluctuations of the cost parameter might lead to the establishment of a quasi-monopoly of perfectly colluding firms. Both cases are possible and illustrated in the following figures.

Figure 1 shows a numerical example with fixed values for c_τ, $\mathbb{E}\left[\tilde{N}^2\right]$, γ and r. The horizontal axis gives values of fictive costs Δ_τ in proportion to c. The solid line represents discounted profits of sustained collusion ('left hand side') as a function of Δ_τ. It first coincides with, then exceeds the dashed line which represents profits of deviation (result of Claim 2), reaches its maximum at Δ_{max}, then decreases and intersects for the second and last time the profit of deviation function at Δ_{IC} (result of Claim 1). No intersection takes place between 0 and Δ_{IC} (result of Claim 3). In this case, the perfectly collusive solution ($\Delta = \Delta_{max}$) would be incentive feasible.

Figure 2 shows the same example as figure 1, but with a significantly higher interest rate (10 times higher than before). One can see, that the order of Δ_{IC} and Δ_{max} is now inverted.

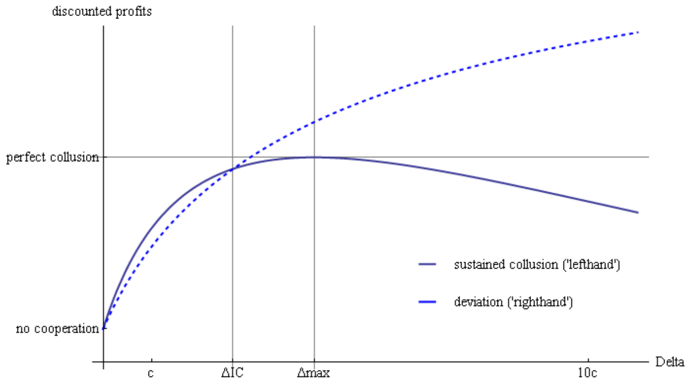

Figure 2: Both sides of the incentive constraint over Δ_τ, high interest rate

5 Conclusion

This article studied the strategic interest of firms in asymmetrically transmitting input prices at a power spot market. It was argued that joint profit maximisation at such power spot markets would potentially yield large extra profits but is hindered by incomplete information about rivals strategies and the lack of a focal point for collusive bidding. Asymmetric transmission of input price shocks, however, may be used as a coordinating mechanism to overcome the collusive dilemma of firms.

In the framework of a symmetric two-firm linear supply function equilibrium model, a theorem on the incentive feasibility of such coordination was established, which shows that the incentive constraint holds continuously for an interval of fictive costs $[0, \Delta_{IC}]$. This implies that firms always have an interest to jointly depart from the non-cooperative equilibrium when coordination is possible. Numerical examples show that even the monopoly solution might be incentive-feasible, depending on the set of parameters. It is assumed that, if the perfect cartel solution is reached without a break of collusion, firms behave like a quasi-monopoly in the following. If the agreement looses incentive feasibility before perfect collusion is implemented, the lack of overt communication is likely to undermine the re-establishment of joint profit maximisation.

The findings add to the literature on asymmetric cost pass-through a model which applies even to the setting of a spot market with supply function bidding. This model represents the first attempt to explain the empirically proved phenomenon that EUA prices have had an asymmetric effect on German power spot prices. It offers an intuitive explanation which is based on the informational structure of the market: the introduction of emissions trading provided an instrument of collusion with the necessary signalling quality which was lacking before. A first result for policy-makers may be phrased as follows: distorting effects of a policy measure (the installation of emissions trading) might not only lie in the economic sphere of goods and money, but also occur through the informational structure in neighbouring markets. The introduction of such measures should therefore regularly be accompanied by enhanced anti-trust screening of industries involved.

The results of this work are limited insofar as asymmetric cost pass-through is modelled as the outcome of a transitory phase, which, however, might last an undetermined period of time. Moreover, the end game situation might not be as clear-cut in reality as it is modelled here. If perfect cartel prices are infeasible due to non-market reasons, these reasons might as well hamper further collusion long before a perfect cartel is reached. Nevertheless, the article does provide a rational basis for understanding the evidence of persistent asymmetric cost pass-through in the German wholesale market for electricity from 2005 to 2007. Finally, the results should serve as a mirror to reflect market outcomes and regulation in similar settings around the world.

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A

A.1 Second order condition for the Best Response and Proof of Proposition 1

For the sufficient condition, consider the profit function given by (1). Its second derivative is

$$\frac{\partial^2 \Pi^i(\beta_i)}{\partial \beta_i^2} = \frac{6\tilde{N}^2\left(\beta_i - \frac{c}{2}\beta_i^2\right)}{(\beta_i+\beta_j+\gamma)^4} - \frac{4\tilde{N}^2(1-c\beta_i)}{(\beta_i+\beta_j+\gamma)^3} - \frac{c\tilde{N}^2}{(\beta_i+\beta_j+\gamma)^2}.$$

Locally, at the best response the second derivative becomes

$$\begin{aligned}\frac{\partial^2 \Pi^i(BR(\beta_j),\beta_j)}{\partial BR(\beta_j)^2} =& \frac{6\,N^2\left(-\frac{c(\beta_j+\gamma)^2}{2(1+c(\beta_j+\gamma))^2} + \frac{\beta_j+\gamma}{1+c(\beta_j+\gamma)}\right)}{\left(\beta_j+\gamma+\frac{\beta_j+\gamma}{1+c(\beta_j+\gamma)}\right)^4} \\ &- \left(\frac{4N^2\left(1-\frac{c(\beta_j+\gamma)}{1+c(\beta_j+\gamma)}\right)}{\left(\beta_j+\gamma+\frac{\beta_j+\gamma}{1+c(\beta_j+\gamma)}\right)^3} + \frac{cN^2}{\left(\beta_j+\gamma+\frac{\beta_j+\gamma}{1+c(\beta_j+\gamma)}\right)^2}\right) \\ =& -\frac{N^2\,(1+c\,(\beta_j+\gamma))^4}{(\beta_j+\gamma)^3\,(2+c\,(\beta_j+\gamma))^3} < 0 \qquad \square\end{aligned}$$

A.2 Second order condition for joint profit maximisation and proof of Proposition 2

The second order condition requires local concavity of joint profits for joint variations of β

$$\begin{aligned}\frac{\partial^2 2\Pi(\beta_{col},\beta_{col})}{\partial\beta_{col}^2} &= -\frac{2cN^2}{(\gamma+2\beta_{col})^2} - \frac{16N^2(1-c\beta_{col})}{(\gamma+2\beta_{col})^3} + \frac{48N^2\left(\beta_{col}-\frac{c}{2}\beta_{col}^2\right)}{(\gamma+2\beta_{col})^4} \\ &= -\frac{2cN^2}{\left(\gamma+\frac{2\gamma}{2+c\gamma}\right)^2} - \frac{16N^2\left(1-\frac{c\gamma}{2+c\gamma}\right)}{\left(\gamma+\frac{2\gamma}{2+c\gamma}\right)^3} + \frac{48N^2\left(\frac{\gamma}{2+c\gamma}-\frac{c\gamma^2}{2(2+c\gamma)^2}\right)}{\left(\gamma+\frac{2\gamma}{2+c\gamma}\right)^4} \\ &= -\frac{2N^2(2+c\gamma)^4}{\gamma^3(4+c\gamma)^3} < 0 \qquad \square\end{aligned}$$

B Completion of the proof of Claim 3, *- Supplementary material -* *(not for journal publication)*

To complete the proof of CLAIM 3, it is sufficient to show concavity, (a) of profits of sustained collusion, and (b) of profits of deviation, each over the interval $\Delta \in [0, \Delta_{max}]$. Showing this for an arbitrary value of c implies concavity of the integral over all c, and therefore of the functions on either side of the incentive constraint (4).

B.1 Proof of concavity of the 'left hand side'

The 'left hand side' of the incentive constraint (4) represents discounted profits of sustained collusion. Both firms play the same strategy of asymmetric price transmission which I will denote β for simplicity:

$$\beta_i = \beta_j = \beta_{APT}(\Delta) \equiv \beta$$

$$\hat{\Pi}_{APT} = \hat{\Pi}(\beta_{APT}, \beta_{APT}) = \hat{\Pi}(\beta)$$

Note that β is monotone decreasing in Δ. One can therefore argue almost equivalently with either variable. The second derivative of collusive profits with respect to Δ is:

$$\frac{\partial^2 \hat{\Pi}(\beta)}{\partial \Delta^2} = \frac{\partial \hat{\Pi}}{\partial \beta} \frac{\partial^2 \beta_{APT}}{\partial \Delta^2} \quad + \quad \frac{\partial^2 \hat{\Pi}}{\partial \beta^2} \left(\frac{\partial \beta_{APT}}{\partial \Delta} \right)^2 \quad < 0 \tag{B.1}$$

which I will have to prove negative. First, let us reduce the terms involved:

$$\frac{\partial \hat{\Pi}}{\partial \beta} = \left(\frac{-4\mathbb{E}[\tilde{N}^2](\beta - \frac{c}{2}\beta^2)}{(\gamma + 2\beta)^3} \right) + \left(\frac{\mathbb{E}[\tilde{N}^2]\,(1 - c\beta)}{(\gamma + 2\beta)^2} \right)$$

$$= \frac{\mathbb{E}[\tilde{N}^2]}{(\gamma + 2\beta)^3} \left(-4(\beta - \frac{c}{2}\beta^2) + (1 - c\beta)\,(\gamma + 2\beta) \right)$$

$$= \frac{\mathbb{E}[\tilde{N}^2]}{(\gamma + 2\beta)^3} \left(\gamma - \beta(2 + c\gamma) \right)$$

$$\frac{\partial^2 \hat{\Pi}}{\partial \beta^2} = -6\, \frac{\mathbb{E}[\tilde{N}^2]\; (\gamma - \beta(2 + c\gamma))}{(\gamma + 2\beta)^4} \quad - \quad \frac{\mathbb{E}[\tilde{N}^2]\,(2 + c\gamma)}{(\gamma + 2\beta)^3}$$

$$= \frac{\mathbb{E}[\tilde{N}^2]}{(\gamma + 2\beta)^3} \left(\frac{-6(\gamma - \beta(2 + c\gamma))}{\gamma + 2\beta} \; - \; (2 + c\gamma) \right)$$

$$= \frac{\mathbb{E}[\tilde{N}^2]}{(\gamma + 2\beta)^3} \left(\frac{4\beta(2 + c\gamma) - c\gamma^2 - 8\gamma}{\gamma + 2\beta} \right)$$

$$\beta' \equiv \frac{\partial \beta_{APT}}{\partial \Delta} = -\frac{\gamma}{(c+\Delta)^2\sqrt{\gamma^2 + \frac{4\gamma}{c+\Delta}}} < 0$$

$$(\beta')^2 \equiv \left(\frac{\partial \beta_{APT}}{\partial \Delta}\right)^2 = \frac{\gamma^2}{(c+\Delta)^4\left(\gamma^2 + \frac{4\gamma}{c+\Delta}\right)} > 0 \tag{B.2}$$

$$\beta'' \equiv \frac{\partial^2 \beta_{APT}}{\partial \Delta^2} = -\frac{2\gamma^2}{(c+\Delta)^4\left(\gamma^2 + \frac{4\gamma}{c+\Delta}\right)^{3/2}} + \frac{2\gamma}{(c+\Delta)^3\sqrt{\gamma^2 + \frac{4\gamma}{c+\Delta}}}$$

$$\frac{(\beta')^2}{\beta''} = \frac{\sqrt{\gamma^2 + \frac{4\gamma}{c+\Delta}}}{2\gamma(c+\Delta)+6} > 0 \tag{B.3}$$

$$\Rightarrow \quad \beta'' > 0$$

With the last result, one can equivalently express the convexity condition (B.1) by

$$\frac{\partial \hat{\Pi}}{\partial \beta} + \frac{(\beta')^2}{\beta''}\,\frac{\partial^2 \hat{\Pi}}{\partial \beta^2} < 0$$

$$\Leftrightarrow$$

$$\frac{\mathbb{E}[\tilde{N}^2]}{(\gamma+2\beta)^3}\left(\gamma - \beta(2+c\gamma)\right) + \frac{(\beta')^2}{\beta''}\,\frac{\mathbb{E}[\tilde{N}^2]}{(\gamma+2\beta)^3}\left(\frac{4\beta(2+c\gamma) - c\gamma^2 - 8\gamma}{\gamma+2\beta}\right) < 0$$

$$\Leftrightarrow \quad \gamma - \beta(2+c\gamma) + \frac{(\beta')^2}{\beta''}\,\frac{4\beta(2+c\gamma) - c\gamma^2 - 8\gamma}{\gamma+2\beta} < 0$$

where I cancelled the common positive factor $\mathbb{E}[\tilde{N}^2]/(\gamma+2\beta)^3$. Using equation (B.3) and the definition for $\beta = \beta_{APT}$ in (8) yields

$$\gamma - \beta\,(2+c\gamma) + \frac{4\beta\,(2+c\gamma) - c\gamma^2 - 8\gamma}{2\gamma(c+\Delta)+6} < 0$$

$$\Leftrightarrow \quad \gamma \quad \underbrace{-\,(2+c\gamma)\,\beta\left(\frac{2\gamma(c+\Delta)+2}{2\gamma(c+\Delta)+6}\right)}_{<0} \quad \underbrace{-\,\frac{c\gamma^2+8\gamma}{2\gamma(c+\Delta)+6}}_{<0\,;\,\nearrow \text{ in } \Delta} < 0 \tag{B.4}$$

The first summand is positive and constant, the two latter are negative, the last term is evidently negative and increasing in Δ. To learn more about the behaviour of this condition, differentiate the second summand with respect to

Δ:

$$\frac{\partial}{\partial \Delta}\left[-\left(2+c\gamma\right)\beta\left(\frac{2\gamma(c+\Delta)+2}{2\gamma(c+\Delta)+6}\right)\right]$$

$$= -\ (2+c\gamma)\ \left(\beta'\,\frac{2\gamma(c+\Delta)+2}{2\gamma(c+\Delta)+6} + \beta\,\frac{8\gamma}{(2\gamma(c+\Delta)+6)^2}\right)$$

Using the definitions in (8) and (B.2) gives

$$-\ (2+c\gamma)\left(\frac{-\gamma(2\gamma(c+\Delta)+2)}{(c+\Delta)^2\sqrt{\gamma^2+\frac{4\gamma}{c+\Delta}}(2\gamma(c+\Delta)+6)} + \frac{4\gamma\left(\sqrt{\gamma^2+\frac{4\gamma}{c+\Delta}}-\gamma\right)}{(2\gamma(c+\Delta)+6)^2}\right)$$

$$= -\ \frac{2+c\gamma}{2\gamma(c+\Delta)+6}\left(\frac{-2\gamma^2(c+\Delta)+2\gamma}{(c+\Delta)^2\sqrt{\gamma^2+\frac{4\gamma}{c+\Delta}}} + \frac{4\gamma\left(\sqrt{\gamma^2+\frac{4\gamma}{c+\Delta}}-\gamma\right)}{2\gamma(c+\Delta)+6}\right)$$

$$= \frac{-\gamma(2+c\gamma)}{2\gamma(c+\Delta)+6}\left(\frac{-12-4\gamma(c+\Delta)^2\sqrt{\gamma^2+\frac{4\gamma}{c+\Delta}}}{(6+2\gamma(c+\Delta))(c+\Delta)^2\sqrt{\gamma^2+\frac{4\gamma}{c+\Delta}}}\right)$$

$$= \left(\frac{\gamma(2+c\gamma)(12+4\gamma(c+\Delta)^2)\sqrt{\gamma^2+\frac{4\gamma}{c+\Delta}}}{(2\gamma(c+\Delta)+6)^2(c+\Delta)^2\sqrt{\gamma^2+\frac{4\gamma}{c+\Delta}}}\right) \quad > \ 0$$

This shows that the term in (B.4) is monotonically increasing in Δ. The term is negative over the whole interval $\Delta \in [0,\, \Delta_{max}]$ if and only if it is negative at the upper bound Δ_{max}. Recall that at $\Delta = \Delta_{max}$ both firms are at the perfectly collusive equilibrium, so due to PROPERTY 2, one has $\partial\Pi/\partial\beta = 0$ and $\partial^2\Pi/\partial\beta^2 < 0$. Consider (B.1) and it becomes clear that the inequality holds at this point, hence, it holds in its reduced form (B.4) and therefore for the whole interval $\Delta \in [0,\, \Delta_{max}]$. □

B.2 Proof of concavity of the 'right hand side'

The 'right-hand-side' of the incentive constraint (4) represents discounted profits of deviation from collusion. Firm i plays the best response to firm j's collusive strategy of asymmetric price transmission:

$$\left|\begin{array}{ll} \beta_i & = \beta_{dev} \equiv BR(\beta_j) \\ \beta_j & = \beta_{APT}(\Delta) \\ \hat{\Pi}^i & = \hat{\Pi}_{dev} \equiv \hat{\Pi}^i(\beta_{dev}, \beta_{APT}) \end{array}\right|$$

Note that β_{APT} is monotone decreasing in Δ, and $0 < BR' < 1$. β_i and β_j therefore vary with the same sign, but a different magnitude in Δ. The second derivative of a deviating firm's profit with respect to Δ is:

$$\frac{\partial^2 \Pi^i_{dev}(\Delta)}{\partial \Delta^2} = \frac{\partial \hat{\Pi}^i}{\partial \beta_j}\frac{\partial^2 \beta_j}{\partial \Delta^2} \quad + \quad \left(\frac{\partial^2 \hat{\Pi}^i}{\partial \beta_j^2} + \frac{\partial^2 \hat{\Pi}^i}{\partial \beta_j \partial \beta_i}\frac{\partial BR}{\partial \beta_j}\right)\left(\frac{\partial \beta_j}{\partial \Delta}\right)^2 \quad < 0 \tag{B.5}$$

which is to be proved negative.

The involved terms have the following functional forms,

$$\frac{\partial \hat{\Pi}^i}{\partial \beta_j} = -\frac{2\mathbb{E}[\tilde{N}^2]\left(\beta_i - \frac{c\beta_i^2}{2}\right)}{(\beta_i + \beta_j + \gamma)^3}$$

$$= -\frac{\mathbb{E}[\tilde{N}^2]}{(\beta_i + \beta_j + \gamma)^3}\left(2BR(\beta_j) - cBR(\beta_j)^2\right)$$

$$= -\frac{\mathbb{E}[\tilde{N}^2]}{(\beta_i + \beta_j + \gamma)^3}\left(2\frac{(\gamma + \beta_j)}{1 + c(\gamma + \beta_j)} - c\left(\frac{(\gamma + \beta_j)}{1 + c(\gamma + \beta_j)}\right)^2\right)$$

$$= -\frac{\mathbb{E}[\tilde{N}^2]}{(\beta_i + \beta_j + \gamma)^3}\left(\frac{(\gamma + \beta_j)(2 + c(\gamma + \beta_j))}{(1 + c(\gamma + \beta_j))^2}\right) \qquad < 0$$

$$\frac{\partial^2 \hat{\Pi}^i}{\partial \beta_j^2} = \frac{6\mathbb{E}[\tilde{N}^2]\left(\beta_i - \frac{c\beta_i^2}{2}\right)}{(\beta_i + \beta_j + \gamma)^4}$$

$$= \frac{\mathbb{E}[\tilde{N}^2]}{(\beta_i + \beta_j + \gamma)^3}\left(\frac{6\,BR(\beta_j) - 3\,c\,BR(\beta_j)^2}{BR(\beta_j) + \beta_j + \gamma}\right)$$

$$= \frac{\mathbb{E}[\tilde{N}^2]}{(\beta_i + \beta_j + \gamma)^3}\left(\frac{6 - 3\,c\,\frac{(\gamma + \beta_j)}{1 + c\,(\gamma + \beta_j)}}{2 + c\,(\beta_j + \gamma)}\right)$$

$$= \frac{\mathbb{E}[\tilde{N}^2]}{(\beta_i + \beta_j + \gamma)^3}\left(\frac{6 + 3\,c\,(\gamma + \beta_j)}{(2 + c\,(\beta_j + \gamma))(1 + c\,(\gamma + \beta_j))}\right)$$

$$= \frac{\mathbb{E}[\tilde{N}^2]}{(\beta_i + \beta_j + \gamma)^3}\left(\frac{3}{1 + c\,(\gamma + \beta_j)}\right) \qquad > 0$$

$$\frac{\partial^2\hat{\Pi}^i}{\partial\beta_j\partial\beta_i} = \frac{6\mathbb{E}[\tilde{N}^2]\left(\beta_i - \frac{c\beta_i^2}{2}\right)}{(\beta_i+\beta_j+\gamma)^4} - \frac{2\mathbb{E}[\tilde{N}^2](1-c\beta_i)}{(\beta_i+\beta_j+\gamma)^3}$$

$$= \frac{\mathbb{E}[\tilde{N}^2]}{(\beta_i+\beta_j+\gamma)^3}\left(\frac{6\,BR(\beta_j)\;-3\,c\,BR(\beta_j)^2}{BR(\beta_j)+\beta_j+\gamma} - 2(1-c\,BR(\beta_j))\right)$$

$$= \frac{\mathbb{E}[\tilde{N}^2]}{(\beta_i+\beta_j+\gamma)^3}\left(\frac{3}{1+c\,(\gamma+\beta_j)} - 2\left(1 - c\frac{(\gamma+\beta_j)}{1+c\,(\gamma+\beta_j)}\right)\right)$$

$$= \frac{\mathbb{E}[\tilde{N}^2]}{(\beta_i+\beta_j+\gamma)^3}\left(\frac{3}{1+c\,(\gamma+\beta_j)} - \frac{2}{1+c\,(\gamma+\beta_j)}\right)$$

$$= \frac{\mathbb{E}[\tilde{N}^2]}{(\beta_i+\beta_j+\gamma)^3}\left(\frac{1}{1+c\,(\gamma+\beta_j)}\right) \qquad > 0$$

So $\partial^2\hat{\Pi}^i/\partial\beta_j\partial\beta_i$ has a positive sign. Consider (B.5): since all factors multiplying $\partial^2\hat{\Pi}^i/\partial\beta_j\partial\beta_i$ are positive, assuming $BR' = 1$ is a stricter condition than the one in (B.5). To simplify calculations, I will assume $BR' = 1$ throughout the rest of this proof.

The stricter condition instead of (B.5) is

$$\beta''_{APT}\left(\frac{\partial\hat{\Pi}^i}{\partial\beta_j}\right) \quad + \quad (\beta'_{APT})^2\left(\frac{\partial^2\hat{\Pi}^i}{\partial\beta_j^2} + \frac{\partial^2\hat{\Pi}^i}{\partial\beta_j\partial\beta_i}\right) < 0 \qquad \text{(B.6)}$$

which can be simplified by cancelling some common positive factors.

$$\text{(B.6)}$$

$$\Leftrightarrow \quad \frac{\beta''_{APT}}{(\beta'_{APT})^2}\left(\frac{-(\gamma+\beta_j)(2\;+\;c\,(\gamma+\beta_j))}{(1+c(\gamma+\beta_j))^2}\right) \quad + \quad \left(\frac{3+1}{1+c\,(\gamma+\beta_j)}\right) \quad < 0$$

$$\Leftrightarrow \quad -\frac{\beta''_{APT}}{(\beta'_{APT})^2}\,(\gamma+\beta_j)(2\;+\;c\,(\gamma+\beta_j)) \quad + \quad 4(1+c(\gamma+\beta_j)) \qquad < 0$$

and finally

$$\frac{-(2\gamma(c+\Delta)+6)(\gamma+\beta_j)(2\;+\;c\,(\gamma+\beta_j))\;+\;4(1+c(\gamma+\beta_j))\sqrt{\gamma^2+\frac{4\gamma}{c+\Delta}}}{\sqrt{\gamma^2+\frac{4\gamma}{c+\Delta}}} \quad < 0$$

Replacing β_j by β_{APT} and cancelling the square root in the denominator yields

$$
\begin{aligned}
&-(\gamma(c+\Delta)+3)\left(\gamma+\sqrt{\gamma^2+\frac{4\gamma}{c+\Delta}}\right)\left(2+c\left(\frac{\gamma+\sqrt{\gamma^2+\frac{4\gamma}{c+\Delta}}}{2}\right)\right)\\
&\quad+\;4\left(1+c\left(\frac{\gamma+\sqrt{\gamma^2+\frac{4\gamma}{c+\Delta}}}{2}\right)\right)\sqrt{\gamma^2+\frac{4\gamma}{c+\Delta}}
\end{aligned}
$$

$$
\begin{aligned}
=\;\; & 2\sqrt{\gamma^2+\frac{4\gamma}{c+\Delta}}\left(2+c\left(\gamma+\sqrt{\gamma^2+\frac{4\gamma}{c+\Delta}}\right)\right)\\
&-3\left(\gamma+\sqrt{\gamma^2+\frac{4\gamma}{c+\Delta}}\right)\left(2+c\left(\frac{\gamma+\sqrt{\gamma^2+\frac{4\gamma}{c+\Delta}}}{2}\right)\right)\\
&\underbrace{-\gamma(c+\Delta)\left(\gamma+\sqrt{\gamma^2+\frac{4\gamma}{c+\Delta}}\right)\left(2+c\left(\frac{\gamma+\sqrt{\gamma^2+\frac{4\gamma}{c+\Delta}}}{2}\right)\right)}_{\equiv K,\, K>0}
\end{aligned}
$$

$$
\begin{aligned}
=\;\; & 2\sqrt{\gamma^2+\frac{4\gamma}{c+\Delta}}\left(2+c\left(\gamma+\sqrt{\gamma^2+\frac{4\gamma}{c+\Delta}}\right)\right)\\
&-3\sqrt{\gamma^2+\frac{4\gamma}{c+\Delta}}\left(2+c\left(\frac{\gamma+\sqrt{\gamma^2+\frac{4\gamma}{c+\Delta}}}{2}\right)\right)\\
&-3\gamma\left(2+c\left(\frac{\gamma+\sqrt{\gamma^2+\frac{4\gamma}{c+\Delta}}}{2}\right)\right)-K
\end{aligned}
$$

$$
\begin{aligned}
=\;\; & \frac{1}{2}\sqrt{\gamma^2+\frac{4\gamma}{(c+\Delta)}}\,c\left(\gamma+\sqrt{\gamma^2+\frac{4\gamma}{c+\Delta}}\right)-2\sqrt{\gamma^2+\frac{4\gamma}{(c+\Delta)}}\\
&-3\gamma\left(2+c\left(\frac{\gamma+\sqrt{\gamma^2+\frac{4\gamma}{c+\Delta}}}{2}\right)\right)-K
\end{aligned}
$$

$$
\begin{aligned}
=\;\; & \frac{1}{2}\,c\,\gamma\sqrt{\gamma^2+\frac{4\gamma}{(c+\Delta)}}+\frac{1}{2}\,c\left(\gamma^2+\frac{4\gamma}{c+\Delta}\right)\\
&-2\sqrt{\gamma^2+\frac{4\gamma}{(c+\Delta)}}-6\gamma-\frac{3}{2}\,c\,\gamma\left(\gamma+\sqrt{\gamma^2+\frac{4\gamma}{c+\Delta}}\right)-K
\end{aligned}
$$

$$
\begin{aligned}
= \quad & \frac{1}{2}\, c \left(\frac{4\gamma}{c+\Delta} \right) \\
& - 2\sqrt{\gamma^2 + \frac{4\gamma}{(c+\Delta)}} - 6\gamma \; - c\,\gamma \left(\gamma + \sqrt{\gamma^2 + \frac{4\gamma}{c+\Delta}} \right) - K
\end{aligned}
$$

$$
= \quad \underbrace{2\gamma \left(\underbrace{\frac{c}{c+\Delta}}_{\leq 1} - 3 \right)}_{<0} - 2\sqrt{\gamma^2 + \frac{4\gamma}{(c+\Delta)}} \; - c\,\gamma \left(\gamma + \sqrt{\gamma^2 + \frac{4\gamma}{c+\Delta}} \right) - K \quad < \; 0
$$

In the last term, all summands have a negative sign. However, this term is only a reduced form of the one in condition (B.6), which implies (B.5). This establishes concavity of deviative profits in Δ at an arbitrary level $c > 0$, and therefore concavity of the right hand side of the incentive constraint (13). $\square$